Research Article

A Mathematical Model and Error Analysis of Shearer Cutting Path Based on Its Attitude

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Abstract

The horizon control system is the key technology in the automation of a shearer. The achievement of accurate shearer cutting path plays an important role for horizon control. A mathematical model of cutting path in the local geographic coordinate frame was built. Error analysis based on genetic algorithm (GA) was studied to guarantee the accuracy of the shearer cutting path. Parameters from a MG1000/2660-WD shearer and data from a working face were used to obtain the shearer cutting path with reference to the local geographic coordinate frame. Also, with error analysis based on GA, the desired sensors were chosen, which allowed coordinate position errors of a shearer’s cutting path to be less than 0.01 m. The desired accuracies of the inertial navigation system and encoders mounted on the different shearers used in thin seam, medium-thickness seam, and thick seam were calculated.

Keywords: Horizon control; Shearer; Cutting path; Error analysis; Genetic algorithm

1. Introduction

Longwall automation can deliver benefits to the industry in terms of increased productivity and improvement in conditions for current on-face workers, particularly by removing them from hazards. Horizon control of a shearer is a vital part of longwall mining automation [4, 5]. The goal of horizon control is to automatically maintain the longwall shearer’s cutting trace within the coal seam so that the mining resource can be optimally extracted [6]. The cutting trace is mainly determined by the height of the drum and the attitude of a shearer. In the practical mining workplace, the height of drum is manually adjusted according to manual observations in order to avoid interference between the drum and the roof and floor of the face. A geometric track cutting-memory method has been applied in several mining shearers, but this approach only achieves the desired in-seam control over a small number of shearer cycles as it cannot take into account the higher than first-order trends and short-term variations in the seam horizon [7]. Some researchers have proposed drum height adjustment based on a coal interface detection method using natural gamma radiation, vibration signal, thermal infrared rays, and other pieces of sensor information to identify coal and rock as mentioned in [6–10]. However, the coal interface detection methods have never been successfully applied because of the complicated and unpredictable geological conditions in a coal-mining face.

Previous automation attempts have in large part been stymied by the inability to accurately determine the three-dimensional path of the longwall shearer as it systematically progresses through the coal panel. Without this information there is no absolute reference for controlling the motion of the equipment, and reliable, sustained automation cannot be achieved [11]. The Australian Coal Association Research Program (ACARP) commissioned a three-year ‘Landmark’ project. In the project, inertial navigation technology has, for the first time, allowed the position of a longwall shearer to be mapped in three dimensions.

It is important to obtain the cutting path of a shearer for maintaining the longwall shearer’s cutting trace within the coal seam. In this paper, a mathematical model based on the attitude of a shearer was built, with which the position of the drums of a shearer relative to the local geographic frame was determined. Considering that the performance of the automated horizon control system is critically dependent on the accuracy of a shearer’s cutting path, the error of this model, induced by the errors of attitude sensors, is
analyzed with genetic algorithm (GA). The error analysis of the model provides the theoretical basis for choosing suitable sensors.

2. Model of the Absolute Position of the Shearer Drums

The variations of coordinates of a shearer, including heading angle $\delta$, pitch angle $\beta$, rolling angle $\gamma$, and the angle between the arms and the mainframe $\theta_1$, $\theta_2$, are the most important factors influencing the cutting path, as shown in Figure 1. An inertial navigation system (INS) mounted onto the mainframe of a shearer not only measured the attitude and heading of a shearer (including heading angle $\delta$, pitch angle $\beta$, and rolling angle $\gamma$) but also measured the three-dimensional position with the aid of an odometer [12]. The angle between the arms and mainframe was measured by axial encoders. The INS and axial encoders fixed to the shearer are shown in Figure 2.

According to the principle of inertial navigation, the shearer frame is referred to as a body frame, as shown in Figure 2. The origin of the shearer frame coincides with the gravity centre of the shearer. The $X_b$-axis, $Y_b$-axis, and $Z_b$-axis are aligned with the pitch, roll, and yaw axes, respectively, of the shearer in which the navigation system is installed. A local geographic frame was built which has its origin at the location of the start point of the longwall mining face. The $X_g$-axis, $Y_g$-axis, and $Z_g$-axis are aligned in the directions of east, north, and the local vertical (up). As illustrated in Figures 1 and 3, the along-face direction is $X_g$(E), the face-advance (retreat) direction is $Y_g$(N), and the vertical direction is $Z_g$(U) [13].

