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Research Article

Adaptive Synchronization for Uncertain Delayed Fractional-Order Hopfield Neural Networks via Fractional-Order Sliding Mode Control

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Adaptive synchronization for a class of uncertain delayed fractional-order Hopfield neural networks (FOHNNs) with external disturbances is addressed in this paper. For the unknown parameters and external disturbances of the delayed FOHNNs, some adaptive estimations are designed. Firstly, a fractional-order switched sliding surface is proposed for the delayed FOHNNs. Then, according to the fractional-order extension of the Lyapunov stability criterion, a fractional-order sliding mode controller is constructed to guarantee that the synchronization error of the two uncertain delayed FOHNNs converges to an arbitrary small region of the origin. Finally, a numerical example of two-dimensional uncertain delayed FOHNNs is given to verify the effectiveness of the proposed method.

1. Introduction

The research of neural networks (NNs) is quite extensive, reflecting the characteristics of multidisciplinary technology. NNs have many successful applications in the fields of associative memories and image processing. Recently, the discussion on NNs has become a hot topic [1–3]. Guo et al. [4] studied the exponential stability analysis for complex-valued memristor-based bidirectional associative memory (BAM) NNs with time delays. Lv et al. [5] used NNs to discuss the adaptive tracking control for a class of uncertain nonlinear systems. Li et al. [6] studied Hopf bifurcation analysis of complex-valued neural networks model.

Fractional calculus (FC) has a long history. As early as 1695, the concept of fractional differential was mentioned in Leibnitz's letter to L'Hospital. For a long time, FC continues to grow. Podlubny's book [7] systematically introduced the concepts and properties of FC. Bai et al. (see [8–13], and the references therein) studied the existence and uniqueness of solutions for fractional differential equations (FDE). Wang et al. [14–16] studied the numerical analysis of FDE. In recent years, fractional-order systems (FOS) have attracted wide attentions. The control problems of all kinds of FOS were studied recently [17–21]. Many researchers focused on fractional-order neural networks (FONNs) [22–27]. Cao et al. [28] investigated the existence and uniqueness of the nontrivial solution of NNs and the uniform stability of the FONNs.

The researches on the stability of NNs, FOS, and stochastic systems have attracted the attention of a large number of researchers, and many achievements have been made [29–41]. The sliding mode control (SMC) is a very popular strategy for a general class of nonlinear uncertain systems, with a very large frame of applications fields. Due to the use of the discontinuous function, its main features are the robustness of closed-loop system and the finite-time convergence. Utkin et al. [42] studied the minimum possible value of control based on adaptation SMC methodology. Efe. et al. [43] discussed the fractional fuzzy adaptive SMC. Aghababa [44] designed a chatter-free terminal sliding mode controller for nonlinear fractional-order dynamical systems. The synchronization problems of FOHNNs have captured more and more researchers' attention [45–48]. Xi et al. [22] have discussed SMC for uncertain FOHNNs. Liu et al.

[24] have researched adaptive synchronization of a class of FONNs. It is well known that time delay is unavoidable due to finite switching speeds of the amplifiers, and it may cause oscillations or instability of dynamic systems. Wang et al. [26] have discussed the stability analysis of FOHNNs with time delay.

However, to the best of our knowledge, there are few attentions to adaptive synchronization for a class of uncertain delayed FOHNNs subject to external disturbances. The SMC technology was used to solve the above problems in the paper. The rest of this paper is organized as follows: some necessary definitions and lemmas are given in Section 2. The main works including the introduction of fractional-order network model, the fractional-order switched sliding mode surface (SMS), the design of adaptive synchronization controller, and stability analysis are included in Section 3. Section 4 presents a simulation example. Finally, the paper is concluded in Section 5.

2. Preliminaries

There are several kinds of definitions for fractional-order derivatives. The definitions of more frequency of use in literatures are Grünwald-Letnikov, Riemann-Liouville, and Caputo definitions [7]. These definitions are generally not equivalent with each other. The Caputo's derivative's Laplace transform requires integer-order derivatives for the initial conditions, which was used in engineering applications frequently. But, the Riemann-Liouville definition's Laplace transform involved fractional-order derivatives for the initial conditions. It was hard to use physically. In the following parts, the Caputo's derivative will be used [24]. Firstly, we give some definitions and lemmas.

