Research Article

A Fast Detection Algorithm for the X-Ray Pulsar Signal

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Received 22 August 2016; Revised 27 November 2016; Accepted 22 December 2016; Published 4 January 2017

Academic Editor: Yakov Strel'niker

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The detection of the X-ray pulsar signal is important for the autonomous navigation system using X-ray pulsars. In the condition of short observation time and limited number of photons for detection, the noise does not obey the Gaussian distribution. This fact has been little considered extant. In this paper, the model of the X-ray pulsar signal is rebuilt as the nonhomogeneous Poisson distribution and, in the condition of a fixed false alarm rate, a fast detection algorithm based on maximizing the detection probability is proposed. Simulation results show the effectiveness of the proposed detection algorithm.

1. Introduction

X-ray pulsars are rapidly rotating neutron stars which could emit X-ray signals periodically, stably, and uniquely [1]. Since Downs first proposed interplanetary navigation using pulsars in 1974 [2], X-ray Pulsar Navigation (XPNV) system has been considered in lots of scenarios for autonomous navigation [3–7]. XPNV gets the spacecraft's position vector from measuring Time Difference of Arrival (TDOA) of the X-ray pulsar signals between the spacecraft and the Solar System Barycenter (SSB). Because of the large distance (thousands of light years) between the X-ray pulsars and the solar system, the received X-ray pulsar signal is always weak and easy to be disturbed by kinds of noise [8]. It is difficult to recover the X-ray pulsar profile [9] and get high precision TDOA, and each observation time for the X-ray pulsars may be as long as hundreds of seconds [3, 10]. However, before using the X-ray pulsar signals for navigation, we should firstly determine whether we have received the desired X-ray pulsar signals, which would help the detector's altitude controller to confirm the right direction of the X-ray pulsar. Unlike measuring TDOA, the signal detection process should be both accurate and fast. Therefore, it is a challenge to detect the weak X-ray pulsar signal with short time and limited photons.

The first widely used X-ray pulsar signal detection algorithm is based on the Fast Fourier Transform (FFT), which has been proved to be invalid when the signal's profile is nonsinusoidal or the noise is nongaussian [11]. The

Gregory-Loredo method parameterizes the number of bins for epoch folding and calculates the average ratio of the probability for the periodic model to the constant model [12]. The Gregory-Loredo method is effective on discovering new pulsars when there is no prior knowledge about the signal's shape; however, it does not use the prior information and is less efficient. The other detection algorithms, like S-transform [13, 14] and Match Filter (MF) [15], consider the noise obeying the Gaussian distribution after epoch folding. However, that happens when the number of photons in the bins of epoch folding is larger than 20 [8]. For the detection problem, the observation time is not long enough to support approximating the Poisson noise by the Gaussian [16], and the algorithms above have this deficiency. Zhang et al. built the Poisson distribution model of X-ray pulsar and proposed a detection method by using the Bayesian theory [16]. However, it simply built the signal and noise by homogeneous Poisson distribution, which is imprecision. Besides that, it did not consider the influences of the unknown signal phase and the Doppler shift, which make it hard to be used in the practical scenario.

For XPNV, the accurate prior knowledge about the X-ray pulsar signal is essential. The prior knowledge includes the X-ray background noise's rate, the signal's flux, period, and profile. By using the prior information, we may build the Probability Density Function (PDF) [10] and get a fast detection algorithm based on Neyman-Pearson (NP) criterion [17]. Even in the condition of few photons for detection,

the proposed algorithm's performance could meet detection requirements of XPNVAV.

The rest of this paper is organized as follows. Section 2 introduces the X-ray pulsar signal model. Section 3 describes the fast detection algorithm in detail. Computer simulations are shown in Section 4 and some conclusions are given in the final part.

