Hindawi
Mathematical Problems in Engineering
Volume 2017, Article ID 9589638, 7 pages
https://doi.org/10.1155/2017/9589638

Research Article

One-Dimensional Vacuum Steady Seepage Model of Unsaturated Soil and Finite Difference Solution

Feng Huang, Jianguo Lyu, Guihe Wang, and Hongyan Liu

School of Engineering & Technology, China University of Geosciences, Beijing 100083, China

Correspondence should be addressed to Feng Huang; huangfengcugb@sina.cn

Received 6 December 2016; Revised 16 February 2017; Accepted 23 February 2017; Published 5 March 2017

Academic Editor: Manuel Pastor

Vacuum tube dewatering method and light well point method have been widely used in engineering dewatering and foundation treatment. However, there is little research on the calculation method of unsaturated seepage under the effect of vacuum pressure which is generated by the vacuum well. In view of this, the one-dimensional (1D) steady seepage law of unsaturated soil in vacuum field has been analyzed based on Darcy's law, basic equations, and finite difference method. First, the gravity drainage ability is analyzed. The analysis presents that much unsaturated water can not be drained off only by gravity effect because of surface tension. Second, the unsaturated vacuum seepage equations are built up in conditions of flux boundary and waterhead boundary. Finally, two examples are analyzed based on the relationship of matric suction and permeability coefficient after boundary conditions are determined. The results show that vacuum pressure will significantly enhance the drainage ability of unsaturated water by improving the hydraulic gradient of unsaturated water.

1. Introduction

In geotechnical engineering, vacuum drainage method has been widely used. In order to improve the efficiency and effectiveness of pumping, vacuum drainage puts negative air pressure in the dewatering wells or drainage pipes to improve the hydraulic gradient of groundwater. The method of vacuum drainage can effectively remove the interstice water and the pore water in weak permeable layer which is difficult to be treated by the traditional drainage method, providing a water-free environment for the underground excavation. In the process of vacuum drainage, the water discharged can be divided into two parts; one is the discharge of saturated water, mainly analyzing the change of free water surface; the other part is the seepage of unsaturated water. For the vacuum seepage of water below the free surface, the emphasis is on the changes of waterhead, hydraulic gradient, and water discharge caused by the vacuum pressure field. For the vacuum seepage of the unsaturated soil, the seepage process is more complex mainly because of its unsteady permeability coefficient. The permeability coefficient of unsaturated soil changes with the change of water content, and it is also affected by soil deformation and other factors. The studies of unsaturated soil in vacuum field mainly include hydraulic boundary conditions and the scope of the vacuum pressure transmission and the estimation method.

At present, many scholars have studied the seepage of unsaturated soil, which is the basis for studying the seepage in vacuum drainage. Cao et al. [1] conducted an experiment to study the hydraulic properties of cracks. The dynamic development of cracks in the expansive soil during drying and wetting has been measured in the laboratory and then the unsaturated seepage in cracked soil is analyzed. Huang and Zhou [2] analyzed the movement of water flow in unsaturated fractured rock with the sandstone sample through experimental research and numerical simulation.

In the aspect of numerical analysis, many methods have been used to solve the unsaturated seepage problems. Lam and Fredlund [3] discussed the saturated-unsaturated transient finite element seepage model for geotechnical engineering. Huang and Jia [4] analyzed the effects of unsaturated transient seepage concerning the stability of an earth dam under rapid drawdown using finite element method. Ng and Shi [5] conducted a parametric study using the finite element method to investigate the influence of various rainfall events and initial ground conditions on transient seepage. Sako and Kitamura [6] proposed a practical numerical model for seepage behavior of unsaturated soil to improve the previous model proposed by Kitamura et al. [7]. Zhang et al. [8] proposed a method to simplify the finite element analysis of seepage to a certain degree with satisfactory results. Pedroso [9] presented a solution to seepage problems in porous media considering the complete time-dependent transition from full saturation to partially unsaturated states. Fu and Jin [10] proposed a numerical model to simulate the unsteady seepage flow through dam, with both saturated and waterhead as variables to describe the seepage domain. Andreea [11] presented stability and seepage analysis performed in unsaturated regime by using steady-state analysis and transient analysis. Based on the theory of porous media, Garcia et al. [12] used a multiphase coupled elastoviscoplastic finite element analysis formulation to describe the rainfall infiltration process into a one-dimensional soil column and obtain the results of generation of pore water pressure and deformations. Liu et al. [13] also analyzed the seepage-deformation in unsaturated soils in numerical way. Wang and Li [14] presented solutions in one-dimensional coupled seepage and deformation of unsaturated soils with arbitrary nonhomogeneous boundary conditions by using finite difference method. The result indicates that the boundary conditions and the coupling effect have a significant influence on the seepage in unsaturated soils. Using eigenfunction expansion and Laplace transformation techniques, Ho et al. [15] discussed the dissipation of excess pore-air and pore water pressure in the two-dimensional (2D) plane strain consolidation of an unsaturated soil stratum. Ho and Fatahi [16] also introduced an exact analytical solution predicting variations in excess pore-air and pore water pressures of unsaturated soil stratum in 2D plane strain condition. Lu and Godt [17] discussed the infinite slope stability under steady unsaturated seepage conditions, and a case study is presented to examine the predictive utility of the framework proposed in research. In the simulation of steady-state subsurface flow, Romano et al. [18] analyzed the unsaturated flow and seepage faces by theory models.

