

## Research Article

# Enhanced Nonlinear Robust Control for TCSC in Power System

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Received 9 September 2017; Revised 6 February 2018; Accepted 15 March 2018; Published 6 May 2018

Academic Editor: Tran H. Linh

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This paper proposes an enhanced robust control method, which is for thyristor controlled series compensator (TCSC) in presences of time-delay nonlinearity, uncertain parameter, and external disturbances. Unlike conventional adaptive control methods, the uncertain parameter is estimated by using system immersion and manifold invariant (I&I) adaptive control. Thus, the oscillation of states caused by the coupling between parameter estimator and system states can be avoided. In addition, in order to overcome the influences of time-delay nonlinearity and external disturbances, backstepping sliding mode control is adopted to design control law recursively. Furthermore, robustness of TCSC control subsystem is achievable provided that dissipation inequality is satisfied in each step. Effectiveness and efficiencies of the proposed control method are verified by simulations. Compared with adaptive backstepping sliding mode control and adaptive backstepping control, the time of reaching steady state is shortened by at least 11% and the oscillation amplitudes of transient responses are reduced by at most 50%.

## 1. Introduction

In modern electrical power industry, security and stability of power system are under constant threat due to increasing power grid capacity, high-voltage transmission lines, and highly complex network configuration. Thyristor controlled series compensator (TCSC) system is often used to improve the stability and transfer capability of power system by scheduling power flow, decreasing voltage asymmetry and net loss, providing voltage support, and damping power oscillation [1–3]. However, some problems generally exist in TCSC system. TCSC shows the characteristics of time-delay nonlinearity due to the time-delay between triggering time and turn-on time of its thyristor controller [4, 5]. As uncertain parameter, damping coefficient is difficult to measure accurately in practice; external disturbances can be mainly caused by interference on generator rotor windings and the fluctuation on susceptance. These problems often deteriorate the performance and increase the difficulty in the design of TCSC robust controller significantly.

The proportional-integral-derivative (PID) control is commonly used to control nonlinear systems based on feedback linearization [6, 7]. PID was applied for TCSC

controller design by assuming that accurate system models are available [8]. However, feedback linearization not only linearizes the useful nonlinearities but also requires an accurate model which is difficult to obtain. Therefore, when the operating conditions of the system fluctuate widely, the transient stability of the TCSC system cannot be guaranteed if models are not obtained precisely.

Recently, adaptive backstepping is an optional method to mitigate the effects of nonlinearities and external disturbances on the system performance without linearization [9, 10]. The core concept of the adaptive backstepping is to design a controller recursively by considering some of the state variables as “virtual controls” and designing intermediate control laws for these variables [11–14]. Some TCSC control methods are proposed on the basis of adaptive backstepping [15, 16]. Nonetheless, this method is flawed with certain shortcomings. The transient stability of a closed-loop system cannot be guaranteed when the parameter estimator is fixed to its limit value and the system running time approaches infinity [17]. In other words, if the estimator is fixed, the estimation error will be accumulated in constructing control Lyapunov function (CLF) and the coupling terms between states and estimation error will be also accumulated as

running time increase. Thus, the transient stability of the close-loop system cannot be guaranteed.

To avoid this oscillation, a new method named system immersion and manifold invariant (I&I) adaptive control can provide a mean of shaping the dynamic response of the estimation error even if estimators reach the limit of its capacity [18, 19]. Unlike conventional adaptive control schemes relying on certainty equivalence principle, I&I adaptive control provides an alternative approach which avoids the cancellation of terms in the derivative of the Lyapunov function [20]. This method was used for SVC of a SMIB system to enhance the stability and improve the transient response [14]. However, I&I adaptive control has not been adopted in estimation of uncertain parameter in nonlinear systems with time-delay nonlinearity and external disturbances.

Sliding mode control can combine with backstepping to design the control law of nonlinear TCSC system, which is insensitive to nonlinearity and external disturbances with matching condition [21–23]. There are two processes in sliding mode motion: making the orbit approach sliding mode and keeping the orbit in sliding mode surface. As the orbit in sliding mode is achieved, the robustness of control system can be guaranteed on account of the advantage of sliding mode control which is insensitive to parameter perturbation and external disturbances, whereas the robustness of control system cannot be guaranteed before the motion orbit reach to sliding mode surface. By combining the advantages of both robust control and sliding control, robust sliding control method was developed to address this problem and achieve good performance in [24, 25]. However, the robust sliding mode control has not been adopted for nonlinear TCSC system when time-delay is involved.

In this paper, an enhanced robust control method is proposed to improve stability and robustness of TCSC system by simultaneously addressing the problems involving the existing nonlinear time-delay, uncertain parameter, and external disturbances. This proposed controller consists of the design of adaptive law and control law. As for adaptive law, the I&I adaptive control is first adopted to estimate uncertain parameter in TCSC system and it achieves excellent result in avoiding the oscillation of states caused by the coupling between parameter estimator and system state. With regard to control law, the influences of time-delay nonlinearity and external disturbances are solved by constructing the control law based on backstepping sliding mode control. Simulation results show that better performance in transient and steady state response is achieved compared with adaptive backstepping sliding mode control and adaptive backstepping control.

## 2. TCSC System Model and Control Objective

Figure 1 shows the single-machine infinite-bus system with TCSC.

The dynamic model of the TCSC system is expressed by the following nonlinear differential equations [5, 26, 27]:

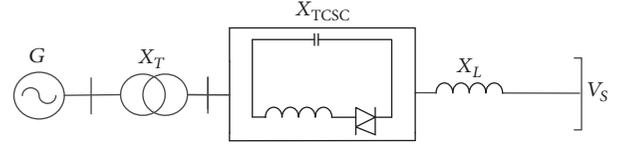


FIGURE 1: Single-machine infinite-bus power system with TCSC.

$$\dot{\delta} = \omega - \omega_0,$$

$$\dot{\omega} = \frac{\omega_0}{H} (P_m - E'_q V_s B_{TCSC} \sin \delta) - \frac{D}{H} (\omega - \omega_0) + \varepsilon_1, \quad (1)$$

$$\dot{B}_{TCSC} = \frac{1}{T_{TCSC}} (-B_{TCSC} (t - d) + B_{TCSC0} + u_B) + \varepsilon_2,$$

where  $\delta$ ,  $\omega$ , and  $B_{TCSC}$  are state variables denoting generator rotor angle, generator rotor angular speed, and equivalent susceptance, respectively. The physical significance of parameters in (1) is shown in Physical Significance of Parameters.

In Physical Significance of Parameters, time-delay  $d$  is caused by the differences between triggering time and turn-on time of its thyristor controller. Damping coefficient  $D$  is viewed as an uncertain parameter, and the relational expression  $\theta = -D/H$  is also the uncertain parameter.  $u_B$  is the equivalence input of SVC regulator. In the equation  $B_{TCSC} = 1/X_1 + X_2 - X_1 X_2 (B_L - B_C)$ ,  $B_C$  is the susceptance of the capacitor in the TCSC.  $B_L$  is the susceptance of the inductor of the TCR.  $X_1 = x'_d + X_T + X_{L1}$  is the total impedance from the generator to the injection of the SVC device with  $x'_d$  being the transient reactance of the generator on the direct axis and  $X_T$  being the reactance of the transformer.  $X_2$  is the total impedance from the injection of the SVC device to the infinite bus. The two external disturbances are as follows:  $\varepsilon_1$  is defined as the interference on generator rotor windings and  $\varepsilon_2$  is the fluctuation on susceptance, respectively [14, 27]. As a result, the controlled TCSC is an uncertain and nonlinear system involving time-delay nonlinearity, uncertain parameter, and external disturbances.

