Research Article

Interval Forecasting in Supply Chain with Small Sample

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To deal with the forecasting with small samples in the supply chain, three grey models with fractional order accumulation are presented. Human judgment of future trends is incorporated into the order number of accumulation. The output of the proposed model will provide decision-makers in the supply chain with more forecasting information for short time periods. The results of practical real examples demonstrate that the model provides remarkable prediction performances compared with the traditional forecasting model.

1. Introduction

The supply chain forecasting can be made more accurate when human judgements are incorporated into the forecast system [1]. The performance of purely quantitative forecasting method can be flawed when historical data is limited [2]. In certain cases, the number of available samples is so scarce that providing reliable estimates is a challenging problem [3]. In order to analyze and predict the small samples systems accurately, a large number of studies on supply chain forecasting using grey models and improved grey models have been reported [4–7]. The GM(1,1) (grey first-order differential equation model) forecasting model is formulated for solving limited time series data [8]. There is no strict hypothesis for the distribution of parent data. However, the reliability and validity of the GM(1,1) have never been discussed. First, without considering other causes when using limited time series data, the forecasting of the GM(1,1) is unreliable and provides insufficient information to a decision-maker. Second, human judgment cannot be incorporated into the forecasting systems. A grey prediction model with fractional-order accumulation is newly proposed and has better performance and more freedom compared with the traditional grey model [9]. Multivariable nonequidistance grey model with fractional order accumulation is discussed [10]. The grey model predictor design is modified by using optimal fractional-order accumulation calculus [11]. The air

quality indicators in the Beijing-Tianjin-Hebei region are predicted by the grey prediction model with fractional-order accumulation [12]. Therefore, in This paper, three grey models with fractional-order accumulation are put forward. The output of the proposed model can obtain a general interval. This interval is not the grey number [13]. It will provide more forecasting information for a short time period. Most importantly, human judgment for future trend is incorporated into the order number of accumulation.

The rest of the paper proceeds as follows. Grey single variable forecasting models with fractional order accumulation are presented in Section 2. The multivariable grey model with fractional order accumulation is proposed in Section 3. Interval forecasting method of grey models for supply chain management is discussed in Section 4. Some conclusions are given in the last section.

2. Grey Single Variable Forecasting Models with Fractional Order Accumulation

For the original data sequence $X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n)\}$, a new sequence $X^{(p/q)} = \{x^{(p/q)}(1), x^{(p/q)}(2), \dots, x^{(p/q)}(n)\}$ can be generated by the fractional order accumulated generating operator (FAGO) as $x^{(p/q)}(k) = \sum_{j=1}^k \binom{k-j+r-1}{k-j} x^{(0)}(j)$, $k = 1, 2, \dots, r$ [14]. $GM^{p/q}(1,1)$

TABLE 1: The predictive values of different models.

Time	resilience performance	GM(1,1)	GM ^{0.7} (1,1)
1	1763	1763	1763
2	2376	2441	2384
3	2763	2685	2740
4	2983	2954	3002
5	3216	3250	3211
MAPE		2.0	0.4

model and grey exponential smoothing are given in the following way.

2.1. GM^{p/q}(1, 1) Model

Definition 1.

$$x^{(p/q)}(k) - x^{(p/q)}(k-1) + az^{(p/q)}(k) = b \quad (1)$$

is referred to as the original form of the GM^{p/q}(1,1) model, where $z^{(p/q)}(k) = (x^{(p/q)}(k) + x^{(p/q)}(k-1))/2$, $k = 2, 3, \dots, n$. It is the traditional GM(1,1) model when $p/q = 1$. The ordinary least squares estimate sequence of the GM^{p/q}(1,1) model satisfies

$$\begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} = (B^T B)^{-1} B^T Y \quad (2)$$

where

$$Y = \begin{bmatrix} x^{(p/q)}(2) - x^{(p/q)}(1) \\ x^{(p/q)}(3) - x^{(p/q)}(2) \\ \vdots \\ x^{(p/q)}(n) - x^{(p/q)}(n-1) \end{bmatrix}, \quad (3)$$

$$B = \begin{bmatrix} -z^{(p/q)}(2) & 1 \\ -z^{(p/q)}(3) & 1 \\ \vdots & \vdots \\ -z^{(p/q)}(n) & 1 \end{bmatrix}.$$

