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Research Article

$\mathscr{H}_\infty$ Filtering for Discrete-Time Nonlinear Singular Systems with Quantization

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This paper investigates the problem of $\mathscr{H}_\infty$ filtering for class discrete-time Lipschitz nonlinear singular systems with measurement quantization. Assume that the system measurement output is quantized by a static, memoryless, and logarithmic quantizer before it is transmitted to the filter, while the quantizer errors can be treated as sector-bound uncertainties. The attention of this paper is focused on the design of a nonlinear quantized $\mathscr{H}_\infty$ filter to mitigate quantization effects and ensure that the filtering error system is admissible (asymptotically stable, regular, and causal), while having a unique solution with a prescribed $\mathscr{H}_\infty$ noise attenuation level. By introducing some slack variables and using the Lyapunov stability theory, some sufficient conditions for the existence of the nonlinear quantized $\mathscr{H}_\infty$ filter are expressed in terms of linear matrix inequalities (LMIs). Finally, a numerical example is presented to demonstrate the effectiveness of the proposed quantized filter design method.

1. Introduction

The study on singular systems (also known as descriptor systems, generalized state-space systems, differential algebraic systems, or implicit systems) has attracted a recurring interest in the past several decades as the singular systems can provide a better description of many physical systems than regular ones such as circuit systems, economic systems, power systems, chemical processes, robotic systems, and networked systems. The problem of stability analysis and controller design for singular systems has achieved many valuable results (see, e.g., [1–7] and references therein). The related problem of robust $\mathscr{H}_\infty$ static output-feedback control for singularly perturbed systems has been addressed in [1]; also the stability, robust stabilization, and $\mathscr{H}_\infty$ control of singular systems with impulsive behavior have been studied in [2]; the problem of observer-based resilient l_2-l_∞ control for singular systems with time-delay has been investigated in [3]; the work in [4] considered the problem of designing stabilizing controllers for singularly perturbed fuzzy systems; by using a new type generalized Lyapunov equation, some new stability conditions for discrete singular systems have been derived in [5]; and [6] investigated the problem state feedback control for uncertain singular systems with time-delay. Due to the fact that the state variables of system are not always available, the problem of state estimation for singular systems has also received considerable attention (see, e.g., [7–17] and references therein). $\mathscr{H}_\infty$ filtering problem for continuous-time linear singular systems has been considered in [8]; in [9], the problem of energy-to-peak filtering for discrete-time linear singular systems has been addressed; $\mathscr{H}_\infty$ unbiased filtering problem for linear descriptor systems has been studied in [10]; the problem of $\mathscr{H}_\infty$ filtering for continuous- and discrete-time singular systems with communication constraints has been investigated in [11] and [12], respectively; the study in [13] considered the problem of robust nonlinear $\mathscr{H}_\infty$ filtering for Lipschitz nonlinear descriptor systems with parametric uncertainties; the problem of robust $\mathscr{H}_\infty$ filter design of uncertain descriptor systems with distributed delays has been studied in [14]; the

paper [15] concerned the problem of generalized nonlinear l_2-l_∞ filtering for discrete-time Markov jump descriptor systems with Lipschitz nonlinearity; the generalized nonlinear $\mathscr{H}_\infty$ filter design problem for discrete-time Lipschitz nonlinear descriptor systems has been considered in [16]; and [17] studied the observer design problem for discrete-time linear descriptor systems which is also applicable to nonlinear descriptor systems with Lipschitz constraints. $\mathscr{H}_\infty$ approach-based fault detection filter design for a continuous-time networked control system and unmanned surface vehicles in network environments has been investigated in [18] and [19], respectively, and some novel filter design criteria have been presented.

On the other hand, due to inherent network-limited bandwidth, quantization effects are unavoidable in practical systems, especially in networked control systems (NCSs); see [20–22] for results in NCSs. As early as 1956, Kalman investigated the effect of quantization in a sampled data control system and pointed out that if a stabilizing controller was quantized using a finite-alphabet quantizer, the feedback system would exhibit limit cycles and chaotic behavior [23]. Over the past several years, significant efforts have been devoted to the study of analysis and synthesis for linear system with quantized feedback [24–29]. The problem of quantized $\mathscr{H}_\infty$ filter design for different systems has been investigated in [30–34]. In [30], the quantized state estimation problem for discrete-time linear systems has been addressed; the work in [31] considered the $\mathscr{H}_\infty$ filtering problem for a class continuous-time polytopic uncertain systems subject to measurement quantization, signal transmission delay, and data packet dropout; the problem of $\mathscr{H}_\infty$ filtering for discrete-time T-S fuzzy systems with measurement quantization and packet dropouts has been studied in [32]; and [33] investigated the problem of $\mathscr{H}_\infty$ filtering for discrete-time polytopic uncertain systems with measurement quantization; in [34], the quantized $\mathscr{H}_\infty$ filtering problem for a class of discrete-time polytopic uncertain systems with packet dropout has been considered. To the best of our knowledge, few attempts have been made on an $\mathscr{H}_\infty$ filter design for singular systems with quantized measurement, especially for the singular systems with Lipschitz nonlinearity, which motivates us for this study.

This paper focuses on the design of $\mathscr{H}_\infty$ filter for a class of discrete-time Lipschitz nonlinear singular systems with measurement quantization. Via introducing auxiliary relaxed variables by Fisher lemma and using the Lyapunov stability theory, some sufficient conditions for the existence of the nonlinear quantized $\mathscr{H}_\infty$ filter are obtained in terms of linear matrix inequalities (LMIs), which can not only ensure that the filtering error system is admissible and has a unique solution but also achieve a prescribed $\mathscr{H}_\infty$ performance. Finally, we will illustrate the effectiveness of our main results by a numerical example.

Notations. The notations that are used throughout this paper are standard. The notation $*$ is used to indicate the terms that can be induced by symmetry. Generally, for a square matrix $\mathscr{A}$, $\mathscr{A}^T$ denotes its transpose and $\mathrm{He}\{\mathscr{A}\}$ denotes $(\mathscr{A}+\mathscr{A}^T)$.

