Research Article

A Novel Method for Interval-Value Intuitionistic Fuzzy Multicriteria Decision-Making Problems with Immediate Probabilities Based on OWA Distance Operators

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The goal of this work is to develop a novel decision-making method which can solve some complex decision problems that include the following three-aspect information: (1) information represented in the form of interval-valued intuitionistic fuzzy values (IVIFVs) not only intuitionistic fuzzy values (IFVs), (2) the probability information and the weighted information, and (3) the importance degree of each concept in the process of decision-making. Firstly, by integrating OWA operator, probabilistic weight (PW), and individual distance of two IVIFNs in the same formulation, we introduce two new distance operators named PIVIFOWAD operator and IPIVIFOWAD operator, respectively. Secondly, satisfaction degree of an alternative is proposed based on the positive ideal IVIFS and the negative ideal IVIFS and applied to MCDM. Finally, we use an illustrative example to show the feasibility and validity of the new method by comparing with the other existing methods.

1. Introduction

Atanassov and Gargov [1, 2] introduced the theory of Interval-Valued Intuitionistic Fuzzy Set (IVIFS), which is a generalization of the Intuitionistic Fuzzy Set (IFS) proposed by Atanassov [3]. The IVIFS has attracted more and more attention since its appearance. Some decision-making methods under IVIF environment have been developed by many scholars. To sum up, there are mainly four aspects on the decision-making under IVIF environment: (1) some decision-making methods are developed based on information measures (specially, distance, similarity, and entropy) because information measures for IVIFSs have great effects on the development of the IVIFS theory and its applications. For example, similarity measures [4–6], inclusion measure [7], entropy measure [8], cross-entropy measure [9], and distance measures [10] are developed and applied to corresponding MCDM and MADM problems; (2) many new aggregation operators are also investigated in the IVIFSs and applied to

some decision-making problems, such as linguistic intuitionistic fuzzy power Bonferroni Mean operators [11], Hamacher aggregation operators [12], fuzzy power Heronian aggregation operators [13], fuzzy generalized aggregation operator [11, 14–18], (fuzzy Einstein) hybrid weighted aggregation operators [19, 20], fuzzy prioritized hybrid weighted aggregation operator [21], and fuzzy Hamacher ordered weighted geometric operator [22]; (3) other methods for decision-making with IVIF information are also explored, such as evidential reasoning methodology [23], particle swarm optimization techniques [4], transform technique [24], nonlinear programming methods [25], and VIKOR methods in IVIFS [26], and others methods [27–32] are also developed for decision-making problems. Distance measure has great effects on obtaining the desirable choice in some decision problems. Motivated by the OWA operator, Xu [33] introduced ordered weighted distance operator based on known Hamming distance. Many extensions of distance operator have been developed; for example, Merigo et al. [34] introduced

a series of aggregation operators related to distance measures [35–38] which were applied to related decision problems [37, 39, 40].

In some real decision problems, many problems are very complex. Aiming at solving some of these complicated decision problems, it is necessary to develop a new kind of decision-making method to solve this kind of problems including the following three-aspect information: (1) information represented in the form of IVIFVs not only IFVs, (2) the weighted information and the probability information, and (3) the degree of importance of each concept in the process of decision-making. Motivated by the ideas of existing operators, we propose new IVIF distance measures by using related weighted operators with probabilistic information and their applications in MCDM in the present work.

The rest of the paper is organized as follows. In Section 2, we review some related definitions on IVIFSs which are in the analysis throughout this paper. Section 3 is focused on PIVPFWAD and IPIVIFOWAD. In Section 4, the concept of satisfaction degree is proposed and the MCDM approach based on the satisfaction degree is also constructed. Section 5, a practical example, is given to explain proposed method and compare and analyze the availability of proposed MCDM methods. This paper is concluded in Section 6.

2. IVIFSs and OWA Distance Operator

In this section, some related basic concepts of VIFSs, OWA operator, and OWAD operator are recapped.

2.1. Interval-Valued Intuitionistic Fuzzy Sets. Assume that $\text{int}([0, 1])$ is the collection of all closed subintervals of $[0, 1]$, and X is a universe of discourse. An IVIFS $[1, 2]$ on X has such a structure

$$\tilde{A} = \{ \langle x, (\mu_{\tilde{A}}(x), \nu_{\tilde{A}}(x)) \rangle \mid x \in X \}. \quad (1)$$

where $\mu_{\tilde{A}} : X \rightarrow \text{int}([0, 1])$ denotes the membership degree and $\nu_{\tilde{A}} : X \rightarrow \text{int}([0, 1])$ denotes the nonmembership degree of the element $x \in X$ to the set $\tilde{A}$, respectively, with the condition that $0 \leq \sup(\mu_{\tilde{A}}(x)) + \sup(\nu_{\tilde{A}}(x)) \leq 1$.

For each $x \in X$, $\mu_{\tilde{A}}(x)$ and $\nu_{\tilde{A}}(x)$ denote $\mu_{\tilde{A}}(x) = [\mu_{\tilde{A}}^-(x), \mu_{\tilde{A}}^+(x)]$, $\nu_{\tilde{A}}(x) = [\nu_{\tilde{A}}^-(x), \nu_{\tilde{A}}^+(x)]$, respectively. Therefore, $\tilde{A}$ may be also expressed as

$$\tilde{A} = \left\{ \langle x, ([\mu_{\tilde{A}}^-(x), \mu_{\tilde{A}}^+(x)], [\nu_{\tilde{A}}^-(x), \nu_{\tilde{A}}^+(x)]) \rangle \mid x \in X \right\}. \quad (2)$$

Eq. (2) satisfies the condition $\mu_{\tilde{A}}^+(x) + \nu_{\tilde{A}}^+(x) \leq 1$.

$$\begin{aligned} \pi_{\tilde{A}}(x) &= [\pi_{\tilde{A}}^-(x), \pi_{\tilde{A}}^+(x)] \\ &= [1 - (\mu_{\tilde{A}}^+(x) - \nu_{\tilde{A}}^+(x)), 1 - (\mu_{\tilde{A}}^-(x) - \nu_{\tilde{A}}^-(x))] \end{aligned} \quad (3)$$

is called the indeterminacy degree. For the convenience, $\tilde{A} = ([\mu_{\tilde{A}}^-, \mu_{\tilde{A}}^+], [\nu_{\tilde{A}}^-, \nu_{\tilde{A}}^+])$ is called an interval-valued intuitionistic fuzzy value (IVIFV).

