Optimization of the Pricing Strategies between Container Terminals under Deregulation

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According to the dual-track system implemented on port tariffs in past years, the vast majority of state-owned container terminals adopt the standard rates specified by China’s Ministry of Transport, while the container terminals of joint ventures are permitted to charge their stevedoring rate with a 20% float ratio up and down. The latest port reform was to improve the port tariff formation mechanism by speeding up the implementation of detailed list and public notice on port pricing. This paper analyses the optimization of the pricing strategies between container terminals under deregulation. Based on a two-stage noncooperative game theoretical model, the Nash equilibria of pricing strategy profiles between container terminals of one port under deregulation are derived. Although the price-matching strategy may be employed by the foreign-owned container terminal, which usually results in a total social welfare loss, the price-matching pricing strategy not being adopted by the state-owned container terminal will avert tacit collusion. Numerical simulation is applied to the case of Shenzhen Port.

1. Instruction

With container transport becoming an essential component of the global supply chain, the importance of the container terminal industry for industrial activity, merchandise trade, globalized production processes, and economic growth cannot be overemphasized. Meanwhile, container terminals also have been under constant pressure to adapt to changes in the economic, institutional, regulatory, and operating landscapes [1].

Since the late 1980s, the ownership structures of container terminals in many countries have been experiencing a transformation from public regulation to private operation. The size of a port or terminal is closely correlated with its efficiency, and some support exists for the claim that the transformation of ownership from the public to the private sector improves economic efficiency [2]. Because of the quantitative relationship that exists between the port ownership structure and port efficiency, a stochastic frontier model has been applied to show whether port privatization is a necessary strategy for ports to gain a competitive advantage [3]. Moreover, a direct correlation between the reform and the change in efficiency has been established to analyse the efficiency of the Spanish Port Authorities during the last three decades [4].

Some studies have addressed the efficiency gains of port privatization from an institutional and operational perspective. The optimization of container terminal operations can be restructured, especially for large ports, thus allowing specialized private entities to concentrate on terminal operation and cargo handling services [5]. The potential efficiency gains from privatization are discussed, thus providing a special reference to the privatization of the transport infrastructure in developing countries [6]. There exist equilibria in which governments choose privatization, and the national welfare of each port country is higher relative to a situation where ports are public [7]. The two port regulation modes are compared, finding that the tariffs, port efficiency level, port service demand, and social welfare are higher under the decentralization mode [8]. The various patterns of port cooperation in connection with the governance structure of ports are compared, introducing a generalization of the port cooperation issue in conclusion [9]. However, in these studies, port tariffs are not centre stage.

In China, many ports have rapidly improved their container-handling performance in recent years. Yap and
Lam [10] unveil the competitive dynamics between the major container ports in East Asia by analysing their extent and intensity, thus concluding that the economic development of China’s interior provinces will continue to stimulate strong container-handling performance in its ports together with investments in terminal capacity and operational efficiency. Yeo et al. [11] present a structure for evaluating port competitiveness, insisting that Chinese ports threaten to oust Busan in Korea as the regional hub following the intensive petitiveness, insisting that Chinese ports threaten to oust Busan in Korea as the regional hub following the intensive investments in port development. Song and Geenhuizen [12] apply panel data analysis in 1999-2010 of China to estimate the output elasticity of port infrastructure through the production function.

The reasons why China can invest heavily in large-scale projects of port and container terminals are also addressed. Yang et al. [13] explore the various paths to find a direction that better suits China’s national conditions during the past 60 years, concluding that the reforms of port governance have had a more lasting positive effect on port throughput than physical investments. Zhuang et al. [14] use alternative duopoly games to model port competition, finding that the interport competition can lead to port specialization. Wu et al. [15] pay attention to how a port would respond if a rival port uses a type of capacity investment strategy, obtaining that investments in port capacity contribute greatly to the local government’s performance. Zhang [16] notes that the optimal incentive scheme is the same in the international landlord port financing model with profit sharing rents or mixed rents.

This paper is motivated by the employment of a price-matching guarantee policy in retailing, which results either in price collusion or price discrimination in a duopoly. Edlin and Emch [17] find that price matching with entry creates greater welfare losses than monopolies in markets with a low ratio of fixed to marginal costs, using parameters from the US wholesale gasoline and air travel markets. Monika and Dhruv [18] examine the process to determine the likelihood that a consumer will claim a price-matching refund when he or she identifies a lower competitive price after the purchase. If consumers are heterogeneous with respect to firm loyalty, and a firm has more loyal customers than the other firm, the equilibrium matching policy and pricing strategy depend on the market conditions (Koh et al., [19]). Under the background of port tariff deregulation, if one port has multiple container terminals, which are state-owned, privately owned or joint-ventured, providing perfectly homogenous services or basically identical services to the shipper, which makes it possible for them to adopt price-matching strategies. However, while the price-matching guarantees seem applicable to container terminals of one port, it is not certain that they will employ price-matching strategies with the deregulation of port capital and port tariff.

The structure of the game employed by this paper is similar to the one used by Dong et al. [20], in which the differential designed capacities of two container terminals of one port are investigated, and two container terminals maximize their profits separately according to whether they adopt the price-matching policy or not, drawing the conclusion that price-matching strategies facilitate tacit collusion between container terminals. This paper comprehensively takes account of the differential designed capacities, capital structures and tariff deregulation of the container terminals. Therefore, the container terminal subjected to foreign investment still maximizes its profits separately according to whether adopts the price-matching policy or not. In contrast, the state-owned container terminal, which can be considered as an economic entity with port authority of local government, should take its own profits and the surplus of shipper as the pricing objective. More importantly, the state-owned container terminal attempts to realize its social welfare maximization under the background of port tariff deregulation. At this moment, we ask whether the price-matching strategy is the only Nash equilibrium for the container terminals of one port. We also address how to avoid tacit collusion among container terminals of one port, which otherwise results in social welfare losses with port tariff deregulation.

We organize the rest of the paper as follows. We first present the model basics under the circumstance of port tariff deregulation in Section 2. The two-stage game analysis among the container terminals of one port is conducted in Section 3. In Section 4, we solve the Nash equilibrium and compare social welfare in the scenarios of adopting the price-matching strategy or not. In Section 5, we will verify some of the analytical results and further explore the managerial insights and policy implications through a case study of Shenzhen Port. Conclusions and directions for future research are summarized in Section 6.