The solution of the whitenization equation $dx^{(p/q)}(t)/dt + ax^{(p/q)}(t) = b$ for GM^{p/q}(1,1) is given by

$$\hat{x}^{(p/q)}(t) = \left[x^{(0)}(1) - \frac{\hat{b}}{\hat{a}} \right] e^{-\hat{a}t} + \frac{\hat{b}}{\hat{a}}, \quad (4)$$

The p/q ($0 < p/q \leq 1$) order inverse accumulated generating operator (IAGO) of $X^{(p/q)}$ is

$$\begin{aligned} \hat{X}^{(0)} &= \alpha^{(p/q)} \hat{X}^{(p/q)} = \alpha^{(1)} \hat{X}^{(1-p/q)}(k) \\ &= \{ \alpha^{(1)} \hat{x}^{(1-p/q)}(1), \alpha^{(1)} \hat{x}^{(1-p/q)}(2), \dots, \\ &\quad \alpha^{(1)} \hat{x}^{(1-p/q)}(n) \} = \{ \hat{x}^{(0)}(1), \hat{x}^{(1-p/q)}(2) - \hat{x}^{(0)}(1), \dots, \\ &\quad \hat{x}^{(1-p/q)}(n) - \hat{x}^{(1-p/q)}(n-1) \}. \end{aligned} \quad (5)$$

The procedures of GM^{p/q}(1,1) can be summarized as follows.

Step 1. For the sequence $x^{(0)} = \{1763, 2376, 2763, 2983, 3216\}$ from [4], 0.7-order accumulation sequence $x^{(0.7)} = \{1763, 3610, 5475, 7275, 9094\}$. B and Y in (2) are

$$B = \begin{bmatrix} -2687 & 1 \\ -4543 & 1 \\ -6375 & 1 \\ -8184 & 1 \end{bmatrix}, \quad (6)$$

$$Y = \begin{bmatrix} 1847 \\ 1865 \\ 1800 \\ 1819 \end{bmatrix}.$$

Step 2. Substituting B and Y into (2), we have the parameters a, b . Then GM^{0.7}(1,1) can be represented as

$$\frac{dx^{(0.7)}(t)}{dt} + 0.008x^{(0.7)}(t) = 1877, \quad t = 1, 2, \dots \quad (7)$$

Step 3. The 0.7-order accumulated generating operation values $\hat{x}^{(0.7)}$ can be obtained by employing (4):

$$\hat{x}^{(0.7)} = \{1763, 3618, 5458, 7283, 9094\}. \quad (8)$$

The 2-order accumulated generating operation values $\hat{x}^{(1)}$ is

$$\hat{x}^{(1)} = \hat{x}^{(0.7)(0.3)} = \{1763, 4147, 6888, 9890, 13101\}. \quad (9)$$

The predicted values $\hat{x}_1^{(0)}$ are $\hat{x}_1^{(0)} = \{1763, 2384, 2740, 3002, 3211\}$, which are listed in Table 1. Mean absolute percentage error (MAPE = $100\%(1/n) \sum_{k=1}^n |(x^{(0)}(k) - \hat{x}^{(0)}(k))/x^{(0)}(k)|$) compares the actual values with the forecasted values to evaluate the precision. The use of intelligent methods is a new trend in the grey models [15, 16]. In this paper, particle swarm optimization is adopted to find the optimal order which produces the minimum MAPE. The experiments are conducted in the MATLAB R2015b.

The results of Table 1 mean that GM^{0.7}(1,1) have better performance than the traditional GM(1,1). To investigate the feasibility of the GM^{p/q}(1,1) model in the supply chain, the following cases are given.

TABLE 2: The periodic resilience performance indicators of a firm [4].

