Research Article

Event-Triggered Output-Feedback Control for Disturbed Linear Systems

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In the last few decades, event-triggered control received considerable attention, because of advantages in reducing the resource utilization, such as communication load and processor. In this paper, we propose an event-triggered output-feedback controller for disturbed linear systems, in order to achieve both better resource utilization and disturbance attenuation properties at the same time. Based on our prior work on state-feedback H_∞ control for disturbed systems, we propose an approach to design an output-feedback H_∞ controller for the system whose states are not completely observable, and a sufficient condition guaranteeing the asymptotic stability and robustness of the system is given in the form of LMIs (Linear Matrix Inequalities).

1. Introduction

The implementation of the feedback law is typically done by the time-triggered scheme, in which sampling the state and computing and transmitting the control input are executed in an equidistant time interval called sampling period. The main problem is to determine the frequency of the execution so that the desired system performance is achieved. In the traditional way, the control task is executed at equidistant sampling time intervals, for example, [1, 2]. However, it is not smart enough to update the control input in a periodic way regardless of the state of the system, especially for systems in which significant changes could happen to the plant rapidly. Meanwhile, the sampling period is chosen according to the worst situation, which can lead to an insufficient utilization of system resource [3].

The event-triggered fashion is such an alternative to the time-triggered paradigm, in which the control task execution is triggered by a so-called “event-condition” usually according to the plant states. The event-triggered control can lead to a remarkable reduction of the system resource, so it was widely used in the networked systems [4, 5] and state estimation [6–9]. Event-triggered control can also improve the overall system performance, which should be attributed to a

better use of the state information, because the control input is transmitted to the plant only when the event condition is violated, which indicates the plant states are in an unexpected condition. In the last decade, research on event-triggered control is successful in both theoretical analysis [10] and applications [11]. A state-feedback approach was applied to event-triggered control in [12]. The study [13] extends its work to the disturbance rejection of input-output linearizable systems with a relative degree. In [14] an event-triggered control for linear disturbed system was studied in which the disturbance is bounded by a linear function of the system state. Reference [15] relaxes the assumption and only assumes that the induced L_2 norm of the disturbance is finite. Other works also drive event-triggered control into disturbed systems meeting with other problems, such as a class of stochastic systems [16], simple nonlinear components [17], and time delay [18]. Some other researches focus on the event-triggered output-feedback control for systems in which the full state information is not observable. In [19] the observer-based event-triggered control is proposed in the continuous-time systems, although in the analysis and examples the full state information is available. In [20] the stability of an event-triggered output-feedback control system with event-triggered state observer was also studied. But the effect of disturbance was

not taken into account in those researches. The observer-based output-feedback approach is studied in [21, 22]. In [23, 24] the problem of output-based event-triggered control is considered in discrete-time framework, from an optimal control perspective in line with the classical Linear Quadratic Gaussian (LQG) setup [25]. In the above papers, the authors either design the controller simply making the closed-loop system matrix Hurwitz [18, 21] or design a robust H_∞ controller in a continuous feedback scheme and achieve the L_2 stability of the system by adjusting the conservatism of the event condition [14, 15]. However, these methods will bring in very large conservatism when designing the controller by solving an optimal program. In this paper, we draw the disturbance into the system and investigate the robust H_∞ output-feedback controller in an event-triggered paradigm and propose a codesign method, in which the event condition and the event-triggered controller can be optimized at the same time.

The outline of the paper is as follows. We review our prior work on state-feedback H_∞ control system based on the event-triggered scheme for the disturbed linear system in Section 2. Based on that, we develop an approach to design an output-feedback H_∞ controller for the system whose states are not completely observable in Section 3. And a sufficient condition guaranteeing the asymptotic stability and robustness of the system is given in the form of LMIs. By solving these LMIs, we can determine an event condition implying the longest sample period on the premise that the stability and the expected disturbance attenuation properties are satisfied. Finally, in Section 4 we illustrate our findings with a numerical example and provide conclusive remarks in Section 5.

2. Problem Statement and Preliminaries

2.1. Notations. We shall use the notation $\mathbb{R}^n$ to denote the n -dimensional Euclidean space and $\mathbb{R}^{m \times n}$ denotes the set of all $m \times n$ -dimensional real matrices. The notation $\|\cdot\|$ denotes the Euclidean norm of the matrix or vector if there is no special description. Notation $X > 0$ ($X < 0$) is used to say that the matrix X is a positive (negative) matrix. Denote by X^T and X^{-1} the transpose and the inverse of X , respectively. I represents the identity matrix of appropriate dimensions.