2. Problem Formulation

Consider the following nonlinear discrete-time singular systems described by

$$\begin{aligned} Ex(t+1) &= \mathscr{A}x(t) + \Psi(t, x(t)) + \mathscr{B}w(t), \\ y(t) &= \mathscr{C}x(t) + \mathscr{D}w(t), \\ z(t) &= \mathscr{L}x(t), \end{aligned} \tag{1}$$

where $x(t) \in \mathscr{R}^n$ is the state variable; $y(t) \in \mathscr{R}^l$ is the measurement output; $z(t) \in \mathscr{R}^q$ is the signal to be estimated; $w(t) \in \mathscr{R}^p$ is the noise signal that is assumed to be the arbitrary signal in $l_2[0, \infty)$; the matrix E may be singular, and we shall assume that $\mathrm{rank}(E) = r < n$; the matrices $\mathscr{A}$, $\mathscr{B}$, $\mathscr{C}$, $\mathscr{D}$, and $\mathscr{L}$ are known matrices with appropriate dimensions.

The nonlinear term we consider here is locally Lipschitz with respect to $x(t)$ in a region $\mathscr{D}$ containing the origin; that is,

$$\begin{aligned} &\Psi(0, x(0)) = 0, \\ &\left\|\Psi(t, x_1(t)) - \Psi(t, x_2(t))\right\| \le \mathscr{F}\left\|\Psi(x_1(t) - x_2(t))\right\|, \\ &\qquad \forall x_1(t), x_2(t) \in \mathscr{D}, \end{aligned} \tag{2}$$

where $\mathscr{F}$ denotes the Lipschitz real matrix of $\Psi(t, x(t))$ with appropriate dimension.

For the unforced discrete-time singular system $Ex(t+1) = \mathscr{A}x(t) + \Psi(t, x(t))$, we have the following definition.

Definition 1 (see [9]). The pair $(E, \mathscr{A})$ is said to be regular if there exists a scalar $s \in \mathscr{C}$ such that $\det(E, \mathscr{A}) \neq 0$, the pair $(E, \mathscr{A})$ is said to be causal if $\deg(\det(E, \mathscr{A})) = \mathrm{rank}(E)$, and the pair $(E, \mathscr{A})$ is said to be stable if all the roots of $\det(E, \mathscr{A})$ lie in the interior of unit disk. We call the pair $(E, \mathscr{A})$ admissible if it is regular, causal, and stable, simultaneously. The unforced discrete-time singular system is said to be regular, causal, and stable (asymptotically stable) if the pair $(E, \mathscr{A})$ is admissible.

The designed full-order nonlinear quantized $\mathscr{H}_\infty$ filter is in the form of

$$\begin{aligned} x_f(t+1) &= \mathscr{A}_f x_f(t) + \lambda_1 \Psi\left(t, x_f(t)\right) + \mathscr{B}_f y_q(t), \\ z_f(t) &= \mathscr{C}_f x_f(t) + \mathscr{D}_f y_q(t), \end{aligned} \tag{3}$$

where $x_f(t) \in \mathscr{R}^n$ is the state of the filter and $z_f(t) \in \mathscr{R}^q$ is the output of the filter; the matrices $\mathscr{A}_f$, $\mathscr{B}_f$, $\mathscr{C}_f$, and $\mathscr{D}_f$ are filter matrices with appropriate dimensions to be designed and $y_q(t)$ is the quantized measurement.

Remark 2. Note that the filter we consider here has a standard form, which is convenient for both theoretical analysis and implementation in practical engineering compared to the singular form.

The quantizer we consider is logarithmic static and time-invariant quantizer given in [30]. The set of quantized levels

is described by $U^{(j)} = \{\pm u_i^j, u_i^j = (\rho^j)^i u_0^j,\ i = \pm 1, \pm 2, \ldots\} \cup \{\pm u_0^j\} \cup \{0\},\ u_0^j > 0$ and the quantizer is defined as follows:

$$q_j(y) = \begin{cases} v_i^{(j)} & 0 \le \dfrac{v_i^{(j)}}{(1+\delta_j)} < y \le \dfrac{v_i^{(j)}}{(1-\delta_j)} \\ 0 & y = 0 \\ -\mathcal{Q}_j(-y) & y < 0, \end{cases} \tag{4}$$

$$\delta_j = \frac{1-\rho_j}{1+\rho_j}, \quad 0 < \rho_j < 1,\ v_i^{(j)} > 0. \tag{5}$$

Then, by using the sector-bound method described in [30], we can obtain that, for any $y(t)$, $y_q(t) = q(y(t)) = (I + \Delta(t))y(t)$, where $\Delta(t) = \mathrm{diag}\{\Delta_1(t), \Delta_2(t), \ldots, \Delta_f(t)\}$, $|\Delta(t)| \le \delta$.

Combining (1) and (3) and defining $\tilde{x}(t) = [x^T(t), x_f^T(t)]^T$ and $e(t) = z(t) - z_f(t)$, one can obtain the filtering error system as follows:

$$\begin{aligned} \tilde{E}\tilde{x}(t+1) &= \widetilde{\mathcal{A}}\tilde{x}(t) + \mathcal{M}\Phi(t, \tilde{x}(t)) + \widetilde{\mathcal{B}}w(t), \\ e(t) &= \widetilde{\mathcal{C}}\tilde{x}(t) + \widetilde{\mathcal{D}}w(t), \end{aligned} \tag{6}$$

where

$$\begin{aligned} \tilde{E} &= \begin{bmatrix} E & 0 \\ 0 & I \end{bmatrix}, \\ \widetilde{\mathcal{A}} &= \overline{\mathcal{A}} + \Delta\overline{\mathcal{A}} \\ &= \begin{bmatrix} \mathcal{A} & 0 \\ \mathcal{B}_f\mathcal{C} & \mathcal{A}_f \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ \mathcal{B}_f\Delta(t)\mathcal{C} & 0 \end{bmatrix}, \\ \mathcal{M} &= \begin{bmatrix} I & 0 \\ 0 & \lambda_1 \end{bmatrix}, \\ \Phi(t, \tilde{x}(t)) &= \begin{bmatrix} \Psi(t, x(t)) \\ \Psi(t, x_f(t)) \end{bmatrix}, \\ \widetilde{\mathcal{B}} &= \overline{\mathcal{B}} + \Delta\overline{\mathcal{B}} = \begin{bmatrix} \mathcal{B} \\ \mathcal{B}_f\mathcal{D} \end{bmatrix} + \begin{bmatrix} 0 \\ \mathcal{B}_f\Delta(t)\mathcal{D} \end{bmatrix}, \\ \widetilde{\mathcal{C}} &= \overline{\mathcal{C}} + \Delta\overline{\mathcal{C}} \\ &= \begin{bmatrix} \mathcal{L} - \mathcal{D}_f\mathcal{C} & -\mathcal{C}_f \end{bmatrix} + \begin{bmatrix} -\mathcal{D}_f\Delta(t)\mathcal{C} & 0 \end{bmatrix}, \\ \widetilde{\mathcal{D}} &= \overline{\mathcal{D}} + \Delta\overline{\mathcal{D}} = -\mathcal{D}_f\mathcal{D} - \mathcal{D}_f\Delta(t)\mathcal{D}. \end{aligned} \tag{7}$$