Hereafter, $\mathcal{IVIFS}$ denotes the collection of all IVIFVs of a IVIFS on X .

For two IVIFVs $\tilde{A}_1 = ([\mu_{\tilde{A}_1}^-, \mu_{\tilde{A}_1}^+], [\nu_{\tilde{A}_1}^-, \nu_{\tilde{A}_1}^+])$ and $\tilde{A}_2 = ([\mu_{\tilde{A}_2}^-, \mu_{\tilde{A}_2}^+], [\nu_{\tilde{A}_2}^-, \nu_{\tilde{A}_2}^+])$, a relation $\leq$ on the IVIFVs is defined as follows:

$$\begin{aligned} \mu_{\tilde{A}_1}^- &\leq \mu_{\tilde{A}_2}^-, \\ \mu_{\tilde{A}_1}^+ &\leq \mu_{\tilde{A}_2}^+, \\ \text{and } \nu_{\tilde{A}_1}^- &\geq \nu_{\tilde{A}_2}^-, \\ \nu_{\tilde{A}_1}^+ &\geq \nu_{\tilde{A}_2}^+. \end{aligned} \quad (4)$$

In order to compare two IVIFVs, score function and accuracy function [41] of an IVIFV are introduced.

For any IVIFV $\tilde{A} = ([\mu_{\tilde{A}}^-, \mu_{\tilde{A}}^+], [\nu_{\tilde{A}}^-, \nu_{\tilde{A}}^+])$, the score function $\tilde{s}(\tilde{A})$ of $\tilde{A}$ is defined as follows:

$$\tilde{s}(\tilde{A}) = \frac{1}{2} ((\mu_{\tilde{A}}^-) + (\mu_{\tilde{A}}^+) - (\nu_{\tilde{A}}^-) - (\nu_{\tilde{A}}^+)), \quad (5)$$

where $\tilde{s}(\tilde{A}) \in [-1, 1]$.

For any IVIFV $\tilde{A} = ([\mu_{\tilde{A}}^-, \mu_{\tilde{A}}^+], [\nu_{\tilde{A}}^-, \nu_{\tilde{A}}^+])$, the accuracy function $\tilde{a}(\tilde{A})$ of $\tilde{A}$ is defined as follows:

$$\tilde{a}(\tilde{A}) = \frac{1}{2} ((\mu_{\tilde{A}}^-) + (\mu_{\tilde{A}}^+) + (\nu_{\tilde{A}}^-) + (\nu_{\tilde{A}}^+)), \quad (6)$$

where $\tilde{a}(\tilde{A}) \in [0, 1]$.

Based on the above definitions, comparison rules are defined as follows.

For any two IVIFVs $\tilde{A}_1, \tilde{A}_2$,

- (1) if $\tilde{s}(\tilde{A}_1) < \tilde{s}(\tilde{A}_2)$, then $\tilde{A}_1 < \tilde{A}_2$;
- (2) if $\tilde{s}(\tilde{A}_1) = \tilde{s}(\tilde{A}_2)$, then
 - (a) if $\tilde{a}(\tilde{A}_1) < \tilde{a}(\tilde{A}_2)$, then $\tilde{A}_1 < \tilde{A}_2$;
 - (b) if $\tilde{a}(\tilde{A}_1) = \tilde{a}(\tilde{A}_2)$, then $\tilde{A}_1 \sim \tilde{A}_2$.

2.2. OWA Distance Operator. In this section, we will review the OWAD operator and then introduce IVIFOWAD operator.

An OWA operator [42] is a function $\text{OWA} : R^n \rightarrow R$ that has an associated weight vector (WV) $\omega = (\omega_1, \dots, \omega_n)$ with $\omega_j \in [0, 1]$ and $\sum_{j=1}^n \omega_j = 1$, such that

$$\text{OWA}(a_1, a_2, \dots, a_n) = \sum_{j=1}^n \omega_j b_j \quad (7)$$

where $(a_1, \dots, a_n) \in R^n$ and b_j is the j th largest of a_i .

Similarly, OWAD operator is introduced as a generalization of OWA operator. Let $A = (a_1, \dots, a_n), B = (b_1, \dots, b_n)$ be two collections of arguments. An OWAD operator [36] is a function $\text{OWAD} : R^n \times R^n \rightarrow R$ that has an associated WV $\omega = (\omega_1, \dots, \omega_n)$ with $\omega_j \in [0, 1]$ and $\sum_{j=1}^n \omega_j = 1$, such that

$$\text{OWAD}(A, B) = \sum_{j=1}^n \omega_j d_j \quad (8)$$

where d_j is the j th largest of $|a_i - b_i|$.

Let $\tilde{A} = \{\tilde{a}_i = (\mu_{\tilde{a}_i}^-, \nu_{\tilde{a}_i}^+)\}$ and $\tilde{B} = \{\tilde{b}_i = (\mu_{\tilde{b}_i}^-, \nu_{\tilde{b}_i}^+)\}$ be two collections of IVIFVs, where $(\mu_{\tilde{a}_i}^-, \nu_{\tilde{a}_i}^+) = ([\mu_{\tilde{a}_i}^-, \mu_{\tilde{a}_i}^+], [\nu_{\tilde{a}_i}^-, \nu_{\tilde{a}_i}^+])$, $(\mu_{\tilde{b}_i}^-, \nu_{\tilde{b}_i}^+) = ([\mu_{\tilde{b}_i}^-, \mu_{\tilde{b}_i}^+], [\nu_{\tilde{b}_i}^-, \nu_{\tilde{b}_i}^+])$, and $i = 1, \dots, n$. We first recall the distance [10] between two IVIFVs $\tilde{a}_1 = ([\mu_{\tilde{a}_1}^-, \mu_{\tilde{a}_1}^+], [\nu_{\tilde{a}_1}^-, \nu_{\tilde{a}_1}^+])$ and $\tilde{a}_2 = ([\mu_{\tilde{a}_2}^-, \mu_{\tilde{a}_2}^+], [\nu_{\tilde{a}_2}^-, \nu_{\tilde{a}_2}^+])$.

$$d(\tilde{a}_1, \tilde{a}_2) = \frac{1}{4} \left(\left| (\mu_{\tilde{a}_1}^-) - (\mu_{\tilde{a}_2}^-) \right| + \left| (\mu_{\tilde{a}_1}^+) - (\mu_{\tilde{a}_2}^+) \right| + \left| (\nu_{\tilde{a}_1}^-) - (\nu_{\tilde{a}_2}^-) \right| + \left| (\nu_{\tilde{a}_1}^+) - (\nu_{\tilde{a}_2}^+) \right| \right) \quad (9)$$

3. IPIVIFOWA Distance Operator

In this subsection, by combining OWA operator, individual distances, and PWs, two new distances named PIVIFOWAD operator and IPIVIFOWAD operator will be introduced. PIVIFOWAD operator is defined as follows.