In the experimental study, many model experiments and field tests have been conducted; for example, Su et al. [19] studied the water seepage and intermittent flow in unsaturated, rough-walled fractures.

Although a lot of work on unsaturated soil seepage has been conducted, the unsaturated seepage in vacuum drainage conditions is not fully studied. The current study of vacuum drainage method is limited to saturated seepage. Pan [20] and Huang et al. [21, 22] analyzed the mechanism of vacuum tube dewatering by laboratory experiment and field test. The lack of research leads to the weak theoretical and computational basis of vacuum drainage, which severely limits the development of the method. In order to obtain the general law of vacuum seepage in unsaturated soil, the finite difference method is used to analyze the vacuum seepage model with flux boundary condition and waterhead boundary condition.

2. Gravity Drainage Ability

When the groundwater level descends under the action of drainage wells, there will be unsaturated zone above the free water surface. When soil is saturated, the pore water will move out because the gravity of water is far greater than the surface tension. In the process of water discharge, the air gradually occupies part of the water's position, which makes saturated soil unsaturated soil. In this situation, the residual water keeps balancing under the action of surface tension and gravity, which makes the pore water stop flowing. The physical model of pore water in unsaturated soils is shown in Figure 1.

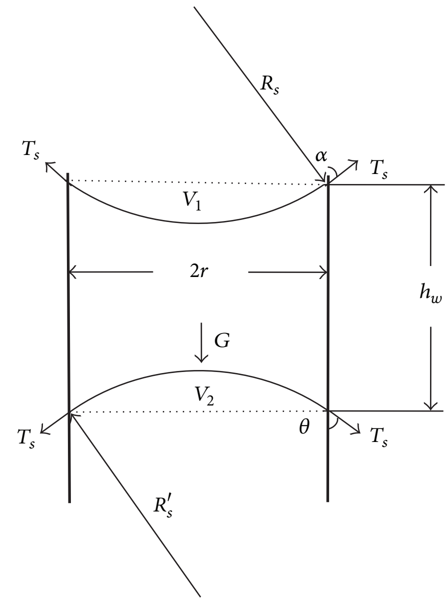

Figure 1: Physical model of pore water in unsaturated soil.

In the physical model, T_s is surface tension; R_s and R_s' are curvature radius of water surface; V_1 and V_2 are the volume of air; V_w is the volume of water; h_w is the height of water element; r is radius of pore; α and θ are the angle between the tangent of water surface and the pore boundary; G is gravity of water element. According to geometric relationship, V_1, V_2, and V_w are presented in

$$
\begin{aligned}
V_1 &= \frac{1}{3}\pi R_s^3 (2+\sin\alpha)(1-\sin\alpha)^2, \\
V_2 &= \frac{1}{3}\pi R_s'^3 (2+\sin\theta)(1-\sin\theta)^2, \\
V_w &= \pi r^2 h_w - \frac{1}{3}\pi \left[R_s^3 (2+\sin\alpha)(1-\sin\alpha)^2 \right. \\
&\quad \left. + \left(\frac{R_s\cos\alpha}{\cos\theta}\right)^3 (2+\sin\theta)(1-\sin\theta)^2 \right].
\end{aligned} \tag{1}
$$

According to the equilibrium condition of statics are as follows:

$$2\pi r T_s \cos\alpha = G + 2\pi r T_s \cos\theta. \tag{2}$$

According to (1), (2) can be expressed as

$$2\pi r T_s \cos\alpha = V_w \rho_w g + 2\pi r T_s \cos\theta. \quad (3)$$

Equation (3) is the equilibrium conditions of gravity drainage.