To this end, we aim to design a quantized $\mathcal{H}_\infty$ filter such that the filtering error system (6) satisfies the following requirements.

(R1) The filtering error system (6) is admissible and of a uniquely discrete solution with $w(t) = 0$.

(R2) The filtering error system (6) has a prescribed level γ of $\mathcal{H}_\infty$ noise attenuation; that is, $\sum_{t=0}^{\infty} e^T(t)e(t) < \gamma^2 \sum_{t=0}^{\infty} w^T(t)w(t)$ is satisfied for any nonzero $w(t) \in l_2[0, \infty)$ with the zero initial condition.

To solve the quantized $\mathcal{H}_\infty$ filtering analysis problem of the considered nonlinear singular system, we need the following preliminary results to prove our main results.

Lemma 3 (see [35]). *Let $\xi \in \mathcal{R}^n$, $\mathcal{Q} \in \mathcal{R}^{n\times n}$, and $\mathcal{B} \in \mathcal{R}^{m\times n}$ with $\mathrm{rank}(\mathcal{B}) < n$ and $\mathcal{B}^\perp$ such that $\mathcal{B}\mathcal{B}^\perp = 0$. Then, the following conditions are equivalent:*

(1) $\xi^T\mathcal{Q}\xi < 0,\ \forall \xi \ne 0 : \mathcal{B}\xi = 0$.

(2) $\mathcal{B}^{\perp T}\mathcal{Q}\mathcal{B}^\perp < 0$.

(3) $\exists \mu \in \mathcal{R} : \mathcal{Q} - \mu\mathcal{B}^T\mathcal{B} < 0$.

(4) $\exists \mathcal{X} \in \mathcal{R}^{n\times m} : \mathcal{Q} + \mathcal{X}\mathcal{B} + \mathcal{B}^T\mathcal{X}^T < 0$.

Lemma 4 (see [16]). *System (6) is regular, causal, and of a unique discrete solution, if there exists a matrix $\mathcal{P} = \mathcal{P}^T$ such that the following inequality holds:*

$$\begin{aligned} &\tilde{E}^T\mathcal{P}\tilde{E} \ge 0, \\ &\begin{bmatrix} \widetilde{\mathcal{A}}^T\mathcal{P}\widetilde{\mathcal{A}} - \tilde{E}^T\mathcal{P}\tilde{E} + \widetilde{\mathcal{F}}^T\widetilde{\mathcal{F}} & * \\ \mathcal{M}^T\mathcal{P}\widetilde{\mathcal{A}} & \mathcal{M}^T\mathcal{P}\mathcal{M} - I \end{bmatrix} < 0. \end{aligned} \tag{8}$$

Lemma 5 (see [36]). *For real matrices $\Omega = \Omega^T$, Γ, Λ, and F with appropriate dimensions and $F^TF \le I$. Then, for any scalar $\epsilon > 0$, the following inequality holds:*

$$\Omega + \Gamma F\Lambda + \Lambda^T F^T \Gamma^T \le \Omega + \epsilon^{-1}\Gamma\Gamma^T + \epsilon\Lambda^T\Lambda. \tag{9}$$

3. Main Results

In this section, the quantized $\mathcal{H}_\infty$ filtering problem for nonlinear singular systems will be considered. First, we will give a new $\mathcal{H}_\infty$ performance analysis criterion based on Fisher lemma given in Lemma 3 such that the filtering error system (6) is admissible and of a unique solution with a prescribed $\mathcal{H}_\infty$ performance γ.

Theorem 6. *Let us consider the nonlinear singular system (1) and the quantized filter (3). Then the quantized filtering error system (6) is admissible and of a unique solution with a prescribed $\mathcal{H}_\infty$ performance $\gamma > 0$, if there exist matrix $\mathcal{P} = \mathcal{P}^T = \begin{bmatrix} \mathcal{P}_1 & \mathcal{P}_2 \\ \mathcal{P}_2^T & \mathcal{P}_3 \end{bmatrix}$, $\mathcal{P}_1 > 0$, matrices $\mathcal{G}$ and $\mathcal{N}$, and scalar λ_1 such that the following matrix inequality holds:*

$$\begin{bmatrix} -3\widetilde{E}^T\mathscr{P}\widetilde{E}+\mathrm{He}\left\{\mathscr{N}\widetilde{\mathscr{A}}\right\}+\widetilde{\mathscr{F}}^T\widetilde{\mathscr{F}} & * & * & * & * \\ \mathscr{M}^T\mathscr{N}^T & -I & * & * & * \\ \widetilde{\mathscr{B}}^T\mathscr{N}^T & 0 & -\gamma^2 I & * & * \\ \mathscr{P}\widetilde{\mathscr{A}}-\mathscr{N}^T+\mathscr{G}\widetilde{\mathscr{A}} & \mathscr{P}\mathscr{M}+\mathscr{G}\mathscr{M} & \mathscr{P}\widetilde{\mathscr{B}}+\mathscr{G}\widetilde{\mathscr{B}} & \mathscr{P}-\mathrm{He}\{\mathscr{G}\} & * \\ \widetilde{\mathscr{C}} & 0 & \widetilde{\mathscr{D}} & 0 & -I \end{bmatrix}<0, \tag{10}$$

where $\widetilde{\mathscr{F}}=\mathrm{diag}\{\mathscr{F}^T\mathscr{F},\mathscr{F}^T\mathscr{F}\}$.