Definition 1. A PIVIFOWAD is a function $\text{PIVIFOWAD} : \mathcal{FV} \times \mathcal{FV} \rightarrow \mathbf{R}$, that has an associated WV $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ with $\omega_i > 0$, $\sum_{i=1}^n \omega_i = 1$ ($i = 1, 2, \dots, n$), such that

$$\text{PIVIFOWAD}(\tilde{A}, \tilde{B}) = \xi \sum_{i=1}^n \omega_i d(\tilde{a}_i, \tilde{b}_i) + (1 - \xi) \sum_{j=1}^n p_j d(\tilde{a}_j, \tilde{b}_j), \quad (10)$$

where $d(\tilde{a}_i, \tilde{b}_i)$ is the i th largest of $d(\tilde{a}_j, \tilde{b}_j)$ and each $d(\tilde{a}_j, \tilde{b}_j)$ has associated a probabilistic $p_j \in [0, 1]$, $\sum_{i=1}^n p_i = 1$.

In Definition 1, if $\xi = 1$, it will be reduced to interval-valued intuitionistic fuzzy ordered weighted distant (IPIVIFOWAD) operator:

$$\text{IPIVIFOWAD}(\tilde{A}, \tilde{B}) = \sum_{i=j}^n \omega_j d(\tilde{a}_j, \tilde{b}_j), \quad (11)$$

where $d(\tilde{a}_j, \tilde{b}_j)$ is the j th largest of $d(\tilde{a}_i, \tilde{b}_i)$ and $d(\tilde{a}_i, \tilde{b}_i)$ is the argument variable represented in the form of individual distance between IVIFVs $\tilde{a}_i, \tilde{b}_i$.

Example 2. Let

$$\begin{aligned} \tilde{A} &= \{([0.3, 0.7], [0.2, 0.3]), ([0.7, 0.8], [0.1, 0.2]), \\ &\quad ([0.2, 0.3], [0.5, 0.6]), ([0.5, 0.6], [0.3, 0.4])\}, \\ \tilde{B} &= \{([0.4, 0.5], [0.2, 0.4]), ([0.5, 0.6], [0.2, 0.3]), \\ &\quad ([0.2, 0.4], [0.5, 0.6]), ([0.4, 0.7], [0.2, 0.3])\} \end{aligned} \quad (12)$$

be two collections of IVIFVs on the set $X = \{x_1, x_2, x_3, x_4\}$ and the WV is $\omega = (0.2, 0.3, 0.1, 0.4)$. Take $\xi = 0.3$; according to Definition 1, we have

$$\begin{aligned} \text{PIVIFOWAD}(\tilde{A}, \tilde{B}) &= 0.3 \times (0.2 \times 0.2 + 0.3 \times 0.175 \\ &\quad + 0.1 \times 0.175 + 0.4 \times 0.05) + 0.7 \times (0.3 \times 0.175 \\ &\quad + 0.2 \times 0.2 + 0.4 \times 0.05 + 0.1 \times 0.175) = 0.13. \end{aligned} \quad (13)$$

Now, we can also develop the IPIVIFOWAD operator by applying IVIF information, individual distance, and immediate probability (IP)[43].

Definition 3. An IPIVIFOWAD is a function $\text{IPIVIFOWAD} : \mathcal{FV} \times \mathcal{FV} \rightarrow \mathbf{R}$ which has an associated WV $\omega = (\omega_1, \omega_2, \dots, \omega_n)^T$ with $\omega_i > 0$, $\sum_{i=1}^n \omega_i = 1$ ($i = 1, 2, \dots, n$), such that

$$\text{IPIVIFOWAD}(\tilde{A}, \tilde{B}) = \sum_{i=1}^n \hat{p}_i d(a_i, b_i), \quad (14)$$

where $d(\tilde{a}_i, \tilde{b}_i)$ is the i th largest of $d(\tilde{a}_j, \tilde{b}_j)$ and $d(\tilde{a}_j, \tilde{b}_j)$ is the argument variable represented in the form of individual distance between IVIFVs $\tilde{a}_j, \tilde{b}_j$ and a PW $p_i > 0$, $\sum_{i=1}^n p_i = 1$. $\hat{p}_i = \omega_i p_i / \sum_{i=1}^n \omega_i p_i$ and p_i is the probabilistic p_j according to $d(\tilde{a}_j, \tilde{b}_j)$, that is, according to the i th largest of the $d(\tilde{a}_j, \tilde{b}_j)$.

It is worth pointing out that IPIVIFOWAD operator is a good approach to unifying probabilities and IVIFOWAD in some particular situations. But it is not always useful. In order to show why this unification does not seem to be a final model, we could also consider other ways of representing $\hat{p}_i$ as in [40].

Example 4. In Example 2, since the following WV $\omega = (0.2, 0.3, 0.1, 0.4)$ and the PW $(0.3, 0.2, 0.4, 0.1)$, now we aggregate this information according to IPIVIFOWAD, as we have calculated $d(a_i, b_i)$ by employing (7) as follows:

$$\begin{aligned} d(a_1, b_1) &= 0.175, \\ d(a_2, b_2) &= 0.2, \\ d(a_3, b_3) &= 0.05, \\ d(a_4, b_4) &= 0.175. \end{aligned} \quad (15)$$

According to the above distance, we reorder the PW $(0.2, 0.3, 0.1, 0.4)$,

$$\sum_{i=1}^5 \omega_i p_i = (0.2, 0.3, 0.1, 0.4) (0.2, 0.3, 0.1, 0.4)^T = 0.3. \quad (16)$$

Therefore, $\hat{p}_1 = \omega_1 p_1 / \sum_{i=1}^5 \omega_i p_i = (0.2 \times 0.2) / 0.3 = 0.133$, and similarly, we have $\hat{p}_2 = 0.3$, $\hat{p}_3 = 0.033$, $\hat{p}_4 = 0.534$. Therefore, we have

$$\begin{aligned} \text{IPIVIFOWAD}(\tilde{A}, \tilde{B}) &= 0.133 \times 0.2 + 0.3 \times 0.175 + 0.033 \times 0.175 \\ &\quad + 0.534 \times 0.05 = 0.112. \end{aligned} \quad (17)$$

Monotonicity is a kind of vital property in the research of aggregation operators. The aggregation operator with monotonicity will be more reliable in decision-making process. The lack of monotonicity may depress the reliability of the final

results. PIVIFOWAD and IPIVIFOWAD are new distance measure and are aggregation operators. We can prove that PIVIFOWAD and IPIVIFOWAD have the properties of boundness, monotonicity, and reflexivity. The proof of these properties is similar to Theorems 1-3 in [44].