2.2. Problem Statement. We first consider an event-triggered state-feedback system for simplicity, and the system was given by

$$\dot{x} = f(x, u, w), \quad (1)$$

in which a feedback controller

$$u = k(x) \quad (2)$$

needs to be designed to guarantee the corresponding closed-loop system

$$\dot{x} = f(x, k(x + e), w) \quad (3)$$

is stable. In (3) $e \in \mathbb{R}^n$ represents the measurement error.

The implementation of the feedback law is typically done by the time-triggered scheme. In this paper, we apply a class of

aperiodic execution schedules instead, and the main problem is to design a proper controller and find the corresponding event condition determining on which time instant the control input $u(t_i)$ should be updated, while making the closed-loop system stable and robust.

2.3. Prior Work. For the sake of simplicity, we will first investigate a state-feedback robust H_∞ controller for perturbed linear systems. Firstly Lemma 1 is given as follows.

Lemma 1 (Schur complement formula [26]). *Consider a symmetric matrix $S \in \mathbb{R}^{n \times n}$*

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}, \quad (4)$$

where $S_{11} \in \mathbb{R}^{r \times r}$, $S_{12} \in \mathbb{R}^{r \times (n-r)}$, $S_{21} \in \mathbb{R}^{(n-r) \times r}$, and $S_{22} \in \mathbb{R}^{(n-r) \times (n-r)}$.

Then, $S < 0$ if and only if $S_{11} < 0$ and $S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$, or, equivalently, $S_{22} < 0$ and $S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0$.

Consider a perturbed linear system given by

$$\begin{aligned} \dot{x} &= Ax + B_1 w + B_2 u, \\ z &= Cx + D_1 w + D_2 u, \end{aligned} \quad (5)$$

where $x \in \mathbb{R}^n$ represents the system state vector, $x(0) = x_0$ is the initial state, $w \in \mathbb{R}^p$ denotes the bounded exogenous disturbance, and $u \in \mathbb{R}^m$ and $z \in \mathbb{R}^q$ are the control input and output signals, respectively.

System (5) is stabilized by a state-feedback controller

$$u = Kx, \quad (6)$$

which is only computed and transmitted on discrete-time instants t_i . The dynamics of the closed-loop system after one control input update $u = Kx(t_i)$ is given by

$$\begin{aligned} \dot{x} &= Ax(t) + B_1 w(t) + B_2 Kx(t_i) \\ &= Ax(t) + B_1 w(t) + B_2 Kx(t) - B_2 Kx(t) \\ &\quad + B_2 Kx(t_i) \\ &= (A + B_2 K)x(t) + B_1 w(t) + B_2 Ke(t), \end{aligned} \quad (7)$$

where the measurement error e is defined by

$$e(t) = y(t_i) - y(t), \quad t \in [t_i, t_{i+1}). \quad (8)$$

Thus we can rewrite system (5):

$$\begin{aligned} \dot{x} &= (A + B_2 K)x + B_1 w + B_2 Ke, \\ z &= (C + D_2 K)x + D_1 w + D_2 Ke. \end{aligned} \quad (9)$$

We set the event condition as

$$|e(t)|^2 < \sigma |x(t)|^2, \quad \sigma > 0, \quad (10)$$

which indicates that if and only if this condition is violated, the feedback loop is closed.

Then the problem is to look for a suitable control law (6) so that system (9) satisfies the demanded disturbance attenuation property. Meanwhile the parameter σ in (10) which decides the sampling frequency is as large as possible.

$$\begin{bmatrix} (A + B_2K)^T X + X(A + B_2K) + \sigma I & XB_2K & XB_1 & (C + D_2K)^T \\ K^T B_2^T X & -I & 0 & K^T D_2^T \\ B_1^T X & 0 & -\gamma^2 I & D_1^T \\ (C + D_2K) & D_2K & D_1 & -I \end{bmatrix} < 0, \quad (11)$$

then we have the following:

(a) System (5) under event condition (10) is asymptotically stable (without regard for the influence of the external disturbance).

(b) The transfer function from disturbance w to output z , denoted by $T_{zw}(s)$, satisfies $\|T_{zw}\|_\infty < \gamma$.