To simplify (1), three state variables are redefined as  $x_1 = \delta - \delta_0$ ,  $x_2 = \omega - \omega_0$ , and  $x_3 = B_{TCSC} - B_{TCSC0}$ , respectively, and we have

$$\dot{x}_1 = x_2,$$

$$\dot{x}_2 = \theta x_2 + k_1 P_m - k_2 (x_3 + B_{TCSC0}) \sin (x_1 + \delta_0) + \varepsilon_1, \quad (2)$$

$$\dot{x}_3 = k_3 (-x_3 (t - d) + u_B) + \varepsilon_2,$$

$$k_1 = \frac{\omega_0}{H},$$

$$k_2 = \frac{\omega_0 E'_q V_s}{H},$$

$$k_3 = \frac{1}{T_{TCSC}},$$

$$\theta = \frac{-D}{H}.$$

(3)

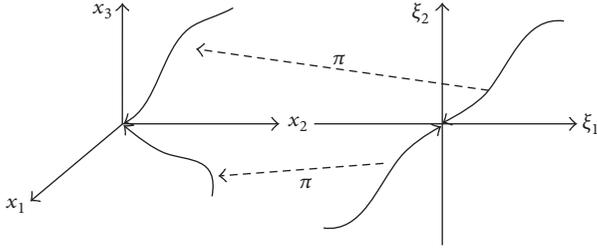


FIGURE 2: Geometric description of immersion and invariance.

The output of TCSC system is expressed as  $\mathbf{y} = [\mathbf{q}_1 \mathbf{x}_1 \quad \mathbf{q}_2 \mathbf{x}_2]^T$ , where  $q_1$  and  $q_2$  are nonnegative weighted coefficients which represent the weighted proportions of the state variables  $x_1$  and  $x_2$  in output.

The proposed method involves an adaptive law ( $\hat{\theta}$ ) and a control law ( $u_B$ ). The objective of designing TCSC robust controller is to guarantee that all the state variables can attain stability and eventually converge to equilibrium points in the nonlinear TCSC system. In addition, by using the proposed robust controller, the robustness of the TCSC robust controller is guaranteed, the performance and speed of transient responses is improved, and the time of reaching steady state is reduced.

### 3. Design of Robust Controller for Nonlinear TCSC

It divided three sections to introduce our proposed method in designing robust controller. In Section 3.1, I&I adaptive control is adopted for adaptive law  $\hat{\theta}$  design, which can ensure that the estimation error of  $\hat{\theta} - \theta$  gradually converged to zero. In Section 3.2, backstepping sliding mode control is used for control law  $u_B$  design recursively. The dissipation inequality is constructed in each step for guaranteeing the robustness of the subsystem. In Section 3.3, the stability and robustness of the TCSC control system are verified.

**3.1. Adaptive Law Design.** Figure 2 shows the mapping between the trajectories of the controlled and target system based on the notions of system immersion and manifold invariance.

The purpose of I&I methodology is to achieve stabilization by immersing the plant dynamics into a stable (lower-order) target system [20, 28]. The I&I adaptive control proposes an alternative approach avoiding the cancellation of terms in the derivative of the Lyapunov function and provides a mean of shaping the dynamic response of the estimation error [29].

Define a manifold

$$Z_e = \hat{\theta} - \theta + \varphi(x_1, x_2) = 0, \quad (4)$$

where  $\theta$  is an uncertain parameter,  $\hat{\theta}$  is the estimation value of  $\theta$ , and  $Z_e$  is an estimation error function.  $\varphi(x_1, x_2)$  is a

continuous function. The dynamics of (4) along with system (2) are

$$\begin{aligned} \dot{Z}_e = \dot{\hat{\theta}} + \sum_{k=1}^2 \frac{\partial \varphi}{\partial x_k} \times \dot{x}_k = \dot{\hat{\theta}} + \frac{\partial \varphi}{\partial x_1} x_2 + \frac{\partial \varphi}{\partial x_2} (\theta x_2 \\ + k_1 P_m - k_2 (x_3 + B_{\text{TCSC}0}) \sin(x_1 + \delta_0) + \varepsilon_1), \end{aligned} \quad (5)$$

where  $\varepsilon_1$  is a bounded function denoting the interference on generator rotor windings.

By cancelling out all the parameter-independent terms, an adaptive law  $\hat{\theta}$  is designed as

$$\begin{aligned} \dot{\hat{\theta}} = -\frac{\partial \varphi}{\partial x_1} x_2 - \frac{\partial \varphi}{\partial x_2} ((\hat{\theta} + \varphi) x_2 + k_1 P_m \\ - k_2 (x_3 + B_{\text{TCSC}0}) \sin(x_1 + \delta_0)). \end{aligned} \quad (6)$$

Substituting (6) into (5), we have

$$\dot{Z}_e = -\frac{\partial \varphi}{\partial x_2} Z_e x_2 + \frac{\partial \varphi}{\partial x_2} \varepsilon_1. \quad (7)$$

Design a CLF as

$$V = \frac{1}{2} Z_e^2 + \frac{1}{4} \rho h_1^2 (h_2 - x_2), \quad (8)$$

where  $h_1$  is the least upper bound of external disturbance  $\varepsilon_1$ , that is,  $h_1 \geq |\varepsilon_1|$ , due to the physical significance of external disturbance on generator rotor windings [14, 27, 30];  $h_2$  is the least upper bound of generator rotor angular  $x_2$  due to the limitation of generator rotor, that is,  $h_2 \geq |x_2|$ .

Let  $\varphi(x_1, x_2) = (1/2)\rho x_2^2$  with  $\rho > 0$ , and the dynamics of (8) is

$$\begin{aligned} \dot{V} = Z_e \left( -\frac{\partial \varphi}{\partial x_2} Z_e x_2 + \frac{\partial \varphi}{\partial x_2} \varepsilon_1 \right) - \frac{1}{4} \rho h_1^2 \\ = -\rho Z_e^2 x_2^2 + \rho Z_e x_2 \varepsilon_1 - \frac{1}{4} \rho h_1^2. \end{aligned} \quad (9)$$

Since  $h_1 \geq |\varepsilon_1|$ , we can obtain  $-(1/4)\rho h_1^2 \leq -(1/4)\rho \varepsilon_1^2$  and then have

$$\begin{aligned} \dot{V} = -\rho Z_e^2 x_2^2 + \rho Z_e x_2 \varepsilon_1 - \frac{1}{4} \rho h_1^2 \\ \leq -\rho Z_e^2 x_2^2 + \rho Z_e x_2 \varepsilon_1 - \frac{1}{4} \rho \varepsilon_1^2 \\ = -\rho \left( Z_e x_2 - \frac{1}{2} \varepsilon_1 \right)^2 \leq 0. \end{aligned} \quad (10)$$

According to Lyapunov stability theorem, the proposed adaptive law  $\hat{\theta}$  and the selected continuous function  $\varphi(x_1, x_2) = (1/2)\rho x_2^2$  can ensure that  $Z_e$  converges to zero in finite time.

*Remark 1.* Unlike the conventional adaptive control based on certainty equivalency principle, the proposed adaptive law can introduce a continuous function  $\varphi(x_1, x_2)$  to compensate

the residual estimation error  $\hat{\theta} - \theta$ . Furthermore, by requiring the estimation error to converge to zero, the stability and convergence of system (7) are guaranteed based on the proposed adaptive law and the selected continuous function. Therefore, the error accumulation of the coupling terms is avoided even if the parameter estimates are fixed.