4. Method for MCDM Based on IPIVIFOWA Operator

4.1. Formal Description of MCDM with IVIFs. The MCDM with IVIF information can be formally presented as follows.

$$R_{m \times n} = \begin{pmatrix} ([\mu_{11}^-, \mu_{11}^+], [\nu_{11}^-, \nu_{11}^+]) & ([\mu_{12}^-, \mu_{12}^+], [\nu_{12}^-, \nu_{12}^+]) & \cdots & ([\mu_{1n}^-, \mu_{1n}^+], [\nu_{1n}^-, \nu_{1n}^+]) \\ ([\mu_{21}^-, \mu_{21}^+], [\nu_{21}^-, \nu_{21}^+]) & ([\mu_{22}^-, \mu_{22}^+], [\nu_{22}^-, \nu_{22}^+]) & \cdots & ([\mu_{2n}^-, \mu_{2n}^+], [\nu_{2n}^-, \nu_{2n}^+]) \\ \vdots & \vdots & \vdots & \vdots \\ ([\mu_{m1}^-, \mu_{m1}^+], [\nu_{m1}^-, \nu_{m1}^+]) & ([\mu_{m2}^-, \mu_{m2}^+], [\nu_{m2}^-, \nu_{m2}^+]) & \cdots & ([\mu_{mn}^-, \mu_{mn}^+], [\nu_{mn}^-, \nu_{mn}^+]) \end{pmatrix} \quad (18)$$

We give the concepts of IVIF-PIS, IVIF-NIS, and satisfaction degree before the decision-making algorithm is given.

Considering the decision information represents the form of IVIFVs, we use (5) and (6) based on comparison approach to identify the IVIF-PIS and the IVIF-NIS. We use O^+ to represent IVIF-PIS and O^- to represent IVIF-NIS; they are determined as follows:

$$O^+ = \left\{ \left\langle C_j, \max_i s(C_j(x_i)) \right\rangle \mid j = 1, 2, \dots, n; i = 1, 2, \dots, m \right\}, \quad (19)$$

$$O^- = \left\{ \left\langle C_j, \min_i s(C_j(x_i)) \right\rangle \mid j = 1, 2, \dots, n; i = 1, 2, \dots, m \right\}. \quad (20)$$

Let D be one of IPIVIFOWAD and PIVIFOWAD, and $D(O^+, O_i)$ and $D(O^-, O_i)$ denote the distance of O^+ and alternative O_i and the O^- and alternative O_i , respectively. Motivated by the well-known TOPSIS, we take both $D(O^+, O_i)$ and $D(O^-, O_i)$ into consideration simultaneously rather than separately. This leads naturally to the concept of **satisfaction degree**.

Definition 5. Let $A = \{A_1, \dots, A_m\}$ be a collection of alternatives. The satisfaction degree $\lambda(O_i)$ of a given alternative x_i over the criteria $C_j (j = 1, 2, \dots, n)$ is defined as

$$\lambda(O_i) = \frac{(1 - \varepsilon) [D(O^-, O_i)]}{\varepsilon [D(O^+, O_i)] + (1 - \varepsilon) [D(O^-, O_i)]}, \quad (21)$$

$i = 1, 2, \dots, m.$

In (21), ε denotes the **risk preference** of the DM and $\varepsilon \in [0, 1]$: $\varepsilon > 0.5$ means that the decision maker is pessimist, while $\varepsilon < 0.5$ means the opposite. $\varepsilon = 0.5$: satisfaction degree is of relative closeness using the classic TOPSIS method. The parameter ε is provided by the DM in advance. It is obvious

Assume that $X = \{x_1, \dots, x_m\}$ is a set of m alternatives, $C = \{C_1, \dots, C_n\}$ is the collection of attributes, and $\omega = (\omega_1, \dots, \omega_n)^T$ are the WV of all attributes, which satisfy $0 \leq \omega_i \leq 1$. Assume that alternative $O_i (i = 1, \dots, m)$ with respect to attribute $C_j (j = 1, \dots, n)$ is evaluated by an IVIFVs $C_j(x_i) = ([\mu_{ij}^-, \mu_{ij}^+], [\nu_{ij}^-, \nu_{ij}^+]) (j = 1, 2, \dots, n; i = 1, 2, \dots, m)$ and $R_{m \times n} = (C_j(x_i))_{m \times n}$ is an IVIF decision matrix. A new kind MCDM approach will be developed based on the distance operators proposed in Section 3.

For a MCDM problem with IVIFVs, the decision matrix $R = (C_j(x_i))_{m \times n}$ and can be constructed or given in advance.

that $\lambda(O_i) \in [0, 1] (i = 1, 2, \dots, m)$. The higher the satisfaction degree, the better the alternative.

4.2. Decision Algorithm for MCDM with IVIF

Step 1. Determine the IVIF-PIS and the IVIF-NIS.

Step 2. Calculate the distance between IVIFVs in A and IVIFVs in $O^+(O^-)$ according to (9).

Step 3. Recalculate the probability according to distance calculated in Step 2.

Step 4. Compute the distance $D(O^+, O_i)$ of the positive ideal IVIFS O^+ and alternative O_i and the distance $D(O^-, O_i)$ of the negative ideal IVIFS O^- and alternative O_i , respectively.

Step 5. Calculate the satisfaction degree $\lambda(O_i)$ according to Definition 5. And get the priority of the alternative $O_i (i = 1, \dots, m)$ by ranking $\lambda(O_i) (i = 1, \dots, m)$; the bigger the satisfaction degree $\lambda(O_i)$, the better the alternation O_i .

Step 6. End.