Proof. Define $V(x) = x^T X x$, where X is one of the positive definite solutions of (11). The time derivative of $V(x)$ along the solution of (9) is

$$\begin{aligned} \dot{V} &= 2x^T X ((A + B_2K)x + B_2Ke) \\ &= x^T (X(A + B_2K) + (A + B_2K)^T X)x \\ &\quad + 2x^T XB_2Ke \\ &\leq x^T (X(A + B_2K) + (A + B_2K)^T X \\ &\quad + XB_2KK^T B_2^T X)x + e^T e. \end{aligned} \quad (12)$$

Since we have $|e(t)|^2 < \sigma |x(t)|^2$, $\dot{V}(x)$ can be reduced to

$$\begin{aligned} \dot{V} &\leq x^T (X(A + B_2K) + (A + B_2K)^T X \\ &\quad + XB_2KK^T B_2^T X + \sigma I)x. \end{aligned} \quad (13)$$

We can see from (11) that

$$\begin{bmatrix} (A + B_2K)^T X + X(A + B_2K) + \sigma I & XB_2K \\ K^T B_2^T X & -I \end{bmatrix} < 0, \quad (14)$$

by the Schur complement formula. Furthermore (14) is equivalent to

$$\begin{aligned} X(A + B_2K) + (A + B_2K)^T X + XB_2KK^T B_2^T X + \sigma I \\ < 0, \end{aligned} \quad (15)$$

which indicates that $\dot{V}(x) < 0$. Therefore system (9) is asymptotically stable.

The result of this event-triggered state-feedback control problem is given by the following theorem in our prior research [27].

Theorem 2. Consider system (5). If there exist a positive definite matrix $X \in \mathbb{R}^{n \times n}$ and a positive scalar $\sigma > 0$, satisfying the following inequality:

Then we define

$$J_T = \int_0^T |z|^2 dt - \gamma^2 \int_0^T |w|^2 dt, \quad (16)$$

and we have

$$\begin{aligned} J_T &= \int_0^T [z^T z - \gamma^2 w^T w] dt \\ &= \int_0^T [z^T z - \gamma^2 w^T w + \dot{V}(x)] dt - V(x(T)) \\ &= \int_0^T [z^T z - \gamma^2 w^T w \\ &\quad + 2x^T X ((A + B_2K)x + B_1w + B_2Ke)] dt \\ &\quad - V(x(T)) \\ &= \int_0^T \begin{bmatrix} x \\ e \\ w \end{bmatrix}^T \left(\begin{bmatrix} (C + D_2K)^T \\ (D_2K)^T \\ D_1^T \end{bmatrix} [C + D_2K \ D_2K \ D_1] \right. \\ &\quad \left. + \begin{bmatrix} X(A + B_2K) + (A + B_2K)^T X & XB_2K & XB_1 \\ B_2^T X & 0 & 0 \\ B_1^T X & 0 & -\gamma^2 I \end{bmatrix} \right) \\ &\quad \cdot \begin{bmatrix} x \\ e \\ w \end{bmatrix} dt + e^T e - e^T e - V(x(T)) \\ &< \int_0^T \begin{bmatrix} x \\ e \\ w \end{bmatrix}^T \left(\begin{bmatrix} (C + D_2K)^T \\ (D_2K)^T \\ D_1^T \end{bmatrix} [C + D_2K \ D_2K \ D_1] \right. \\ &\quad \left. + \begin{bmatrix} X(A + B_2K) + (A + B_2K)^T X & XB_2K & XB_1 \\ B_2^T X & 0 & 0 \\ B_1^T X & 0 & -\gamma^2 I \end{bmatrix} \right) \end{aligned}$$

$$\begin{aligned}
& \cdot \begin{bmatrix} x \\ e \\ w \end{bmatrix} dt + \sigma x^T x - e^T e - V(x(T)) \\
& = \int_0^T \begin{bmatrix} x \\ e \\ w \end{bmatrix}^T \left(\begin{bmatrix} (C + D_2 K)^T \\ (D_2 K)^T \\ D_1^T \end{bmatrix} [C + D_2 K \quad D_2 K \quad D_1] \right. \\
& \quad \left. + \begin{bmatrix} X(A + B_2 K) + (A + B_2 K)^T X + \sigma I & XB_2 K & XB_1 \\ B_2^T X & -I & 0 \\ B_1^T X & 0 & -\gamma^2 I \end{bmatrix} \right) \\
& \cdot \begin{bmatrix} x \\ e \\ w \end{bmatrix} dt - V(x(T)). \tag{17}
\end{aligned}$$