As the reference point of the position of a shearer mainframe, $O_b$, is represented by, with respect to the local geographic frame $g$,

$$P^g_0 = [X_O \ Y_O \ Z_O]^T$$

$p^b_1$ and $p^b_2$ are taken as the centre position of the drums of a shearer with respect to the shearer frame $b$. They are described as follows:

$$P^b_1 = \begin{bmatrix} B \\ L \cos \theta_1 + \frac{l}{2} \\ L \sin \theta_1 \end{bmatrix} \quad (2)$$

$$P^b_2 = \begin{bmatrix} B \\ -L \cos \theta_2 - \frac{l}{2} \\ L \sin \theta_2 \end{bmatrix}$$

where $B$ is the centre distance between shearer drum and shearer mainframe, $l$ is the length of the mainframe of a shearer, and $L$ is the length of the arm, as presented in Figure 1. The subscripts 1 and 2 are the left and right drums, respectively.

According to the principle of transformation between body frame and local geographic frame mentioned in [14], the relationship between the shearer frame $b$ and the local geographic frame $g$ is given as follows:

$$P^g_i = P^g_0 + R^b_b \cdot P^b_i \quad (i = 1, 2)$$

where the transformation matrix from the shearer frame $b$ to the local geographic frame $g$ is

$$R^g_b = [R^b_g]^{-1} = \begin{bmatrix} \cos \delta & \sin \delta & 0 \\ -\sin \delta & \cos \delta & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \cos \gamma & 0 & \sin \gamma \\ 0 & 1 & 0 \\ -\sin \gamma & 0 & \cos \gamma \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \beta & -\sin \beta \\ 0 & \sin \beta & \cos \beta \end{bmatrix}.$$
Combining (1)–(4), we get the coordinates of the positions of
the shearer drums with respect to the local geographic frame
\( g \), as follows:

\[
P_{g1} = \begin{cases}
X_O + B \cos \delta \cos \gamma + \left( \frac{1}{2} + L \cos \theta_1 \right) \left( \sin \delta \cos \beta + \cos \delta \sin \gamma \sin \beta \right) \\
+ L \sin \theta_1 \left( - \sin \delta \sin \beta + \cos \delta \sin \gamma \cos \beta \right) \\
Y_O - B \sin \delta \cos \gamma + \left( \frac{1}{2} + L \cos \theta_1 \right) \left( \cos \delta \cos \beta - \sin \delta \sin \gamma \sin \beta \right) \\
- L \sin \theta_1 \left( \cos \delta \sin \beta + \sin \delta \sin \gamma \cos \beta \right) \\
Z_O - B \sin \gamma + \left( \frac{1}{2} + L \cos \theta_1 \right) \sin \beta \cos \gamma + L \sin \theta_1 \cos \beta \cos \gamma
\end{cases}
\]

\[
P_{g2} = \begin{cases}
X_O + B \cos \delta \cos \gamma - \left( \frac{1}{2} + L \cos \theta_2 \right) \left( \sin \delta \cos \beta + \cos \delta \sin \gamma \sin \beta \right) \\
+ L \sin \theta_2 \left( - \sin \delta \sin \beta + \cos \delta \sin \gamma \cos \beta \right) \\
Y_O - B \sin \delta \cos \gamma - \left( \frac{1}{2} + L \cos \theta_2 \right) \left( \cos \delta \cos \beta - \sin \delta \sin \gamma \sin \beta \right) \\
- L \sin \theta_2 \left( \cos \delta \sin \beta + \sin \delta \sin \gamma \cos \beta \right) \\
Z_O - B \sin \gamma - \left( \frac{1}{2} + L \cos \theta_2 \right) \sin \beta \cos \gamma + L \sin \theta_2 \cos \beta \cos \gamma
\end{cases}
\]