5. Case Study

In this section, we will give a practical example about the optimal invest strategy to show the application of the proposed IPIVIFOWAD and PIVIFOWAD.

Technological innovation not only is directly related to the survival and development of an enterprise, but also affects the economic development of a region or even a country. As we all know, the management of an enterprise's technological innovation activities is an important manifestation of its technological innovation capability. In evaluating the technological innovation capability of enterprises, the following evaluation index system should be considered:

(1) G_1 : innovation system construction, attitude to innovation failure, and incentives for innovation by the enterprise distribution system;

(2) G_2 : establishment and implementation of technological innovation strategy, the formation, and maintenance of enterprise innovation culture;

(3) G_3 : the feasibility of research and development project feasibility report;

(4) G_4 : the completeness of the monitoring;

(5) G_5 : evaluation system and innovation awareness of leaders and staff.

DM assesses the technical innovation management of 5 large enterprises $O_i (i = 1, \dots, 5)$ by questionnaires survey and discussion and uses the IVIFVs for evaluation and constructs the IVIF decision matrix as shown in Table 1.

To find the desirable alternative, the experts give the probabilistic weight information as follows: $p = (0.3, 0.3, 0.2, 0.1, 0.1)$. They assume that the importance degree of each characteristic is $w = (0.2, 0.3, 0.1, 0.3, 0.1)$.

5.1. Decision-Making Using IPIVIFOWAD Operator

Step 1. Determine the IVIF-PIS O^+ and the IVIF-NIS O^- by (5) and (6) which are shown in Table 2.

We can see from Table 2 that $s_j(G_1) (j = 1, 2, 3, 4, 5)$ all are different, so do $s_j(G_2), s_j(G_3), s_j(G_4), s_j(G_5) (j = 1, 2, 3, 4, 5)$. Therefore, we do not need to compute the accuracy function. And so, IVIF-PIS O^+ and IVIF-NIS O^- are obtained, respectively, and shown as follows:

$$\begin{aligned} O^+ &= \{ \langle G_1, ([0.6, 0.7], [0.2, 0.3]) \rangle, \\ &\quad \langle G_2, ([0.6, 0.7], [0.1, 0.2]) \rangle, \\ &\quad \langle G_3, ([0.6, 0.7], [0.2, 0.3]) \rangle, \\ &\quad \langle G_4, ([0.7, 0.8], [0.1, 0.2]) \rangle, \\ &\quad \langle G_5, ([0.6, 0.7], [0.2, 0.3]) \rangle \}, \\ O^- &= \{ \langle G_1, ([0.3, 0.4], [0.5, 0.6]) \rangle, \\ &\quad \langle G_2, ([0.2, 0.3], [0.6, 0.7]) \rangle, \\ &\quad \langle G_3, ([0.4, 0.5], [0.3, 0.4]) \rangle, \\ &\quad \langle G_4, ([0.2, 0.3], [0.5, 0.6]) \rangle, \\ &\quad \langle G_5, ([0.3, 0.4], [0.4, 0.5]) \rangle \}. \end{aligned} \quad (22)$$

Step 2. Denote $O^+ = \{\tilde{\gamma}_1, \dots, \tilde{\gamma}_5\}$, $O^- = \{\tilde{\tau}_1, \dots, \tilde{\tau}_5\}$, $O_i = \{\tilde{a}_{i1}, \dots, \tilde{a}_{i5}\} (i = 1, 2, \dots, 5)$. Now we calculate the distance $d(\tilde{\gamma}_j, \tilde{a}_{ij}) (i, j = 1, 2, \dots, 5)$ between IVIFVs $\tilde{\gamma}_j, \tilde{a}_{ij}$, $d(\tilde{\tau}_j, \tilde{a}_{ij}) (i, j = 1, 2, \dots, 5)$ between the IVIFVs $\tilde{\tau}_j, \tilde{a}_{ij}$, respectively. The results can be found in Tables 3 and 4.

Step 3. Calculate IP by using the above probabilities and weights according to Tables 3 and 4. The results are shown in Tables 5 and 6.

Step 4. Calculate the IPIVIFOWAD(O_i, O^+) and IPIVIFOWAD(O_i, O^-) according to Step 2 and Step 3. For convenience, we denote IPIVIFOWAD(O_i, O^+) and IPIVIFOWAD(O_i, O^-) as $D(O_i, O^+)$ and $D(O_i, O^-) (i = 1, 2, \dots, 5)$, respectively. The results are as follows.

$$\begin{aligned} D(O_1, O^+) &= 0.0579, \\ D(O_2, O^+) &= 0.2979, \\ D(O_3, O^+) &= 0.1842, \\ D(O_4, O^+) &= 0.1957, \\ D(O_5, O^+) &= 0.1167, \\ D(O_1, O^-) &= 0.2474, \\ D(O_2, O^-) &= 0.1105, \\ D(O_3, O^-) &= 0.2214, \\ D(O_4, O^-) &= 0.1905, \\ D(O_5, O^-) &= 0.2816. \end{aligned} \quad (23)$$

Step 5. Calculate the satisfaction degree according to the distance in Step 4. The results can be found in Table 7 under different risk preference ϵ .

It follows from Table 7 that the order of alternatives is consistent with results by using IPIVIFOWAD when parameter changes. We can obtain the ranking $O_1 > O_5 > O_3 > O_4 > O_2$. All of the results show that O_1 is the desirable alternative. Such a conclusion can be drawn directly from Figure 1.

5.2. Decision-Making Using PIVIFOWAD. Now we use the PIVIFOWAD to this decision-making problem. According to the distance matrices in Tables 3 and 4, note that the WA has an importance of 40 percent and the probabilistic information has an importance of 60 percent. We can rearrange the probabilistic information according the distance. The results can be found in Tables 8 and 9.

Therefore, we can calculate PIVIFOWAD distances $D(O_i, O^+), D(O_i, O^-) (i = 1, \dots, 5)$ as follows:

$$\begin{aligned} D(O_1, O^+) &= 0.076, \\ D(O_2, O^+) &= 0.281, \\ D(O_3, O^+) &= 0.208, \\ D(O_4, O^+) &= 0.192, \\ D(O_5, O^+) &= 0.123, \\ D(O_1, O^-) &= 0.228, \\ D(O_2, O^-) &= 0.14, \\ D(O_3, O^-) &= 0.179, \end{aligned}$$

TABLE 1: IVIF decision matrix.