Since the pressure gradient is the driving force of the gas and water movement, it is possible to increase the pressure gradient by applying a negative pressure difference across the pore water to promote the movement of unsaturated water. Therefore, according to equilibrium conditions of gravity drainage, when the air pressure difference, Δp, is generated by vacuum pumping on both ends of the pore water, (3) can be changed into

$$2\pi r T_s \cos\alpha = V_w \rho_w g + 2\pi r T_s \cos\theta + \Delta p. \quad (4)$$

Equation (4) is the balance equation of the pore water under the pressure difference condition. It is can be seen from the equation that when

$$\Delta p > 2\pi r T_s \cos\alpha - V_w \rho_w g - 2\pi r T_s \cos\theta \quad (5)$$

the equilibrium of the equation will be broken and the pore water begins to move under the pressure difference. At the same time, a conclusion can be drawn that the larger the pressure difference is, the more the pore water can be driven. Therefore, in order to pump the unsaturated water, the negative pressure can be applied to the drainage well to increase the pressure difference.

3. Darcy's Law of Unsaturated Seepage

Unsaturated seepage can be studied using Darcy's law [23]. Darcy's law is

$$v_w = -k_w \frac{\partial h_w}{\partial y}, \quad (6)$$

where v_w is the velocity of flow; k_w is permeability coefficient; $\partial h_w/\partial y$ is hydraulic gradient. For unsaturated soils, the permeability coefficient is not constant and is a function of water content or matric suction $(u_a - u_w)$. Therefore, it is necessary to consider the change of permeability coefficient in studying unsaturated seepage.

For an unsaturated soil element, there is a one-dimensional steady flow in the x-direction. The soil element is shown in Figure 2.

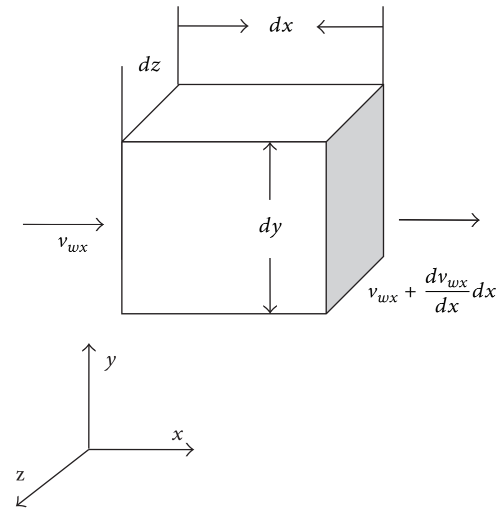

Figure 2: Unsaturated seepage element.

In Figure 2, v_{wx} is the volume of water flowing through the unit area of soil in x-direction. According to the continuity of water flow, the amount of water entering the soil element is equal to the amount of water flowing out, which can be shown in

$$\left(v_{wx} + \frac{dv_{wx}}{dx}dx\right)dydz - v_{wx}dydz = 0. \quad (7)$$

The net flow can be worked out from (7):

$$\frac{dv_{wx}}{dx}dxdydz = 0. \quad (8)$$

Substituting Darcy's law into (8),

$$\frac{d\{-k_{wx}(u_a - u_w)\,dh_w/dx\}}{dx}dxdydz = 0. \quad (9)$$

$k_{wx}(u_a - u_w)$ is the permeability coefficient of the unsaturated soil, which is related to the matric suction and changes in the x-direction. k_{wx} represents $k_{wx}(u_a - u_w)$ in the following, conveniently. dh_w/dx is hydraulic gradient in x-direction; h_w is waterhead. Equation (9) is one-dimensional steady seepage equation of unsaturated soil. After being simplified, (9) can be expressed as

$$k_{wx}\frac{d^2 h_w}{dx^2} + \frac{dk_{wx}}{dx}\frac{dh_w}{dx} = 0. \quad (10)$$

In this study, only the seepage law of unsaturated soil in horizontal direction is considered, so the position head is steady along x-direction and the pore water pressure head is variable.

4. Finite Difference Solution of Vacuum Seepage in Waterhead Boundary Conditions

For one-dimensional steady vacuum seepage of unsaturated soil, the water pressure head h_{p0} at the boundary of the vacuum seepage field is measured as a boundary condition. The water pressure head h_{v0} is measured at the wall of the vacuum well as another boundary condition. The one-dimensional seepage area is equidistantly separated into n points with the pitch of Δx. The calculation model is shown in Figure 3.

The boundary condition is

$$\begin{aligned} f(x_1) &= h_{p_0}, \quad x = 0, \\ f(x_n) &= h_{v_0}, \quad x = l. \end{aligned} \quad (11)$$

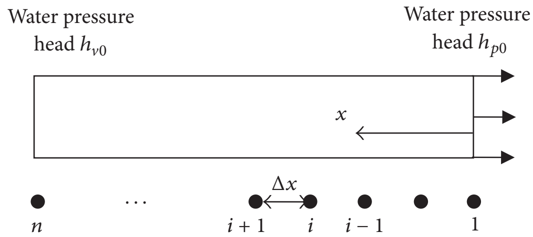

FIGURE 3: Finite difference calculation model of one-dimensional steady vacuum seepage in waterhead boundary condition.