2. Methods

In this section, we construct an analytical framework model for the two container terminals in the same port area. With the gradual deregulation of foreign investment into the port sector, the container terminals of one port are built by different capital, such as state-owned, foreign-owned, bank financing, self-collected funds, IPOs and so on. In addition, the number of major container terminals in mainland China further increased to 99, among which 59 container terminals involved foreign investment, and the proportion of foreign investment container terminals increased to 59.6% in 2016 (China Port Yearbook, [21]). For simplicity, we consider a port with two kinds of capital sources, which are represented by container terminals 1 and 2, and are denoted as being foreign-owned and state-owned, respectively.

Considering an economy with a monopolistic sector with two firms, each one producing a differentiated good, and the consumers of the same type with a utility function separable and linear, following Singh and Vives [22], the quadratic and strictly concave utility function gives rise to a linear demand structure, the inverse demands of the container terminals are given by,

$$ f_1 + t \cdot \frac{q_1}{s_1} = \alpha_1 - \beta_1 q_1 - \gamma q_2 \tag{1} $$

$$ f_2 + t \cdot \frac{q_2}{s_2} = \alpha_2 - \beta_2 q_2 - \gamma q_1 \tag{2} $$

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where $f$ is the tariff of the container terminal of one port, $t$ is the parameter of waiting costs in the container terminal, $s$ is the designed capacity of the container terminal, $q$ is the output volume of the container terminal. $\alpha$ is the intercept of the reservation price for the container terminal's service. $\beta$ and $\gamma$ are the sensitivity of the container terminal's demand to its own generalized price and the other's generalized price. The values of $\alpha_i$ and $\beta_i$ for $i = 1, 2$ are positive. Since two container terminals are located in the same port, we assume that the container terminals are perfectly substitutable; i.e., $\alpha_1 = \alpha_2 = \alpha$ and $\beta_1 = \beta_2 = \gamma = \beta$. Without the loss of generality, we also assume that $0 < s_1 \leq s_2$.

Located in the same region, the two container terminals face ocean carriers' selection decisions to meet the original demand of the shipper or freight forwarder, such as terminal tariffs, congestion cost as well as total transport chain's cost, connectivity, availability, immediacy, etc. For example, prompt response, 24 hour a day; seven days a week service and less waiting time service. In addition, professional and skilled labour in the operation of container terminal, network connectivity of feeder ports and transfer stations, the area and policy of free trade port, which can be divided into two categories: hardware factors include infrastructure and facility, denoted by designed capacity, the congestion cost of liner company starts to rise sharply when the actual container throughput exceeds 80% of rated capacity. The software factors contain the quality and skill of terminal worker and management personnel, terminal operation process system and automated terminal technology, as well as custom clearance process and maritime administrative inspection, reflected by the parameter $t$.

In addition, the generalized price that is assumed to take the sum of the container terminal's tariff and waiting time costs, with the latter expressed by the ratio of container throughput to the designed capacity that indicates the container terminal's efficiency, has been adopted in some studies to analyse port congestion costs (see De Borger and Van Dender, [23]; Basso and Zhang, [24]; Bae et al. [25]; Chen and Liu, [26]; Sheng et al. [27]).

Therefore, the demands for container terminals' services can be solved by Eqs. (1) and (2):

$$q_1 = \frac{s_1 \left[ \beta s_2 f_2 - (\beta s_2 + t) f_1 + at \right]}{t (\beta s_1 + \beta s_2 + t)}$$

$$q_2 = \frac{s_2 \left[ \beta s_1 f_1 - (\beta s_1 + t) f_2 + at \right]}{t (\beta s_1 + \beta s_2 + t)}$$

Contrary to the Cournot model, the Bertrand model is a good approximation of duopoly competition under the circumstance that it is more difficult to adjust the price than it is to adjust capacity/output (Cabral, [28]). The dual-track system has been implemented on port tariffs in China in past years. According to this pricing policy, the vast majority of state-owned container terminals adopt the standard rates specified by China's Ministry of Transport, while the container terminals of joint ventures are permitted to charge their stevedoring rate with a 20% float ratio up and down. In order to transform the container terminal's tariffs from mainly depending on government pricing to being based on market regulations, charging method of port tariffs was formulated by China's Ministry of Transport and National Development and Reform Commission, which come into effect on March 1, 2016, and valid for 5 years. According to the method, the port tariffs of market regulation include operation lump sum rate, storage fee, custody fee, as well as the service charges of ship material, oil and gas supply, power supply, garbage receiving process and sewage disposal. Moreover, the container terminal can set different lump sum rates in accordance with providing differential service.

The game is assumed to be played in two stages through backwards-induction. In the second stage, the two container terminals have to make a decision on their tariffs independently according to the pricing strategy that is adopted. Then, in the first stage, both container terminals decide simultaneously whether to adopt a price-matching strategy or not.

### 3. Optimizing Strategy


Both container terminals have identical and constant variable costs in consideration of locating within the same port, and the variable costs are normalized to zero on the supposition that basically the same equipment and related fees of energy consumption, truck drayage, handling labour, etc. exist for the two container terminals in one port area.

Since neither the foreign-owned or state-owned container terminal has a conjecture about the other one, the container terminal owned by foreign capital will maximize its own profits as follows:

$$\max_{q_1} \pi_1 = f_1 - \frac{s_1 \left[ \beta s_2 f_2 - (\beta s_2 + t) f_1 + at \right]}{t (\beta s_1 + \beta s_2 + t)}$$

By taking the derivative, the first-order conditions (FOCs) and the second-order conditions (SOCs) are given by

$$\frac{\partial \pi_1}{\partial f_1} = -2 \left( \frac{\beta s_2 + t}{t (\beta s_1 + \beta s_2 + t)} \right) s_1 f_1 + \left( \frac{\beta s_2 s_1}{t (\beta s_1 + \beta s_2 + t)} \right) f_2$$

$$\frac{\partial^2 \pi_1}{\partial f_1^2} = \frac{2 (\beta s_2 + t) s_1}{t (\beta s_1 + \beta s_2 + t)}$$

To solve container terminal 1's optimization problem, we set the first-order condition equal to zero as

$$-2 \left( \frac{\beta s_2 + t}{t (\beta s_1 + \beta s_2 + t)} \right) s_1 f_1 + \left( \frac{\beta s_2 s_1}{t (\beta s_1 + \beta s_2 + t)} \right) f_2 + \frac{\alpha s_1}{\beta s_1 + \beta s_2 + t} = 0$$

which leads to the following solution of the container terminal invested by foreign capital:

$$f_1 = \frac{\beta s_2 s_2 + \alpha t}{2 (\beta s_2 + t)}$$
With regard to the container terminal established by state-owned capital, which can be considered as an economic entity where port authority is controlled by the local government, the pricing objectives of the state-owned container terminal not only include its own profits, similar to the foreign-owned container terminal, but also contain the surplus of the shippers.