	G_1	G_2	G_3	G_4	G_5
O_1	$([0.6, 0.7], [0.2, 0.3])$	$([0.6, 0.7], [0.1, 0.2])$	$([0.5, 0.6], [0.3, 0.4])$	$([0.4, 0.5], [0.2, 0.3])$	$([0.5, 0.6], [0.3, 0.4])$
O_2	$([0.3, 0.4], [0.5, 0.6])$	$([0.2, 0.3], [0.6, 0.7])$	$([0.4, 0.6], [0.2, 0.3])$	$([0.7, 0.7], [0.1, 0.2])$	$([0.4, 0.6], [0.1, 0.2])$
O_3	$([0.5, 0.6], [0.2, 0.4])$	$([0.5, 0.7], [0.2, 0.3])$	$([0.4, 0.5], [0.3, 0.4])$	$([0.2, 0.3], [0.5, 0.6])$	$([0.3, 0.4], [0.4, 0.5])$
O_4	$([0.3, 0.4], [0.4, 0.5])$	$([0.5, 0.6], [0.3, 0.4])$	$([0.5, 0.6], [0.2, 0.3])$	$([0.5, 0.6], [0.2, 0.3])$	$([0.5, 0.6], [0.2, 0.3])$
O_5	$([0.4, 0.6], [0.2, 0.4])$	$([0.5, 0.6], [0.2, 0.3])$	$([0.6, 0.7], [0.2, 0.3])$	$([0.3, 0.4], [0.5, 0.6])$	$([0.6, 0.7], [0.2, 0.3])$

TABLE 2: The results by using score function.

	G_1	G_2	G_3	G_4	G_5
s_1	0.4	0.5	0.2	0.2	0.2
s_2	-0.2	-0.4	0.25	0.6	0.35
s_3	0.25	0.35	0.1	-0.3	-0.1
s_4	-0.1	0.2	0.3	0.3	0.3
s_5	0.2	0.3	0.4	-0.2	0.4

TABLE 3: The distance between $\tilde{a}_{ij}$ and $\tilde{y}_j$.

	$\tilde{y}_1$	$\tilde{y}_2$	$\tilde{y}_3$	$\tilde{y}_4$	$\tilde{y}_5$
O_1	0	0	0.1	0.3	0.1
O_2	0.3	0.5	0.15	0	0.25
O_3	0.1	0.1	0.2	0.5	0.3
O_4	0.3	0.2	0.1	0.2	0.1
O_5	0.15	0.1	0	0.4	0

TABLE 4: The distance between $\tilde{a}_{ij}$ and $\tilde{\tau}_j$.

	$\tilde{\tau}_1$	$\tilde{\tau}_2$	$\tilde{\tau}_3$	$\tilde{\tau}_4$	$\tilde{\tau}_5$
O_1	0.3	0.5	0.1	0.3	0.2
O_2	0	0	0.1	0.5	0.3
O_3	0.25	0.4	0	0	0
O_4	0.1	0.3	0.1	0.3	0.2
O_5	0.25	0.4	0.2	0.1	0.3

TABLE 5: The IP according to Table 3.

	G_1	G_2	G_3	G_4	G_5
IP_1	0.1053	0.1579	0.1053	0.4737	0.1579
IP_2	0.2609	0.3913	0.0435	0.2609	0.0435
IP_3	0.1053	0.1579	0.1053	0.4737	0.1579
IP_4	0.2607	0.3913	0.0435	0.2607	0.0435
IP_5	0.0952	0.4286	0.1429	0.2857	0.0476

TABLE 6: The IP according to Table 4.

	G_1	G_2	G_3	G_4	G_5
IP_1	0.3158	0.1579	0.0526	0.3158	0.1579
IP_2	0.1053	0.1579	0.1053	0.4737	0.1579
IP_3	0.2857	0.4286	0.0952	0.1429	0.0476
IP_4	0.2857	0.1429	0.0476	0.4286	0.0952
IP_5	0.3158	0.1579	0.1579	0.3158	0.0526

TABLE 7: Satisfaction degree obtained by IPIVIFOWAD under different risk preference parameter ε .

	$\lambda(O_1)$	$\lambda(O_2)$	$\lambda(O_3)$	$\lambda(O_4)$	$\lambda(O_5)$	Ranking
$\varepsilon = 0.1$	0.9747	0.7696	0.9154	0.8976	0.9560	$O_1 > O_5 > O_3 > O_4 > O_2$
$\varepsilon = 0.3$	0.9088	0.4641	0.7372	0.6943	0.8492	$O_1 > O_5 > O_3 > O_4 > O_2$
$\varepsilon = 0.5$	0.8103	0.2707	0.5459	0.4933	0.7070	$O_1 > O_5 > O_3 > O_4 > O_2$
$\varepsilon = 0.7$	0.6468	0.1372	0.3400	0.2944	0.5084	$O_1 > O_5 > O_3 > O_4 > O_2$
$\varepsilon = 0.9$	0.3219	0.0396	0.1178	0.0976	0.2115	$O_1 > O_5 > O_3 > O_4 > O_2$

TABLE 8: The probabilities according to Table 3.

	G_1	G_2	G_3	G_4	G_5
$\hat{p}_1$	0.14	0.18	0.16	0.3	0.22
$\hat{p}_2$	0.26	0.3	0.1	0.24	0.1
$\hat{p}_3$	0.14	0.18	0.16	0.3	0.22
$\hat{p}_4$	0.26	0.3	0.1	0.24	0.1
$\hat{p}_5$	0.14	0.3	0.22	0.24	0.1

TABLE 9: The probabilities according to Table 4.

	G_1	G_2	G_3	G_4	G_5
$\hat{p}_1$	0.26	0.18	0.1	0.24	0.22
$\hat{p}_2$	0.14	0.18	0.16	0.3	0.22
$\hat{p}_3$	0.26	0.3	0.16	0.18	0.1
$\hat{p}_4$	0.26	0.18	0.1	0.3	0.16
$\hat{p}_5$	0.26	0.18	0.22	0.24	0.1

FIGURE 1: Satisfaction degree obtained by IPIVIFOWAD under different ε .

$$D(O_4, O^-) = 0.198,$$

$$D(O_5, O^-) = 0.271.$$

(24)

Therefore, we can obtain the satisfaction degree under difference risk preference parameters ε ; please refer to Table 10.