2. X-Ray Pulsar Signal Model

Unlike the radio signal, the X-ray pulsar signal shows the particle property, which means the output of X-ray detector on the spacecraft is not an analog signal but a discrete arriving time sequence of individual photons. Assuming the observation time is T_{obs} , the number of received photons is $N_{T_{\text{obs}}}$, and the PDF of $N_{T_{\text{obs}}}$ can be described by nonhomogeneous Poisson process [10]:

$$P(N_{T_{\text{obs}}} = k) = \frac{\left(\int_0^{T_{\text{obs}}} \lambda(t) dt\right)^k \exp\left(-\int_0^{T_{\text{obs}}} \lambda(t) dt\right)}{k!}, \quad (1)$$

where $\lambda(t)$ is the arriving rate of the photons as

$$\lambda(t) = \lambda_b + \lambda_s h(\varphi(t)). \quad (2)$$

Here λ_b is the X-ray background noise's rate which is time-invariant in the cosmos environment, λ_s is the X-ray pulsar signal's flux, $h(\varphi(t))$ is the X-ray pulsar signal's normalization standard profile template, $\varphi(t) = f_r \times t + \phi_0$ is the X-ray pulsar signal's phase, f_r is the received frequency, and ϕ_0 is the initial phase. Since the X-ray pulsar signal is periodic, $h(\varphi(t) + T) = h(\varphi(t))$, where T is the period.

3. The Fast Detection Algorithm

3.1. The Optimal NP Algorithm. We divide the observation timeline into N equal bins, which is shown in Figure 1. The duration time of each of the bins is Δt . When Δt is small enough, the arriving rate in the n th bin $\lambda_n(\theta)$ could be treated as a determined variable:

$$\lambda_n(\theta) \approx \frac{1}{\Delta t} \int_{t_0 + (n-1)\Delta t}^{t_0 + n\Delta t} \lambda(t; \theta) dt, \quad (3)$$

where $\theta = [f_r \ \phi_0]^T$; the supper index T represents the transpose of a vector. The number of photons in n th bin x_n is a random variable with the PDF:

$$p(x_n = k; \theta) = \frac{(\lambda_n(\theta) \Delta t)^k}{k!} \exp(-\lambda_n(\theta) \Delta t). \quad (4)$$

Since the random variables in the different bins are independent, the Joint Probability Density Function (JPDF) could be expressed as

$$\begin{aligned} p(\mathbf{X}; \theta) &= \prod_{n=1}^N p(x_n; \theta) \\ &= \prod_{n=1}^N \frac{(\lambda_n(\theta) \Delta t)^{x_n}}{x_n!} \exp(-\lambda_n(\theta) \Delta t), \end{aligned} \quad (5)$$

where $\mathbf{X} = (x_1, x_2, \dots, x_N)$.

FIGURE 1: The observation timeline.

If the X-ray pulsar signal exists in the received photons, $\lambda_n(\theta) = \lambda_b + \lambda_s h(\theta)$, then the JPDF of $\mathbf{X}$ could be expressed as

$$\begin{aligned} H1: p_1(\mathbf{X}; \theta) &= \prod_{n=1}^N \frac{((\lambda_b + \lambda_s h(\theta)) \Delta t)^{x_n}}{x_n!} \exp(-(\lambda_b + \lambda_s h(\theta)) \Delta t). \end{aligned} \quad (6)$$

If the X-ray pulsar signal does not exist in the received photons, $\lambda_n(\theta) = \lambda_b$, then the JPDF of $\mathbf{X}$ could be expressed as

$$H0: p_0(\mathbf{X}; \theta) = \prod_{n=1}^N \frac{(\lambda_b \Delta t)^{x_n}}{x_n!} \exp(-\lambda_b \Delta t). \quad (7)$$

In the condition of a fixed probability of false alarm p_{FA} , NP criterion maximizes the detection probability [17]. Based on NP criterion, we could do the Likelihood Ratio Test (LRT):

$$L(\mathbf{X}; \theta) = \frac{p_1(\mathbf{X}; \theta)}{p_0(\mathbf{X}; \theta)} = \gamma. \quad (8)$$

If γ is larger than the threshold γ_{th} , we can determine that we have received the X-ray pulsar signal. Otherwise, the X-ray pulsar signal does not exist in the received photons. Equation (8) could be simplified as follows:

$$L(\mathbf{X}; \theta) = \frac{p_1(\mathbf{X}; \theta)}{p_0(\mathbf{X}; \theta)} \quad (9)$$

$$= \prod_{n=1}^N \left(\frac{\lambda_n(\theta)}{\lambda_b} \right)^{x_n} \exp((\lambda_b - \lambda_n(\theta)) \Delta t),$$

$$\ln(L(\mathbf{X}; \theta))$$

$$= \sum_{n=1}^N \ln \left(\frac{\lambda_n(\theta)}{\lambda_b} \right)^{x_n} + \sum_{n=1}^N (\lambda_b - \lambda_n(\theta)) \Delta t, \quad (10)$$

$$\ln(L(\mathbf{X}; \theta)) + \sum_{n=1}^N (\lambda_s(\theta)) \Delta t \quad (11)$$

$$= \sum_{n=1}^N x_n (\ln(\lambda_n(\theta)) - \ln(\lambda_b)).$$

Since $\sum_{n=1}^N (\lambda_s(\theta)) \Delta t$ is independent of the measurement variable x_n , the left side of (11) could be added to the threshold, and then

$$\sum_{n=1}^N x_n (\ln(\lambda_n(\theta)) - \ln(\lambda_b)) = \gamma'. \quad (12)$$