3. Error Analysis

It is known from (5) and (6) that the position error of the
shearer drums was mainly affected by the angle error of
\( \delta, \beta, \gamma, \theta_1 \), and \( \theta_2 \) which was determined by the sensors
including the INS and axial encoders. Usually, the price
of these sensors increases with the improvement in their
accuracy. Hence, INS and axial encoders with reasonable
accuracy were chosen according to the tolerance limits of
three-dimensional coordinates with absolute error.
GA is an artificial search method motivated by natural principles and the concept of survival of the fittest. Each independent variable in the GA is represented by a gene sequence and each solution is described as a chromosome. Compared with the general optimization algorithm, GA is a global search algorithm, which searches for multiple points with probability mechanism. So that a better global optimization value can be obtained. The mathematical model established in this paper is a kind of multiparameter constrained optimization and nonmonotonic problem. So GA algorithm was selected for analyzing the relation between the error in the cutting path and the accuracy of the sensors. The workflow of the error analysis is shown in Figure 4.

### 3.1. Objective Functions and Constraint Conditions

The error of absolute positions of shearer drums can be analyzed by derivation with an error transferring formula \[15, 16\] as follows:

\[
\Delta P^g_i = \begin{bmatrix} \frac{\partial P^g_i}{\partial \delta} & \frac{\partial P^g_i}{\partial \beta} & \frac{\partial P^g_i}{\partial \gamma} & \frac{\partial P^g_i}{\partial \theta_l} \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta \beta \\ \Delta \gamma \\ \Delta \theta_l \end{bmatrix} \tag{7}
\]

\[(i = 1, 2)\]

where \( \Delta P^g_i, \Delta \delta, \Delta \beta, \Delta \gamma, \) and \( \Delta \theta_l \) are errors in \( P^g_i \), \( \delta, \beta, \gamma \), and \( \theta_l \), respectively. The angles \( \beta \) and \( \gamma \) are measured by the INS, so we can assume that the errors of \( \beta \) and \( \gamma \) are equal; namely, \( \Delta \beta = \Delta \gamma \). The angles of \( \theta_1 \) and \( \theta_2 \) are measured by two axial encoders, so the errors in \( \theta_1 \) and \( \theta_2 \) are equal; namely, \( \Delta \theta_1 = \Delta \theta_2 = \Delta \theta \). So (7) can be expressed in the form of

\[
\Delta P^g_i = \begin{bmatrix} \frac{\partial P^g_i}{\partial \delta} & \frac{\partial P^g_i}{\partial \beta} & \frac{\partial P^g_i}{\partial \gamma} & \frac{\partial P^g_i}{\partial \theta_l} \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta \beta \\ \Delta \gamma \\ \Delta \theta_l \end{bmatrix} \tag{8}
\]

\[(i = 1, 2)\]

where

\[
K_i^{max} = \begin{bmatrix} \frac{\partial P^g_i}{\partial \delta} & \frac{\partial P^g_i}{\partial \beta} & \frac{\partial P^g_i}{\partial \gamma} & \frac{\partial P^g_i}{\partial \theta_l} \end{bmatrix} \begin{bmatrix} \frac{\partial X^g_i}{\partial \delta} & \frac{\partial X^g_i}{\partial \beta} & \frac{\partial X^g_i}{\partial \gamma} & \frac{\partial X^g_i}{\partial \theta_l} \\ \frac{\partial Y^g_i}{\partial \delta} & \frac{\partial Y^g_i}{\partial \beta} & \frac{\partial Y^g_i}{\partial \gamma} & \frac{\partial Y^g_i}{\partial \theta_l} \\ \frac{\partial Z^g_i}{\partial \delta} & \frac{\partial Z^g_i}{\partial \beta} & \frac{\partial Z^g_i}{\partial \gamma} & \frac{\partial Z^g_i}{\partial \theta_l} \end{bmatrix} \tag{9}\]

\[(i = 1, 2)\]

where \( K_i^{max} \) is the error coefficient matrix, whose elements will be treated as objective functions. Maximum values of the objective functions will be yielded with GA.