Time	RPI 1	RPI 2	RPI 3	RPI 4	RPI 5
1	2386	1345	674	6784	1763
2	2399	1876	567	5647	2376
3	3453	2345	453	4563	2763
4	3645	2784	363	3743	2983
5	4064	3764	278	3345	3216

TABLE 3: The error analysis of different models.

	RPI 1	RPI 2	RPI 3	RPI 4	RPI 5
MAPE of GM(1,1)	6	3	1	3	2
MAPE of $GM^{p/q}(1,1)$	4.1	2.3	0.4	1.2	0.4

TABLE 4: Actual data of the sales volume of registered printers in Taiwan by year.

Year	2002	2003	2004	2005	2006	2007	2008
Sales volume	1047957	1138440	1091705	1083829	981984	838822	820161

TABLE 5: The forecasting values and MAPE of different models.

Forecasting technique	Forecasting value for 2008	Actual value	MAPE
Moving average	910403	820161	11.0
Exponential smoothing	854168		4.1
Quadratic trend	646699		21.2
Cubic trend	665629		18.8
Non-linear trend	876998		6.9
ANP	830797		1.3
$GM^{0.4}(1,1)$	817142		0.4
GM(1,1)	836018		1.9

TABLE 6: The predictive values of different models (units:ten thousand).

Year	LCD TV demand	single exponential model	AR(2) model	$GM^{0.9}(1,1)$
2001	81		59	81
2002	150	151	95	197
2003	393	149	183	423
2004	970	567	505	898
2005	2018	1086	1092	1903
2006	4300	2682	2551	4026
MAPE		37.6	41.1	10.9

The Supply Chain Performance Resilience Forecasting Example [4]. The periodic resilience performance indicators of case firm are listed in Table 2. By $GM^{p/q}(1,1)$, respectively, the forecasting values and MAPE of different models are given in Table 3.

In Table 3, the error analysis shows that $GM^{p/q}(1,1)$ ensure the best fit of the data to achieve strong prediction capability. $GM^{p/q}(1,1)$ can predict the supply chain resilience of an Indian electronics manufacturer.

The Sales Volume of Printers Forecasting Example [20]. Shih et al used the sales volume of printers from 2002 to 2007 as

training data. The volume of 2008 is the testing data. Actual values are listed in Table 4 and the errors of eight models are shown in Table 5. The results show that the forecasting accuracy of the $GM^{0.4}(1,1)$ is better than traditional GM(1,1).

The LCD TV Demand Forecasting Example [19]. Tsaur collected the LCD TV demand data from 2001 to 2006 as shown in Table 6. It was obvious that the collected data are limited time series data, and therefore the possible methods such as GM(1,1) and single exponential smoothing model were used for forecasting. The forecasting results are shown in Table 6. In Table 6, we find that the $GM^{p/q}(1,1)$ has

FIGURE 1: The line of $X^{(0)}$. FIGURE 2: The line of $X^{(0.81)}$.

better forecasting performance of MAPE than the single exponential smoothing model.

2.2. Grey Exponential Smoothing Model. FAGO is widely used in grey models for its ability to smooth the randomness of original data [9]. Through FAGO, the disorderly data may be converted into regular form. Actually, by using FAGO, the disorderly sequence may be converted into an approximately increased sequence. For example, $X^{(0)} = \{16, 3, 7, 2, 4, 5\}$, the 0.81-AGO sequence is $X^{(0.81)} = \{16, 15.96, 21.16, 20.86, 23.28, 26.54\}$. The lines of these sequences are illustrated in Figures 1 and 2, respectively. Comparing the two lines, it is clear that the trend of sequence $X^{(0.81)}$ in Figure 2 is more obvious than the sequence $X^{(0)}$ in Figure 1.

An irregular and increased sequence can be predicted by using double exponential smoothing (GDES). Then we give the following definition.