According to the classic quadratic utility function (see Singh and Vives, [22]), the continuum of shippers of the same type with a utility function can be expressed as follows:

\[
U(q_1, q_2) = \alpha_1 q_1 + \alpha_2 q_2 - \frac{\beta_1 q_1^2 + 2\gamma q_1 q_2 + \beta_2 q_2^2}{2}
\]

(9)

To avoid social welfare loss with port tariff deregulation, the state-owned container terminal should charge its tariff to realize the social welfare maximization:

\[
\max_{f_1} w(f_2) = \alpha (q_1 + q_2) - \frac{\beta (q_1 + q_2)^2}{2} + \pi_2
\]

(10)

\[
\pi_1 = \frac{4\alpha^2 s_1 t (\beta s_1 + t)^3 (\beta s_2 + t) (\beta s_1 + \beta s_2 + t)}{[2\beta^2 s_1^2 (\beta s_2 + 2t) + \beta^2 s_2 (7s_1 + 2s_2) + 2t^2 (4\beta s_1 + 3\beta s_2 + 2t)]^2}
\]

\[
w_2 = \alpha^2 (4\beta^5 s_1^4 s_2^2 + 16\beta^4 s_1^4 s_2 t + 28\beta^4 s_1^3 s_2^3 t + 8\beta^4 s_1^2 s_2^4 t + 12\beta^3 s_1^3 s_2^5 t + 72\beta^3 s_1^4 s_2^4 t + 73\beta^3 s_1^3 s_2^5 t + 28\beta^3 s_1^2 s_2^6 t + 4\beta^3 s_1 s_2^7 t + 4\beta^3 s_1 s_2^7 t + 110\beta^2 s_1 ^2 s_2^8 t + 76\beta^2 s_1 s_2^9 t + 16\beta^2 s_2^10 t + 44\beta s_1^2 s_2^{11} t + 64\beta s_1 s_2^{12} t + 20\beta s_2^{13} t + 16s_1^5 + 8s_2^5) (2\beta^3 s_1^2 s_2^2 + 4\beta^3 s_1 s_2^3 t + 7\beta^3 s_1 s_2^4 t + 2\beta^3 s_2^5 t + 8\beta s_1 t^4 + 6\beta s_2 t^5 + 4t^6)^{-2}
\]

(14)

3.2. Price-Matching is Employed by Both Container Terminals.

According to price-matching strategies employed by foreign-owned and state-owned container terminals, the two container shippers should charge the same tariff to the shipper. Under these circumstances, each container terminal forecasts that one unilateral tariff cut below the other’s counterpart will be matched by the other.

Therefore, the profits of a foreign-owned container terminal can be rewritten as

\[
\min \{f_1, f_2\} \left[ -\frac{(\beta s_2 + t) s_1}{t (\beta s_1 + \beta s_2 + t)} \min \{f_1, f_2\} + \frac{\beta s_1 s_2}{t (\beta s_1 + \beta s_2 + t)} \min \{f_1, f_2\} + \frac{\alpha s_1}{\beta s_1 + \beta s_2 + t} \right]
\]

(15)

In consideration of a fixed value of the state-owned container terminal’s tariff, the foreign-owned container terminal’s strategy \(f_1 > f_2\) has the same result as the strategy

\[
\frac{\partial \omega_2}{\partial f_2} = -\frac{\alpha (s_1 + s_2)}{\beta s_1 + \beta s_2 + t} - \frac{\beta (s_1 + s_2) (a - f_2)}{\beta s_1 + \beta s_2 + t} \times \left( -\frac{s_1 + s_2}{\beta s_1 + \beta s_2 + t} \right) - 2t \times \left( -\frac{a - f_2}{\beta s_1 + \beta s_2 + t} \right)
\]

(11)

Similarly, the first-order condition can be rewritten as

\[
\frac{\partial \omega_2}{\partial f_1} = -\frac{\alpha s_1}{\beta s_1 + \beta s_2 + t} - \frac{\beta (s_1 + s_2) (a - f_2)}{\beta s_1 + \beta s_2 + t} \times \left( -\frac{s_1 + s_2}{\beta s_1 + \beta s_2 + t} \right) - 2t \times \left( -\frac{a - f_2}{\beta s_1 + \beta s_2 + t} \right)
\]

(12)

Therefore, the profits and social welfare for both container terminals are given, respectively, by

\[
f_1 = f_2. \text{ For this reason, we only analyse } f_1 \leq f_2, \text{ for which the foreign-owned container terminal’s optimization problem becomes}
\]

\[
\max_{f_1 : f_1 \leq f_2} \pi_1 (f_1) = f_1 \left[ -\frac{s_1}{\beta s_1 + \beta s_2 + t} f_1 + \frac{\alpha s_1}{\beta s_1 + \beta s_2 + t} \right]
\]

(16)

which is solved at \(f_1^* = \alpha / 2\). Otherwise, there exists \(f_1^* = f_2\).