Definition 1 (Riemann-Liouville fractional-order integral [7]). The Riemann-Liouville fractional integral of order α for a function $f(t)$ is defined as

$$ {}_{t_0}D_t^{-\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_{t_0}^{t} \frac{f(\tau)}{(t-\tau)^{1-\alpha}} d\tau. \quad (1)$$

where $\alpha > 0$, $t \geq t_0$. $\Gamma(\alpha)$ is Euler's gamma function.

The gamma function $\Gamma(\alpha)$ is defined for all complex numbers except the nonpositive integers. For complex numbers with a positive real part, it is defined via a convergent infinite integral:

$$\Gamma(\alpha) = \int_0^{+\infty} \tau^{\alpha-1} e^{-\tau} d\tau. \quad (2)$$

Definition 2 (Caputo fractional-order derivative [7]). The Caputo fractional derivative of order α for a function $f(t)$ is defined as

$$ {}_{t_0}^{C}D_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_{t_0}^{t} (t-\tau)^{n-\alpha-1} f^{(n)}(\tau) d\tau. \quad (3)$$

where $\alpha > 0$, n is an integer satisfying $n-1 \leq \alpha < n$. Particularly, for $0 < \alpha < 1$ case, one can get

$$ {}_{t_0}^{C}D_t^{\alpha} f(t) = \frac{1}{\Gamma(1-\alpha)} \int_{t_0}^{t} (t-\tau)^{-\alpha} f'(\tau) d\tau. \quad (4)$$

According to Definition 2, for any constants $L_1 \in \mathbf{R}$ and $L_2 \in \mathbf{R}$, the linearity of Caputo's fractional derivative is described by

$$ {}_{t_0}^{C}D_t^{\alpha} (L_1 f(t) + L_2 g(t)) = L_1 {}_{t_0}^{C}D_t^{\alpha} f(t) + L_2 {}_{t_0}^{C}D_t^{\alpha} g(t). \quad (5)$$

In nonlinear control systems, Lyapunov second method gives a way to analyze the stability of the system without explicitly solving the differential equations. The Lyapunov stability theory for FOS has been developed by Li et al. [33]. One of the main contributions of [33] is the following lemma.

Consider the fractional-order nonlinear system:

$$ \begin{gathered} {}_{t_0}^{C}D_t^{\alpha} x(t) = f(x,t), \\ x(t_0) = x_{t_0}, \end{gathered} \quad (6)$$

where $x(t) \in \mathbf{R}^n$ is the state vector and $f(x,t) \in \mathbf{R}^n$ is a Lipschitz continuous nonlinear function.

Lemma 3 (see [49]). *Let $G(t)$ be a continuous function on $[0, +\infty)$, if there exist constants $\kappa_1 > 0$ and $\kappa_2 > 0$, such that*

$$ {}_{t_0}^{C}D_t^{\alpha} G(t) \leq -\kappa_1 G(t) + \kappa_2, \quad t \geq 0. \quad (7)$$

Then,

$$ G(t) \leq G(0) E_{\alpha}(-\kappa_1 t^{\alpha}) + \kappa_2 t^{\alpha} E_{\alpha,\alpha+1}(-\kappa_1 t^{\alpha}), \quad t \geq 0. \quad (8)$$

where $0 < \alpha < 1$, $E_{\alpha}(\cdot)$ and $E_{\alpha,\alpha+1}(\cdot)$ are one-parameter Mittag-Leffler function and two-parameter Mittag-Leffler function, respectively.

Remark 4. Mittag-Leffler stability means asymptotical stability [32].

Lemma 5 (see [7]). *If $x(t) \in C^1[0,T]$, for $\alpha > 0$ and $T > 0$, then the following equations hold:*

$$ {}_{0}^{C}D_t^{\alpha} ({}_{0}D_t^{-\alpha} x(t)) = x(t). \quad (9)$$

and

$$ {}_{0}D_t^{-\alpha} ({}_{0}^{C}D_t^{\alpha} x(t)) = x(t) - \sum_{k=0}^{n-1} \frac{x^{(k)}(t)}{k!} t^k. \quad (10)$$

In particular, for $0 < \alpha < 1$,

$$ {}_{0}D_t^{-\alpha} ({}_{0}^{C}D_t^{\alpha} x(t)) = x(t) - x(0). \quad (11)$$

Lemma 6 (see [21]). *Let $x(t) \subset \mathbf{R}^n$ be a continuous and derivable function. Then for any $t > 0$ the following inequality holds:*

$$ \frac{1}{2} {}_{0}^{C}D_t^{\alpha} x^T(t) x(t) \leq x^T(t) {}_{0}^{C}D_t^{\alpha} x(t) \quad (12)$$