Definition 2 (see [21]). For the original time series $X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n)\}$, r -AGO is given in Definition 1. GDES follows the equations

$$\begin{aligned} S'(k) &= \alpha x^{(r)}(k) + (1 - \alpha) S'(k-1) \\ S''(k) &= \alpha S'(k) + (1 - \alpha) S''(k-1) \\ b_k &= \frac{\alpha}{1 - \alpha} (S'(k) - S''(k)) \\ a_k &= 2S'(k) - S''(k) \end{aligned} \quad (10)$$

where $0 \leq \alpha \leq 1$, $S'(k)$ and $S''(k)$ are the single and double exponential smoothing values for time k , respectively. The forecasting form is $x^{(r)}(k) = a_k + kb_k$.

If $r = 0$, GDES is the traditional double exponential smoothing. The process of calculating GDES can be summarized as follows.

Step 1. Set the order number r and obtain the r -FAGO sequence of $X^{(0)}$ according to Definition 1.

Step 2. Calculate the parameters (a_k and b_k) by using Definition 2.

Step 3. Compute the predictive value using $\hat{x}^{(r)}(k+m) = a_k + mb_k$, where m is the out-of-sample size.

Step 4. Transform the prediction value back to the original sequence by means of IAGO [21].

In this paper, the predictive values are calculated by using different α 's. The α that produces a small mean square error for the fitted values and shows an expected future growth is chosen. We assumed that $S'(0)$ and $S''(0)$ are equal to the initial historical values.

In this case, the data from [17] is the spare demand of a firm. The accuracy of demand forecast significantly affects the firm's sustainability and profitability. To compare the GDES with traditional double exponential smoothing, the smoothing constant of GDES and traditional double exponential smoothing is set to $\alpha = 0.69$. The predictive values of two models are shown in Table 7.

As shown in Table 7, GDES can enhance the accuracy of small data forecasting.

3. The Multivariable Grey Model with Fractional Order Accumulation

The existing multivariable grey models (GMC(1,N)) all used first-order accumulated generating operation sequence [22–26]. In this section, the fractional order accumulated generating operation sequence is introduced into the GMC(1,N) model.

Definition 3. It is assumed that $X_1^{(0)} = \{x_1^{(0)}(1), x_1^{(0)}(2), \dots, x_1^{(0)}(r)\}$ is the sequence of system characteristic and $X_i^{(0)} = \{x_i^{(0)}(1), x_i^{(0)}(2), \dots, x_i^{(0)}(r)\}$ ($i = 2, 3, \dots, N$) are the sequences of relevant factors. Then

$$\begin{aligned} \frac{dx_1^{(p/q)}(t)}{dt} + b_1 x_1^{(p/q)}(t) \\ = b_2 x_2^{(p/q)}(t) + b_3 x_3^{(p/q)}(t) + \dots + b_n x_n^{(p/q)}(t) + u \end{aligned} \quad (11)$$

is the GMC $^{p/q}$ (1,N) model, where $x_i^{(p/q)}(k)$ is the p/q order accumulation of $x_i^{(0)}(k)$, $x_i^{(p/q)}(k) = \sum_{j=1}^k \binom{k-j+r-1}{k-j} x_i^{(0)}(j)$, $k = 1, 2, \dots, r$ [14]. It is the traditional GMC(1,N) model when $p/q = 1$. Using ordinary

TABLE 7: The predictive values of different models.

Year	Actual demand [17]	GDES	traditional double exponential smoothing
2006	16	16	16
2007	3	3	-2
2008	7	9	4
2009	2	2	-1
2010	4	4	3
2011	5	6	5
MAPE		8.5	62.4

TABLE 8: The customer perception indicators of engine product [18].