Proof. Firstly, we shall show the filtering error system is regular and causal with $w(t)=0$. Equation (10) implies

$$-3\widetilde{E}^T\mathscr{P}\widetilde{E}+\mathscr{N}\widetilde{\mathscr{A}}+\widetilde{\mathscr{A}}^T\mathscr{N}^T+\widetilde{\mathscr{F}}^T\widetilde{\mathscr{F}}<0. \tag{11}$$

Without loss of generality, we assume that $\widetilde{E}=\begin{bmatrix} I_{n+r} & 0 \\ 0 & 0\end{bmatrix}$ and $\mathscr{N}=\begin{bmatrix}\mathscr{N}_1 & 0 \\ 0 & \mathscr{K}\end{bmatrix}$, and then (11) can be rewritten as

$$\begin{bmatrix} -3\mathscr{P}_1+\mathrm{He}\left\{\mathscr{N}_1\mathscr{A}\right\} & * \\ \mathscr{K}\mathscr{B}_f\left(I+\Delta(t)\right)\mathscr{C} & \mathrm{He}\left\{\mathscr{K}\mathscr{A}_f\right\}\end{bmatrix}<0. \tag{12}$$

Form (12) we have $\mathrm{He}\{\mathscr{K}\mathscr{A}_f\}<0$ that implies $\mathscr{A}_f$ is nonsingular.

$$\det\left(s\widetilde{E}-\widetilde{\mathscr{A}}\right)=\det\left(sI_{n+r}-\widetilde{\mathscr{A}}\right)\cdot\det\left(-\mathscr{A}_f\right). \tag{13}$$

By choosing nonzero scalar s, we have $\det(s\widetilde{E}-\widetilde{\mathscr{A}})\neq 0$; that is, $\det(s\widetilde{E}-\widetilde{\mathscr{A}})\neq 0$. Furthermore, we can obtain

$$\deg\det\left(s\widetilde{E}-\widetilde{\mathscr{A}}\right)=n+r=\mathrm{rank}\,\widetilde{E}. \tag{14}$$

Then, we have that the pair $(\widetilde{E},\widetilde{\mathscr{A}})$ is regular and causal. Based on Definition 1, we can obtain the fact that filtering error system (6) is regular and causal when condition (10) holds.

Now, by constructing a Lyapunov function as $V(\widetilde{x}(t))=3\widetilde{x}^T(t)\widetilde{E}^T\mathscr{P}\widetilde{E}\widetilde{x}(t)$, $\mathscr{P}=\mathscr{P}^T=\begin{bmatrix}\mathscr{P}_1 & \mathscr{P}_2 \\ \mathscr{P}_2^T & \mathscr{P}_3\end{bmatrix}$, $\mathscr{P}_1>0$, we have

$$\begin{aligned}
\Delta V(\widetilde{x}(t)) &= V(\widetilde{x}(t+1))-V(\widetilde{x}(t)) \\
&= \left(\widetilde{\mathscr{A}}\widetilde{x}(t)+\mathscr{M}\Phi(t,\widetilde{x}(t))\right)^T\mathscr{P}\widetilde{E}\widetilde{x}(t+1) \\
&\quad+\widetilde{x}^T(t+1)\widetilde{E}^T\mathscr{P}\left(\widetilde{\mathscr{A}}\widetilde{x}(t)+\mathscr{M}\Phi(t,\widetilde{x}(t))\right) \\
&\quad+\widetilde{x}^T(t+1)\widetilde{E}^T\mathscr{P}\widetilde{E}\widetilde{x}(t+1) \\
&\quad-3\widetilde{x}^T(t)\widetilde{E}^T\mathscr{P}\widetilde{E}\widetilde{x}(t) \\
&\le \left(\widetilde{\mathscr{A}}\widetilde{x}(t)+\mathscr{M}\Phi(t,\widetilde{x}(t))\right)^T\mathscr{P}\widetilde{E}\widetilde{x}(t+1) \\
&\quad+\widetilde{x}^T(t+1)\widetilde{E}^T\mathscr{P}\left(\widetilde{\mathscr{A}}\widetilde{x}(t)+\mathscr{M}\Phi(t,\widetilde{x}(t))\right) \\
&\quad+\widetilde{x}^T(t+1)\widetilde{E}^T\mathscr{P}\widetilde{E}\widetilde{x}(t+1) \\
&\quad-3\widetilde{x}^T(t)\widetilde{E}^T\mathscr{P}\widetilde{E}\widetilde{x}(t)+\widetilde{x}^T(t)\widetilde{\mathscr{F}}^T\widetilde{\mathscr{F}}\widetilde{x}(t) \\
&\quad-\Phi^T(t,\widetilde{x}(t))\Phi(t,\widetilde{x}(t))=\xi^T(t)\Gamma\xi(t),
\end{aligned} \tag{15}$$

where $\xi(t)=\left[\widetilde{x}^T(t)\ \ \Phi^T(t,\widetilde{x}(t))\ \ \widetilde{x}^T(t+1)\widetilde{E}^T\right]$ and

$$\Gamma=\begin{bmatrix} -3\widetilde{E}^T\mathscr{P}\widetilde{E}+\widetilde{\mathscr{F}}^T\widetilde{\mathscr{F}} & * & * \\ 0 & -I & * \\ \mathscr{P}\widetilde{\mathscr{A}} & \mathscr{P}\mathscr{M} & \mathscr{P}\end{bmatrix}. \tag{16}$$

Form the filtering error system (6), we have

$$\underbrace{\left[\widetilde{\mathscr{A}}\ \ \widetilde{\mathscr{B}}\ \ -I\right]}_{\mathscr{B}}\underbrace{\xi(t)}_{\xi}=0, \tag{17}$$

and, by using conditions (1) and (4) of Lemma 3 with $\mathscr{X}^T=\left[\mathscr{N}^T\ \ 0\ \ \mathscr{G}^T\right]^T$, we can obtain $\Gamma<0$ if and only if (10) is true. Therefore, we have $\Delta V(\widetilde{x}(t))<0$ for any nonzero $\xi(t)$, and one can infer that the filtering error system (6) is asymptotically stable with $w(t)=0$.