3. Main Results

In this section, considering a system of the uncertain FOHNNs with delay (as a master system)

$$\begin{aligned} {}_0^C D_t^{\alpha} x_i(t) = -a_i(t) x_i(t) + \sum_{j=1}^{n} b_{ij} f_j\left(x_j(t)\right) \\ + \sum_{j=1}^{n} c_{ij} g_j\left(x_j(t-\tau)\right) + I_i. \end{aligned} \tag{13}$$

where $i = 1, 2, \ldots, n$, $0 < \alpha < 1$, n is the number of units in a neural network, $x_i(t)$ is the state of the ith unit at time t, f_j, g_j denotes the activation function of the jth neuron, b_{ij}, c_{ij} denotes the constant connection weight of the jth neuron on the ith neuron, $a_i > 0$ represents the rate with which the ith neuron resets its potential to the resting state when disconnected from the network and $a_i > 0$ is unknown, I_i denotes the constant external inputs, and τ is the transmission constant delay.

Let us discuss the synchronization results, assuming that (13) is a master system and the slave system is defined by the following equation:

$$\begin{aligned} {}_0^C D_t^{\alpha} y_i(t) = -a_i(t) y_i(t) + \sum_{j=1}^{n} b_{ij} f_j\left(y_j(t)\right) \\ + \sum_{j=1}^{n} c_{ij} g_j\left(y_j(t-\tau)\right) + I_i + d_i(t) \\ + u_i(t). \end{aligned} \tag{14}$$

where $y_i(t)$ is the state of the ith unit at time t, $d_i(t)$ is the unknown external disturbance, and $u_i(t)$ is the control input which will be given later.

Defining the synchronization error $e_i(t)$ as

$$e_i(t) = y_i(t) - x_i(t) \tag{15}$$

then the error dynamics between the master system (13) and the slave system (14) can be written as

$$\begin{aligned} {}_0^C D_t^{\alpha} e_i(t) = -a_i(t) e_i(t) \\ + \sum_{j=1}^{n} b_{ij}\left(f_j\left(y_j(t)\right) - f_j\left(x_j(t)\right)\right) \\ + \sum_{j=1}^{n} c_{ij}\left(g_j\left(y_j(t-\tau)\right) - g_j\left(x_j(t-\tau)\right)\right) \\ + d_i(t) + u_i(t). \end{aligned} \tag{16}$$

Assumption 7. Assuming that the nonlinear functions f_j and g_j $(j = 1, 2, \ldots, n)$ satisfy local Lipschitz conditions, and existing positive constants L_j^1 and L_j^2 such that

$$\begin{aligned} \left|f_j\left(y_j(t)\right) - f_j\left(x_j(t)\right)\right| \le L_j^1 \left|y_j(t) - x_j(t)\right|, \\ \left|g_j\left(y_j(t-\tau)\right) - g_j\left(x_j(t-\tau)\right)\right| \\ \le L_j^2 \left|y_j(t-\tau) - x_j(t-\tau)\right|. \end{aligned} \tag{17}$$

Assumption 8. Let the external disturbance $d_i(t)$ $(i = 1, 2, \ldots, n)$ be a bounded continuous function, so there exists an unknown positive constant ρ_i such that

$$\left|d_i(t)\right| \le \rho_i \tag{18}$$

For the sake of simplicity, this article only discusses the constant unknown disturbance.

3.1. Controller Design. Generally, designing the process of SMC has two steps. Firstly, an appropriate SMS is designed, which represents the required system dynamic characteristics. In this paper, a switching fractional-order SMS is given as

$$s_i(t) = {}_0D_t^{-1}\left({}_0^C D_t^{\alpha} e_i(t) + p_i e_i(t) + q_i \operatorname{sign}\left(e_i(t)\right)\right), \tag{19}$$

where $i = 1, 2, \ldots, n$, $e_i(t)$ is the state of the error system (16), and p_i and q_i are positive constants. sign$(\cdot)$ is the symbolic function.

$$\operatorname{sign}\left(e_i(t)\right) = \begin{cases} 1 & e_i(t) > 0, \\ -1 & e_i(t) < 0, \\ \in [-1, 1] & e_i(t) = 0. \end{cases} \tag{20}$$

According to the SMC theory, when the system operates in SMS, the SMS and its derivative must satisfy

$$\begin{aligned} s_i(t) = 0, \\ \dot{s}_i(t) = 0. \end{aligned} \tag{21}$$

As a result, considering ((19)-(21)), one obtains

$$\dot{s}_i(t) = {}_0^C D_t^{\alpha} e_i(t) + p_i e_i(t) + q_i \operatorname{sign}\left(e_i(t)\right) = 0. \tag{22}$$

