Research Article

Multispeed Lattice Boltzmann Model with Space-Filling Lattice for Transcritical Shallow Water Flows

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Inspired by the recent success of applying multispeed lattice Boltzmann models with a non-space-filling lattice for simulating transcritical shallow water flows, the capabilities of their space-filling counterpart are investigated in this work. Firstly, two lattice models with five integer discrete velocities are derived by using the method of matching hydrodynamics moments and then tested with two typical 1D problems including the dam-break flow over flat bed and the steady flow over bump. In simulations, the derived space-filling multispeed models, together with the stream-collision scheme, demonstrate better capability in simulating flows with finite Froude number. However, the performance is worse than the non-space-filling model solved by finite difference scheme. The stream-collision scheme with second-order accuracy may be the reason since a numerical scheme with second-order accuracy is prone to numerical oscillations at discontinuities, which is worthwhile for further study.

1. Introduction

The shallow water equations (SWEs) have been used to model free surface flows in rivers and coastal areas under the assumption of the hydrostatic pressure [1]. Numerically, the SWEs can be solved by using conventional numerical method such as finite difference methods [2], finite element methods [3], and finite volume methods [4]. Alternatively, the lattice Boltzmann method (LBM) [5–8] can be used to model SWEs at mesoscopic level [1, 9–18]. In particular, one-dimensional shallow water flows have been studied by Frandsen [19], Thang et al. [20], and Chopard et al. [21].

A major limitation of LBM is its inability to model supercritical flows. For this reason, Chopard et al. [21] developed an asymmetric lattice Boltzmann model for one-dimensional flow flows which can simulate flows with Froude numbers larger than 1. La Rocca et al. [22] proposed a multispeed model with a non-space-filling lattice which is solved by finite difference scheme and successfully simulated supercritical flows. Here, by using the word “multispeed,” it means lattices that have more than one nonzero speed in one dimension; see, for example, La Rocca et al. [22] and Brownlee et al. [23]. For instance, the commonly used D1Q3, D2Q9, and

D3Q19 lattices are not classified as multispeed lattice. Also, “non-space-filling”/“space-filling” means a lattice model that cannot/can fit into the standard stream-collision scheme.

Inspired by the success of the non-space-filling multispeed model, the capability of their space-filling counterpart will be investigated on simulating flows with finite Froude number. Specifically, two space-filling models will be derived by matching hydrodynamic moments (see, e.g., [1, 22, 24]) and then tested by using two typical shallow water problems.

2. Multispeed Lattice Boltzmann Models for 1D Shallow Water Equation

The one-dimensional SWEs read

$$\begin{aligned} \frac{\partial h}{\partial t} + \frac{\partial (hu)}{\partial x} &= 0, \\ \frac{\partial (hu)}{\partial t} + \frac{\partial (hu^2)}{\partial x} &= -g \frac{\partial}{\partial x} \left(\frac{h^2}{2} \right) + \nu \frac{\partial^2 (hu)}{\partial x^2} - gh \frac{\partial z_b}{\partial x} \quad (1) \\ &+ F, \end{aligned}$$

which describe the evolution of water depth h and depth-averaged velocity u . The force term F is used to model various interesting effects such as wind-induced surface stress and the bed gradient.

SWEs can be modelled by using a mesoscopic evolution rule

$$f_\alpha(x + e_\alpha \Delta t, t + \Delta t) - f_\alpha(x, t) = -\frac{1}{\tau} (f_\alpha - f_\alpha^{\text{eq}}) + \Delta t S_\alpha \left(x + \frac{1}{2} e_\alpha \Delta t, t \right) \quad (2)$$

which describes the fluid motion using a distribution function f . Important factors, such as wind-induced surface stress, are also modelled by a force term at the right hand side; that is, $S_\alpha = w_\alpha e_{\alpha i} F_i / (cs)^2$ and $F_i = F_i(x + (1/2)e_\alpha \Delta t, t)$. The weight factor is denoted by w_α for a discrete velocity e_α , and the sound speed cs can be calculated by using e/λ , where λ is a reference quantity varying with lattice.