Next, we will prove that the filtering system (6) is of unique solution. The difference of the Lyapunov function can be rewritten as follows:

$$\begin{aligned}
\Delta V(\widetilde{x}(t)) &= V(\widetilde{x}(t+1))-V(\widetilde{x}(t)) \\
&\le \left(\widetilde{\mathscr{A}}\widetilde{x}(t)+\mathscr{M}\Phi(t,\widetilde{x}(t))\right)^T\mathscr{P}\widetilde{E}\widetilde{x}(t+1) \\
&\quad+\widetilde{x}^T(t+1)\widetilde{E}^T\mathscr{P}\left(\widetilde{\mathscr{A}}\widetilde{x}(t)+\mathscr{M}\Phi(t,\widetilde{x}(t))\right) \\
&\quad+\widetilde{x}^T(t+1)\widetilde{E}^T\mathscr{P}\widetilde{E}\widetilde{x}(t+1)-3\widetilde{x}^T(t)\widetilde{E}^T\mathscr{P}\widetilde{E}\widetilde{x}(t) \\
&\quad+\widetilde{x}^T(t)\widetilde{\mathscr{F}}^T\widetilde{\mathscr{F}}\widetilde{x}(t)-\Phi^T(t,\widetilde{x}(t))\Phi(t,\widetilde{x}(t)) \\
&= 3\left(\widetilde{\mathscr{A}}\widetilde{x}(t)+\mathscr{M}\Phi(t,\widetilde{x}(t))\right)^T \\
&\quad\cdot\mathscr{P}\left(\widetilde{\mathscr{A}}\widetilde{x}(t)+\mathscr{M}\Phi(t,\widetilde{x}(t))\right)-3\widetilde{x}^T(t)\widetilde{E}^T\mathscr{P}\widetilde{E}\widetilde{x}(t) \\
&\quad+\widetilde{x}^T(t)\widetilde{\mathscr{F}}^T\widetilde{\mathscr{F}}\widetilde{x}(t)-\Phi^T(t,\widetilde{x}(t))\Phi(t,\widetilde{x}(t)) \\
&= 3\delta^T(t)\Theta\delta(t),
\end{aligned} \tag{18}$$

where $\xi(t)=\left[\widetilde{x}^T(t)\ \ \Phi^T(t,\widetilde{x}(t))\right]$ and

$$\Theta=\begin{bmatrix}\widetilde{\mathscr{A}}^T\mathscr{P}\widetilde{\mathscr{A}}-\widetilde{E}^T\mathscr{P}\widetilde{E}+\widetilde{\mathscr{F}}^T\widetilde{\mathscr{F}} & * \\ \mathscr{M}^T\mathscr{P}\widetilde{\mathscr{A}} & -I+\mathscr{M}^T\mathscr{P}\mathscr{M}\end{bmatrix}, \tag{19}$$

and then from (10) we can obtain $\Delta V(\widetilde{x}(t))<0$; that is, $\Theta<0$. Therefore, based on Lemma 4, we can prove that the filtering error system (6) is of unique solution.

Finally, let us consider the $\mathscr{H}_\infty$ performance of the filtering system (6). For any nonzero $w(t) \in l_2[0, \infty)$ and zero initial condition, we have

$$\begin{aligned}
&\Delta V(\tilde{x}(t)) + e^T(t)e(t) - \gamma^2 w^T(t)w(t) \\
&\quad = \left(\widetilde{\mathscr{A}}\tilde{x}(t) + \mathscr{M}\Phi(t,\tilde{x}(t)) + \widetilde{\mathscr{B}}w(t)\right)^T \\
&\quad \cdot \mathscr{P}\tilde{E}\tilde{x}(t+1) + \tilde{x}^T(t+1) \\
&\quad \cdot \tilde{E}^T\mathscr{P}\left(\widetilde{\mathscr{A}}\tilde{x}(t) + \mathscr{M}\Phi(t,\tilde{x}(t)) + \widetilde{\mathscr{B}}w(t)\right) \\
&\quad + \tilde{x}^T(t+1)\tilde{E}^T\mathscr{P}\tilde{E}\tilde{x}(t+1) - 3\tilde{x}^T(t)\tilde{E}^T\mathscr{P}\tilde{E}\tilde{x}(t) \\
&\quad + \left(\widetilde{\mathscr{C}}\tilde{x}(t) + \widetilde{\mathscr{D}}w(t)\right)^T\left(\widetilde{\mathscr{C}}\tilde{x}(t) + \widetilde{\mathscr{D}}w(t)\right) \\
&\quad - \gamma^2 w^T(t)w(t) \\
&\quad \le \left(\widetilde{\mathscr{A}}\tilde{x}(t) + \mathscr{M}\Phi(t,\tilde{x}(t)) + \widetilde{\mathscr{B}}w(t)\right)^T \\
&\quad \cdot \mathscr{P}\tilde{E}\tilde{x}(t+1) + \tilde{x}^T(t+1) \\
&\quad \cdot \tilde{E}^T\mathscr{P}\left(\widetilde{\mathscr{A}}\tilde{x}(t) + \mathscr{M}\Phi(t,\tilde{x}(t)) + \widetilde{\mathscr{B}}w(t)\right) \\
&\quad + \tilde{x}^T(t+1)\tilde{E}^T\mathscr{P}\tilde{E}\tilde{x}(t+1) - 3\tilde{x}^T(t)\tilde{E}^T\mathscr{P}\tilde{E}\tilde{x}(t) \\
&\quad + \left(\widetilde{\mathscr{C}}\tilde{x}(t) + \widetilde{\mathscr{D}}w(t)\right)^T\left(\widetilde{\mathscr{C}}\tilde{x}(t) + \widetilde{\mathscr{D}}w(t)\right) \\
&\quad - \gamma^2 w^T(t)w(t) + \tilde{x}^T(t)\widetilde{\mathscr{F}}^T\widetilde{\mathscr{F}}\tilde{x}(t) - \Phi^T(t,\tilde{x}(t)) \\
&\quad \cdot \Phi(t,\tilde{x}(t)) = \eta^T(t)\Lambda\eta(t),
\end{aligned} \tag{20}$$

where $\eta(t) = \left[\tilde{x}^T(t) \;\; \Phi^T(t,\tilde{x}(t)) \;\; \tilde{w}^T(t) \;\; \tilde{x}^T(t+1)\tilde{E}^T\right]$ and

$$\Lambda = \begin{bmatrix} -3\tilde{E}^T\mathscr{P}\tilde{E} + \widetilde{\mathscr{C}}^T\widetilde{\mathscr{C}} + \widetilde{\mathscr{F}}^T\widetilde{\mathscr{F}} & * & * & * \\ 0 & -I & * & * \\ \widetilde{\mathscr{D}}^T\widetilde{\mathscr{C}} & 0 & -\gamma^2 I + \widetilde{\mathscr{D}}^T\widetilde{\mathscr{D}} & * \\ \mathscr{P}\widetilde{\mathscr{A}} & \mathscr{P}\mathscr{M} & \mathscr{P}\widetilde{\mathscr{B}} & \mathscr{P} \end{bmatrix}. \tag{21}$$