Then, we have the sliding mode equation (SME)

$${}_0^C D_t^{\alpha} e_i(t) = -\left(p_i e_i(t) + q_i \operatorname{sign}\left(e_i(t)\right)\right). \tag{23}$$

In the next parts, we construct the SMC law $u_i(t)$ to make sure the state trajectories of system (16) reach the SMS $s_i(t) = 0$ and keep on it forever by the SMC method. The fractional-order SMC law is presented as

$$\begin{aligned} u_i(t) = -\left(-\widehat{a}_i(t) + p_i\right) e_i(t) - \widehat{d}_i(t) - \left|s_i(t)\right| \\ \cdot \left(\sum_{j=1}^{n} \left|b_{ij}\right| L_j^f \left|e_j(t)\right| + \sum_{j=1}^{n} \left|c_{ij}\right| L_j^g \left|e_j(t-\tau)\right|\right) \\ - q_i \operatorname{sign}\left(e_i(t)\right) - \zeta_i^a s_i(t) - \zeta_i^b \operatorname{sign}\left(s_i(t)\right), \end{aligned} \tag{24}$$

where $\zeta_i^a > 0$ and $\zeta_i^b > 0$ are constant gains, $\widehat{a}_i(t)$ is the estimation of $a_i(t)$, $\widehat{d}_i(t)$ is the estimation of $d_i(t)$, and the unknown parameters $a_i(t)$ and $d_i(t)$ are estimated as

$$\begin{aligned} \dot{\widehat{a}}_i(t) = -\eta_i^a s_i(t) e_i(t), \\ \dot{\widehat{d}}_i(t) = \eta_i^d s_i(t). \end{aligned} \tag{25}$$

where $i = 1, 2, \ldots, n$, $\eta_i^a > 0$ and $\eta_i^d > 0$ are adaptation gains.

In order to realize SMC, two steps are required. Firstly, the system trajectories are controlled to reach the SMS $s_i(t) = 0$, which is shown in Theorem 9. Secondly, once the system operates in SMS, we should get the stability of the error system (16) and make sure SMS converge to zero in finite time, which is shown in Theorem 12.

Theorem 9. *For the uncertain delayed FOHNNs (16), if the system is controlled by the SMC law (24) and (25), then the system trajectories will converge to the SMS $s_i(t) = 0$ in finite time.*

Proof. Choose the positive definite Lyapunov function candidate

$$V_i(t) = \frac{1}{2}s_i(t)^2 + \frac{1}{2\eta_i^a}\left(\hat{a}_i(t) - a_i(t)\right)^2 + \frac{1}{2\eta_i^d}\left(\hat{d}_i(t) - d_i\right)^2. \quad (26)$$

Taking the integer-order derivative of $V_i(t)$, we have

$$\dot{V}_i(t) = s_i(t)\dot{s}_i(t) + \frac{1}{\eta_i^a}\left(\hat{a}_i(t) - a_i\right)\dot{\hat{a}}_i(t) + \frac{1}{\eta_i^d}\left(\hat{d}_i(t) - d_i\right)\dot{\hat{d}}_i(t). \quad (27)$$

Inserting $\dot{s}_i(t)$ from (22) into the above equation, one has

$$\dot{V}_i(t) = s_i(t)\left({}_0^C D_t^\alpha e_i + p_i e_i + q_i \operatorname{sign}(e_i)\right) + \frac{1}{\eta_i^a}\left(\hat{a}_i(t) - a_i\right)\dot{\hat{a}}_i(t) + \frac{1}{\eta_i^d}\left(\hat{d}_i(t) - d_i\right)\dot{\hat{d}}_i(t). \quad (28)$$

Based on (16), we get

$$\begin{aligned}\dot{V}_i(t) = s_i(t)\Bigg(&-a_i e_i(t) + \sum_{j=1}^n b_{ij}\left(f_j\left(y_j(t)\right) - f_j\left(x_j(t)\right)\right) \\ &+ \sum_{j=1}^n c_{ij}\left(g_j\left(y_j(t-\tau)\right) - g_j\left(x_j(t-\tau)\right)\right) + d_i \\ &+ u_i(t) + p_i e_i(t) + q_i \operatorname{sign}(e_i(t))\Bigg) + \frac{1}{\eta_i^a}\left(\hat{a}_i(t) - a_i\right)\dot{\hat{a}}_i(t) + \frac{1}{\eta_i^d}\left(\hat{d}_i(t) - d_i(t)\right)\dot{\hat{d}}_i(t).\end{aligned} \quad (29)$$