By Schur complement formula, (11) is equivalent to

$$\begin{aligned}
& \begin{bmatrix} (C + D_2 K)^T \\ (D_2 K)^T \\ D_1^T \end{bmatrix} [C + D_2 K \quad D_2 K \quad D_1] \\
& + \begin{bmatrix} X(A + B_2 K) + (A + B_2 K)^T X + \sigma I & XB_2 K & XB_1 \\ B_2^T X & -I & 0 \\ B_1^T X & 0 & -\gamma^2 I \end{bmatrix} \tag{18} \\
& < 0.
\end{aligned}$$

So we get

$$\int_0^T [z^T z - \gamma^2 w^T w + \dot{V}(x)] dt < 0; \tag{19}$$

that is,

$$x^T(T) X x(T) + \int_0^T |z|^2 dt < \gamma^2 \int_0^T |w|^2 dt. \tag{20}$$

Notice that the system is asymptotically stable and let $T \rightarrow \infty$, and we have

$$\|z\|_2^2 < \gamma^2 \|w\|_2^2. \tag{21}$$

□

3. Design of Event-Triggered Output-Feedback Controller

In last section, we propose an LMI condition that helps us to design a robust H_∞ state-feedback controller when the states of the system are completely observable. In this section, we will investigate how to design an output-feedback controller for systems in which the states of the system are not completely observable.

The system is given by

$$\begin{aligned}
\dot{x} &= Ax + B_1 w + B_2 u, \\
z &= C_1 x + D_{11} w + D_{12} u, \\
y &= C_2 x + D_{21} w,
\end{aligned} \tag{22}$$

where the system state $x \in \mathbb{R}^n$ is not completely observable and $z \in \mathbb{R}^q$ and $y \in \mathbb{R}^r$ are the controllable output and observable output of the system, respectively.

The main aim of this section is to find an output-feedback controller described as

$$\begin{aligned}
\dot{\hat{x}} &= A_k \hat{x} + B_k y, \\
u &= C_k \hat{x} + D_k y,
\end{aligned} \tag{23}$$

where $\hat{x}$ is the state of the controller and A_k , B_k , C_k , and D_k are the parameters of the controller to be determined. We will also apply an event-triggered feedback strategy which brings in the measurement error e defined in last section, and the controller should be rewritten as

$$\begin{aligned}
\dot{\hat{x}} &= A_k \hat{x} + B_k y + B_k C_2 e, \\
u &= C_k \hat{x} + D_k y + D_k C_2 e.
\end{aligned} \tag{24}$$

Apply controller (24) to system (22), and we can get the closed-loop system

$$\begin{aligned}
\dot{\xi} &= A_c \xi + B_c w + B_e e, \\
z &= C_c \xi + D_c w + D_e e
\end{aligned} \tag{25}$$

in which

$$\begin{aligned}
\xi &= \begin{bmatrix} x \\ \hat{x} \end{bmatrix}, \\
A_c &= \begin{bmatrix} A + B_2 D_k C_2 & B_2 C_k \\ B_k C_2 & A_k \end{bmatrix}, \\
B_c &= \begin{bmatrix} B_1 + B_2 D_k D_{21} \\ B_k D_{21} \end{bmatrix}, \\
B_e &= \begin{bmatrix} B_2 D_k C_2 \\ B_k C_2 \end{bmatrix}, \\
C_c &= [C_1 + D_{12} D_k C_2 \quad D_{12} C_k], \\
D_c &= D_{11} + D_{12} D_k D_{21}, \\
D_e &= D_{12} D_k.
\end{aligned} \tag{26}$$

According to the result of Theorem 2, the expected output-feedback controller (23) can be designed if there exists a symmetric positive definite matrix X_c , which makes the following inequality hold:

$$\begin{bmatrix} A_c^T X_c + X_c A_c + \sigma I & X_c B_e & X_c B_c & C_c^T \\ & -I & 0 & D_e^T \\ & & -\gamma^2 I & D_c^T \\ & & & -I \end{bmatrix} < 0. \tag{27}$$

In (27), A_c , B_c , and B_e are unknowns based on the parameters of the controllers A_k , B_k , C_k , and D_k , and they all appear in the inequality in nonlinear forms. In the rest of this section, we will investigate how to transform (27) into several LMIs that are all linear and algorithmically solvable.