Year	sales volume	energy efficiency	product safety	cost performance	service promptness
2005	850.15	8.2112	1	1	1
2006	972.18	8.5027	1.0326	1.0786	1.0405
2007	1078.47	8.2711	1.0352	1.1029	1.007
2008	1069.28	8.0806	1.0521	1.0624	0.9818
2009	1259.26	8.1915	1.0846	1.0277	1.0028
2010	1454.51	9.0849	1.0911	1.0705	1.1229
2011	1413.92	8.7934	1.0404	0.9908	1.0796
2012	1672.11	8.9042	1.0612	0.9769	1.0992
2013	1948.23	9.3961	1.1289	1.0243	1.1704
2014	2063.35	9.938	1.2214	0.9711	1.3701

least squares, the parameters of $\text{GMC}^{p/q}(1, N)$ model are estimated:

$$[\hat{b}_1, \hat{b}_2, \dots, \hat{b}_n, \hat{u}]^T = (B^T B)^{-1} B^T Y \quad (12)$$

where

$$Y = \begin{bmatrix} x_1^{(p/q)}(2) - x_1^{(p/q)}(1) \\ x_1^{(p/q)}(3) - x_1^{(p/q)}(2) \\ \vdots \\ x_1^{(p/q)}(n) - x_1^{(p/q)}(n-1) \end{bmatrix}, \quad (13)$$

$$B = \begin{bmatrix} -0.5(x_1^{(p/q)}(2) + x_1^{(p/q)}(1)) & x_2^{(p/q)}(2) & \dots & x_n^{(p/q)}(2) & 1 \\ -0.5(x_1^{(p/q)}(3) + x_1^{(p/q)}(2)) & x_2^{(p/q)}(3) & \dots & x_n^{(p/q)}(3) & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -0.5(x_1^{(p/q)}(r-1) + x_1^{(p/q)}(r)) & x_2^{(p/q)}(r) & \dots & x_n^{(p/q)}(r) & 1 \end{bmatrix}.$$

Set $\hat{x}_1^{(0)}(1) = x_1^{(0)}(1)$; when $t \geq 2$, the convolution integral of the right hand of (12) can also be discretised as

$$x_1^{(p/q)}(t) = x_1^{(0)}(1) e^{-b_1(t-1)} + \sum_{\tau=2}^t \left\{ e^{-b_1(t-\tau+0.5)} \frac{f(\tau) + f(\tau-1)}{2} \right\}, \quad (14)$$

where $f(t) = b_2 x_2^{(p/q)}(t) + b_3 x_3^{(p/q)}(t) + \dots + b_n x_n^{(p/q)}(t) + u$. Then the p/q ($0 \leq p/q \leq 1$) order inverse accumulated generating operator of the predicted value is the same as (5).

For example, the customer perception indicators of engine product are listed in Table 8 from [18]. To obtain the sales volume forecasting value, the engine sales volume is $X_1^{(0)}$. Energy efficiency, product safety, cost performance, and service promptness are $X_2^{(0)}$, $X_3^{(0)}$, $X_4^{(0)}$, and $X_5^{(0)}$, respectively. This case has limited data. Thus we can build $\text{GMC}^{p/q}(1, 5)$.

TABLE 9: The predictive values of different models.

Year	sales volume	Quadratic polynomial model [18]	GMC ^{0.15} (1,5)
2005	850.15	850.15	850.15
2006	972.18	861.7971	965.79
2007	1078.47	929.5540	1055.96
2008	1069.28	1058.6918	1144.18
2009	1259.26	1103.4097	1264.41
2010	1454.51	1285.5118	1399.63
2011	1413.92	1525.6792	1555.71
2012	1672.11	1567.0815	1743.53
2013	1948.23	1786.3352	1934.37
2014	2063.35	2095.0247	2100.82
MAPE		8.24	3.0

GMC^{0.15}(1,5) has the smaller MAPE than the GMC^{p/q}(1,5) with the other accumulated order numbers. Thus the results of GMC^{0.15}(1,5) are given in Table 9.

By comparing the MAPE in Table 9, the example of demand forecasting for engine products demonstrated the applicability and validity of the GMC^{p/q}(1,5).

Actually, $X_1^{(0)}$ is taken as the supply chain demand. $X_i^{(0)} = \{x_i^{(0)}(1), x_i^{(0)}(2), \dots, x_i^{(0)}(r)\}$ ($i = 2, 3, \dots, N$) are the sequences of relevant factors. GMC^{p/q}(1,N) are suitable to predict the supply chain demand, because GMC^{p/q}(1,N) can depict the model more precisely with new degrees of freedom and performances.