The finite difference equation at the point i is

$$k_{wxi}\left\{\frac{h_{w(i+1)}+h_{w(i-1)}-2h_{wi}}{(\Delta x)^2}\right\} + \left\{\frac{k_{wx(i+1)}-k_{wx(i-1)}}{2\Delta x}\right\}\left\{\frac{h_{w(i+1)}-h_{w(i-1)}}{2\Delta x}\right\}=0, \tag{12}$$

where $k_{wx(i-1)}$, k_{wxi}, and $k_{wx(i+1)}$ are the permeability coefficients at points $i-1$, i, and $i+1$, respectively. $h_{wx(i-1)}$, h_{wxi}, and $h_{wx(i+1)}$ are the water pressure head at points $i-1$, i, and $i+1$, respectively.

Equation (12) can be changed as follows:

$$-\left\{8k_{wxi}\right\}h_{wi}+\left\{4k_{wxi}+k_{wx(i+1)}-k_{wx(i-1)}\right\}h_{w(i+1)} + \left\{4k_{wxi}+k_{wx(i-1)}-k_{wx(i+1)}\right\}h_{w(i-1)}=0. \tag{13}$$

In the differential calculation, the saturated permeability coefficient is used to calculate water pressure head at every point in the first iteration process. Then, the matric suction will be determined according to the water pressure head worked out before. After finding out the matric suction, the permeability coefficient of unsaturated soil can be determined according to the relationship between matric suction and permeability coefficient. At this time, the first iterative process is completed. Furthermore, next iterative process begins by submitting the calculated permeability coefficient to (13). The iterative process will continue until the water pressure head and permeability coefficient become stable. The permeability coefficient keeps stable during each iterative process, which means the equation for each iteration is linear equation.

The corresponding relationship between permeability coefficient and matric suction of unsaturated soil should be solved according to soil water characteristic curve (SWCC). The permeability coefficient of unsaturated soil varies with the water content, so the permeability coefficient is a function of water content. Meanwhile, the volume water content θ_w can be written as a function of the matric suction (u_a-u_w), and the SWCC can be obtained according to the relationship between θ_w and (u_a-u_w). Therefore, the permeability coefficient of unsaturated soil can also be expressed by the matric suction and unsaturated soil permeability coefficient can be obtained through the soil water characteristic curve.

First, the soil water characteristic curve is equally divided into m parts along the volume water content. Then the

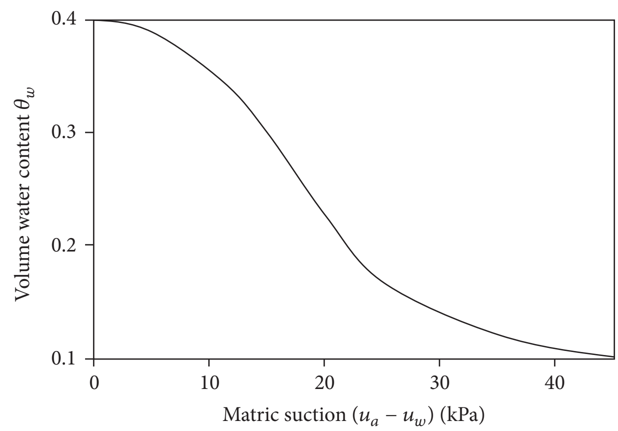

FIGURE 4: Soil water characteristic curve.

permeability coefficient is calculated using the matric suction corresponding to each intermediate point [20]. The permeability coefficient corresponding to the water content in i zone is

$$k_w(\theta_w)_i=\frac{k_s}{k_{sc}}A_d\sum_{j=i}^{m}\left[(2j+1-2i)(u_a-u_w)_j^{-2}\right] \qquad i=1,2,\ldots,m, \tag{14}$$

where $A_d=(T_s^2\rho_w g/2\mu_w)(\theta_s^p/N^2)$; i is zone number; j is the count from i to m; k_s is measured saturated permeability coefficient; k_{sc} is calculated saturated permeability coefficient; ρ_w is density of water; g is gravitational acceleration; μ_w is absolute viscosity of water; θ_s is saturated volume water content; p is interaction constants of pores with different sizes, which is assumed to be 2; N is the total number of discontinuities between saturated volume water content and zero volume water content, $N=m[\theta_s/(\theta_s-\theta_L)]$, $m\le N$, $m=n$, when $\theta_L=0$; $(u_a-u_w)_j$ is matric suction corresponding to the ith discontinuous midpoint.