Form the filtering error system (6), we have

$$\underbrace{\left[\widetilde{\mathscr{A}} \;\; \mathscr{M} \;\; \widetilde{\mathscr{B}} \;\; -I\right]}_{\mathscr{B}}\underbrace{\eta(t)}_{\xi} = 0, \tag{22}$$

and, by using conditions (1) and (4) of Lemma 3 with $\mathscr{X} = \left[\mathscr{N} \;\; 0 \;\; 0 \;\; \mathscr{G}\right]$ and Schur complement lemma, we can obtain $\Lambda < 0$ if and only if (10) is true. Then for any $\eta(t) \neq 0$ we have $V(\tilde{x}(t+1)) - V(\tilde{x}(t)) + e^T(t)e(t) - \gamma^2 w^T(t)w(t) < 0$, which implies that $V(\tilde{x}(\infty)) - V(\tilde{x}(0)) + \sum_{t=0}^{\infty} e^T(t)e(t) - \gamma^2\sum_{t=0}^{\infty} w^T(t)w(t) < 0$. Consider the zero initial condition and we obtain $\sum_{t=0}^{\infty} e^T(t)e(t) < \gamma^2\sum_{t=0}^{\infty} w^T(t)w(t)$ for any nonzero $w(t) \in l_2[0, \infty)$. □

Based on the $\mathscr{H}_\infty$ performance analysis criterion given in Theorem 6, we will present a sufficient condition for designing the quantized $\mathscr{H}_\infty$ filter in the form of (3), that is, to determine the filter gain matrices in (3) such that the prescribed $\mathscr{H}_\infty$ performance $\gamma > 0$ is guaranteed for the quantized filtering error system (6).

Theorem 7. *Let us consider the nonlinear singular system (1) and the quantized filter (3). For given quantization density $\rho > 0$, the quantized filtering error system (6) is admissible and of a unique solution with a prescribed $\mathscr{H}_\infty$ performance $\gamma > 0$, if there exist matrices $\mathscr{P}_1 = \mathscr{P}_1^T > 0$, $\mathscr{G}_1$, $\mathscr{G}_2$, $\mathscr{N}_1$, and nonsingular matrix $\mathscr{K}$ and some scalars λ_1, a_1, a_2, a_3, and $\varepsilon > 0$ such that the following matrix inequality holds:*

$$\begin{bmatrix} \Upsilon_{11} & * & * & * & * & * & * & * & * \\ \Upsilon_{21} & -I & * & * & * & * & * & * & * \\ \Upsilon_{31} & 0 & -\gamma^2 I & * & * & * & * & * & * \\ \Upsilon_{41} & \Upsilon_{42} & \Upsilon_{43} & \Upsilon_{44} & * & * & * & * & * \\ \overline{\mathscr{C}} & 0 & \overline{\mathscr{D}} & 0 & -I & * & * & * & * \\ \Upsilon_{61} & 0 & 0 & \Upsilon_{64} & -\overline{\mathscr{D}}_f^T & -\varepsilon I & * & * & * \\ \Upsilon_{71} & 0 & 0 & \Upsilon_{74} & 0 & 0 & -\varepsilon I & * & * \\ \varepsilon\delta\mathscr{H}_2 & 0 & \varepsilon\delta\mathscr{D} & 0 & 0 & 0 & 0 & -\varepsilon I & * \\ \varepsilon\delta\mathscr{H}_2 & 0 & \varepsilon\delta\mathscr{D} & 0 & 0 & 0 & 0 & 0 & -\varepsilon I \end{bmatrix} < 0, \tag{23}$$

where

$$\Upsilon_{11} = \begin{bmatrix} -3E^T\mathscr{P}_1E + \mathrm{He}\{\mathscr{N}_1\mathscr{A}\} + \mathscr{F}^T\mathscr{F} & * \\ \mathscr{K}^TE + \overline{\mathscr{B}}_f\mathscr{C} & \mathrm{He}\{\mathscr{A}_f\} + \mathscr{F}^T\mathscr{F} + a_1\mathscr{K} \end{bmatrix},$$

$$\Upsilon_{21} = \begin{bmatrix} \mathscr{N}_1^T & 0 \\ 0 & \lambda_1^T\mathscr{K}^T \end{bmatrix},$$

$$\Upsilon_{31} = \left[\mathscr{B}^T\mathscr{N}_1^T \;\; \mathscr{D}^T\overline{\mathscr{B}}_f^T\right],$$

$$\Upsilon_{41} = \begin{bmatrix} \mathscr{P}_1\mathscr{A} + \mathscr{G}_1\mathscr{A} + (1+a_2)\overline{\mathscr{B}}_f\mathscr{C} - \mathscr{N}_1^T & (1+a_2)\overline{\mathscr{A}}_f \\ \mathscr{K}^T\mathscr{A} + \mathscr{G}_2\mathscr{A} + (a_1+a_3)\overline{\mathscr{B}}_f\mathscr{C} & (a_1+a_3)\overline{\mathscr{A}}_f - \mathscr{K}^T \end{bmatrix},$$

$$\Upsilon_{42} = \begin{bmatrix} \mathscr{P}_1 + \mathscr{G}_1 & (a_2+1)\lambda_1\mathscr{K} \\ \mathscr{K}^T + \mathscr{G}_2 & (a_1+a_3)\lambda_1\mathscr{K} - \mathscr{K}^T \end{bmatrix},$$