According to Assumption 7, we will obtain

$$\begin{aligned}&s_i(t)\sum_{j=1}^n b_{ij}\left(f_j\left(y_j(t)\right) - f_j\left(x_j(t)\right)\right) \\ &\leq \left|s_i(t)\right|\sum_{j=1}^n \left|b_{ij}\right|\left|f_j\left(y_j(t)\right) - f_j\left(x_j(t)\right)\right| \\ &\leq \left|s_i(t)\right|\sum_{j=1}^n \left|b_{ij}\right| L_j^f \left|e_j(t)\right|\end{aligned} \quad (30)$$

Correspondingly, we have

$$\begin{aligned}&s_i(t)\sum_{j=1}^n c_{ij}\left(g_j\left(y_j(t-\tau)\right) - g_j\left(x_j(t-\tau)\right)\right) \\ &\leq \left|s_i(t)\right|\sum_{j=1}^n \left|c_{ij}\right| L_j^g \left|e_j(t-\tau)\right|\end{aligned} \quad (31)$$

Combining ((29)-(31)), we can get the following conclusion:

$$\begin{aligned}&\dot{V}_i(t) \\ &\leq s_i(t)\left(\left(-a_i + p_i\right)e_i(t) + q_i \operatorname{sign}\left(e_i(t)\right) + d_i + u_i(t)\right) \\ &\quad + \left|s_i(t)\right|\sum_{j=1}^n \left|b_{ij}\right| L_j^f \left|e_j(t)\right| \\ &\quad + \left|s_i(t)\right|\sum_{j=1}^n \left|c_{ij}\right| L_j^g \left|e_j(t-\tau)\right| \\ &\quad + \frac{1}{\eta_i^a}\left(\hat{a}_i(t) - a_i\right)\dot{\hat{a}}_i(t) + \frac{1}{\eta_i^d}\left(\hat{d}_i(t) - d_i\right)\dot{\hat{d}}_i(t).\end{aligned} \quad (32)$$

Substituting $u_i(t)$ from (24) into (32), it yields

$$\begin{aligned}\dot{V}_i(t) \leq s_i(t)\Bigg(&\left(-a_i + p_i\right)e_i(t) + q_i \operatorname{sign}\left(e_i(t)\right) + d_i \\ &- \left(-\hat{a}_i + p_i\right)e_i(t) - \hat{d}_i(t) - \left|s_i(t)\right| \\ &\cdot\left(\sum_{j=1}^n \left|b_{ij}\right| L_j^f \left|e_j(t)\right| + \sum_{j=1}^n \left|c_{ij}\right| L_j^g \left|e_j(t-\tau)\right|\right) \\ &- q_i \operatorname{sign}\left(e_i(t)\right) - \zeta_i^a s_i(t) - \zeta_i^b \operatorname{sign}\left(s_i(t)\right) + \left|s_i(t)\right| \\ &\cdot\sum_{j=1}^n \left|b_{ij}\right| L_j^f \left|e_j(t)\right| + \left|s_i(t)\right|\sum_{j=1}^n \left|c_{ij}\right| L_j^g \left|e_j(t-\tau)\right|\Bigg) \\ &+ \frac{1}{\eta_i^a}\left(\hat{a}_i(t) - a_i\right)\dot{\hat{a}}_i(t) + \frac{1}{\eta_i^d}\left(\hat{d}_i(t) - d_i\right)\dot{\hat{d}}_i(t).\end{aligned} \quad (33)$$

Through operation, we get

$$\begin{aligned}\dot{V}_i(t) &\leq s_i(t)\Big((\hat{a}_i(t)-a_i)e_i(t)-\big(\hat{d}_i(t)-d_i\big)\\&\quad-\zeta_i^a s_i(t)-\zeta_i^b\operatorname{sign}(s_i(t))\Big)+\frac{1}{\eta_i^a}(\hat{a}_i(t)-a_i)\dot{\hat{a}}_i(t)\\&\quad+\frac{1}{\eta_i^d}\big(\hat{d}_i(t)-d_i\big)\dot{\hat{d}}_i(t).\end{aligned}\tag{34}$$