To successfully simulate shallow water flows, the key is to define an appropriate local equilibrium function f_α^{eq} and the associated lattice. Here, the equilibrium distribution function in Zhou [1] is generalized to

$$f_\alpha^{\text{eq}} = \begin{cases} w_0 h \left(\frac{1}{w_0} - A \frac{gh}{e^2} - \frac{u_i u_i}{2cs^2} \right) & \alpha = 0, \\ w_\alpha h \left(B \frac{gh}{e^2} + \frac{e_{\alpha i} u_i}{cs^2} + \frac{1}{2} \left[\frac{e_{\alpha i} e_{\alpha j} u_i u_j}{cs^4} - \frac{u_i u_i}{cs^2} \right] \right) & \alpha \neq 0, \end{cases} \quad (3)$$

where the coefficients A and B can be adjusted to satisfy the conservation of mass and momentum, i.e.,

$$h = \sum_\alpha f_\alpha^{\text{eq}}, \quad (4)$$

$$hu^2 + \frac{gh^2}{2} = \sum_\alpha e_\alpha e_\alpha f_\alpha^{\text{eq}}.$$

By substituting (3) into (4), the following equations can be obtained:

$$w_0 A = \sum_\alpha w_\alpha B \quad (\alpha \neq 0), \quad (5)$$

$$\sum_\alpha \frac{e_\alpha e_\alpha w_\alpha B}{e^2} = \frac{1}{2} \quad (\alpha \neq 0).$$

The weights w_α in (3) will depend on the choice of lattice. Here two sets of velocity lattices will be adopted, that is, discrete velocities $(0, \pm 1, \pm 2)$ derived by Qian and Zhou [25] and $(0, \pm 1, \pm 3)$ derived by Chikatamarla and Karlin [26]. They are named as DIQ5A and DIQ5B, respectively, and the relevant parameters are listed in Table 1. For convenience, the standard $(0, \pm 1)$ (DIQ3) lattice also is listed and it will be used for comparison in the following simulations.

3. Case 1: 1D Dam-Break Flow

To test the ability of the derived space-filling models, firstly a simple but challenging 1D dam-break flow over flat bed

TABLE 1: Parameters of three lattice Boltzmann models. The reference quantity and the weights for DIQ5A and DIQ5B are adopted from Qian and Zhou [25] and Chikatamarla and Karlin [26], respectively.

	DIQ3	DIQ5A	DIQ5B
A	3/4	1/2	$(110 + 23\sqrt{10})/48$
B	3/2	1/2	$(5 - \sqrt{10})/6$
λ	$\sqrt{3}$	1	$\sqrt{(5 - \sqrt{10})/3}$
$w(0)$	2/3	1/2	$(16 - 4\sqrt{10})/45$
$w(\pm 1)$	1/6	1/6	$(24 + 3\sqrt{10})/80$
$w(\pm 2)$ or $w(\pm 3)$	—	1/12	$(16 + 5\sqrt{10})/720$

is used. Initially, the water is stationary but there is a discontinuity in the middle of computational region ($x = 1000$ m), described as follows:

$$h(x) = \begin{cases} h_1, & 0 \leq x \leq 1000 \text{ m} \\ h_2, & 1000 \text{ m} < x \leq 2000 \text{ m}. \end{cases} \quad (6)$$

The discontinuity will then propagate stream-wise and change its shape. The sharp gradient at the wave front is difficult to be captured smoothly.

In simulations, 2000 cells are used and therefore Δx is 1 m and Δt is 0.1 s. The initial upstream water depth is set to be 5 m for all tests. Firstly, in order to verify the two multispeed lattice models, the water depth of 3 m is specified in the downstream, where the maximum Froude (Fr) number is 0.26. In Figure 1, the predictions of water depth and velocity of two models and the DIQ3 model are compared with the corresponding analytical solutions [27]. It is shown that all three models are satisfactory, which is as expected since the Fr is relatively low.