Lemma 3 (projection theorem). *P, Q, and H are matrices given in proper dimension, and H is symmetric. N_P and N_Q are orthogonal complements of P and Q, respectively. Then there exists a matrix X such that*

$$H + P^T X^T Q + Q^T X P < 0 \quad (28)$$

if and only if

$$\begin{aligned} N_P^T H N_P &< 0, \\ N_Q^T H N_Q &< 0. \end{aligned} \quad (29)$$

Now we define

$$K = \begin{bmatrix} A_k & B_k \\ C_k & D_k \end{bmatrix} \quad (30)$$

which is to be determined to design the controller, and

$$\begin{aligned} A_0 &= \begin{bmatrix} A & 0 \\ 0 & 0 \end{bmatrix}, \\ B_0 &= \begin{bmatrix} B_1 \\ 0 \end{bmatrix}, \end{aligned}$$

$$C_0 = [C_1 \ 0],$$

$$\bar{B} = \begin{bmatrix} 0 & B_2 \\ I & 0 \end{bmatrix},$$

$$\bar{C} = \begin{bmatrix} 0 & I \\ C_2 & 0 \end{bmatrix},$$

$$\bar{D}_{12} = [0 \ D_{12}],$$

$$\bar{D}_{21} = \begin{bmatrix} 0 \\ D_{21} \end{bmatrix},$$

$$\bar{C}_2 = \begin{bmatrix} 0 \\ C_2 \end{bmatrix}.$$

(31)

Then we get

$$\begin{aligned} A_c &= A_0 + \bar{B}K\bar{C}, \\ B_c &= B_0 + \bar{B}K\bar{D}_{21}, \\ C_c &= C_0 + \bar{D}_{12}K\bar{C}, \\ D_c &= D_{11} + \bar{D}_{12}K\bar{D}_{21}, \\ B_e &= \bar{B}K\bar{C}_2, \\ D_e &= \bar{D}_{12}K\bar{C}_2. \end{aligned} \quad (32)$$

Substitute (32) into (27), and we get

$$\begin{bmatrix} (A_0 + \bar{B}K\bar{C})^T X_c + X_c (A_0 + \bar{B}K\bar{C}) + \sigma I & X_c \bar{B}K\bar{C}_2 & X_c (B_0 + \bar{B}K\bar{D}_{21}) & (C_0 + \bar{D}_{12}K\bar{C})^T \\ & -I & 0 & (\bar{D}_{12}K\bar{C}_2)^T \\ & & -\gamma^2 I & (D_{11} + \bar{D}_{12}K\bar{D}_{21})^T \\ & & & -I \end{bmatrix} < 0. \quad (33)$$

According to the Schur complement formula, (33) is equivalent to

$$\begin{bmatrix} (A_0 + \bar{B}K\bar{C})^T X_c + X_c (A_0 + \bar{B}K\bar{C}) & X_c \bar{B}K\bar{C}_2 & X_c (B_0 + \bar{B}K\bar{D}_{21}) & (C_0 + \bar{D}_{12}K\bar{C})^T & I \\ & -I & 0 & (\bar{D}_{12}K\bar{C}_2)^T & 0 \\ & & -\gamma^2 I & (D_{11} + \bar{D}_{12}K\bar{D}_{21})^T & 0 \\ & & & -I & 0 \\ & & & & -\frac{1}{\sigma} I \end{bmatrix} < 0. \quad (34)$$

We define

$$H_{X_c} = \begin{bmatrix} A_0^T X_c + X_c A_0 & 0 & X_c B_0 & C_0^T & I \\ & -I & 0 & 0 & 0 \\ & & -\gamma^2 I & D_{11}^T & 0 \\ & & & -I & 0 \\ & & & & -\frac{1}{\sigma} I \end{bmatrix}, \quad (35)$$

$$P_{X_c} = [\bar{B}^T X_c \ 0 \ 0 \ \bar{D}_{12}^T \ 0],$$

$$Q = [\bar{C} \ \bar{C}_2 \ \bar{D}_{21} \ 0 \ 0],$$

and then (34) can be rewritten as

$$H_{X_c} + P_{X_c}^T K Q + Q^T K P_{X_c} < 0. \quad (36)$$

According to Lemma 3, (36) is solvable if and only if

$$\begin{aligned} N_{P_{X_c}}^T H_{X_c} N_{P_{X_c}} &< 0, \\ N_Q^T H_{X_c} N_Q &< 0, \end{aligned} \quad (37)$$

where $N_{P_{X_c}}$ and N_Q are orthogonal complements of P_{X_c} and Q , respectively.