As an example, Figure 4 is a soil water characteristic curve.

For Figure 4, the curve is divided into 6 parts. The values of 6 points are shown in Figure 5.

Since the value of A_d does not affect the final value of k_w [20], the value of A_d is assumed to be 1 for calculation. In this example, the saturated permeability coefficient $k_s=1\times10^{-8}$ m/s, so k_{sc} is worked out as follows:

$$k_{sc}=A_d\sum_{j=i}^{m}\left[(2j+1-2i)(u_a-u_w)_j^{-2}\right] \qquad i=0,1,2,\ldots,m. \tag{15}$$

Substituting the matric suction values in Figure 5 into (15) yields $k_{sc}=3.6\times10^{-3}$ m/s, so $k_s/k_{sc}=2.8\times10^{-6}$ in (14). According to the obtained parameter values, the permeability coefficient can be found when the matric suction value in Figure 5 is substituted into (15). The calculated results of the example are shown in Table 1.

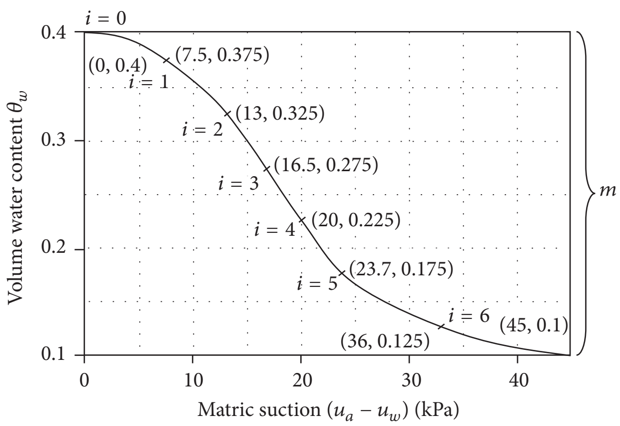

Figure 5: The distribution of discrete points of soil water characteristic curve.

Table 1: Calculated results of matric suction and permeability coefficient.

Point	$(u_a - u_w)$/kPa	k_{sc}/(m/s)
0	0	1×10^{-8}
1	7.5	3.47×10^{-9}
2	13	2.66×10^{-9}
3	16.5	1.92×10^{-9}
4	20	1.23×10^{-9}
5	23.7	6.30×10^{-10}
6	36	1.79×10^{-10}

It can be seen from Table 1 that the water permeability coefficient decreases as the matric suction increases. This is because when the matric suction increases, the water content decreases and gas occupies more volume, which leads to reduced water permeability coefficient.

The calculation process of the example is a step of the difference method. Using the method above, assume the pore water pressure is −40 kPa, the vacuum influence range is 5 m, and the pore water pressure at the distance of 5 m is −10 kPa in the one-dimensional unsaturated soil shown in Figure 3. Then the water pressure of the one-dimensional steady seepage under vacuum effect is shown in Figure 6.

It can be seen from Figure 6 that the water pressure at every point is equal when unsaturated soil is in the absence of vacuum pressure, which will not produce seepage. When the vacuum is applied to the unsaturated soil, the water pressure at each point is reduced. The closer the distance is from the vacuum well, the lower the pore water pressure value is. Thus the unsaturated water under the action of vacuum negative pressure began to seep by increasing the hydraulic gradient. The average hydraulic gradient between every two points is shown in Figure 7.

Figure 7 shows that, under the effect of vacuum pressure, the hydraulic gradient decreases when the distance increases. The hydraulic gradient is related to the transfer of vacuum degree.

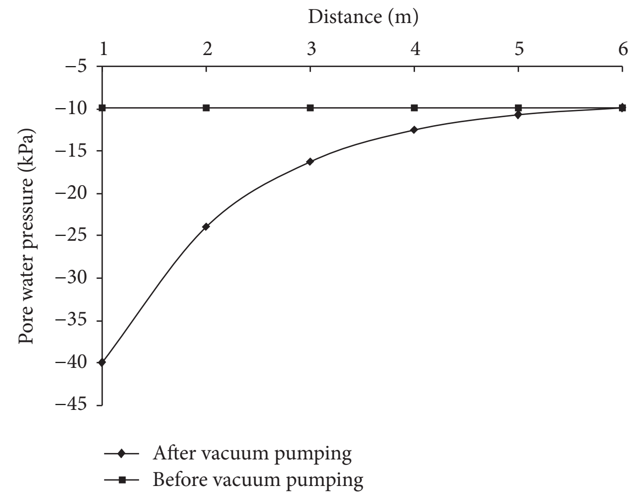

Figure 6: Water pressure distribution of one-dimensional unsaturated seepage in waterhead boundary condition.