Similarly, the first-order condition the state-owned container terminal of can be rewritten as

\[
\frac{\partial \omega_2}{\partial f_2} = -\frac{\alpha (s_1 + s_2)}{\beta s_1 + \beta s_2 + t} - \frac{\beta (s_1 + s_2) (a - f_2)}{\beta s_1 + \beta s_2 + t} \times \left( -\frac{s_1 + s_2}{\beta s_1 + \beta s_2 + t} \right) - 2t \times \left( -\frac{a - f_2}{\beta s_1 + \beta s_2 + t} \right)
\]

(13)
which is solved as

$$f_1^* = \frac{\alpha t (s_2 - s_1)}{\beta (s_1 + s_2)^2 + 2s_2t}$$

If the following inequality holds:

$$\left(\sqrt{2s_2t + t} - \beta s_2 - t\right) \leq s_1 \leq s_2$$

Therefore, we obtain $f_1^* = f_2^* = \alpha/2$. Otherwise, we can get $f_1^* = f_2^* = at(s_2 - s_1)/[\beta(s_1 + s_2)^2 + 2s_2t]$

In the meantime, the profits and social welfare for both container terminals are given, respectively, by

$$\pi_1 = \frac{s_1}{\beta s_1 + \beta s_2 + t} \cdot \left(\frac{\alpha}{2}\right)^2$$

$$w_2 = \frac{3\beta (s_1 + s_2)^2 + 2t (2s_1 + s_2)}{2(\beta s_1 + \beta s_2 + t)^2} \cdot \left(\frac{\alpha}{2}\right)^2$$

3.3. Price-Matching Is Only Employed by Foreign-Owned Container Terminal. At this very moment, the foreign-owned container terminal employs the price-matching strategy, while the state-owned container terminal does not.

The profits of both container terminals can be expressed as

$$\pi_1 (f_1) = \min \{f_1, f_2\}$$

$$\cdot \left[ -\frac{(\beta s_2 + t)s_1}{t(\beta s_1 + \beta s_2 + t)} \min \{f_1, f_2\} \right.$$

$$+ \frac{\beta s_1 s_2}{t(\beta s_1 + \beta s_2 + t)} f_2 + \frac{\alpha s_1}{\beta s_1 + \beta s_2 + t} \left] \right.$$  

$$\pi_2 (f_2) = f_2 \left[ -\frac{(\beta s_1 + t)s_2}{t(\beta s_1 + \beta s_2 + t)} f_2 \right.$$

$$+ \frac{\beta s_1 s_2}{t(\beta s_1 + \beta s_2 + t)} \min \{f_1, f_2\} + \frac{\alpha s_2}{\beta s_1 + \beta s_2 + t} \left] \right.$$  

Proposition 1. It is necessary to have $f_1 = f_2$ in any equilibrium of price-matching only employed by the foreign-owned container terminal.

Proof. On the supposition of $f_1 > f_2$, by taking the price-matching strategy employed by the foreign-owned container terminal into consideration, the social welfare of state-owned container terminal can also be rewritten as

$$\max w (f_2) = \alpha \left[ \frac{(s_1 + s_2)(a - f_2)}{\beta s_1 + \beta s_2 + t} \right.$$  

$$- \frac{\beta [(s_1 + s_2)(a - f_2) / (\beta s_1 + \beta s_2 + t)]^2}{2}$$  

$$\frac{s_2(a - f_2) / (\beta s_1 + \beta s_2 + t)}{s_2}$$

The first-order condition can be obtained by

$$-\frac{a(s_1 + s_2)}{\beta s_1 + \beta s_2 + t} \cdot \frac{\beta(s_1 + s_2)(a - f_2)}{\beta s_1 + \beta s_2 + t}$$  

$$\cdot \left( -\frac{s_1}{\beta s_1 + \beta s_2 + t} - 2t \cdot \frac{(a - f_2)}{\beta s_1 + \beta s_2 + t} \right) = 0$$

Thus, the second-order condition of it is given by

$$-\frac{\beta (s_1 + s_2)}{\beta s_1 + \beta s_2 + t} \cdot \frac{(s_1 + s_2)}{\beta s_1 + \beta s_2 + t} - \frac{2t}{\beta s_1 + \beta s_2 + t}$$

$$\cdot \frac{s_2}{\beta s_1 + \beta s_2 + t} < 0$$

Evidently, the social welfare function of the state-owned container terminal is concave and maximized at

$$f_2 = \frac{\alpha t (s_2 - s_1)}{2s_2 + (\beta(s_1 + s_2)^2)}$$

Given the state-owned container terminal 2’s optimal tariff is $\alpha t(s_2 - s_1)/(2s_2 + \beta(s_1 + s_2)^2)$, we can obtain the foreign-owned container terminal’s profits as follows:

$$\min \{f_1, f_2\} \left[ -\frac{(\beta s_2 + t)s_1}{t(\beta s_1 + \beta s_2 + t)} \min \{f_1, f_2\} \right.$$  

$$+ \frac{\beta s_1 s_2}{t(\beta s_1 + \beta s_2 + t)} \cdot \frac{\alpha t (s_2 - s_1)}{2s_2 + \beta(s_1 + s_2)^2} \right]$$

which is maximized at

$$f_1 = \frac{\beta s_2 \left( \alpha t (s_2 - s_1) / (2s_2 + \beta(s_1 + s_2)^2) \right)}{2(\beta s_2 + t)} + \alpha t$$

Therefore,

$$\Delta f = f_2 - f_1 = -\frac{\alpha s_1 t (\beta s_1 + 3\beta s_2 + 2t)}{2(\beta s_2 + t) \left[ 2s_2 + \beta(s_1 + s_2)^2 \right]} < 0$$
It is a contradiction to the supposition of \( f_1 > f_2 \) in the equilibrium of the price-matching that is only employed by the foreign-owned container terminal.

Similarly, suppose that \( f_1 < f_2 \) in equilibrium of price-matching is only employed by the foreign-owned container terminal. The state-owned container terminal will maximize social welfare at

\[
f_2 = \frac{(2\beta s_1 + t) \left( \beta s_1 f_1 + \alpha t \right)}{t \beta s_2 + 2 (\beta s_1 + t)^2}
\]

Consequently, the foreign-owned container terminal's profits are maximized at

\[
f_1 = \frac{\beta s_2 f_2 + \alpha t}{2 (\beta s_2 + t)}
\]

Therefore, we can obtain the following inequality expression:

\[
\Delta f = f_1 - f_2
= \frac{\alpha \beta s_1 t (2\beta s_1 + t)}{2\beta^2 s_1^2 (\beta s_2 + 2t) + \beta^2 s_2 t (7\beta s_1 + 2\beta s_2) + 2t^2 (4\beta s_1 + 3\beta s_2 + 2t)} \geq 0
\]

In conclusion, the price-matching strategy only employed by the foreign-owned container terminal has no effect on the equilibrium outcome in this game.