Insert (25)

$$\begin{aligned}\dot{V}_i(t) &\leq s_i(t)\Big((\hat{a}_i(t)-a_i)e_i(t)-\big(\hat{d}_i(t)-d_i\big)\\&\quad-\zeta_i^a s_i(t)-\zeta_i^b\operatorname{sign}(s_i(t))\Big)-\frac{1}{\eta_i^a}(\hat{a}_i(t)-a_i)\eta_i^a s_i(t)\\&\quad\cdot e_i(t)+\frac{1}{\eta_i^d}\big(\hat{d}_i(t)-d_i\big)\eta_i^d s_i(t).\end{aligned}\tag{35}$$

Then, one obtains

$$\dot{V}_i(t) \leq -\zeta_i^a s_i^2(t)-\zeta_i^b\operatorname{sign}(s_i(t))s_i(t).\tag{36}$$

Using $\operatorname{sign}(s_i(t))s_i(t) = |s_i(t)|$ and property of inequality, we get

$$\dot{V}_i(t) \leq -\zeta_i^b|s_i(t)| < 0,\tag{37}$$

where $\zeta_i^b > 0$. Therefore, according to Lyapunov theory, the system states will converge to SMS $s_i(t) = 0$. Hence, the proof is achieved completely. □

Remark 10. Theorem 9 gets the error systems trajectories to reach the sliding surface $s_i(t) = 0$ in finite time.

Remark 11. Time delay and external disturbance have little influence on the error system (16).

3.2. Stability of Sliding Mode. For the SME (23), we choose the positive definite Lyapunov function

$$V_i(t) = \frac{1}{2}e_i^2(t).\tag{38}$$

Taking the fractional-order derivative of $V_i(t)$ and using Lemma 6, we get

$$\begin{aligned}{}_0^C D_t^\alpha V_i(t) &\leq e_i(t)\,{}_0^C D_t^\alpha e_i(t)\\&= e_i(t)(-p_i e_i - q_i\operatorname{sign}(e_i(t)))\\&= -p_i e_i^2(t) - q_i e_i(t)\operatorname{sign}(e_i(t))\\&= -p_i e_i^2(t) - q_i|e_i(t)| \leq -q_i|e_i(t)|\\&= -\sqrt{2}q_i V_i^{1/2}(t),\end{aligned}\tag{39}$$

where $q_i > 0$. As a result, according to Lemma 3 and Remark 4, e_i will converge to 0 asymptotically.

Therefore, the state trajectories of system (23) will converge to 0, so one has the following conclusion.

Theorem 12. *The sliding mode dynamics system (23) is asymptotically stable, and its states $e_i(t)$ converge to 0.*

Corollary 13. *By Theorems 9 and 12, system (16) is asymptotically stable, which means that system (14) can synchronize system (13).*

4. Numerical Simulations

The effectiveness of the obtained theoretical results is shown by the example. Considering the two-dimensional uncertain delayed FOHNNs (as the Master system)

$$\begin{aligned}{}_0^C D_t^\alpha x_1(t) &= -x_1 + 0.5\sin(x_1(t)) + \sin(x_2(t))\\&\quad+0.5\tanh(x_1(t-0.8))\\&\quad+\tanh(x_2(t-0.8))+0.2,\\{}_0^C D_t^\alpha x_2(t) &= -0.5x_2 + \sin(x_1(t)) - 0.5\sin(x_2(t))\\&\quad-0.5\tanh(x_1(t-0.8))\\&\quad-\tanh(x_2(t-0.8))+0.3\end{aligned}\tag{40}$$

where $\alpha = 0.9$, the initial conditions are $x_1(0) = -5, x_2(0) = 5$.

The form of the slave system is given by

$$\begin{aligned}{}_0^C D_t^\alpha y_1(t) &= -y_1 + 0.5\sin(y_1(t))\\&\quad+\sin(y_2(t))0.5\tanh(y_1(t-0.8))\\&\quad+\tanh(y_2(t-0.8))+0.2+0.1\\&\quad+u_1(t),\\{}_0^C D_t^\alpha y_2(t) &= -0.5y_2 - \sin(y_1(t)) - 0.5\sin(y_2(t))\\&\quad-0.5\tanh(y_1(t-0.8))\\&\quad-\tanh(y_2(t-0.8))+0.3+0.15\\&\quad+u_2(t)\end{aligned}\tag{41}$$

Assume that the initial conditions are $y_1(0) = -3, y_2(0) = 3$, and $\alpha = 0.9$.