TABLE 1: The parameters of X-ray pulsar sources.

ID	Name	RA(J2000) (hh:mm:ss)	DE(J2000) (dd:mm:ss)	Flux (photons/cm ² /s)	Period (s)
1	B0540-69	05:39:39	-69:44:36	$5.15e \times 10^{-3}$	0.0505
2	B1744-24A	17:48:02	-24:46:38	1.09×10^{-3}	0.0116
3	B1823-13	18:26:13	-13:34:47	2.63×10^{-3}	0.1015

If $\Delta t \rightarrow 0$, there is only one or none photon in each bin, and then (12) could be converted as

$$G(\boldsymbol{\varphi}) = \sum_{i=1}^M (\ln(\lambda(\varphi_i)) - \ln(\lambda_b)) = \gamma', \quad (13)$$

where M is the total number of the photons, $\boldsymbol{\varphi} = [\varphi_1 \ \varphi_2 \ \cdots \ \varphi_M]^T$, φ_i is the phase of i th photons, which could be calculated by

$$\varphi_i = f_r \cdot t_i + \phi_0. \quad (14)$$

From (13), the optimal detection variable is $G(\boldsymbol{\varphi})$. Thus, the steps of the optimal NP detection algorithm for the X-ray pulsar signal are as below:

- (I) The X-ray detector on-board receives the photons and notes each photons' arriving time t_i .
- (II) Convert the transferred arriving time t_i into φ_i using (14).
- (III) Search the arriving rate $\lambda(\varphi_i)$ of each arriving time from the XPNV database (since $\lambda(\varphi_i)$ is periodic, the XPNV database is a table which stores one period of the function $\lambda(\varphi_i)$), and calculate the logarithm $\ln(\lambda(\varphi_i))$.
- (IV) Add up all $\ln(\lambda(\varphi_i))$, and compare the summation $G(\boldsymbol{\varphi})$ with γ_{th} .

The prior knowledge before the detection process includes the XPNV database about the arriving rate functions $\lambda(t)$ and γ_{th} . The XPNV database could be built by observing the X-ray pulsars for months and some explorers have tried to do that, such as the Rossi X-Ray Timing Explorer (RXTE) from the National Aeronautics and Space Administration (NASA) and the Röntgen Satellite (ROSAT) from the German Aerospace Center (DLR). It is difficult to get an analytical expression for γ_{th} . Monte Carlo simulations would be used to calculate γ_{th} . The calculation process is as below:

- (I) Set a fixed probability of false alarm p_{FA} .
- (II) Generate the arriving time of the photons based on the Poisson distribution with the arriving rate λ_b .
- (III) Calculate $G(\boldsymbol{\varphi})$ based on the above steps of the optimal NP detection algorithm.
- (IV) Repeat step (II) and step (III) for K times; K is the times of the Monte Carlo simulations.
- (V) Rank each simulation result in descending order, and set a threshold γ_{th} to make the proportion of $G(\boldsymbol{\varphi})$ greater than γ_{th} equal p_{FA} . γ_{th} is the threshold we pursue.

3.2. The Suboptimal NP Algorithm. Besides the prior knowledge, how to convert the arriving time t_i into the phase φ_i is another important issue. ϕ_0 is different due to the spacecraft's position, and f_r could be affected by the Doppler shift due to the spacecraft's velocity:

$$\varphi_i = f_r \times t_i + \phi_0 = (f_0 + f_d) \times t_i + \phi_0, \quad (15)$$

$$f_d = \frac{\mathbf{v} \cdot \mathbf{r}}{c} f_0, \quad (16)$$

where f_0 is the fixed frequency of the X-ray pulsar, f_d is the Doppler frequency, $\mathbf{v}$ is the spacecraft's velocity vector, $\mathbf{r}$ is the X-ray pulsar's direction vector, and c is the light velocity. Supposing there are no prior distributions of ϕ_0 and f_d , we could use the two-dimension search based on the Maximum Likelihood (ML) estimation [10] to deal with the issue.