3.4. Price-Matching Is Only Employed by State-Owned Container Terminal. In contrast to the game of price-matching only employed by foreign-owned container terminal, the profits of both container terminals are, respectively, given by

\[
\pi_1 (f_1) = f_1 \left[ -\frac{(\beta s_2 + t) s_1}{t (\beta s_1 + \beta s_2 + t)} f_1 + \frac{\beta s_1 s_2}{t (\beta s_1 + \beta s_2 + t)} \min \{ f_1, f_2 \} + \frac{\alpha s_1}{\beta s_1 + \beta s_2 + t} \right]
\]

\[
w(f_2) = \alpha \left( q_1 + q_2 \right) - \frac{\beta (q_1 + q_2)^2}{2} + \pi_2 - \left( \min \{ f_1, f_2 \} + t \cdot \frac{q_1}{s_2} \right) q_2
\]

The above functions can also be rewritten as

\[
\pi_1 (f_1) = f_1 \left[ -\frac{(\beta s_2 + t) s_1}{t (\beta s_1 + \beta s_2 + t)} f_1 + \frac{\beta s_1 s_2}{t (\beta s_1 + \beta s_2 + t)} f_2 + \frac{\alpha s_1}{\beta s_1 + \beta s_2 + t} \right]
\]

\[
w(f_2) = \alpha \left( s_1 + s_2 \right) - f_1 s_1 - f_2 s_2 + \pi_2
\]

Proposition 2. No price-matching equilibrium only employed by the state-owned container terminal satisfies \( f_2 < f_1 \).

Proof. On the supposition that there exists an equilibrium satisfying \( f_2 < f_1 \), the state-owned container terminal's objective function is as follows:

\[
\max_{f_1, f_2 \leq s_1} \left[ \frac{\alpha (s_1 + s_2) - f_1 s_1 - f_2 s_2}{\beta s_1 + \beta s_2 + t} \right]
\]

\[
\frac{\beta}{2} \left( \frac{(\alpha (s_1 + s_2) - f_1 s_1 - f_2 s_2)}{(\beta s_1 + \beta s_2 + t)} \right)^2
\]

which is maximized at

\[
\min \left\{ \frac{(2\beta s_1 + t)}{t \beta s_2 + 2 (\beta s_1 + t)}, f_1 \right\}
\]

If \( f_1 > \alpha (2\beta s_1 + t)/(3\beta s_1 + \beta s_2 + 2t), f_2 = (2\beta s_1 + t)/(\beta s_1 + t)^2 < f_1 \). Given \( f_2 < f_1 \), foreign-owned container terminal's profits can be rewritten as

\[
f_1 \left[ -\frac{(\beta s_2 + t) s_1}{t (\beta s_1 + \beta s_2 + t)} f_1 + \frac{\beta s_1 s_2}{t (\beta s_1 + \beta s_2 + t)} f_2 + \frac{\alpha s_1}{\beta s_1 + \beta s_2 + t} \right]
\]

which is maximized at

\[
f_1 = \frac{\beta s_2 f_2 + \alpha t}{2 (\beta s_2 + t)}
\]

Thus, we have

\[
f_1 = \frac{\beta s_2 ((2\beta s_1 + t)/(\beta s_1 + \beta s_2 + t)) + \alpha t}{2 (\beta s_2 + t)}
\]

which is solved at

\[
f_1 = \frac{2\beta s_2 (2\beta s_1 + t)/(\beta s_1 + \beta s_2 + t) + \alpha t}{2 (\beta s_2 + t)}
\]

That is,
The value of \( f_1 \) above is less than \( \alpha (2\beta s_1 + t) / (3\beta s_1 + \beta s_2 + 2t) \), which is a contradiction. \( \square \)

**Proposition 3.** Any tariff combination satisfying \( \alpha t / (\beta s_2 + 2t) \leq f_1 = f_2 \leq \alpha (2\beta s_1 + t) / (3\beta s_1 + \beta s_2 + 2t) \) is a Nash equilibrium of the price-matching only employed by the state-owned container terminal.

**Proof.** The state-owned container terminal’s objective function

\[
\max_{t_1 \leq t_2 \leq t} \left[ \frac{\alpha (s_1 + s_2) - f_1 s_1 - f_2 s_2}{(s_1 + s_2 + t)} - \frac{\beta ((\alpha (s_1 + s_2) - f_1 s_1 - f_2 s_2) / (s_1 + s_2 + t))^2}{2} - t \cdot \frac{((\alpha (s_1 + s_2) - f_1 s_1 - f_2 s_2) / (s_1 + s_2 + t))^2}{s_2} \right]
\]

is maximized at

\[
\min \left\{ \frac{(2\beta s_1 + t) (\beta s_1 f_1 + \alpha t)}{t \beta s_2 + 2 (\beta s_1 + t)^2}, f_1 \right\}.
\]

If the following inequality holds,

\[
\frac{(2\beta s_1 + t) (\beta s_1 f_1 + \alpha t)}{t \beta s_2 + 2 (\beta s_1 + t)^2} \geq f_1
\]

\[\Rightarrow \frac{f_1}{\alpha (2\beta s_1 + t) / (3\beta s_1 + \beta s_2 + 2t)} \leq \frac{\alpha (2\beta s_1 + t)}{3\beta s_1 + \beta s_2 + 2t}.
\]

Under the present circumstances, the state-owned container terminal has no motivation to deviate from \( f_2 = f_1 \) unilaterally.

On the other hand, the profit function of the foreign-owned container terminal is given by

\[
f_1 = \beta s_2 f_2 + \alpha t / 2 (\beta s_2 + t)
\]

With some algebra, the function is concave and is maximized at

\[
f_1 = \frac{\beta s_2 f_2 + \alpha t}{2 (\beta s_2 + t)}
\]

If \( \alpha t / (\beta s_2 + 2t) \leq f_2 = f_1 = \alpha (2\beta s_1 + t) / (3\beta s_1 + \beta s_2 + 2t) \), the foreign-owned container terminal will not raise its tariff. Otherwise, the profits of foreign-owned container terminal become

\[
f_1 = \frac{s_1}{\beta s_1 + \beta s_2 + t} + \frac{\alpha s_1}{\beta s_1 + \beta s_2 + t}
\]

which is increasing when \( f_1 \leq \alpha (2\beta s_1 + t) / (3\beta s_1 + \beta s_2 + 2t) < \alpha / 2 \).