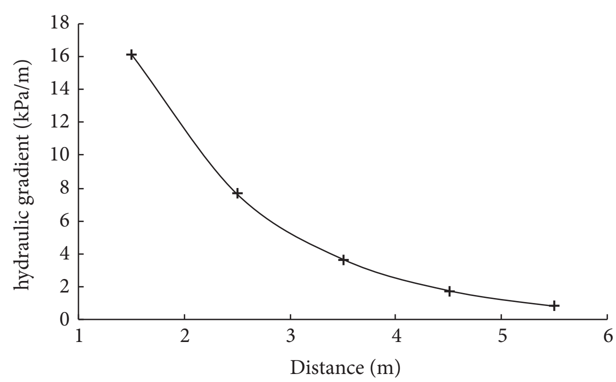

Figure 7: Average hydraulic gradient between every two points in waterhead boundary condition.

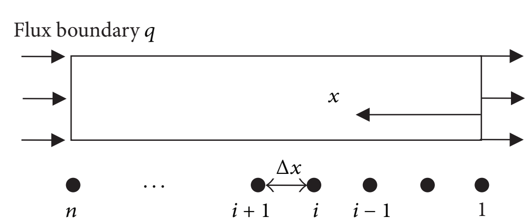

Figure 8: Finite difference calculation model of one-dimensional steady vacuum seepage in flux boundary condition.

5. Finite Difference Solution of Vacuum Seepage in Flux Boundary Conditions

The calculation model of finite difference solution of vacuum seepage in flux boundary conditions is shown in Figure 8.

The flow at point i is represented with the head at $i+1$ and $i-1$ by Darcy's law:

$$q_{wx} = k_{wxi} \frac{h_{w(i+1)} - h_{w(i-1)}}{2\Delta x} A, \tag{16}$$

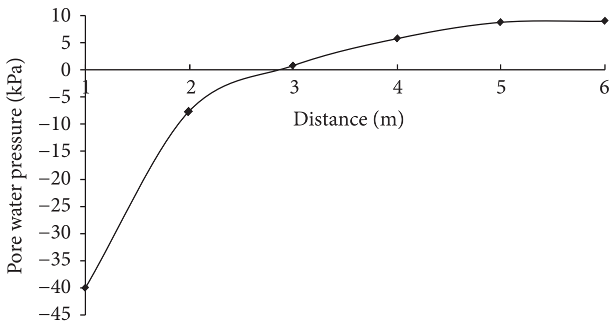

Figure 9: Water pressure distribution of one-dimensional unsaturated seepage in flux boundary condition.

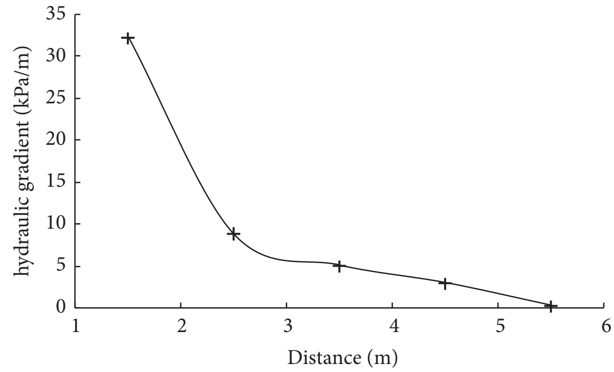

Figure 10: Average hydraulic gradient between every two points in flux boundary condition.

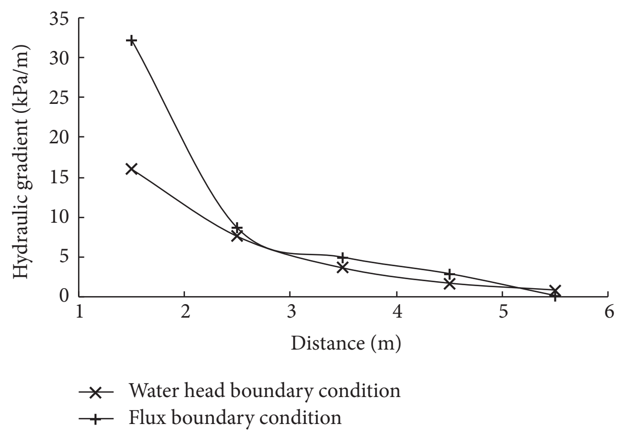

Figure 11: Comparison of variations of hydraulic gradient along the drainage distance.