4. Interval Forecasting of Grey Models for Supply Chain Management

Fractional derivatives accumulate the whole history of the system in weighted form and it is referred to as the memory effect. As we know, big samples forecasting models depend on statistical laws. In this section, the small samples forecasting models which depend on the memory effect are a new path. For example, $x^{(1)}(k)$ in grey forecasting models denotes the weight of $x^{(0)}(i)$ ($i = 1, 2, \dots, k$) as 1. The larger p/q of $x^{(p/q)}(k)$ is, the larger the weight of old data is. The smaller p/q of $x^{(p/q)}(k)$ is, the smaller the weight of old data is. Reducing p/q can reduce the weights of old data, which can put more emphasis on the newer data.

Although the grey models (including GM^{p/q}(1,1), GDES^{p/q}, and GMC^{p/q}(1,N)) have better forecasting performance, the input data is too limited and the forecasting values are point estimations which supply too limited information for a decision-maker. In this section, in order to obtain a better fitted model with smaller estimated errors and a smaller support forecasting interval, a novel fractional grey modelling mechanism is applied for solving limited time series data using the following steps.

Actually, for many forecasting cases in the supply chain, the accumulated order number often satisfied $0 < p/q < 1$. If the future trend is similar to the newer data, the setting value of p/q is smaller. If the future trend is similar to the order data,

the setting value of p/q is bigger. In the supply chain, for many forecasting cases with limited information, if it is difficult to judge the future trend, the forecasting value must be in a forecasting interval. The upper value and the lower value of forecasting interval can be obtained by different fractional grey models.

For the original data sequence $X^{(0)} = \{x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n)\}$, the forecasting values of GM^{0.1}(1,1) model (or GDES^{0.1} and GMC^{0.1}(1,N)) are denoted as

$$\widehat{X}_{0.1}^{(0)} = \{\widehat{x}_{0.1}^{(0)}(1), \widehat{x}_{0.1}^{(0)}(2), \dots, \widehat{x}_{0.1}^{(0)}(n)\}, \quad (15)$$

and the forecasting values of GM^{0.9}(1,1) model (or GDES^{0.9} and GMC^{0.9}(1,N)) are denoted as

$$\widehat{X}_{0.9}^{(0)} = \{\widehat{x}_{0.9}^{(0)}(1), \widehat{x}_{0.9}^{(0)}(2), \dots, \widehat{x}_{0.9}^{(0)}(n)\}. \quad (16)$$

For the same point k , the smaller of $\widehat{x}_{0.9}^{(0)}(k)$ and $\widehat{x}_{0.1}^{(0)}(k)$ is set to the lower value of the forecasting interval. The bigger of $\widehat{x}_{0.9}^{(0)}(k)$ and $\widehat{x}_{0.1}^{(0)}(k)$ is set to the upper value of the forecasting interval. Thus we can obtain a grey forecasting interval. For example,

$$X^{(0)} = \{1763, 2376, 2763, 2983, 3216\}, \quad (17)$$

the forecasting values of GM^{0.1}(1,1) model (or GDES^{0.1} and GMC^{0.1}(1,N)) are

$$\widehat{X}_{0.1}^{(0)} = \{1763, 2353, 2756, 3023, 3197, 3307\}, \quad (18)$$

and the forecasting values of GM^{0.9}(1,1) model (or GDES^{0.9} and GMC^{0.9}(1,N)) are

$$\widehat{X}_{0.9}^{(0)} = \{1763, 2418, 2709, 2973, 3233, 3497\}. \quad (19)$$

Therefore, the grey forecasting interval values are

$$\begin{aligned} & \{[1763, 1763], [2353, 2418], [2709, 2756], \\ & [2973, 3023], [3197, 3233], [3307, 3497]\}. \end{aligned} \quad (20)$$

The above method is called the grey interval model (GIM). To investigate the feasibility of GIM in supply chain forecasting, seven cases are listed as follows.