Therefore, foreign-owned container terminal has no motivation to reduce its tariff either. \( \square \)

**Proposition 4.** There are no other price-matching equilibria only employed by the state-owned container terminal satisfying \( f_1 = f_2 \).

**Proof.** In consideration of an equilibrium satisfying \( f_1 = f_2 \), then \( (2\beta s_1 + t) (\beta s_1 f_1 + \alpha t) / (t \beta s_2 + 2 (\beta s_1 + t)^2) \) becomes smaller than \( f_1 = f_2 \). Thus, the proof of Proposition 3 above, we understand that the state-owned container terminal has a motivation to deviate to \( (2\beta s_1 + t) (\beta s_1 f_1 + \alpha t) / (t \beta s_2 + 2 (\beta s_1 + t)^2) \) unilaterally.

In the other hand, if \( f_1 = f_2 = \alpha t / (\beta s_2 + 2t), (\beta s_2 f_2 + \alpha t) / (2 (\beta s_2 + t)) > f_2 = f_1 \).

According to the profit function of the foreign-owned container terminal, which is concave, it is maximized at

\[
\frac{\beta s_2 f_2 + \alpha t}{2 (\beta s_2 + t)}
\]

Therefore, the foreign-owned container terminal would be better off if it increases its tariff from \( f_1 \) to \( (\beta s_2 f_2 + \alpha t) / (2 (\beta s_2 + t)) \).

In brief, the equilibrium of \( f_1 = f_2 = \alpha (2\beta s_1 + t) / (3\beta s_1 + \beta s_2 + 2t) \) incurs the highest profits and social welfare among all equilibria for both container terminals, which are given as follows:

\[
\pi_1^4 = \frac{\alpha^2 s_1 (2\beta s_1 + t)}{(3\beta s_1 + \beta s_2 + 2t)^2}
\]

\[
w_2^4 = \frac{a^2 (5\beta s_1^2 + 6\beta s_1 s_2 + \beta s_2^2 + 4s_1 t + 2s_2 t)}{2 (3\beta s_1 + \beta s_2 + 2t)^2}
\]

**4. Comparison**

**Theorem 5.** The Nash equilibria are price-matching strategies employed by neither container terminal or only by the foreign-owned container terminal.

**Proof.** According to the payoffs of both container terminals for each game that are given above, as well as the assumption
of (\(\sqrt{2(2\beta s_2 + t) - \beta s_2 - t}/\beta \leq s_1 < s_2\), it is sufficient to show that \(\pi_1^2 > \pi_1^4\), \(w_1^2 < w_1^3\), and \(w_2^2 < w_2^4\).

These inequalities can be shown as follows:

\[
\pi_1^2 - \pi_1^4 = \frac{\alpha^2 \beta^2 s_1 (s_1 - s_2)^2}{4(\beta s_1 + \beta s_2 + t)(3\beta s_1 + \beta s_2 + 2t)^2} > 0
\]

\[
w_2^2 - w_1^4 = -\alpha^2 \beta s_2 \left(4\beta^6 s_1^6 s_2 + 8\beta^8 s_1^5 s_2^2 + 16\beta^8 s_1^4 s_2^3 + 76\beta^8 s_1^3 s_2^4 t + 104\beta^8 s_1^2 s_2^5 t + 8\beta^8 s_1 s_2^6 t + 88\beta^8 s_1^4 s_2^3 t + 302\beta^8 s_1^3 s_2^4 t^2 + 133\beta^8 s_1^2 s_2^5 t^2 + 366\beta^8 s_1 s_2^6 t^2 + 200\beta^8 s_1^4 s_2^3 t^3 + 366\beta^8 s_1^3 s_2^4 t^3 + 124\beta^8 s_1^2 s_2^5 t^3 + 232\beta^8 s_1^3 s_2^4 t^4 + 376\beta^8 s_1^2 s_2^5 t^4 + 20\beta^8 s_1 s_2^6 t^4 + 136\beta^8 s_1^2 s_2^5 t^5 + 112\beta^8 s_1 s_2^6 t^5 + 8\beta^8 s_1^4 s_2^3 t^5 + 32\beta^8 s_1^3 s_2^4 t^5\right)
\]

\[
\leq 0
\]

\[
\Delta w = w_2^4 - w_1^2 = -\alpha^2 \beta s_1 \left(8\beta^8 s_1^2 s_2^2 + 32\beta^8 s_1^4 s_2 t + 72\beta^8 s_1^2 s_2 t^2 + 20\beta^8 s_1^4 s_2 t^2 + 14\beta^8 s_1^4 s_2 t^3 + 156\beta^8 s_1^2 s_2^3 t^2 + 192\beta^8 s_1^2 s_2^3 t^3 + 60\beta^8 s_1^2 s_2^3 t^4 + 8\beta^8 s_1^4 s_2 t^4 + 60\beta^8 s_1^2 s_2^3 t^4 + 279\beta^8 s_1^2 s_2^3 t^4 + 214\beta^8 s_1^2 s_2^3 t^4 + 44\beta^8 s_1^2 s_2^3 t^4 + 94\beta^8 s_1^2 s_2^3 t^4 + 220\beta^8 s_1^2 s_2^3 t^4 + 84\beta^8 s_1^2 s_2^3 t^4 + 64\beta s_1 t^5 + 16\beta s_1 t^5\right)
\]

\[
\leq 0
\]

**Theorem 6. The price-matching not being adopted by the state-owned container terminal averts tacit collusion.**

**Proof.** It is sufficient to show that the collusive tariffs are equal to the equilibrium tariffs under the price-matching adopted by both container terminals, i.e., \(f_1 = f_2 = \alpha/2\), which also solves the following optimization problem:

\[
\begin{align*}
\max_{f_1,f_2} & \quad f_1 \left[ \beta s_1 f_2 - (\beta s_1 + t) f_1 + \alpha t \right] \\
& \quad + f_2 \left[ \beta s_1 f_1 - (\beta s_1 + t) f_2 + \alpha t \right] \\
& \quad \text{subject to } t (\beta s_1 + \beta s_2 + t)
\end{align*}
\]

**Theorem 7. The price-matching strategies employed by both container terminals result in total social welfare loss.**

**Proof.** The total social welfare is the sum of both foreign-owned and state-owned container terminals’ profits and the consumer surplus, which can be written as

\[
tw = w_2 + \pi_1 - \left(f_1 + t \cdot \frac{q_1}{s_1}\right) q_1
\]

If the price-matching strategy is employed by neither container terminal, the total social welfare under this circumstance is denoted as \(tw^1\). However, if price-matching strategy is employed by both container terminals, we have \(q_1 = s_1 / (\beta s_1 + \beta s_2 + t) \cdot \alpha/2\) and \(q_2 = s_2 / (\beta s_1 + \beta s_2 + t) \cdot \alpha/2\), and the total social welfare in this game is labelled as \(tw^2\).