where A is the cross-sectional area of soil element. Then $h_{w(i+1)}$ can be expressed as

$$h_{w(i+1)} = h_{w(i-1)} + \frac{2q_{wx}\Delta x}{Ak_{wxi}}. \quad (17)$$

It can be obtained by submitting (17) into (13) that

$$\begin{aligned} &-\{8k_{wxi}\} h_{wi} \\ &+ \{4k_{wxi} + k_{wx(i+1)} - k_{wx(i-1)}\}\left\{h_{w(i-1)} + \frac{2q_{wx}\Delta x}{Ak_{wxi}}\right\} \\ &+ \{4k_{wxi} - k_{wx(i+1)} + k_{wx(i-1)}\} h_{w(i-1)} = 0. \end{aligned} \quad (18)$$

h_{wi} can be worked out as

$$\begin{aligned} h_{wi} &= h_{w(i-1)} \\ &+ \left\{\frac{4k_{wxi} + k_{wx(i+1)} - k_{wx(i-1)}}{8k_{wxi}{}^2}\right\}\frac{2q_{wx}\Delta x}{A}. \end{aligned} \quad (19)$$

Water pressure head at each point can be solved using calculation method presented in Section 3. The permeability coefficient is assumed $k_n = k_{n-1}$ when calculating the waterhead at the boundary.

In the one-dimensional model of unsaturated seepage shown in Figure 8, assuming that the pore water pressure at the well wall is −40 kPa, the influence distance of vacuum pressure is 5 m, the cross-sectional area A is 2 m^2, and q_{wx} is 5×10^{-9} m^3/s. The water pressure of the one-dimensional steady seepage under vacuum effect is shown in Figure 9.

The results presented in Figure 9 that vacuum pressure can also increase the hydraulic gradient in flux boundary condition, which will accelerate the seepage of unsaturated water. The closer the distance is from the vacuum well, the lower the pore water pressure value is. For this example, the calculated pore water pressure becomes 0 kPa in the vicinity of 3 m and then becomes positive later. This is because the pumping capacity is not sufficient in this example, which leads to the result that the complement of pore water near the boundary is timely and enough and the soil becomes saturated there. However, the pore pressure near the well is still negative because of the vacuum effect, so the pore water pressure will become positive at a certain location. It can be calculated in the same method that if the vacuum pressure in the well or the boundary flux is drastically reduced, the point at which the pore water pressure is 0 kPa will be further away from the vacuum well and the pore water pressure values may also be all negative. In summary, the pore water pressure becoming positive is related to the specific conditions of the example and different boundary conditions lead to different calculation results.

The average hydraulic gradient between every two points is shown in Figure 10.

Figure 10 shows that, under the effect of vacuum pressure, the average hydraulic gradient decreases as the distance increases. The hydraulic gradient is related to the distribution of vacuum degree.

The comparison between variations of hydraulic gradient along the drainage distance in the waterhead boundary condition and flux boundary condition is shown in Figure 11.

It can be found from the curves that the hydraulic gradient in the flux boundary condition tends to decrease faster than that from the waterhead boundary case in this example. This is because, in the waterhead boundary condition, the water continuously flows to the border to ensure that the waterhead value is constant and negative. However, the water supply at the border in flux boundary condition is sufficient for this example, which makes the pore pressure positive and the waterhead is much higher than the beginning when the dynamic equilibrium of seepage is reached. Thus the hydraulic gradient in the flux boundary condition decreases faster than that from the waterhead boundary case in this example.

6. Conclusions

(1) Due to the surface tension, much unsaturated water can not be drained off by gravity drainage method. This part of the water will seep by applying vacuum pressure difference.

(2) According to the relationship between matric suction and permeability coefficient, a one-dimensional model of vacuum seepage of unsaturated soil is established. According to the head boundary condition and flux boundary condition, the finite difference method is used to solve 2 examples. The results show that the vacuum pressure acting on the well wall can increase the hydraulic gradient in the seepage area of unsaturated soil, which makes the residual unsaturated water flow.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This study is supported by "The Fundamental Research Funds for the Central Universities of China" (no. 2-9-2015-082).

References

[1] L. Cao, Z. Wang, and Y. Chen, "Unsaturated seepage analysis of cracked soil including development process of cracks," *Advances in Materials Science and Engineering*, vol. 2016, Article ID 2684297, 13 pages, 2016.

[2] Y. Huang and Z. F. Zhou, "Effect of a vertical fracture on horizontal unsat-urated seepage flow," *Journal of Hydrodynamics*, vol. 23, no. 1, pp. 499–504, 2011.

[3] L. Lam and D. G. Fredlund, "Saturated-unsaturated transient finite element seepage model for geotechnical engineering," *Advances in Water Resources*, vol. 7, no. 3, pp. 132–136, 1984.