TABLE 10: The fitting values of different models.

Month	sales volume	$GM^{0.1}(1,1)$	$GM^{0.9}(1,1)$
1	103148	103148	103148
2	119066	119269	124066
3	137468	139992	143842
4	155892	162736	164238
5	186370	187063	186099
6	216230	212887	209924
7	249776	240225	236108
8	266829	269135	265023
9	294475	299698	297046
10	334595	332004	332578
11	365988	366158	372051
MAPE		1.4	2.4

TABLE 11: The predictive results of different models.

model	sales volume	predictive values	interval value
simple linear regression (Xue 2015)	414032	436402	[425763, 494570]
GIM	414032		[402269, 415940]

TABLE 12: The predictive results of number of end-of-life vehicles.

Year	number of end-of-life vehicles	interval value
2008	1977	[1977, 1977]
2009	2455	[2407, 2567]
2010	2818	[2460, 2527]
2011	2266	[2422, 2432]
2012	2241	[2319, 2369]
2013	2265	[2199, 2320]

Case 1. The data are from [27]. The data come from the sales volume of semiconductor components in a firm. To compare the proposed model with the other model, the data before November are the training set. The fitting values of different models are given in Table 10.

The predictive results (December) of different models are given in Table 11.

We can clearly see from the results given in Table 10 that $GM^{p/q}(1,1)$ can obtain higher fitting accuracy. From the results given in Table 11, we can see that GIM can obtain shorter intervals including the actual value. But the simple linear regression derives a wide forecasting interval which provides too little information for the decision-maker.

Case 2. The data come from [28]. The data refer to the numbers of end-of-life vehicles in the West Anatolia. Forecasting the return flow of an end-of-life product is important for all decision levels of the reverse supply chain. In this paper, the predictive results of GIM are given in Table 12. The results of interval value can provide guidance to the managers and practitioners of recovery and recycling systems.

Case 3. The data come from [29]. In recent years, China's high production of thin film transistor liquid crystal display (TFT-LCD) panels has led to intense competition in this

industry, causing its supply to be greater than the actual demand. Under such circumstances, reducing inventory levels and inventory turnover is a critical issue faced by panel manufacturers. The production quantity is maintained at an appropriate balance point considering the total cost. To achieve this balance in production marketing coordination, an accurate short-term demand forecast is essential. Because the demand for TFT-LCD panels is affected tremendously by the global business cycle, whereas the business cycle has substantially changed in recent years, the use of numerous long-term historical observations does not satisfy the needs of the short-term forecasts. Therefore, this study applied data produced by a leading Taiwanese TFT-LCD panel manufacturer to verify the forecasting performance of GIM [29]. In this paper, the predictive results of GIM are given in Table 13. The actual value is in the forecasting interval. It demonstrates that GIM can provide more information with predictive values.

Case 4. The data come from [30]. They are the return quantity for a third party e-waste firm in Turkey. The predictive results are listed in Table 14.

The predictive results in Table 14 indicate that $GM^{0.1}(1,1)$ and $GM^{0.9}(1,1)$ can obtain small fitting errors. By GIM, the obtained forecasting interval indeed includes the actual value.

TABLE 13: The predictive results of demand of TFT-LCD panels.

Months	Actual value	$GM^{0.1}(1,1)$	$GM^{0.9}(1,1)$	interval value
4	1.718	1.718	1.718	[1.718, 1.718]
5	1.728	1.714	1.715	[1.714, 1.715]
6	1.714	1.735	1.740	[1.735, 1.740]
7	1.753	1.746	1.741	[1.741, 1.746]
MAPE		0.6	0.7	
8	1.742	1.748	1.729	[1.729, 1.748]

TABLE 14: The predictive results of return quantity for third party.

Time	Actual value	$GM^{0.1}(1,1)$	$GM^{0.9}(1,1)$	interval value
1	536.4	536.4	536.4	[536.4, 536.4]
2	538.61	538.2	535.8	[535.8, 538.2]
3	569.35	569.6	554	[554, 569.6]
4	613	612.8	565.3	[565.3, 612.8]
MAPE		0.03	2.7	
5	633.32	664.6	572.6	[572.6, 664.6]

TABLE 15: The forecasting interval value of the proposed model.