$$\Upsilon_{43} = \begin{bmatrix} \mathscr{P}_1\mathscr{B} + \mathscr{G}_1\mathscr{B} + (a_2+1)\overline{\mathscr{B}}_f\mathscr{D} \\ \mathscr{K}^T\mathscr{B} + \mathscr{G}_2\mathscr{B} + (a_1+a_3)\overline{\mathscr{B}}_f\mathscr{D} \end{bmatrix},$$

$$\Upsilon_{44} = \begin{bmatrix} \mathscr{P}_1 - \mathrm{He}\{\mathscr{G}_1\} & * \\ \mathscr{K}^T - \mathscr{G}_2 - a_2\mathscr{K}^T & a_1\mathscr{K} - \mathrm{He}\{a_3\mathscr{K}\} \end{bmatrix},$$

$$\Upsilon_{61} = \Upsilon_{71} = \begin{bmatrix} 0 & \overline{\mathscr{B}}_f^{\,T} \end{bmatrix},$$

$$\Upsilon_{64} = \begin{bmatrix} \overline{\mathscr{B}}_f^{\,T} & a_1\overline{\mathscr{B}}_f^{\,T} \end{bmatrix},$$

$$\Upsilon_{74} = \begin{bmatrix} a_2\overline{\mathscr{B}}_f^{\,T} & a_3\overline{\mathscr{B}}_f^{\,T} \end{bmatrix}. \tag{24}$$

The $\mathscr{H}_\infty$ filter gain matrices in (3) can be obtained from (23) as $\mathscr{A}_f = \mathscr{K}^{-1}\overline{\mathscr{A}}_f$, $\mathscr{B}_f = \mathscr{K}^{-1}\overline{\mathscr{B}}_f$, $\mathscr{C}_f = \overline{\mathscr{C}}_f$, and $\mathscr{D}_f = \overline{\mathscr{D}}_f$.

Proof. The condition of (10) can be rewritten as follows:

$$\Pi + \mathrm{He}\left\{\Pi_1\frac{\Delta(k)}{\delta}\Pi_2\right\}, \tag{25}$$

where

$$\Pi = \begin{bmatrix} -3\widetilde{E}^T\mathscr{P}\widetilde{E} + \mathrm{He}\left\{\mathscr{N}\overline{\mathscr{A}}\right\} + \widetilde{\mathscr{F}}^T\widetilde{\mathscr{F}} & * & * & * & * \\ \mathscr{M}^T\mathscr{N}^T & -I & * & * & * \\ \overline{\mathscr{B}}^T\mathscr{N}^T & 0 & -\gamma^2 I & * & * \\ \mathscr{P}\overline{\mathscr{A}} - \mathscr{N}^T + \mathscr{G}\overline{\mathscr{A}} & \mathscr{P}\mathscr{M} + \mathscr{G}\mathscr{M} & \mathscr{P}\overline{\mathscr{B}} + \mathscr{G}\overline{\mathscr{B}} & \mathscr{P} - \mathrm{He}\{\mathscr{G}\} & * \\ \overline{\mathscr{C}} & 0 & \overline{\mathscr{D}} & 0 & -I \end{bmatrix},$$

$$\Pi_1 = \begin{bmatrix} \mathscr{H}_1^T\mathscr{N}^T & 0 & 0 & \mathscr{H}_1^T\mathscr{P}^T & -\mathscr{D}_f^T \\ \mathscr{H}_1^T\mathscr{N}^T & 0 & 0 & \mathscr{H}_1^T\mathscr{G}^T & 0 \end{bmatrix}^T,$$

$$\Pi_2 = \begin{bmatrix} \delta\mathscr{H}_2 & 0 & \delta\mathscr{D} & 0 & 0 \\ \delta\mathscr{H}_2 & 0 & \delta\mathscr{D} & 0 & 0 \end{bmatrix}, \tag{26}$$

and $\mathscr{H}_1 = \begin{bmatrix} 0 & \mathscr{B}_f^T \end{bmatrix}^T$ and $\mathscr{H}_2 = \begin{bmatrix} \delta\mathscr{C} & 0 \end{bmatrix}$.

Then, by Lemma 5, we have that there exists a constant scalar $\varepsilon > 0$ such that

$$\begin{bmatrix} \Pi & * \\ \Sigma_1 & \Sigma_2 \end{bmatrix} < 0, \tag{27}$$

where

$$\Sigma_1 = \begin{bmatrix} \mathscr{H}_1^T\mathscr{N}^T & 0 & 0 & \mathscr{H}_1^T\mathscr{P}^T & -\mathscr{D}_f^T \\ \mathscr{H}_1^T\mathscr{N}^T & 0 & 0 & \mathscr{H}_1^T\mathscr{G}^T & 0 \\ \varepsilon\delta\mathscr{H}_2 & 0 & \varepsilon\delta\mathscr{D} & 0 & 0 \\ \varepsilon\delta\mathscr{H}_2 & 0 & \varepsilon\delta\mathscr{D} & 0 & 0 \end{bmatrix},$$

Table 1: The minimum $\mathscr{H}_\infty$ filtering performances $\gamma_{\min}$ under different quantization density ρ.

ρ	0.3	0.4	0.5	0.6	0.7	0.8
δ	0.5385	0.4286	0.3333	0.2500	0.1765	0.1111
$\gamma_{\min}$	0.1951	0.1924	0.1820	0.1586	0.1363	0.1177

$$\Sigma_2 = \begin{bmatrix} -\varepsilon I & * & * & * \\ 0 & -\varepsilon I & * & * \\ 0 & 0 & -\varepsilon I & * \\ 0 & 0 & 0 & -\varepsilon I \end{bmatrix}. \tag{28}$$

We can design the filter if (27) is solvable. Here, assume that the matrix variables involved in (27) have the following form:

$$\{\mathscr{P}, \mathscr{G}, \mathscr{N}\} = \left\{ \begin{bmatrix} \mathscr{P}_1 & * \\ \mathscr{K}^T & a_1\mathscr{K} \end{bmatrix}, \begin{bmatrix} \mathscr{G}_1 & a_2\mathscr{K} \\ \mathscr{G}_2 & a_3\mathscr{K} \end{bmatrix}, \begin{bmatrix} \mathscr{N}_1 & 0 \\ 0 & \mathscr{K} \end{bmatrix} \right\}. \tag{29}$$

With the aforementioned related matrices considered and letting $\overline{\mathscr{A}}_f = \mathscr{K}\mathscr{A}_f$, $\overline{\mathscr{B}}_f = \mathscr{K}\mathscr{B}_f$, $\overline{\mathscr{C}}_f = \mathscr{C}_f$, $\overline{\mathscr{D}}_f = \mathscr{D}_f$, Theorem 7 can be obtained from (27). □

Remark 8. For given a_1, a_2, a_3, and δ, Theorem 7 is strictly LMIs. The quantization error bound δ can be calculated according to the given quantization density ρ by (5) and the optimal values of a_1, a_2, and a_3 can be obtained by using fminsearch function in optimization toolbox of MATLAB (see [34, 37] for more details). Then Theorem 7 can be easily solved with the help of LMI control box in MATLAB [38].