The total social welfare loss can be rewritten as

\[
\Delta tw = tw^1 - tw^2
\]

**5. Simulation Example**

In this section, we will verify some of the analytical results and further explore the managerial insights and policy
implications through a simulation example of Shenzhen Port, which reflects some different scenarios.

5.1. Background Information about the Case Study. Shenzhen Port is located in the Pearl River Delta and is spread along Shenzhen city’s 260 km coastline. It is separated by the New Territories and the Kowloon Peninsula of Hong Kong into two areas: the western port area and the eastern port area. The former mainly contains Shekou Container Terminal (SCT), and the latter refers to the Yantian International Container Terminal (YICT). As an important gateway for the import and export trade of China, Shenzhen Port is one of the hubs in the Pearl River Delta. It handles a container throughput of 25.21 (million TEUs) in 2017, with its volume increasing by 5.13 percent, thus ranking it as the third container port in the world for five consecutive years.

Firstly, SCT was founded in 1989 with the registered capital of 0.6 billion Hong Kong dollars. As a wholly foreign-owned container terminal, China Merchants Holdings (International) Company Limited (CMHI) owns 100% of SCT. Headquartered in Hong Kong, CMHI is the flagship company of China Merchants Group. CMHI was listed on the Hong Kong Stock Exchange in 1992. Upon robust growth and increased market recognition, it became a Hong Kong Hang Seng Index constituent stock in September 2004. In 2017, the designed capacity of SCT reached 3.8 (million TEUs) and it handled a container throughput of 5.3 (million TEUs).

Secondly, YICT was established in 1993. As a joint venture by the Shenzhen Yantian Port Group and Hutchison Port Holdings Trust (HPH Trust), the former is a state-owned enterprise affiliated with State-owned Assets Supervision and the Administration Commission of Shenzhen Municipal People’s Government. The latter is the first publicly traded container port business trust listed on the Singapore Exchange. It is affiliated with Hutchison Ports, who is the global leader in the container port industry based on throughput, and a subsidiary of CK Hutchison Holdings Limited. HPH Trust owns interests in the world class deep-water container port assets located in two of the world’s busiest container port cities by throughput at Hong Kong and Shenzhen. In 2017, the designed capacity of YICT is 8.8 (million TEUs), and it handled a container throughput of 12.7 (million TEUs).

5.2. Assumptions about the Parameters of the Model. First, according to background information about the case study, the designed capacities of container terminals are set at

\[ s_1 = 3.8 \text{ (million TEUs)} \] and \[ s_2 = 8.8 \text{ (million TEUs)} \], respectively.

Second, following the survey conducted by Zheng and Negenborn (2014) and the related data of the port of Shenzhen in 2017, the average container dwell time in the port of Shenzhen is approximately 6 days, and the daily storage charge for a 20 feet dry container is 11 RMB. Considering the largest value that the volume-capacity can take is 3, the value of \( t \) is set at 22 RMB (11 \times 2).

Third, the cross-price parameter is a measure of the sensitivity of the demand for one container port to changes in the other container port’s price. Luo et al. [29] set this value at 0.1 million TEUs/HK$. Combined with (3) and (4), we have

\[ \alpha/ (\beta_1 + \beta_s + t) = 3 \] and \[ \beta_1 s_2 / (\beta_1 + \beta_s + t) = 0.1 \], which means that \( \alpha = 385.85 \) and \( \beta = 8.46 \).

All parameters values used in the application are summarized in Table 1.

5.3. Comparison Analysis. After applying the values in Table 1 to the model, we obtain Table 2.

From Table 2, we find some interesting observations as follows.

(i) A higher terminal tariff causes a lower container throughput. In particular, the container throughput of the foreign-owned terminal falls faster.

(ii) A higher terminal tariff also causes more profit for the state-owned terminal and less profit for the foreign-owned terminal.

(iii) A higher terminal tariff eventually leads to the total social welfare decreasing more quickly.

5.4. Sensitivity Analysis. We perform a sensitivity analysis to gain further managerial insights. Specifically, the sensitivity analysis can be used to judge the effect trends of the container terminal throughput and the whole throughput as well as the total social welfare of the container terminals in one port if the designed capacity expands.

We present the results in Table 3.

In Table 3, we increase the capacity of foreign-owned container terminal by 0.5 million TEUs at a time while keeping all other parameters the same as the base case that was illustrated in Table 1. From the base case (Case I) to Case II, with the designed capacity of foreign-owned container terminal expanding by 131.6%, the container throughput and profit of the foreign-owned container terminal continually increase 74.2% and 74.3%, respectively. In addition, the total social welfare of the port remains positive, thus increasing by 6.7%.

Table 3 shows that without external competition from other ports, the foreign-owned container terminal has an incentive to expand its capacity. If all other parameters are maintained, we find some interesting results as follows:

(i) A larger capacity of the foreign-owned container terminal leads to a higher container throughput for the container terminal, while that of the state-owned container terminal drops slightly. Consequently, the total container throughput remains increasing.

(ii) A larger capacity of the foreign-owned container terminal also leads to more profit for it and lower profit for the state-owned container terminal.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Concept</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Potential demand</td>
<td>385.85</td>
</tr>
<tr>
<td>s_1</td>
<td>Designed capacity</td>
<td>3.8</td>
</tr>
<tr>
<td>s_2</td>
<td>Designed capacity</td>
<td>8.8</td>
</tr>
<tr>
<td>t</td>
<td>Waiting costs</td>
<td>22</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Tariff sensitivity</td>
<td>8.46</td>
</tr>
</tbody>
</table>
### Table 2: Equilibrium of no price-matching and price-matching strategies.