[4] M. S. Huang and C. Q. Jia, "Strength reduction FEM in stability analysis of soil slopes subjected to transient unsaturated seepage," *Computers and Geotechnics*, vol. 36, no. 1-2, pp. 93–101, 2009.

[5] C. W. W. Ng and Q. Shi, "A numerical investigation of the stability of unsaturated soil slopes subjected to transient seepage," *Computers and Geotechnics*, vol. 22, no. 1, pp. 1–28, 1998.

[6] K. Sako and R. Kitamura, "A practical numerical model for seepage behavior of unsaturated soil," *Soils and Foundations*, vol. 46, no. 5, pp. 595–604, 2006.

[7] R. Kitamura, S. Fukuhara, K. Uemura, G. Kisanuki, and M. Seyama, "A numerical model for seepage through unsaturated soil," *Soils and Foundations*, vol. 38, no. 4, pp. 261–265, 1998.

[8] J. Zhang, Q. Xu, and Z. Chen, "Seepage analysis based on the unified unsaturated soil theory," *Mechanics Research Communications*, vol. 28, no. 1, pp. 107–112, 2001.

[9] D. M. Pedroso, "A solution to transient seepage in unsaturated porous media," *Computer Methods in Applied Mechanics and Engineering*, vol. 285, pp. 791–816, 2015.

[10] J.-F. Fu and S. Jin, "A study on unsteady seepage flow through dam," *Journal of Hydrodynamics*, vol. 21, no. 4, pp. 499–504, 2009.

[11] C. Andreea, "Unsaturated slope stability and seepage analysis of a dam," *Energy Procedia*, vol. 85, pp. 93–98, 2016.

[12] E. Garcia, F. Oka, and S. Kimoto, "Numerical analysis of a one-dimensional infiltration problem in unsaturated soil by a seepage-deformation coupled method," *International Journal for Numerical and Analytical Methods in Geomechanics*, vol. 35, no. 5, pp. 544–568, 2011.

[13] E. Liu, H.-S. Yu, G. Deng, J. Zhang, and S. He, "Numerical analysis of seepage–deformation in unsaturated soils," *Acta Geotechnica*, vol. 9, no. 6, pp. 1045–1058, 2014.

[14] H. B. Wang and J. W. Li, "Analytical solutions to the one-dimensional coupled seepage and deformation of unsaturated soils with arbitrary nonhomogeneous boundary conditions," *Transport in Porous Media*, vol. 108, no. 2, pp. 481–496, 2015.

[15] L. Ho, B. Fatahi, and H. Khabbaz, "A closed form analytical solution for two-dimensional plane strain consolidation of unsaturated soil stratum," *International Journal for Numerical and Analytical Methods in Geomechanics*, vol. 39, no. 15, pp. 1665–1692, 2015.

[16] L. Ho and B. Fatahi, "Analytical solution for the two-dimensional plane strain consolidation of an unsaturated soil stratum subjected to time-dependent loading," *Computers and Geotechnics*, vol. 67, pp. 1–16, 2015.

[17] N. Lu and J. Godt, "Infinite slope stability under steady unsaturated seepage conditions," *Water Resources Research*, vol. 44, no. 11, Article ID W11404, pp. 2276–2283, 2008.

[18] C. G. Romano, E. O. Frind, and D. L. Rudolph, "Significance of unsaturated flow and seepage faces in the simulation of steady-state subsurface flow," *Ground Water*, vol. 37, no. 4, pp. 625–632, 1999.

[19] G. W. Su, J. T. Geller, K. Pruess, and F. Wen, "Experimental studies of water seepage and intermittent flow in unsaturated, rough-walled fractures," *Water Resources Research*, vol. 35, no. 4, pp. 1019–1037, 1999.

[20] X. M. Pan, *Research on application of compound dewatering technique of vacuum tube well [Ph.D. thesis]*, China University of Geosciences, Beijing, China, 2009 (Chinese).

[21] F. Huang, G. Wang, C. Wang, and X. Pan, "Application analysis of large-diameter vacuum tube dewatering well-with a certain range of beijing subway project as an example," *Electronic Journal of Geotechnical Engineering*, vol. 18, pp. 2281–2296, 2013.

[22] F. Huang and G. Wang, "One-dimensional experimental research on vacuum degree transmission law in unsaturated soil," *Electronic Journal of Geotechnical Engineering*, vol. 17, pp. 3313–3322, 2012.

[23] D. G. Fredlund, *Soil Mechanics for Unsaturated Soils*, China Architecture & Building Press, Beijing Shi, China, 1997 (Chinese).