Choosing $p_1 = p_2 = 0.5, q_1 = q_2 = 1.5, L_1^f = L_2^f = L_1^g = L_1^g = 1, \eta_1^a = \eta_2^a = 1.1, \eta_1^d = \eta_2^d = 1.2, \zeta_1^a = \zeta_2^a = 1.1, \zeta_1^b = \zeta_2^b = 1.2$, one gets

$$\begin{aligned}\dot{\hat{a}}_1(t) &= -1.1e_1(t)s_1(t)\\\dot{\hat{a}}_2(t) &= -1.1e_2(t)s_2(t)\end{aligned}\tag{42}$$

and

$$\begin{aligned}\dot{\hat{d}}_1(t) &= 1.2s_1(t)\\\dot{\hat{d}}_2(t) &= 1.2s_2(t)\end{aligned}\tag{43}$$

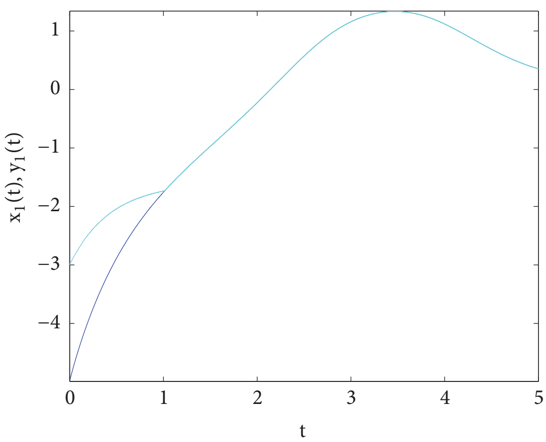

FIGURE 1: Synchronization between $x_1(t)$ (blue line) and $y_1(t)$ (green line).

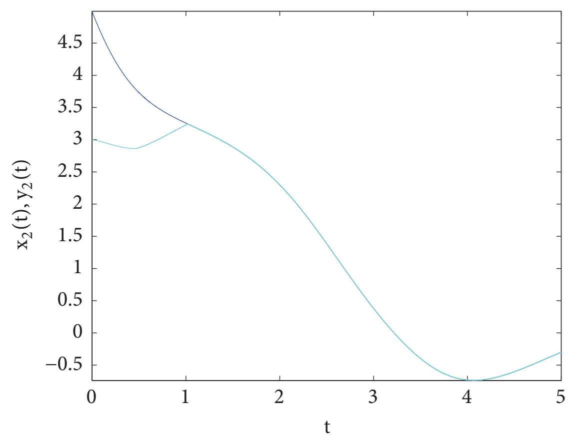

FIGURE 2: Synchronization between $x_2(t)$ (blue line) and $y_2(t)$ (green line).

We use (19) and design the SMS

$$\begin{aligned} s_1(t) &= {}_0D_t^{-1}\left({}_0^CD_t^{0.9}e_1(t) + 0.5e_1(t) + 1.5\operatorname{sign}(e_1(t))\right) \\ s_2(t) &= {}_0D_t^{-1}\left({}_0^CD_t^{0.9}e_2(t) + 0.5e_2(t) + 1.5\operatorname{sign}(e_2(t))\right). \end{aligned} \tag{44}$$

Thus, according to (24), the control inputs are obtained as

$$\begin{aligned} u_1(t) &= (\hat{a}_1(t) - 0.5)e_1(t) - \hat{d}_1(t) - 1.5\operatorname{sign}(e_1(t)) \\ &\quad - 1.1s_1(t) - 1.2\operatorname{sign}(s_1(t)) - |s_1(t)|(0.5|e_1(t)| \\ &\quad + |e_2(t)| + 0.5|e_1(t-0.8)| + |e_2(t-0.8)|), \\ u_2(t) &= (\hat{a}_2(t) - 0.5)e_2(t) - \hat{d}_2(t) - 1.5\operatorname{sign}(e_2(t)) \\ &\quad - 1.1s_2(t) - 1.2\operatorname{sign}(s_2(t)) - |s_2(t)|(-|e_1(t)| \\ &\quad - 0.5|e_2(t)| - 0.5|e_1(t-0.8)| - 1|e_2(t-0.8)|). \end{aligned} \tag{45}$$

The simulation results are depicted in Figures 1–6. Figures 1–3 show the synchronization between two fractional-order neural networks and the time response of the synchronization errors. The time response of the updated parameters and the sliding surfaces are included in Figures 4, 5, and 6, respectively. From the results, we can see that the synchronization errors converge to origin rapidly, and favorable synchronization performance has been achieved.