<table>
<thead>
<tr>
<th>Game</th>
<th>$f_1$ (RMB)</th>
<th>$f_2$ (RMB)</th>
<th>$q_1$ (million TEUs)</th>
<th>$q_2$ (million TEUs)</th>
<th>$\pi_1$ (million RMB)</th>
<th>$\pi_2$ (million RMB)</th>
<th>Total welfare (million RMB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Subgame 1</td>
<td>99</td>
<td>133</td>
<td>11.88</td>
<td>13.91</td>
<td>1175</td>
<td>1849</td>
<td>5837</td>
</tr>
<tr>
<td>Subgame 2</td>
<td>193</td>
<td>193</td>
<td>5.70</td>
<td>13.20</td>
<td>1100</td>
<td>2547</td>
<td>5158</td>
</tr>
</tbody>
</table>

### Table 3: Sensitivity analysis of increasing container terminal capacity.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$s_1$ (million TEUs)</th>
<th>$q_1$ (million TEUs)</th>
<th>$q_2$ (million TEUs)</th>
<th>$\pi_1$ (million RMB)</th>
<th>$\pi_2$ (million RMB)</th>
<th>Total welfare (million RMB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1</td>
<td>3.8</td>
<td>5.70</td>
<td>13.20</td>
<td>1100</td>
<td>2547</td>
<td>5158</td>
</tr>
<tr>
<td>Case 2</td>
<td>4.3</td>
<td>6.25</td>
<td>12.78</td>
<td>1205</td>
<td>2466</td>
<td>5202</td>
</tr>
<tr>
<td>Case 3</td>
<td>4.8</td>
<td>6.76</td>
<td>12.39</td>
<td>1304</td>
<td>2390</td>
<td>5244</td>
</tr>
<tr>
<td>Case 4</td>
<td>5.3</td>
<td>7.24</td>
<td>12.02</td>
<td>1396</td>
<td>2318</td>
<td>5283</td>
</tr>
<tr>
<td>Case 5</td>
<td>5.8</td>
<td>7.69</td>
<td>11.67</td>
<td>1484</td>
<td>2251</td>
<td>5319</td>
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<tr>
<td>Case 6</td>
<td>6.3</td>
<td>8.12</td>
<td>11.34</td>
<td>1566</td>
<td>2187</td>
<td>5354</td>
</tr>
<tr>
<td>Case 7</td>
<td>6.8</td>
<td>8.52</td>
<td>11.03</td>
<td>1644</td>
<td>2127</td>
<td>5387</td>
</tr>
<tr>
<td>Case 8</td>
<td>7.3</td>
<td>8.90</td>
<td>10.73</td>
<td>1717</td>
<td>2070</td>
<td>5418</td>
</tr>
<tr>
<td>Case 9</td>
<td>7.8</td>
<td>9.26</td>
<td>10.45</td>
<td>1787</td>
<td>2016</td>
<td>5448</td>
</tr>
<tr>
<td>Case 10</td>
<td>8.3</td>
<td>9.61</td>
<td>10.19</td>
<td>1854</td>
<td>1965</td>
<td>5476</td>
</tr>
<tr>
<td>Case 11</td>
<td>8.8</td>
<td>9.93</td>
<td>9.93</td>
<td>1917</td>
<td>1917</td>
<td>5503</td>
</tr>
</tbody>
</table>

(iii) A larger capacity of the foreign-owned container terminal eventually causes the total social welfare to continually increase.

### 6. Conclusions

With gradual deregulation of foreign investment into its port sector, Sino-foreign joint ventures and even wholly foreign-owned terminals are permitted in China. In the meantime, the latest reform of China was to deregulate the port tariffs according to China’s Ministry of Transport and National Development and Reform Commission, which issued the method of port tariffs on July 12, 2017. Under this background, the container terminal operator of China can independently formulate specific tariffs no more than the upper bound of the government’s guidance, such as cargo dues, pilotage fees, tug fees, parking fees, special trimming charges, and oil fence charges. Moreover, the competitive service is charged based on governmental guidance and uniform pricing from market regulation. At this moment, although price-matching guarantees seem available to container terminals according to the port deregulation of the pricing policy, it is not clear whether they will employ price-matching policies or not, or whether the price-matching strategy is the only Nash equilibrium for the two container terminals of one port. We are also concerned with how to avoid tacit collusion between container terminals, which would otherwise result in social welfare losses in the process of port deregulation.

In this paper, we focus on port deregulation’s influence on the price-matching strategy of container terminals in one port, which is mainly financed by foreign or state capital, using a two-stage noncooperative game theoretical model, and then solve the game through backwards-induction. The main findings are different from previous conclusions in that the only Nash equilibrium available is for both terminals to adopt the price-matching policy if the capacities of the two terminals are not equal, as well as price-matching strategies facilitating tacit collusion between container terminals. To address the differential designed capacity and port deregulation of the two container terminals in one port comprehensively, the main contribution of this paper to the intraport competition literature is that price-matching adopted by both container terminals is not the Nash equilibrium, even though the foreign-owned container terminal maximizes its own profit. Therefore, as an economic entity with port authority, the state-owned container terminal formulated its pricing strategy from the perspective of the local government, comprehensively considering its own profits and the surplus of the container shipper. The policy recommendation is that the price-matching strategy not employed by the state-owned container terminal can avert tacit collusion between the container terminals of one port. Due to raising the equilibrium container port tariff, the price-matching strategy employed by the container terminals of one port will eventually result in potential social welfare losses.

The author recognizes that there is still space to further study this topic. Potential directions for future study include but are not limited to the following. First, we should further enrich the current optimizing model to consider the different marginal operation cost and technological spillover, as well as trade facilitation, revenue maximization and other objectives. Second, the case study could be better linked with various aspects of the model when further detailed data are able to be collected in the future, as it is usually a combination of commercial, governance, institutional, and social decisions. Third, more quantitative tests and calculations of the game.
in this case remain. Finally, the optimization of the pricing strategy of container terminal should be taken into the global supply chain consideration in future study.

Data Availability
The data used to support the findings of this study are included within the article.

Conflicts of Interest
The author declares that there are no conflicts of interest regarding the publication of this paper.

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