*Remark 2.* Theoretically, there is a large flexibility in selecting  $\varphi(x_1, x_2)$  which can guarantee that  $\lim_{t \rightarrow \infty} Z_e(t) = 0$ . For simplicity, we let  $\varphi(x_1, x_2) = (1/2)\rho x_2^2$ , which is the lowest order and simplest form of  $\varphi(x_1, x_2)$ .

*Remark 3.* The selected  $\varphi(x_1, x_2)$  and the designed adaptive law are not only guarantee that the estimation error  $Z_e$  converges to zero but also ensure that the parametric manifold  $I_e = \{(x, \hat{\theta}) \in R^3 \times R^1 \mid \hat{\theta} - \theta + \varphi(x_1, x_2) = 0\}$  is invariant and attractive [28, 29].

**3.2. Control Law Design.** In this section, the control law is designed by using backstepping sliding mode control method in TCSC system with time-delay nonlinearity and external disturbances. Three steps are constructed to design the control law recursively. In each step, the dissipation inequality is satisfied to guarantee the robustness of TCSC control system.

Based on dissipation theory, an inequation external disturbances is constructed as

$$V(x(T)) - V(x(0)) \leq \int_0^T s(\Delta) dt; \quad (11)$$

$V(x)$  is an energy storage function.  $s(\Delta) = \gamma \|\Delta\|^2 - \|y\|^2$  is an energy supply function, where  $\Delta = (\varepsilon_1, \varepsilon_2)^T$  is external disturbance,  $\gamma$  is a nonnegative constant, and  $y = [q_1 x_1 \ q_2 x_2]^T$  is the output of the TCSC system. From (11), the dissipation inequality is achievable provided that the  $L_2$  gain from the disturbance to the output of the system is smaller than or equal to  $\gamma$ , where  $\gamma$  is disturbance attenuation constant.

According to the definitions above, the control law is designed as follows.

Define error state variables  $z_i$  ( $i = 1, 2, 3$ )

$$\begin{aligned} z_1 &= x_1, \\ z_2 &= x_2 - x_2^*, \\ z_3 &= x_3 - x_3^*, \end{aligned} \quad (12)$$

where  $x_2^*$  and  $x_3^*$  are virtual control law. The derivative of  $z_1$  along with (2) is

$$\dot{z}_1 = x_2 = z_2 + x_2^*. \quad (13)$$

*Step 1.* The first CLF is

$$V_1 = \frac{1}{2} z_1^2, \quad (14)$$

and the derivative of  $V_1$  along with (13) is

$$\dot{V}_1 = z_1 z_2 + z_1 x_2^*. \quad (15)$$

To satisfy Lyapunov stability theorem, the virtual control law  $x_2^*$  is

$$x_2^* = -c_1 z_1, \quad (16)$$

where  $c_1$  is a positive constant. Substituting (16) into (15), it can be seen clearly that  $\dot{V}_1 \leq 0$  when  $z_2 = 0$ . However, when  $z_2 \neq 0$ , we can construct the second CLF in Step 2.

*Step 2.* The second CLF is

$$V_2 = V_1 + \frac{1}{2} z_2^2, \quad (17)$$

and the derivative of  $V_2$  along with (15) is

$$\dot{V}_2 = \dot{V}_1 + z_2 \dot{z}_2. \quad (18)$$

Define a function  $M_1$  as

$$M_1 = \dot{V}_2 + \frac{1}{2} (\|y\|^2 - \gamma^2 \|\varepsilon_1\|^2). \quad (19)$$

Substituting  $y = [q_1 x_1 \ q_2 x_2]^T$  into (19), (19) can be rewritten as

$$\begin{aligned} M_1 &= -\alpha z_1^2 - \left( \frac{\gamma \varepsilon_1}{2} - \frac{z_2}{\gamma} \right)^2 - \frac{1}{4} \gamma^2 \varepsilon_1^2 + z_2 [\eta_1 x_1 \\ &+ \eta_2 x_2 + \theta x_2 + k_1 P_m \\ &- k_2 (x_3 + B_{\text{TCSC}0}) \sin(x_1 + \delta_0)], \end{aligned} \quad (20)$$

where

$$\begin{aligned} \alpha &= c_1 - \frac{1}{2} q_1^2 - \frac{1}{2} q_2^2 c_1^2, \\ \eta_1 &= \frac{c_1}{\gamma^2} - \frac{c_1}{2} q_2^2 + 1, \\ \eta_2 &= \frac{1}{\gamma^2} + \frac{1}{2} q_2^2 - c_1. \end{aligned} \quad (21)$$

To guarantee that  $M_1 \leq 0$ ,  $x_3^*$  is

$$x_3^* = \frac{\eta_1 x_1 + \eta_2 x_2 + (\hat{\theta} + \varphi) x_2 + k_1 P_m}{k_2 \sin(x_1 + \delta_0)} - B_{\text{TCSC}0}, \quad (22)$$

substituting  $x_3 = x_3^* + z_3$  and (22) into (20), we have

$$\begin{aligned} M_1 &= -\alpha z_1^2 - \left( \frac{\gamma}{2} \varepsilon_1 - \frac{z_2}{\gamma} \right)^2 - \frac{1}{4} \gamma^2 \varepsilon_1^2 - z_2 x_2 Z_e \\ &- k_2 \sin(x_1 + \delta_0) z_2 z_3. \end{aligned} \quad (23)$$

According to Remark 3, we can obtain  $M_1 \leq 0$  when  $z_3 = 0$ . But, when  $z_3 \neq 0$ , we can get into Step 3.

*Step 3.* Design sliding surface  $s = \lambda_1 z_1 + \lambda_2 z_2 + z_3 = 0$ , where  $\lambda_1$  and  $\lambda_2$  are designed parameters. To ensure the

whole TCSC system is globally asymptotically stable, the third CLF is

$$V_3 = \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2 + \frac{1}{2}s^2 + \int_{t-d}^t q(x(\tau)) d\tau, \quad (24)$$

where  $q(x(\tau))$  is a nonnegative function. The derivative of  $V_3$  is

$$\dot{V}_3 = z_1\dot{z}_1 + z_2\dot{z}_2 + s\dot{s} + q(x(t)) - q(x(t-d)); \quad (25)$$

$M_2$  is designed as

$$M_2 = \dot{V}_3 + \frac{1}{2}(\|y\|^2 - \gamma^2\|\varepsilon_1\|^2 - \gamma^2\|\varepsilon_2\|^2). \quad (26)$$

Substituting the control output and (25) into (26), we obtain

$$\begin{aligned} M_2 = & \dot{V}_3 + \frac{1}{2}(\|y\|^2 - \gamma^2\|\varepsilon_1\|^2 - \gamma^2\|\varepsilon_2\|^2) = -\alpha z_1^2 \\ & - \left(\frac{\gamma}{2}\varepsilon_1 - \frac{z_2}{\gamma}\right)^2 - \frac{1}{4}\gamma^2\varepsilon_1^2 - z_2x_2Z_e - \frac{1}{2}\gamma^2\varepsilon_2^2 - k_2 \\ & \cdot \sin(x_1 + \delta_0)z_2z_3 + s(\lambda_1x_2 + \lambda_2\dot{x}_2 + \lambda_2c_1x_2 \\ & + k_3(-x_3(t-d) + u_B) + \varepsilon_2 - \dot{x}_3^*) + q(x(t)) \\ & - q(x(t-d)). \end{aligned} \quad (27)$$