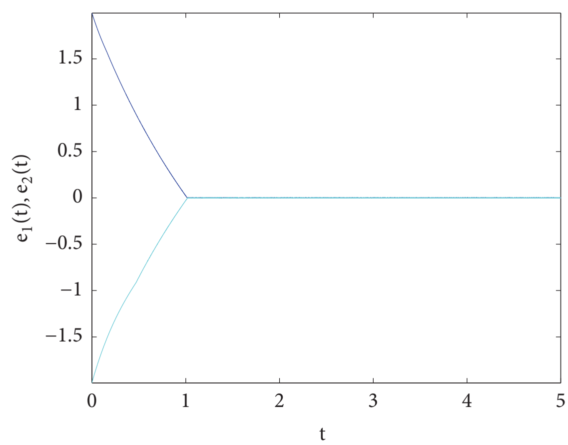

FIGURE 3: Synchronization errors $e_1(t)$ (blue line), $e_2(t)$ (green line).

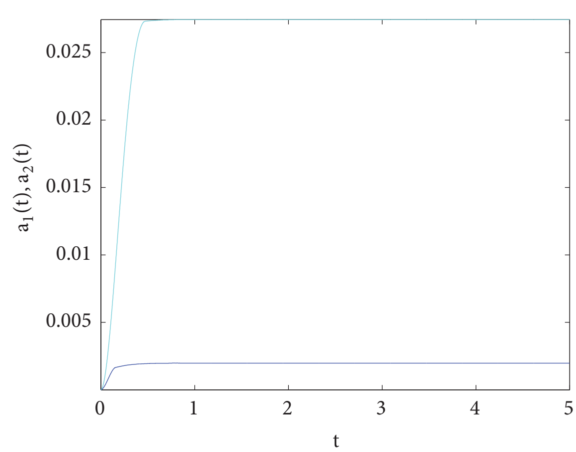

FIGURE 4: The time response of control parameters: $\hat{a}_1$ (blue line) and $\hat{a}_2$(green line).

5. Conclusion

The adaptive synchronization problem for FOHNNs with system uncertainties, time delay, and external disturbances has been studied by SMC. Some estimations for system uncertainties and external disturbances are made. Firstly, establishing a switched SMS, the finite-time stability of the SMS to origin is proved according to the fractional-order Lyapunov theory. Secondly, an adaptive synchronization fractional-order sliding mode controller is designed to force the error systems trajectories to reach the switching SMS and remain on it forever. The effectiveness and feasibility of theoretical results are verified by the numerical simulations.

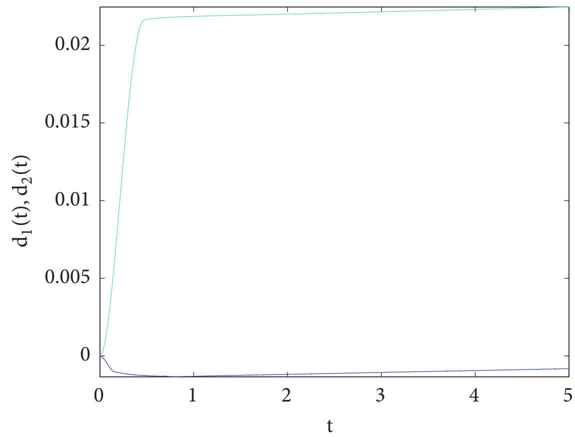

Figure 5: The time response of control parameters: $\hat{d}_1$ (blue line) and $\hat{d}_2$ (green line).

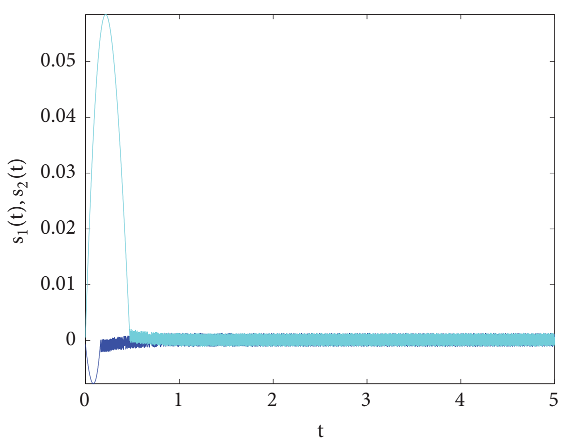

Figure 6: The sliding surfaces: $s_1(t)$ (blue line) and $s_2(t)$ (green line).

Furthermore, taking advantage of the SMC theory, the stability problems of FOHNNs with both multiple time delays and impulses will be discussed in future works.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

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