Now we get two matrix inequalities including only one matrix variable X_c instead of a matrix inequality about two different variables X_c and K , between which the relationship is nonlinear.

However, the variable X_c not only appears in H_{X_c} but also appears in $N_{P_{X_c}}$, so the first inequality in (37) is not an LMI. The following task is to transform it to an equivalent LMI.

Given X_c , we can define

$$T_{X_c} = \begin{bmatrix} A_0 X_c^{-1} + X_c^{-1} A_0^T & 0 & B_0 & X_c^{-1} C_0^T & X_c^{-1} \\ & -I & 0 & 0 & 0 \\ & & -\gamma^2 I & D_{11}^T & 0 \\ & & & -I & 0 \\ & & & & -\frac{1}{\sigma} I \end{bmatrix}, \quad (38)$$

$$P = [\bar{B}^T \ 0 \ 0 \ \bar{D}_{12}^T \ 0]. \quad (39)$$

Then we can get the following lemma.

Lemma 4. If we have $X_c > 0$ and $T_{X_c} P$ are defined as (38) and (39), then we have

$$\begin{aligned} N_{P_{X_c}}^T H_{X_c} N_{P_{X_c}} &< 0 \iff \\ N_P^T T_{X_c} N_P &< 0. \end{aligned} \quad (40)$$

Proof. We define

$$S = \text{diag}(X_c, I, I) \quad (41)$$

and then we have $P_{X_c} = PS$ and $\text{Ker}(P_{X_c}) = S^{-1} \text{Ker}(P)$, in which $\text{Ker}(P_{X_c})$ and $\text{Ker}(P)$ are the kernel space of P_{X_c} and P , respectively.

Furthermore, notice the definition of $N_{P_{X_c}}$ and N_P , and we have $N_{P_{X_c}} = S^{-1} N_P$, which leads to

$$\begin{aligned} N_{P_{X_c}}^T H_{X_c} N_{P_{X_c}} &< 0 \iff \\ N_P^T (S^{-1})^T H_{X_c} S^{-1} N_P &< 0 \end{aligned} \quad (42)$$

and also notice the fact that

$$(S^{-1})^T H_{X_c} S^{-1} = T_{X_c} \quad (43)$$

and we can finally get

$$\begin{aligned} N_{P_{X_c}}^T H_{X_c} N_{P_{X_c}} &< 0 \iff \\ N_P^T T_{X_c} N_P &< 0. \end{aligned} \quad (44)$$

□

According to Lemma 4, we can design an output-feedback controller (23) for system (22) if there exists a symmetric matrix $X_c > 0$ such that

$$\begin{aligned} N_P^T T_{X_c} N_P &< 0, \\ N_Q^T H_{X_c} N_Q &< 0. \end{aligned} \quad (45)$$

The first inequality in (45) is about variable X_c^{-1} while the second is about X_c , so verifying the existence of X_c satisfying both the inequalities in (45) is a nonconvex optimal problem. Next we will analyze how to transform this problem into an LMI.

As X_c is symmetrical, we can write X_c and X_c^{-1} in the following block form:

$$\begin{aligned} X_c &= \begin{bmatrix} X & X_2 \\ X_2^T & X_3 \end{bmatrix}, \\ X_c^{-1} &= \begin{bmatrix} Y & Y_2 \\ Y_2^T & Y_3 \end{bmatrix}, \end{aligned} \quad (46)$$

where X and Y are n -dimensional symmetric submatrix.

The following lemma will indicate that the inequalities in (45) only have restraints of X , Y , and Y_2 .