Forecasting technique	Actual value	interval value
GIM	820161	[813065, 963371]

TABLE 16: The forecasting interval value of the proposed model.

Year	Actual demand [17]	$GDES^{0.1}$	$GDES^{0.9}$	interval value
2006	16	16	16	
2007	3	-1	4	[-1,4]
2008	7	5	9	[5,9]
2009	2	0	2	[0,2]
2010	4	3	4	[3,4]
2011	5	5	6	[5,6]

Thus, we can conclude that GIM is effective for limited sample forecasting problems.

Case 5. Shih et al. used the sales volume of printers from 2002 to 2007 as the training data. The volume of 2008 is the testing data [20]. Actual values are listed in Table 4 and the results of GIM are shown in Table 15. The results show that the actual value of 2008 is in the forecasting interval. Thus, we can conclude that GIM is effective for the limited sample forecasting problem.

Case 6. Actual values are listed in Table 7 and the results of GIM are shown in Table 16. As shown in Table 16, the results show that the actual demand values are all in the forecasting interval. It also indicates that highly volatile demand can obtain volatile intervals. These results can provide more information for the supply chain manager. Thus, we can conclude that GIM is effective for limited sample forecasting problems.

Case 7 (see [19]). Tsaur collected the LCD TV demand data from 2001 to 2006 as shown in Table 6. The forecasting results are shown in Table 17. In Table 17, we find that the

interval of GIM is shorter than that of the fuzzy autoregressive model.

5. Conclusion

In theory, $GM^{p/q}(1,1)$, $GDES^{p/q}$, and $GMCP^{p/q}(1,N)$ models are discussed in this paper, respectively. Their application scopes are listed in Table 18. The results show that these models outperform the traditional grey models in the prediction precision.

Using traditional grey models, the forecasting values of limited data are point estimations which supply too limited information for a decision-maker. It is easy to arouse suspicion for this kind of point estimations. Thus, a novel interval forecasting method is put forward. From the empirical results, we found that the forecasting capability of the GIM is quite encouraging, but those of the time series models are not. In addition, the interval of GIM is smaller than that of the simple linear regression and fuzzy autoregressive model. In the real world, the environment is uncertain and we can only use a limited amount of data to provide future forecasts for a

TABLE 17: The predictive results of different models (units:ten thousand).

Year	LCD TV demand	the proposed model	fuzzy autoregressive model [19]
2001	81		
2002	150	[197,210]	[82,276]
2003	393	[423,476]	[318,532]
2004	970	[898,1024]	[687,969]
2005	2018	[1903,2156]	[1748,2163]
2006	4300	[4026,4493]	[4041,4779]

TABLE 18: The application scopes summary.

Situation	Method
$GM^{p/q}(1,1)$	exponential trend
$GDES^{p/q}$	volatile trend
$GMC^{p/q}(1,N)$	multi-variable limited data

TABLE 19: The proposed model summary.

Situation Judgement	Method
the future trend is similar to the newer data	p/q is smaller
the future trend is similar to the order data	p/q is bigger
difficult to judge the future trend	interval forecasting

short time period. In this situation, GIM is more satisfactory than the time series model.

In practice, GIM is summarized in Table 19. This paper contributes to the literature with an interval model for small samples forecasting in supply chain. It significantly improves small sample forecasting due to the interval result carrying more information. This paper demonstrates that grey modelling can be successfully applied to the forecasting problem in supply chain. Moreover, the proposed forecasting system can be used as a strategic tool for forecasting under uncertain conditions with a small amount of recent data.

For future research, in order to investigate the feasibility of the novel model in supply chain, it may be used for other real world cases for forecasting and the performances of the methods can be compared.

Data Availability

All the data are from the references in this paper.

Conflicts of Interest

The authors declare no conflicts of interest.

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