Assuming ϕ_0 's searching range is $[0, 1)$, dividing the range into p segments, the j th searching phase is ϕ_0^j . Dividing the f_d 's searching range into q segments, the k th searching frequency is f_d^k . Then, take ϕ_0^j and f_d^k into step (II) of the optimal NP detection algorithm and calculate $G_{j,k}(\boldsymbol{\varphi}; \phi_0^j, f_d^k)$. ϕ_0 and f_d in $\max(G_{j,k}(\boldsymbol{\varphi}; \phi_0^j, f_d^k))$ have the maximum probability equaling the real ones, where $\max(\cdot)$ is the maximum function. Therefore, we could use $\max(G_{j,k}(\boldsymbol{\varphi}; \phi_0^j, f_d^k))$ as the detection variable to be compared with γ_{th} . We may call the algorithm resolving the unknown ϕ_0 and f_d as the suboptimal NP detection algorithm.

For the practical navigation application, ϕ_0 and f_d are always unknown, so the suboptimal NP detection algorithm is more useful than the optimal one.

4. Computer Simulations

Three weak X-ray pulsars (B0540-69, B1744-24A, and B1823-13) are selected to simulate the proposed detection algorithm's performance. The parameters of the X-ray pulsars are shown in Table 1, and the normalization standard profile templates [18] are shown in Figure 2. The X-ray background noise's rate λ_b is 0.005 photons/s/cm². The X-ray pulsar signal's flux λ_s could be obtained from Table 1 [1]. The X-ray detector's area is 1 m², and the fixed probability of false alarm p_{FA} is set as 0.001.

The initial phase ϕ_0 obeys the uniform distribution with range $[0, 1)$. The unknown velocity v obeys the uniform distribution with range $(-7.9, 7.9)$ km/s, in which 7.9 km/s is the first cosmic velocity. The Doppler frequency f_d could be calculated by (16). The method to generate the photons refers to reference [10].

The MF algorithm calculates the correlation peak for the X-ray pulsar signal's profile with the standard signal

FIGURE 2: The normalization standard profile templates of three X-ray pulsars.

FIGURE 3: The detection probability for B0540-69.

template [15]. Since the MF algorithm is optimal when the noise is obeying the Gaussian distribution, we choose the MF algorithm for comparison.

We divide the simulation algorithm into three groups: (I) the optimal NP detection algorithm; (II) the suboptimal NP detection algorithm, and the number of segments for searching both ϕ_0 and f_d is set as 10; (III) the MF algorithm with unknown information about ϕ_0 and f_d , and the number of bins for epoch folding is 64. We perform the Monte Carlo simulations for 1000 times to calculate the detection probability, and the results are shown from Figures 3–5.

From Figures 3–5, the detection probability increases with the increasing observation time. Meanwhile, in the

FIGURE 4: The detection probability for B1744-24A.

FIGURE 5: The detection probability for B1823-13.

same simulation conditions, the suboptimal NP detection algorithm's detection probability is better than the MF's. The MF algorithm is effective when the noise is obeying the Gaussian distribution. However, in the condition of short observation time and limited photons, the noise obeys Poisson distribution, which makes the MF algorithm less effective than the proposed detection algorithm.

Since the limited number of photons, the estimation for ϕ_0 and f_d based on ML may be imprecise, and the detection probability of the suboptimal NP detection algorithm is lower than the optimal NP detection algorithm. Therefore, the optimal NP detection algorithm is a performance bound.

5. Conclusion

The detection for the X-ray pulsar signal is important for XPNV and some other navigation scenarios using X-ray pulsars. In this paper, we propose a fast detection algorithm for the X-ray pulsar signal based on NP criteria. Computer simulations show that, in the condition of short observation time and limited number of photons, the detection probability of the proposed algorithm is still high. Besides that, the fast detection algorithm may be extended to other detection issues when the signals obey the Poisson distribution.

Competing Interests

The authors declare that they have no competing interests.

Acknowledgments

This work was supported by National Natural Science Foundation of China (61271265 and 61671263) and Tsinghua University Initiative Scientific Research Program (2013089244 and 20161080057).

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