It follows from Table 10 that the ranking is consistent with the results by using the PIVIFOWAD operators when parameter changes. We can obtain the order of alternatives: $O_1 > O_3 > O_5 > O_4 > O_2$. All of the results show that O_1 is the desirable one. Such a conclusion can be also drawn directly from Figure 2.

If we change the weight important degree ξ , we can obtain other satisfaction degrees listed in Table 11. From Table 11, we can see that the desirable alternative is consistent with the ranking obtained by PIVIFOWAD when parameter ξ changes although the ranking of alternatives is not the same. All of the results show that O_1 is the desirable one. Such a conclusion can be drawn directly from Figure 3.

5.3. Effectiveness Test of the Proposed Method. For MCDM problems, Wang and Triantaphyllou [45] established assessing criteria (please refer to [45]) to assess the effectiveness of MCDM methods. In what follows, we will use the above MCDM criteria to test our proposed methods in Section 4. As far as the proposed method based on IPIVIFOWAD is concerned, we choose the satisfactory degree $\varepsilon = 0.5$ to analyze the above criteria.

Validity Test for Criterion 1. In Section 5.2, we obtained that O_1 is the desirable one and the order of alternatives is

TABLE 10: Modified IVIF decision matrix.

	G_1	G_2	G_3	G_4	G_5
O_1	$([0.6,0.7], [0.2,0.3])$	$([0.6,0.7], [0.1,0.2])$	$([0.5,0.6], [0.3,0.4])$	$([0.4,0.5], [0.2,0.3])$	$([0.5,0.6], [0.3,0.4])$
O_2	$([0.3,0.4], [0.5,0.6])$	$([0.2,0.3], [0.6,0.7])$	$([0.4,0.6], [0.2,0.3])$	$([0.7,0.7], [0.1,0.2])$	$([0.4,0.6], [0.1,0.2])$
O_3	$([0.2,0.4], [0.5,0.6])$	$([0.2,0.3], [0.5,0.7])$	$([0.3,0.4], [0.4,0.5])$	$([0.5,0.6], [0.2,0.3])$	$([0.4,0.5], [0.3,0.4])$
O_4	$([0.4,0.5], [0.3,0.4])$	$([0.3,0.4], [0.5,0.6])$	$([0.2,0.3], [0.5,0.6])$	$([0.2,0.3], [0.5,0.6])$	$([0.2,0.3], [0.5,0.6])$
O_5	$([0.4,0.6], [0.2,0.4])$	$([0.5,0.6], [0.2,0.3])$	$([0.6,0.7], [0.2,0.3])$	$([0.3,0.4], [0.5,0.6])$	$([0.6,0.7], [0.2,0.3])$

TABLE 11: Satisfaction degree obtained by PIVIFOWAD under different ξ .

	$\lambda(O_1)$	$\lambda(O_2)$	$\lambda(O_3)$	$\lambda(O_4)$	$\lambda(O_5)$	Ranking
$\xi = 0.1$	0.7762	0.2740	0.5053	0.4923	0.7008	$O_1 > O_5 > O_3 > O_4 > O_2$
$\xi = 0.3$	0.7584	0.3136	0.4766	0.5027	0.6921	$O_1 > O_5 > O_4 > O_3 > O_2$
$\xi = 0.5$	0.7419	0.3509	0.4487	0.5128	0.6835	$O_1 > O_5 > O_4 > O_2 > O_3$
$\xi = 0.7$	0.7267	0.3859	0.4217	0.5231	0.6751	$O_1 > O_5 > O_4 > O_3 > O_2$
$\xi = 0.9$	0.7126	0.4190	0.3955	0.5333	0.6667	$O_1 > O_5 > O_4 > O_2 > O_3$

TABLE 12: Modified IVIF decision matrix.

	G_1	G_2	G_3	G_4	G_5
O_1	$([0.6,0.7], [0.2,0.3])$	$([0.6,0.7], [0.1,0.2])$	$([0.5,0.6], [0.3,0.4])$	$([0.4,0.5], [0.2,0.3])$	$([0.5,0.6], [0.3,0.4])$
O_2	$([0.3,0.4], [0.5,0.6])$	$([0.2,0.3], [0.6,0.7])$	$([0.4,0.6], [0.2,0.3])$	$([0.7,0.7], [0.1,0.2])$	$([0.4,0.6], [0.1,0.2])$
O_3	$([0.2,0.4], [0.5,0.6])$	$([0.2,0.3], [0.5,0.7])$	$([0.3,0.4], [0.4,0.5])$	$([0.5,0.6], [0.2,0.3])$	$([0.4,0.5], [0.3,0.4])$
O_4	$([0.4,0.5], [0.3,0.4])$	$([0.3,0.4], [0.5,0.6])$	$([0.2,0.3], [0.5,0.6])$	$([0.2,0.3], [0.5,0.6])$	$([0.2,0.3], [0.5,0.6])$
O_5	$([0.4,0.6], [0.2,0.4])$	$([0.5,0.6], [0.2,0.3])$	$([0.6,0.7], [0.2,0.3])$	$([0.3,0.4], [0.5,0.6])$	$([0.6,0.7], [0.2,0.3])$

FIGURE 2: Satisfaction degree obtained by PIVIFOWAD under different ε and $\xi = 0.4$.

$O_1 > O_5 > O_3 > O_4 > O_2$. In order to test the effectiveness of the developed IPIVIFOWAD method under criterion 1, we construct the modified IVIF decision matrix (Table 12) by interchanging the membership and nonmembership of alternatives O_3 and O_4 in Table 1.

FIGURE 3: Satisfaction degree obtained by PIVIFOWAD under different ξ and $\varepsilon = 0.5$.