Let  $z_3 = s - \lambda_1z_1 - \lambda_2z_2$ ; (27) is rewritten as

$$\begin{aligned} M_2 = & -(\alpha - \lambda_1^2)z_1^2 - \left(\frac{\gamma}{2}\varepsilon_1 - \frac{z_2}{\gamma}\right)^2 - \left[\lambda_2k_2 \right. \\ & \cdot \sin(x_1 + \delta_0) - \frac{1}{4}k_2^2\sin^2(x_1 + \delta_0)]z_2^2 - \left[\lambda_1z_1 \right. \\ & - \frac{1}{2}k_2\sin(x_1 + \delta_0)z_2] \left. - \left[\frac{\gamma}{2}\varepsilon_1 - \frac{s}{\gamma}\left(\lambda_2 \right. \right. \right. \\ & \left. \left. - \frac{\eta_2 + \hat{\theta} + \varphi}{k_2\sin(x_1 + \delta_0)}\right)\right]^2 - \frac{1}{2}\left(\gamma\varepsilon_2 - \frac{s}{\gamma}\right)^2 - z_2Z_ex_2 \\ & + s \left\{ \frac{s}{\gamma^2}\left(\lambda_2 - \frac{\eta_2 + \hat{\theta} + \varphi}{k_2\sin(x_1 + \delta_0)}\right)^2 - k_2 \right. \\ & \cdot \sin(x_1 + \delta_0)z_2 + (\lambda_1 + \lambda_2\hat{\theta} + \lambda_2c_1)x_2 - \lambda_2k_2 \\ & \cdot \sin(x_1 + \delta_0)(B_{\text{TCSC0}} + x_3) + \frac{s}{2\gamma^2} + \lambda_2k_1P_m \\ & - k_3x_3(t-d) + k_3u_B - \frac{1}{k_2\sin(x_1 + \delta_0)}\left[\eta_1x_2 \right. \\ & + (\hat{\theta} + \dot{\varphi})x_2(\hat{\theta} + \varphi + \eta_2) \\ & \left. \left. \left. \cdot (\hat{\theta}x_2 + k_1P_m - k_2\sin(x_1 + \delta_0)(x_3 + B_{\text{TCSC0}}))\right] \right\} \end{aligned}$$

$$\begin{aligned} & + \frac{\cos(x_1 + \delta_0)x_2}{k_2\sin^2(x_1 + \delta_0)}(\eta_1x_1 + \eta_2x_2 + (\hat{\theta} + \varphi)x_2 \\ & + k_1P_m) \left. \right\} + q(x(t)) - q(x(t-d)). \end{aligned} \quad (28)$$

Define a nonnegative function  $q(x(t)) = |k_3sx_3(t)|$ , and then we can obtain  $q(x(t-d)) = |k_3sx_3(t-d)|$ . An inequation is established as

$$-|k_3sx_3(t-d)| \leq k_3sx_3(t-d). \quad (29)$$

Moreover, to ensure the robustness and stability of the nonlinear TCSC control system, the control law is then designed as

$$\begin{aligned} u_B = & -\frac{1}{k_3} \left\{ \frac{s}{\gamma^2} \left( \lambda_2 - \frac{\eta_2 + \hat{\theta} + \varphi}{k_2\sin(x_1 + \delta_0)} \right)^2 - k_2 \right. \\ & \cdot \sin(x_1 + \delta_0)z_2 + (\lambda_1 + \lambda_2\hat{\theta} + \lambda_2c_1)x_2 - \lambda_2k_2 \\ & \cdot \sin(x_1 + \delta_0)(B_{\text{TCSC0}} + x_3) + \frac{s}{2\gamma^2} + \lambda_2k_1P_m \\ & - \frac{1}{k_2\sin(x_1 + \delta_0)} \left[ \eta_1x_2 + (\hat{\theta} + \dot{\varphi})x_2 \right. \\ & + (\hat{\theta} + \varphi + \eta_2) \\ & \left. \left. \left. \cdot (\hat{\theta}x_2 + k_1P_m - k_2\sin(x_1 + \delta_0)(x_3 + B_{\text{TCSC0}})) \right] \right\} \\ & + \frac{\cos(x_1 + \delta_0)x_2}{k_2\sin^2(x_1 + \delta_0)}(\eta_1x_1 + \eta_2x_2 + (\hat{\theta} + \varphi)x_2 \\ & + k_1P_m) - \mu|k_3x_3(t)| + \sigma s \left. \right\}, \end{aligned} \quad (30)$$

where  $\sigma$  is a nonnegative sliding mode observer gain and  $\mu$  is a sign function, which is defined as  $\mu = -1$  when  $s \rightarrow 0^+$  and  $\mu = 1$  when  $s \rightarrow 0^-$ .

Substituting  $q(x(t)) = |k_3sx_3(t)|$ ,  $q(x(t-d)) = |k_3sx_3(t-d)|$ , (29), and (30) into (28), (28) is rewritten as

$$\begin{aligned} M_2 & \leq -(\alpha - \lambda_1^2)z_1^2 - \left(\frac{\gamma}{2}\varepsilon_1 - \frac{z_2}{\gamma}\right)^2 \\ & - \left[\lambda_2k_2\sin(x_1 + \delta_0) - \frac{1}{4}k_2^2\sin^2(x_1 + \delta_0)\right]z_2^2 \\ & - \left[\lambda_1z_1 - \frac{1}{2}k_2\sin(x_1 + \delta_0)z_2\right]^2 \\ & - \left[\frac{\gamma}{2}\varepsilon_1 - \frac{s}{\gamma}\left(\lambda_2 + \frac{\eta_2 + \hat{\theta} + \varphi}{k_2\sin(x_1 + \delta_0)}\right)\right]^2 \\ & - \frac{1}{2}\left(\gamma\varepsilon_2 - \frac{s}{\gamma}\right)^2 - z_2Z_ex_2 - \sigma s^2 \leq 0. \end{aligned} \quad (31)$$

To guarantee  $\lambda_2 k_2 \sin(x_1 + \delta_0) - (1/4)k_2^2 \sin^2(x_1 + \delta_0) \geq 0$  with  $k_2 \geq 0$ , we can get  $\lambda_2 \geq (1/4)k_2$  if  $\sin(x_1 + \delta_0) \geq 0$  and  $\lambda_2 < -(1/4)k_2$  if  $\sin(x_1 + \delta_0) < 0$ . Furthermore, the parameters  $\lambda_1$ ,  $\lambda_2$ ,  $k_2$ , and  $c_1$  are selected to ensure that  $\alpha - \lambda_1^2 \geq 0$  and  $\lambda_2 k_2 \sin(x_1 + \delta_0) - (1/4)k_2^2 \sin^2(x_1 + \delta_0) \geq 0$ . Therefore, according to Remark 3, we can obtain  $M_2 \leq 0$  as time approaches to infinity.