Lemma 5. $X_c > 0$ is a symmetric matrix, and X , Y , and Y_2 are defined in (46). Then

$$\begin{aligned} N_P^T T_{X_c} N_P &< 0, \\ N_Q^T H_{X_c} N_Q &< 0 \end{aligned} \quad (47)$$

holds if and only if

$$\begin{bmatrix} N_c & 0 \\ 0 & I \end{bmatrix}^T \cdot \begin{bmatrix} AY + YA^T & YC_1^T & 0 & B_1 & Y & Y_2 \\ & -I & 0 & D_{11} & 0 & 0 \\ & & -I & 0 & 0 & 0 \\ & & & -\gamma^2 I & 0 & 0 \\ & & & & -\sigma I & 0 \\ & & & & & -\sigma I \end{bmatrix} \begin{bmatrix} N_c & 0 \\ 0 & I \end{bmatrix} \quad (48)$$

$$< 0,$$

$$\begin{bmatrix} N_o & 0 \\ 0 & I \end{bmatrix}^T \cdot \begin{bmatrix} A^T X + XA & 0 & XB_1 & C_1^T & I & 0 \\ & -I & 0 & 0 & 0 & 0 \\ & & -\gamma^2 I & D_{11}^T & 0 & 0 \\ & & & -I & 0 & 0 \\ & & & & -\sigma I & 0 \\ & & & & & -\sigma I \end{bmatrix} \begin{bmatrix} N_o & 0 \\ 0 & I \end{bmatrix} \quad (49)$$

$$< 0,$$

where

$$\begin{aligned} N_c &= \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}, \\ N_o &= \begin{bmatrix} I & 0 \\ -C_1 & -D_{21} \\ 0 & I \end{bmatrix}, \end{aligned} \quad (50)$$

and V_1 and V_2 are orthogonal complements of B_2 and D_{12} respectively.

Proof. First we will prove that $N_P^T T_{X_c} N_P < 0$ is equivalent to (48). Notice the definition of $X_c, A_0, B_0, C_0, \bar{B}$, and $\bar{D}_{12}$, and we have

$$T_{X_c} = \begin{bmatrix} AY + YA^T & AY_2 & 0 & B_1 & YC_1^T & Y & Y_2 \\ & 0 & 0 & 0 & Y_2^T C_1^T & Y_2^T & Y_3 \\ & & -I & 0 & 0 & 0 & 0 \\ & & & -\gamma^2 I & D_{11}^T & 0 & 0 \\ & & & & -I & 0 & 0 \\ & & & & & -\sigma I & 0 \\ & & & & & & -\sigma I \end{bmatrix},$$

$$P = \begin{bmatrix} 0 & I & 0 & 0 & 0 & 0 & 0 \\ B_2^T & 0 & 0 & 0 & D_{12}^T & 0 & 0 \end{bmatrix}. \quad (51)$$

So we have

$$N_P = \begin{bmatrix} V_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ V_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix}. \quad (52)$$

Notice that the second row of N_P is zero vector; therefore $N_P^T T_{X_c} N_P < 0$ is equivalent to

$$\begin{bmatrix} V_1 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ V_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix}^T \cdot \begin{bmatrix} AY + YA^T & 0 & B_1 & YC_1^T & Y & Y_2 \\ & -I & 0 & 0 & 0 & 0 \\ & & -\gamma^2 I & D_{11}^T & 0 & 0 \\ & & & -I & 0 & 0 \\ & & & & -\sigma I & 0 \\ & & & & & -\sigma I \end{bmatrix} \begin{bmatrix} V_1 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ V_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix} \quad (53)$$

$$< 0.$$

Also notice that

$$\begin{bmatrix} V_1 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ V_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix} = \begin{bmatrix} I & 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I \\ 0 & 0 & 0 & 0 & I \end{bmatrix} \begin{bmatrix} N_c & 0 \\ 0 & I \end{bmatrix}. \quad (54)$$

Then we can finally get LMI (48). $\square$

We can similarly get LMI (49); just consider

$$Q = \begin{bmatrix} 0 & I & 0 & 0 & 0 & 0 & 0 \\ C_2 & 0 & I & D_{21} & 0 & 0 & 0 \end{bmatrix},$$

$$N_Q = \begin{bmatrix} I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -C_2 & -D_{21} & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix}. \quad (55)$$

Till now we can get X , Y , and Y_2 from the result of Lemma 5. Furthermore, it can be easily proved that X_c can always be determined by X , Y , and Y_2 if and only if $I - XY$ is reversible or equivalently

$$\begin{bmatrix} I & X \\ Y & I \end{bmatrix} < 0. \quad (56)$$

According to the analyses above, we can obtain a sufficient condition for designing the output-feedback controller (23), as shown in the following theorem.