Repeating the same steps 1-2 in Section 5.1, we can obtain the modified IVIF-PIS O^+ and the IVIF-NIS O^- which are listed as follows:

$$\begin{aligned}
O^+ = \{ & \langle G_1, ([0.7, 0.8], [0.2, 0.3]) \rangle, \\
& \langle G_2, ([0.7, 0.8], [0.2, 0.3]) \rangle, \\
& \langle G_3, ([0.8, 0.9], [0.3, 0.4]) \rangle, \\
& \langle G_4, ([0.7, 0.8], [0.3, 0.5]) \rangle, \\
& \langle G_5, ([0.7, 0.8], [0.4, 0.5]) \rangle \}, \\
O^- = \{ & \langle G_1, ([0.2, 0.4], [0.7, 0.8]) \rangle, \\
& \langle G_2, ([0.3, 0.4], [0.6, 0.7]) \rangle, \\
& \langle G_3, ([0.3, 0.4], [0.6, 0.8]) \rangle, \\
& \langle G_4, ([0.3, 0.5], [0.5, 0.6]) \rangle, \\
& \langle G_5, ([0.3, 0.4], [0.5, 0.7]) \rangle \}.
\end{aligned} \tag{25}$$

Using Step 3-Step 5 of IPIVIFOWAD method, the IPIV-IFOWAD distances $D(O_i, O^+)$ between alternatives O_i and O^+ and the IPIVIFOWAD distances $D(O_i, O^-)$ between alternatives O_i and O^- are calculated, respectively, where $i = 1, 2, \dots, 5$.

$$\begin{aligned}
D(O_1, O^+) &= 0.0579, \\
D(O_2, O^+) &= 0.2978, \\
D(O_3, O^+) &= 0.3452, \\
D(O_4, O^+) &= 0.3789, \\
D(O_5, O^+) &= 0.1167, \\
D(O_1, O^-) &= 0.3786, \\
D(O_2, O^-) &= 0.3526, \\
D(O_3, O^-) &= 0.0974, \\
D(O_4, O^-) &= 0.1, \\
D(O_5, O^-) &= 0.3243.
\end{aligned} \tag{26}$$

According to the satisfaction degree formula when $\varepsilon = 0.5$, we have

$$\begin{aligned}
\lambda(O_1) &= 0.8674, \\
\lambda(O_2) &= 0.5421, \\
\lambda(O_3) &= 0.2200, \\
\lambda(O_4) &= 0.2088, \\
\lambda(O_5) &= 0.7355.
\end{aligned} \tag{27}$$

We can see from the above satisfaction degrees that the rank is $O_1 > O_5 > O_2 > O_3 > O_4$; that is, O_1 is the best one. Therefore, the best alternative coincides with the best alternative obtained in Section 5.1 by the same method, and the relative orders of the rest of the unchanged alternatives

keep constant. That is, Criterion 1 is suitable for the proposed method.

Validity Test for Criterion 2 and Criterion 3. According to the requirements of criterion 2 and test criterion 3 introduced in [45], the original problem should be decomposed into two smaller MCDM problems, such as $\{O_1, O_2, O_3, O_4\}$ and $\{O_1, O_3, O_4, O_5\}$. For the subproblem $\{O_1, O_2, O_3, O_4\}$, we can obtain the satisfaction degree by repeating Step 1 to Step 6 as follows:

$$\begin{aligned}
\lambda(O_1) &= 0.7988, \\
\lambda(O_2) &= 0.2859, \\
\lambda(O_3) &= 0.56048, \\
\lambda(O_4) &= 0.48488.
\end{aligned} \tag{28}$$

Therefore, the ranking of the subproblem is $O_1 > O_3 > O_4 > O_2$. For the subproblem $\{O_1, O_3, O_4, O_5\}$, we can obtain the satisfaction degree by repeating Step 1 to Step 6 as follows:

$$\begin{aligned}
\lambda(O_1) &= 0.8577, \\
\lambda(O_3) &= 0.38, \\
\lambda(O_4) &= 0.2740, \\
\lambda(O_5) &= 0.6066.
\end{aligned} \tag{29}$$

The ranking of the subproblem $\{O_1, O_3, O_4, O_5\}$ is $O_1 > O_5 > O_3 > O_4$.

We obtain the final ranking $O_1 > O_5 > O_3 > O_4 > O_2$ by combining the order of alternatives of subproblems $\{O_1, O_2, O_3, O_4\}$ and $\{O_1, O_3, O_4, O_5\}$, and the final order is the same as the order of original decision problem and it also exhibits transitive property. Criterion 2 and criterion 3 proposed in [45] are also suitable for the proposed method.

5.4. Comparison with Existing Work. For the comparison with Hadi-Vencheh and Mirjaberi's method [46], in the classical TOPSIS method, we often need to compute the relative closeness of the alternative O_i with respect to the PIS O^+ as below:

$$RC(O_i) = \frac{D(O_i, O^-)}{D(O_i, O^-) + D(O_i, O^+)} \tag{30}$$

$D(\cdot)$ is a distance measure. The ranking of all alternatives can be determined according to the closeness index $RC(O_i)$. If $\varepsilon = 0.5$ in our proposed equation (21), then (21) will be (30).

However, Hadi-Vencheh and Mirjaberi[46] suggested that one may use the following formula instead of the relative closeness index:

$$\zeta(O_i) = \frac{D(O_i, O^-)}{D_{\max}(O_i, O^-)} - \frac{D(O_i, O^+)}{D_{\min}(O_i, O^+)} \tag{31}$$

$D_{\max}(O_i, O^-) = \max_{1 \leq i \leq m} \{D(O_i, O^-)\}$ and $D_{\min}(O_i, O^+) = \min_{1 \leq i \leq m} \{D(O_i, O^+)\}$. Equation (31) is called the revised

closeness used to measure the extent to which the alternative O_i is close to the PIS O^+ and is far away from the NIS O^+ , simultaneously. By (31),

$$\begin{aligned}\zeta(O_1) &= -0.1213, \\ \zeta(O_2) &= -4.751, \\ \zeta(O_3) &= -2.3954, \\ \zeta(O_4) &= -2.7029, \\ \zeta(O_5) &= -1.0151\end{aligned}\quad (32)$$

Therefore, the ranking of $\{O_1, O_2, O_3, O_4, O_5\}$ is arranged $O_1 > O_5 > O_3 > O_4 > O_2$ which coincides with our proposed method.

6. Conclusion

IVIFSs, which are a generalization of the IFs, have been used widely in decision problems. IVIFS permits the membership degrees and nonmembership degrees to a given set to have an interval value in $[0, 1]$ and can be considered as a powerful tool to express complex information in the human decision-making process. In this paper, we introduced some new distance measures, namely, PIVIFOWAD operator and IPIV-IFOWAD operator, while, with respect to probabilistic decision-making problems with IVIF information, some new probabilistic decision-making analysis methods are developed. The new distance operators such as IVIFOWAD operator, PIVIFOWAD operator, and IPIVIFOWAD operator have been developed in this paper, while the concept of satisfaction degree of alternatives has been introduced based on some distance measures and applied to MCDM problem with IVIF information.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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