**3.3. Proof of System Stability.** The dynamics of closed-loop error system are expressed as

$$\begin{aligned}\dot{z}_1 &= \dot{x}_1, \\ \dot{z}_2 &= \dot{x}_2 - \dot{x}_2^*, \\ \dot{z}_3 &= \dot{x}_3 - \dot{x}_3^*.\end{aligned}\quad (32)$$

Substituting virtual control input  $x_2^*$  and  $x_3^*$  into (32), we can obtain

$$\begin{aligned}\dot{z}_1 &= \dot{x}_1 = z_2 - c_1 z_1, \\ \dot{z}_2 &= \dot{x}_2 - \dot{x}_2^* \\ &= \theta x_2 + k_1 P_m - k_2 (x_3 + B_{\text{TCSC0}}) \sin(\delta_0 + x_1) + \varepsilon_1 \\ &\quad + c_1 z_2 - c_1^2 z_1, \\ \dot{z}_3 &= \dot{x}_3 - \dot{x}_3^* = k_3 (-x_3(t-d) + u_B) + \varepsilon_2 - \dot{x}_3^*.\end{aligned}\quad (33)$$

The asymptotic stability of the closed-loop error system (33) is discussed in two different conditions.

Firstly, when the external disturbances  $\varepsilon_1 \neq 0$  and  $\varepsilon_2 \neq 0$ , a relationship according to (26) and (31) is constructed as

$$\begin{aligned}M_2 &= \dot{V}_3 + \frac{1}{2} (\|y\|^2 - \gamma^2 \|\varepsilon_1\|^2 - \gamma^2 \|\varepsilon_2\|^2) \\ &\leq -(\alpha - \lambda_1^2) z_1^2 - \left(\frac{\gamma}{2} \varepsilon_1 - \frac{z_2}{\gamma}\right)^2 \\ &\quad - \left[\lambda_2 k_2 \sin(x_1 + \delta_0) - \frac{1}{4} k_2^2 \sin^2(x_1 + \delta_0)\right] z_2^2 \\ &\quad - \left[\lambda_1 z_1 - \frac{1}{2} k_2 \sin(x_1 + \delta_0) z_2\right]^2 \\ &\quad - \left[\frac{\gamma}{2} \varepsilon_1 - \frac{s}{\gamma} \left(\lambda_2 + \frac{\eta_2 + \hat{\theta} + \varphi}{k_2 \sin(x_1 + \delta_0)}\right)\right]^2 \\ &\quad - \frac{1}{2} \left(\gamma \varepsilon_2 - \frac{s}{\gamma}\right)^2 - z_2 Z_e x_2 - \sigma s^2 \leq 0.\end{aligned}\quad (34)$$

Let  $V(x) = 2V_3(x)$ ; an inequation with regard to  $\dot{V}_3$  is expressed as

$$\dot{V} = 2\dot{V}_3 \leq \gamma^2 \|\varepsilon_1\|^2 + \gamma^2 \|\varepsilon_2\|^2 - \|y\|^2. \quad (35)$$

By integrating both sides of (35), the dissipation inequality is

$$\begin{aligned}V(x(t)) - V(x(0)) \\ \leq \int_0^T (\gamma^2 \|\varepsilon_1\|^2 + \gamma^2 \|\varepsilon_2\|^2 - \|y\|^2) dt.\end{aligned}\quad (36)$$

The dissipation inequality (34) is satisfied and the robustness of TCSC controller is guaranteed by using the proposed method. This implies that all the increased energy of the system from  $t = 0$  to  $T$  is always equal to or less than the ones from outside. Therefore, the energy of power systems has been decreasing. Obviously, the proposed method consisting of the designed law  $u_B$  and the adaptive law  $\hat{\theta}$  can guarantee that the closed-loop error system is globally asymptotically stable.

Secondly, when the external disturbances are not involved, we can construct Lemma 4 and prove it as below.

**Lemma 4.** *Based on the above derivations, one can conclude that the dynamics of closed-loop error system are asymptotically stable when  $\varepsilon_1 = 0$  and  $\varepsilon_2 = 0$ .*

*Proof.* Substituting  $\varepsilon_1 = 0$  and  $\varepsilon_2 = 0$  into (34), we have

$$\begin{aligned}\dot{V}_3 + \frac{1}{2} \|y\|^2 \\ \leq -(\alpha - \lambda_1^2) z_1^2 - \left(\frac{z_2}{\gamma}\right)^2 \\ - \left[\lambda_2 k_2 \sin(x_1 + \delta_0) - \frac{1}{4} k_2^2 \sin^2(x_1 + \delta_0)\right] z_2^2 \\ - \left[\lambda_1 z_1 - \frac{1}{2} k_2 \sin(x_1 + \delta_0) z_2\right]^2 \\ - \left[\frac{s}{\gamma} \left(\lambda_2 - \frac{\eta_2 + \hat{\theta} + \varphi}{k_2 \sin(x_1 + \delta_0)}\right)\right]^2 - \frac{1}{2} \left(\frac{s}{\gamma}\right)^2 \\ - \sigma s^2 \leq 0.\end{aligned}\quad (37)$$

According to  $\dot{V}_3 + (1/2)\|y\|^2 \leq 0$  and  $(1/2)\|y\|^2 = (1/2)(q_1^2 x_1^2 + q_2^2 x_2^2) > 0$ , we can obtain  $\dot{V}_3 \leq 0$  and have

$$V_3(t) \leq V_3(0), \quad (38)$$

where  $t \geq 0$ . Furthermore, according to  $V_3 = (1/2)z_1^2 + (1/2)z_2^2 + (1/2)s^2 + \int_{t-d}^t q(x(\tau))d\tau$  with  $q(x(t)) = |k_3 s x_3(t)|$ , we have  $V_3(t) \geq 0$ .

Since  $V_3(0)$  is bounded,  $V_3(t) \geq 0$ , and  $\dot{V}_3(t) \leq 0$ , then we know that  $V_3(t)$  is nonincreasingly bounded and  $z_1$ ,  $z_2$ ,  $s$ ,  $x_1$ , and  $x_2$  are all bounded.

Define  $Q(t) = \dot{V}_3(t)$ , and the integral of  $Q(t)$  is

$$\int_0^t Q(\tau) d\tau = V_3(t) - V_3(0). \quad (39)$$

According to  $V_3(t)$  which is nonincreasingly bounded,  $\dot{V}_3(t) \leq 0$ , and  $V_3(t) \geq 0$ , we obtain the fact that  $\lim_{t \rightarrow \infty} \int_0^t Q(\tau) d\tau$  exists. Finally,  $\lim_{t \rightarrow \infty} \dot{V}_3(t) = 0$  is achievable due to Barbalat's Lemma (Barbalat, 1959).

Moreover, due to  $\lim_{t \rightarrow \infty} \dot{V}_3(t) = 0$  and  $z_1$ ,  $z_2$ ,  $x_2$ ,  $x_1$ , and  $s$ , which are all bounded,  $V_3 \geq 0$ , we have  $z_1 \rightarrow 0$ ,  $z_2 \rightarrow 0$ ,  $x_1 \rightarrow 0$ ,  $x_2 \rightarrow 0$ , and  $s \rightarrow 0$  when  $t \rightarrow \infty$ . In addition, the sliding surface is defined as  $s = \lambda_1 z_1 + \lambda_2 z_2 + z_3$ , and then we can get  $z_3 = s - \lambda_1 z_1 - \lambda_2 z_2$ . Obviously, it is obtained that  $z_3 \rightarrow 0$  as  $t \rightarrow \infty$ . Lemma 4 holds.  $\square$