To examine the capability of simulating flow with higher Froude number, a series of tests will be carried out by adjusting the downstream depth. It is found, as shown in Figure 2, that the DIQ5B model can simulate flows with $Fr = 1.14$ if the relaxation time is set to be 1.99, although there are errors near the discontinuity perhaps due to the numerical oscillations. The DIQ5A is able to simulate flows with $Fr = 1.07$, while the DIQ3 model fails to simulate Froude number beyond unity. The difference between two multispeed models shows that the capability can also be impacted by the lattice structure.

4. Case 2: 1D Steady Flow over Bump

A steady flow over a bump bed will be utilized to further test the multispeed models. For this flow problem, the subcritical flow over the same bump has been studied by Goutal and Maurel [28], Zhou [1], and Peng et al. [18].

For testing, the bed bottom is defined as

$$Z_b(x) = \begin{cases} 0.2 - 0.05(x - 6)^2, & 8 \text{ m} < x < 12 \text{ m} \\ 0, & \text{else.} \end{cases} \quad (7)$$

The inlet boundary condition is specified by setting a constant discharge of $Q = 4.42 \text{ m}^2/\text{s}$. The length of channel is 25 m.

FIGURE 1: Water depth and velocity profiles predicted by D1Q3, D1Q5A, and D1Q5B models at $t = 80$ s ($h_1 = 5$ m and $h_2 = 3$).

A fixed water depth h_0 will be used at the outlet boundary, which will be reduced gradually to find the maximum Fr that can be simulated. The space step Δx is set to be 0.1 m to get the grid-independent solution according to Peng et al. [18] and the time step $\Delta t = 0.002$ s. A relaxation time of 1.99 is employed for all simulations except for the cases shown in Figure 3. Besides, the bed friction is ignored here.

First, the case with a downstream water depth of 2 m is tested to verify the multispeed models. As shown in Figure 3, all models agree well with the analytical solution, which again confirms capability of multispeed model in the relative low Froude number.

Then the downstream water depth is adjusted to test the capability of higher Froude number, that is, to find the minimal downstream water depth. The corresponding results are presented in Figure 4. It is found that the maximum Fr for D1Q3 is 0.69, which is consistent with the previous findings by Zhou [1]. The D1Q5B model can simulate flows with a Froude number of 1.09, while the D1Q5A model fails to simulate Froude number beyond 0.91, which also shows the importance of lattice structure.

FIGURE 2: Water depth and velocity profiles predicted by D1Q5B models at $t = 80$ s with $h_2 = 0.61$ m and maximum Fr = 1.14 [(a) water depth, (b) water velocity].

FIGURE 3: Water surface profiles with $h_0 = 2$ m.

5. Concluding Remarks

In this study, the space-filling multispeed lattice Boltzmann model has been investigated to simulate the transcritical shallow water flows. By matching hydrodynamics moments, two lattice models with five integer lattice velocities have been derived, which are tested together with the standard model with three discrete velocities for a 1D dam-break and a 1D steady flow over bump.

It is encouraging to find that the space-filling multispeed model can improve the capability of simulating flows with

FIGURE 4: Water surface and Froude number profiles with minimum water depth for three models ($h_0 = 1.93, 1.77$, and 1.71 and $Fr = 0.69, 0.91$, and 1.09 for D1Q3, D1Q5A, and D1Q5B models, resp.).

higher Fr , particularly compared with the standard D1Q3 model. The different performances between two multispeed models show that capability can be impacted by the lattice structure.

In comparison with the non-space-filling model proposed by La Rocca et al. [22], the space-filling versions here demonstrate limited capability in terms of finite Froude number although similar numbers of discrete velocities are used in these two studies. This is potentially due to the fact that the stream-collision scheme is a second-order scheme. It is well known that a numerical scheme of second-order accuracy is prone to numerical oscillations at discontinuities. The impact of the stream-collision scheme will be studied further in the future.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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