4. A Numerical Example

In this section, a numerical example will be presented to illustrate the effectiveness and applicability of the proposed method. Let us consider the following nonlinear system in form of (1) borrowed from [16]:

$$\begin{aligned} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t+1) \\ x_2(t+1) \end{bmatrix} &= \begin{bmatrix} -0.5 & 0 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \\ &\quad + \begin{bmatrix} 0.5 \\ -0.5 \end{bmatrix} w(t) \\ &\quad + \begin{bmatrix} \frac{1}{\sqrt{10}} \sin(x_1(t)) \\ \frac{1}{\sqrt{10}} \sin(x_2(t)) \end{bmatrix}, \\ y(t) &= \begin{bmatrix} 0.1 & 0.2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} - w(t), \\ z(t) &= \begin{bmatrix} 0.2 & -0.1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}, \end{aligned} \tag{30}$$

and then we have $\mathscr{F} = \left[\begin{smallmatrix} 0.1 & 0 \\ 0 & 0.1 \end{smallmatrix}\right]$ from (2).

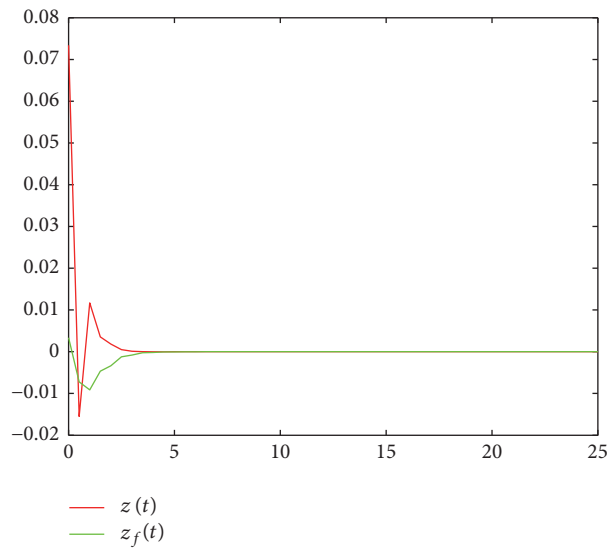

Figure 1: Responses of $z(t)$ and $z_f(t)$.

Table 1 shows the minimum $\mathscr{H}_\infty$ attention level obtained by using Theorem 7 with $a_1 = -1.20$, $a_2 = 0.11$, $a_3 = 3.50$, and $\lambda_1 = 1$, under different quantization density. As expected, we can obtain that $\gamma_{\min}$ increases as the quantization density ρ decreases from this table. Moreover, when $\gamma_{\min} = 0.1820$, the corresponding filter parameters of Theorem 7 can be calculated as follows:

$$\begin{aligned} \mathscr{A}_f &= \begin{bmatrix} 0.2688 & 0.5305 \\ -0.2087 & -0.9779 \end{bmatrix}, \\ \mathscr{B}_f &= \begin{bmatrix} 0.4268 \\ -0.5199 \end{bmatrix} \\ \mathscr{C}_f &= \begin{bmatrix} 0.0456 & 0.0620 \end{bmatrix}, \\ \mathscr{D}_f &= 0.0114. \end{aligned} \tag{31}$$

We assume the external disturbance $w(t) = \sin(t)e^{-2t}$, and the simulation results of signals $z(t)$ and $z_f(t)$ are shown in Figure 1. Figure 2 shows the response of the filtering error $e(t)$. From Figures 1 and 2, we can see that the designed $\mathscr{H}_\infty$ filter is effective.

5. Conclusion

The quantized $\mathscr{H}_\infty$ filtering problem for a class of discrete-time Lipschitz nonlinear singular systems has been addressed in this paper, where the system measurement output is quantized by a static, memoryless, and logarithmic quantizer.

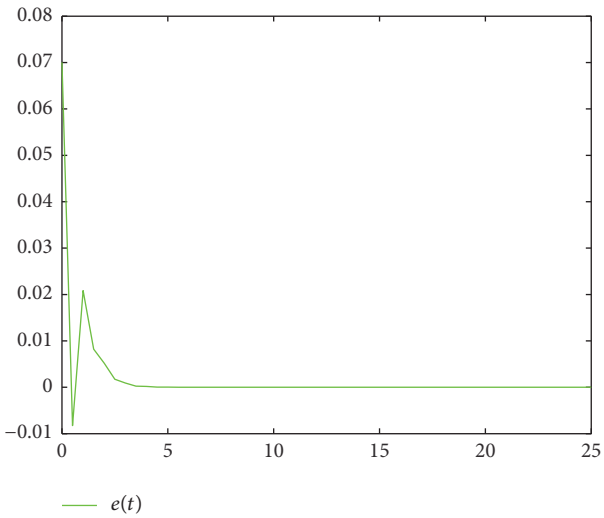

Figure 2: Filtering error $e(t)$.

By introducing some variables and applying Lyapunov stability theory, sufficient conditions for designing the quantized $\mathscr{H}_\infty$ filter are presented in linear matrix inequalities (LMIs), which ensure the quantized filtering error systems to be admissible and has a unique solution with a prescribed $\mathscr{H}_\infty$ performance. A numerical example is given to show the effectiveness of the proposed design method. The results proposed in this paper can be further developed by the novel LMI decoupling approach presented in [39] which has also been used to deal with the $\mathscr{H}_\infty$ filtering problem; see [40] for more details.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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