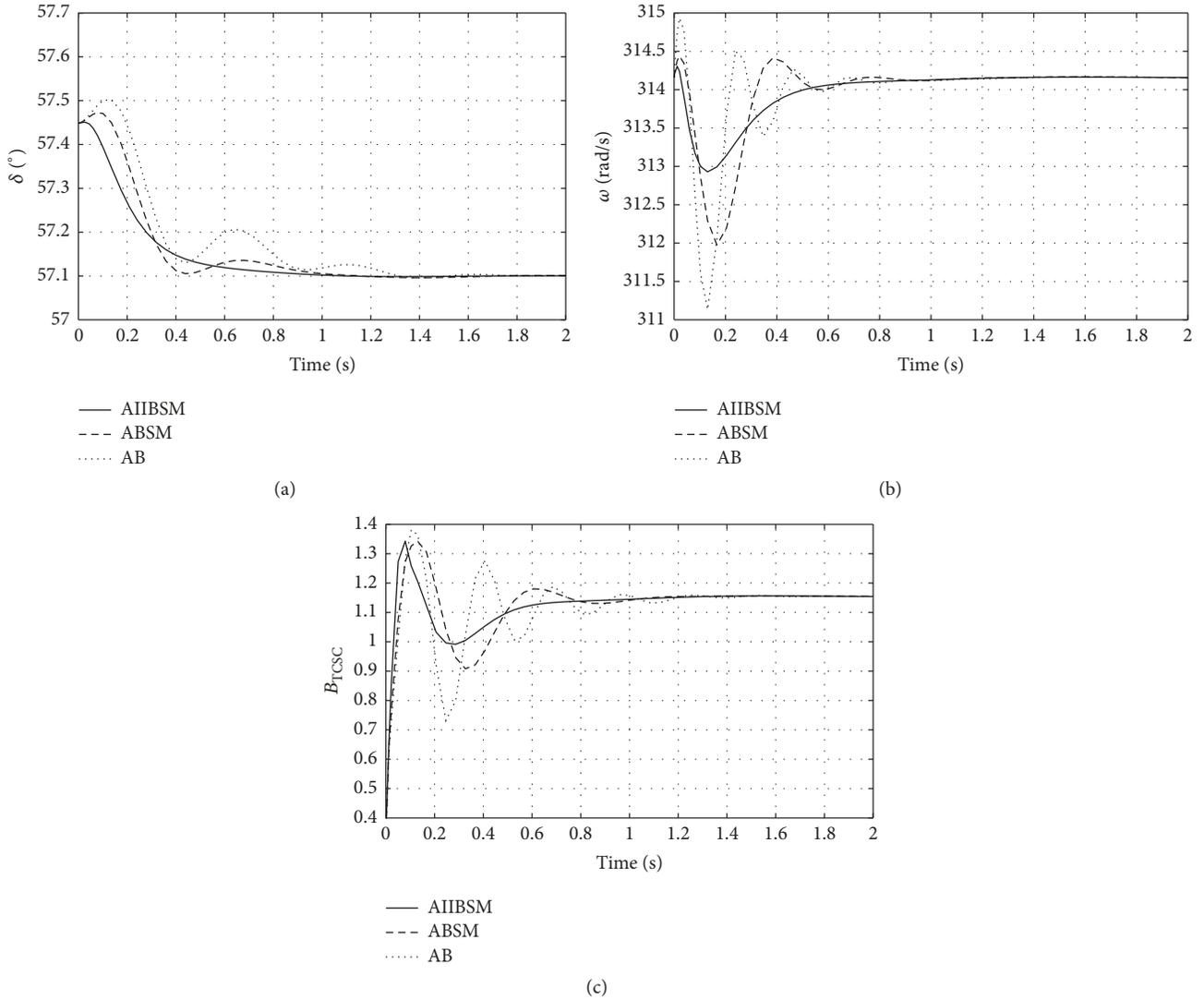


FIGURE 3: Transient responses in case 1: (a) transient responses of rotor angle; (b) transient responses of rotor angular speed; (c) transient responses of equivalent susceptance.

As a result, the control objective is achieved by using the designed law  $u_B$  and the adaptive law  $\hat{\theta}$ . This implies that all the responses of state variables attain stability and eventually converge to equilibrium points.

#### 4. Simulations

The MATLAB software is performed in TCSC simulations. According to operating characteristics of TCSC system, the time-delay between the triggering time and turn-on time is equal to or less than 0.04 s. The damping coefficient is viewed as the uncertain parameter. The external disturbances in the system are  $\varepsilon_1 = e^{-2t} \sin(5t)$  and the unknown fluctuation on susceptance is  $\varepsilon_2 = e^{-2t} \cos(5t)$ , respectively [14, 27]. The parameters used in the simulation are given in Table 1.

The generator rotor angle  $\delta$ , generator rotor angular speed  $\omega$ , and equivalent susceptance  $B_{TCSC}$  are the state variables whose transient response trajectories are tracked. The initial

operating values are  $\delta_0 = 57.1^\circ$ ,  $\omega_0 = 314.159$  rad/s, and  $B_{TCSC0} = 0.4$  pu.

Four case studies are performed to simulate the nonlinear TCSC systems with different delay times  $d$  or attenuation coefficient  $\gamma$ . Cases 1–3 aim to compare the stability and robustness of TCSC control systems designed by the proposed adaptive immersion and invariance backstepping sliding mode (AIIBSM) method, adaptive backstepping sliding mode (ABSM) method, and adaptive backstepping method (AB).  $d$  and  $\gamma$  are set to 0.01 s and 2 in case 1, 0.02 s and 2 in case 2, and 0.04 s and 2 in case 3. In case 4, simulations of our designed TCSC system with  $d_4 = 0.02$  s are performed at  $\gamma_1 = 2$  and  $\gamma_2 = 5$ , respectively.

Figure 3 shows the comparisons between the proposed method and the two existing methods in case 1, where  $d_1 = 0.01$  and  $\gamma_1 = 2$ . The transient responses of three state variables, that is, rotor angle, rotor angular speed, and equivalent susceptance, are investigated, respectively. It can