Theorem 6. For a perturbed system (22), N_o and N_c were defined in (50); then a robust H_∞ output-feedback controller can be designed for system (22) if there exist symmetric matrices X and Y and a matrix Y_2 , satisfying the following LMIs:

(1)

$$\begin{bmatrix} N_c & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} AY + YA^T & YC_1^T & 0 & B_1 & Y & Y_2 \\ & -I & 0 & D_{11} & 0 & 0 \\ & & -I & 0 & 0 & 0 \\ & & & -\gamma^2 I & 0 & 0 \\ & & & & -\sigma I & 0 \\ & & & & & -\sigma I \end{bmatrix} \begin{bmatrix} N_c & 0 \\ 0 & I \end{bmatrix} \quad (57)$$

< 0 .

(2)

$$\begin{bmatrix} N_o & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} A^T X + XA & 0 & XB_1 & C_1^T & I & 0 \\ & -I & 0 & 0 & 0 & 0 \\ & & -\gamma^2 I & D_{11}^T & 0 & 0 \\ & & & -I & 0 & 0 \\ & & & & -\sigma I & 0 \\ & & & & & -\sigma I \end{bmatrix} \begin{bmatrix} N_o & 0 \\ 0 & I \end{bmatrix} \quad (58)$$

< 0 .

(3)

$$\begin{bmatrix} I & X \\ Y & I \end{bmatrix} < 0. \quad (59)$$

4. Simulation

In this section, we propose a simulation example to illustrate the efficiency of our theoretical results.

We consider a second-order perturbed system of (22), in which

$$A = \begin{bmatrix} 5 & 10 \\ 1 & 0 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad (60)$$

$$D_{11} = D_{12} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$D_{21} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

The initial state of the system is

$$x(0) = [-1 \ 2]^T \quad (61)$$

and the disturbance signal w is given as

$$w(t) = w = \sin(8\pi t), \quad t \in [0.5, 1]. \quad (62)$$

By solving LMIs in Theorem 6, we get the maximum value of $\bar{\sigma}_{\max} = 0.042$. And the corresponding optimal controller is

$$K_{\text{opt}} = [-81, -56]. \quad (63)$$

Then we apply optimal controller (63) as the event-triggered controller based on Theorem 6. The performance of the event-triggered controlled system and the sample time are shown in Figures 1 and 2. We can see from Figure 1 that the trajectory of the closed-loop system converges rapidly and smoothly by the robust H_∞ controller (63) and good capacity of disturbance attenuation property is also guaranteed.

5. Conclusions

In this paper, we address the use of event-triggered control for disturbed systems, and the event condition given to determine the feedback frequency is decided in order to achieve purpose of disturbance rejection (in the H_∞ sense). We first propose a state-feedback case, and a sufficient condition which guarantees the asymptotic stability and robustness of the system is given in the form of LMI (Linear

FIGURE 1: System dynamics and sampling time.

FIGURE 2: Disturbance attenuation property.

Matrix Inequality). Based on that, we develop an approach to design an output-feedback H_∞ controller for the system whose states are not completely observable. The condition to design the controller is given in three LMIs. By solving these LMIs, we can determine an event condition implying the optimal feedback frequency on the premise that the stability and the expected disturbance attenuation properties are satisfied.

A codesign method for determining the event-triggered controller and event condition simultaneously is proposed in this paper. Compared with the existing research on the event-triggered control, in which only the event conditions are to be determined while the controllers are fixed in the context of continuous-time framework, the codesign principal is less conservable and makes it possible to optimize both the controller and the event condition. The Linear Matrix Inequalities obtained in this paper provide a flexible parameter, which can be used to improve the conservatism of the method. A dynamical H_∞ controller is proposed to alleviate

the effect on the system outputs caused by the external disturbance and obtain the expected disturbance attenuation property. From the simulation example, we can see that the trajectory of the closed-loop system converges rapidly and smoothly by the dynamical H_∞ controller, and good capacity of disturbance attenuation property is also guaranteed.

Following this paper, some cone optimization methods could be applied to improve the optimization program of the matrix inequality for further reduction of the conservatism of the stability condition. And other absolute-error based event condition can be studied instead of the relative-error based event condition, to improve the triggering efficiency of the event-triggered control system. Moreover, we are also considering event-triggered control for networked systems, in which time delay and package drop may occur and high efficiency communication methods like event-triggered communication are more necessary.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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