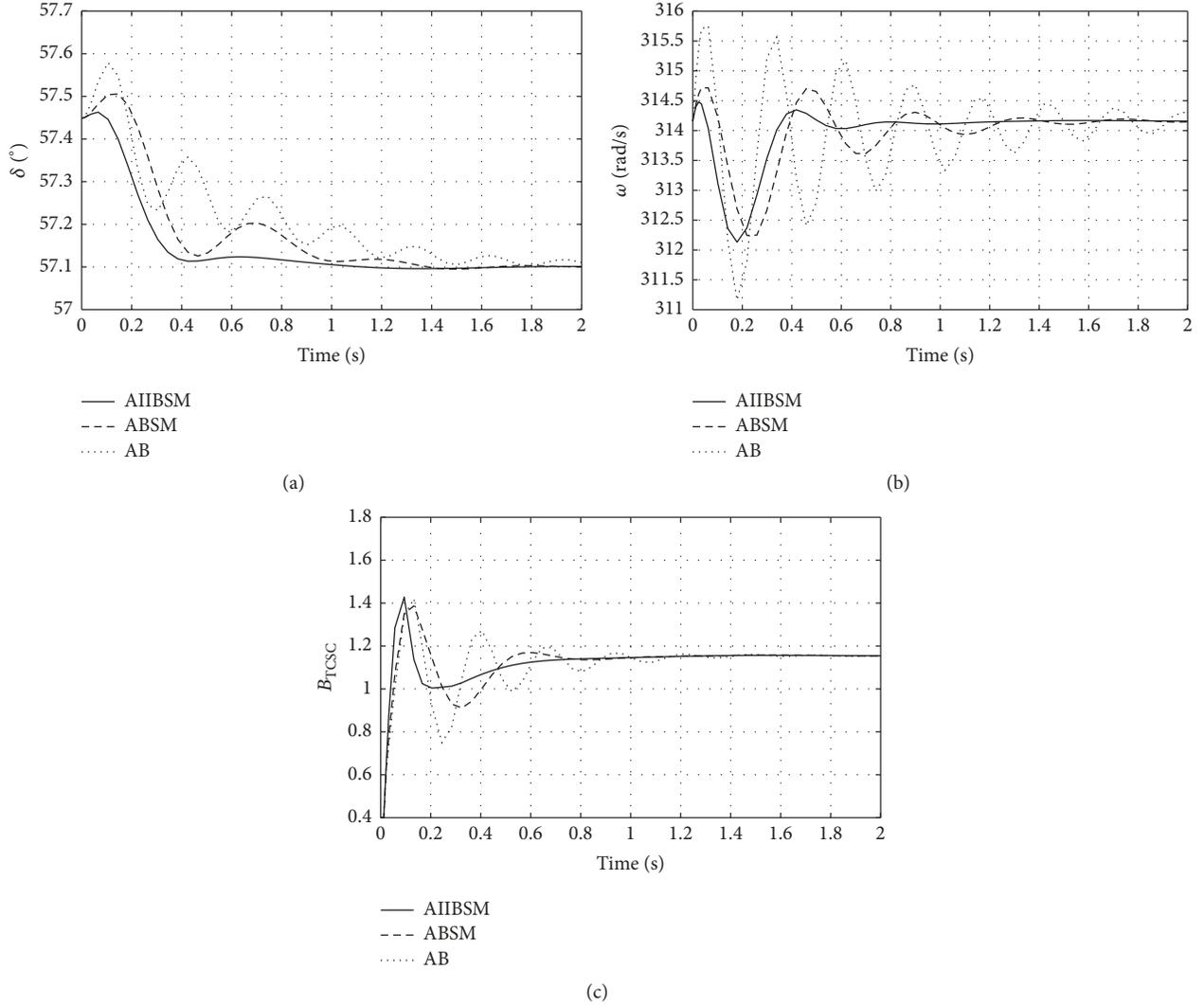


FIGURE 4: Transient responses in case 2: (a) transient responses of rotor angle; (b) transient responses of rotor angular speed; (c) transient responses of equivalent susceptance.

TABLE 1: Parameters for nonlinear TCSC system simulations.

System parameters	Values and units
Generator	$H = 8$ (p.u.); $V_s = 1.0$ (p.u.); $E'_q = 1.08$ (p.u.); $P_m = 1.0$ (p.u.);
Transmission line	$X_1 = 0.84$ (p.u.); $X_2 = 0.52$ (p.u.);
TCSC	$B_{TCSC0} = 0.814$ (p.u.); $B_L + B_C = 0.3$ (p.u.); $T_{TCSC} = 0.02$ (s);
Output weighted coefficient	$q_1 = 0.4$ ; $q_2 = 0.6$ ;
Sliding mode gain	$\lambda_1 = 1$ ; $\lambda_2 = 1$ ; $\sigma = 100$ ;
Unfixed parameter	$d = 0.01, 0.02, 0.04$ ; $\gamma = 2, 5$ ;

be seen that the transient responses of the state variables of the AIIBSM controller are faster and the AIIBSM system tends to attain stability more rapidly, whereas the transient responses of the ABSM and AB controllers fluctuate faster

and the ABSM and AB systems tend to reach stability in a longer finite time. Especially, three state variables cannot achieve the steady state in finite time.

Figure 4 shows the transient responses of the three state variables in case 2, where  $d_2 = 0.02$  s and  $\gamma_1 = 2$ . The transient trajectories of the AIIBSM controller fluctuate less strongly and quickly converge to steady state than compared to those of the ABSM and AB controllers in finite time, suggesting that the proposed AIIBSM method results in better system performance.

Figure 5 shows the transient responses of the three state variables in case 3, where  $d_3 = 0.04$  and  $\gamma_1 = 2$ . The result of case 3 reveals that the proposed method can guarantee that the state variables of the nonlinear TCSC system are globally bounded and transient responses will eventually converge to a stable value regardless of what delay time is considered. A comparison of the transient responses of the AIIBSM controller in the three cases shows that the transient

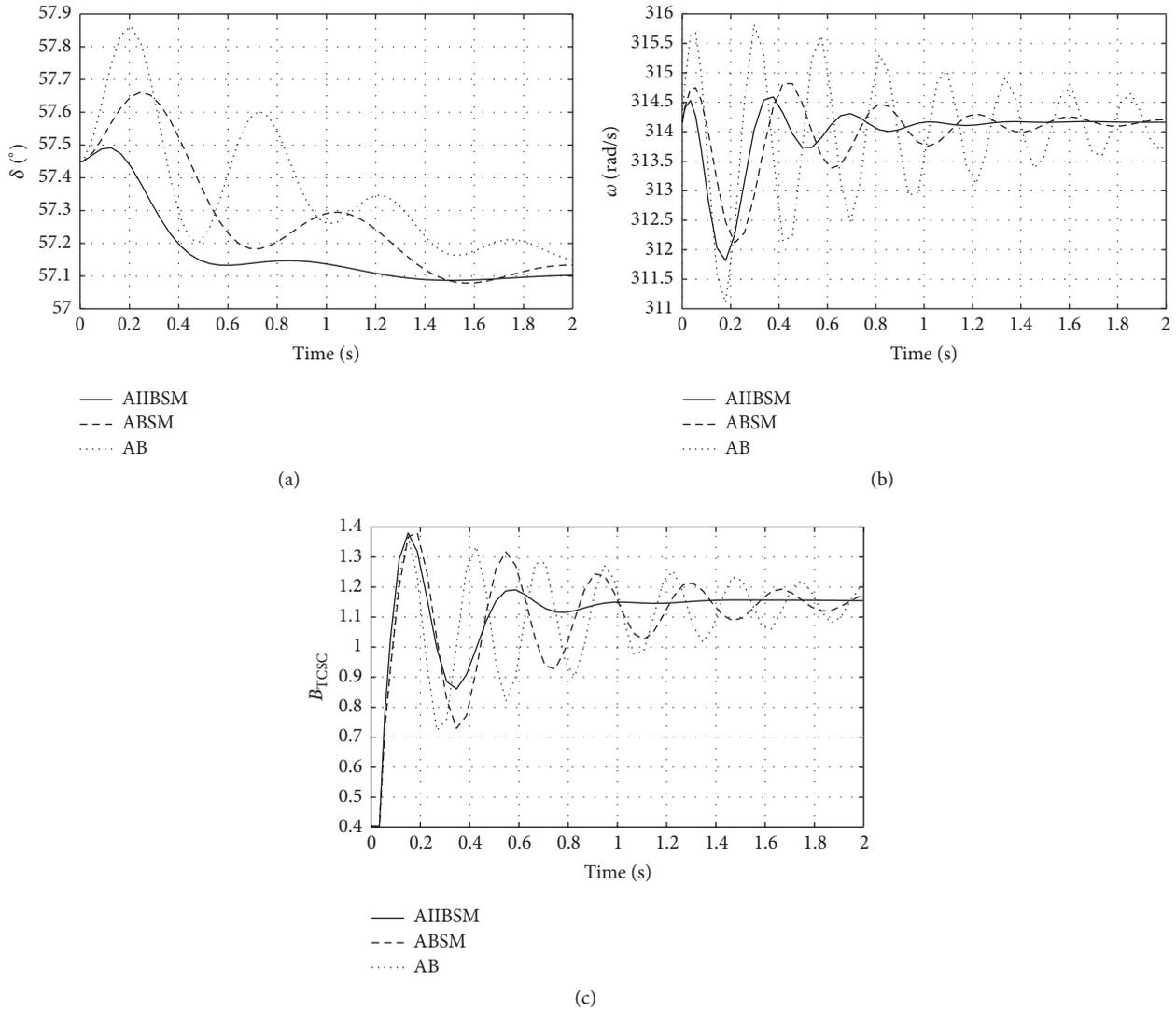


FIGURE 5: Transient responses in case 3: (a) transient responses of rotor angle; (b) transient responses of rotor angular speed; (c) transient responses of equivalent susceptance.

responses in case 3 converge more slowly and the system stability is attained in a longer finite time. Similar comparative results are obtained for the ABSM and AB controllers.

Figure 6 shows transient responses of the three state variables in case 4. The simulation results are compared to investigate the effect of the attenuation coefficient  $\gamma$  on the stability and robustness of TCSC control systems. It can be seen that the AIIBSM controller attains more rapid speed responses and stronger robustness at  $\gamma_1 = 2$  than at  $\gamma_2 = 5$ . Therefore, the delay time  $d$  is a crucial nonlinear factor impacting the transient and steady performance of the TCSC system. A smaller  $d$  can lead to better transient stability and stronger robustness. Furthermore, the attenuation coefficient  $\gamma$  is also a key factor impacting the performance of the control system. A smaller  $\gamma$  tends to result in stronger robustness and smaller oscillation. This result is consistent with the theoretical analysis.

## 5. Conclusions

This paper investigates the problems involving time-delay nonlinearity, uncertain parameter, and external disturbances in the controller design of TCSC system. An uncertain parameter is estimated in the adaptive law design based on I&I adaptive control. The oscillation of states caused by the coupling between parameter estimator and system states is avoided, due to the fact that the errors of parameter estimation can gradually converge to zero by using the designed adaptive law. In addition, backstepping sliding mode control is adopted to solve the problems caused by time-delay nonlinearity and external disturbances in the control law design. Moreover, by satisfying the dissipation inequality, the robustness of the proposed TCSC control system is achieved. Simulation results have confirmed that, by using the proposed method, all state variables are globally

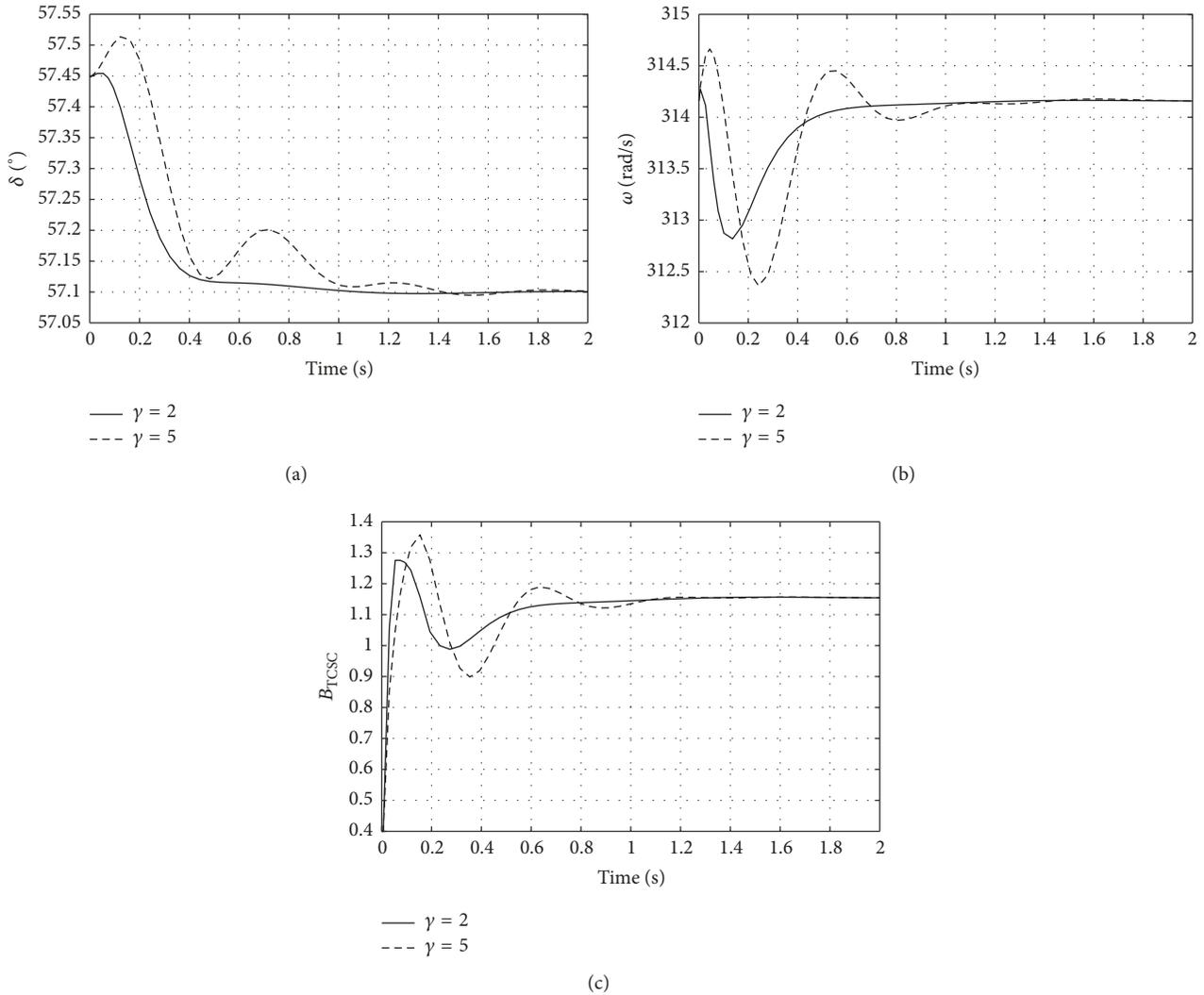


FIGURE 6: Transient responses in case 3: (a) transient responses of rotor angle (b); transient responses of rotor angular speed; (c) transient responses of equivalent susceptance.

bounded and transient responses eventually converge to equilibrium points regardless of time-delay.

### Physical Significance of Parameters

$\delta$ :	Generator rotor angle
$H$ :	Inertia constant
$P_m$ :	Mechanical power
$d$ :	Time-delay
$T_{TCS}$ :	Time constant
$\epsilon_{1,2}$ :	External disturbances
$\omega$ :	Generator rotor angular speed
$E'_q$ :	Transient electromotive force
$D$ :	Damping coefficient
$B_{TCS}$ :	Equivalent susceptance
$V_s$ :	Infinite bus voltage
$u_B$ :	Equivalence input.

### Conflicts of Interest

The authors declare that they have no conflicts of interest.

### Acknowledgments

This work was supported in part by the NSF of China under Grant 51707140, in part by the Fundamental Research Funds for the Central Universities under Grant JBX170416, JB160415, and XJS16020, in part by the Natural Science Basic Research Plan, in Shaanxi Province of China (2017JQ5119), and in part by China Postdoctoral Science Foundation (2017M613066).

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