\[
\begin{bmatrix} \Delta \delta \\ \Delta \beta \\ \Delta \gamma \\ \Delta \theta_l \end{bmatrix} = \left[ \frac{\partial P^g_i}{\partial \delta} \right]^{-1} \begin{bmatrix} \frac{\partial P^g_i}{\partial \delta} & \frac{\partial P^g_i}{\partial \beta} & \frac{\partial P^g_i}{\partial \gamma} & \frac{\partial P^g_i}{\partial \theta_l} \end{bmatrix} \tag{10}\]

\[(i = 1, 2)\]

Suppose \( \Delta X = \Delta Y = \Delta Z \); if we want to achieve tolerant error limits of \( \delta, \beta, \gamma \), \( \left[ \frac{\partial P^g_i}{\partial \delta} \right]^{-1} \left[ \frac{\partial P^g_i}{\partial \delta} \right] \left[ \frac{\partial P^g_i}{\partial \beta} \right] \left[ \frac{\partial P^g_i}{\partial \gamma} \right] \left[ \frac{\partial P^g_i}{\partial \theta_l} \right] \) must be the minimum value.

The attitude of a shearer corresponds to the geological conditions of a mining area, that is, the pitch angle \( \beta \) corresponds to the floor gradient along the working face and the roll angle \( \gamma \) corresponds to the floor gradient along the face-advance (retreat) direction. The heading angle corresponds to the angle between open-off cut and north. Suppose the heading angle of a shearer varies within a certain range. The angle between the arms and mainframe \( \theta_l \) \((i = 1, 2)\) is determined by the shearer itself. It can be concluded that the range of variables such as \( \delta, \beta, \gamma, \) and \( \theta_l \) are available from the geological survey report and the mechanical structure of a shearer, which will be treated as constraint conditions of this GA.
3.2. The Calculation of Error Coefficients Based on GA.
Standard optimization process of GA can be divided into five key steps: encoding, selecting, crossover, mutation, and fitness judgment.

Encoding is defined as the mapping relation from problem space to encoding space. The selecting process is choosing individuals with higher fitness value to form a mating pool. Probability of being selected for each chromosome is determined by the proportion of the individual fitness and the total. And the roulette method is mostly applied in the selecting process. For a given group $P = \{a_1, a_2, \ldots, a_n\}$, where $n$ is the scale number of the group, the fitness value of $a_j$ can be described as $f(a_j)$ and the probability $p_s(a_j)$ can be calculated as follows:

$$p_s(a_j) = \frac{f(a_j)}{\sum_{i=1}^{n} f(a_i)}$$  \hspace{1cm} (11)
where \( j = 1, 2, \ldots, n \). The expectation number \( P(a_j) \) of previous chromosome in offspring is determined as follows:

\[
P(a_j) = n \cdot p_i(a_j)
\]  

(12)

The crossover in GA is transmitting favorable gene to the next generation by simulating the gene recombinant process of sexual reproduction. Assume that \( (V^l, V^m) \) were parent chromosomes and the crossover operation was performed on \( i \)-th position. So the crossover position of the corresponding child chromosomes \( (V^l, V^m) \) can be presented as follows:

\[
v^l_i = \alpha v^l_i + (1 - \alpha) v^m_i
\]

\[
v^m_i = \alpha v^m_i + (1 - \alpha) v^l_i
\]  

(13)

where \( \alpha \) is a random value in the range of (0, 1) and \( i \) is an integer.

Structure and physical properties of the chromosomes were changed through the mutation operation. Assume that \( V^m \) was the selected chromosome and the mutation process was performed on \( i \)-th position, which can be presented as follows:

\[
v^l_i = \beta v^l_i + (1 - \beta) v^l_i
\]

\[
v^m_i = \beta v^m_i + (1 - \beta) v^m_i
\]  

(14)

where \( \beta \) is a random value in the range of (0, 1).

As an important evolution algorithm form, GA is used to yield maximum values of error coefficients based on objective functions and constraint conditions as mentioned above. The detailed calculation process with GA is shown in Figure 4. Firstly, these parameters \( \delta, \beta, \gamma, \theta_1 \) are encoded as double-string, and an initial population is generated at random within the constraint conditions. Secondly, objective functions and constraint conditions as mentioned in (9) are treated as fitness functions, and values of these fitness functions are calculated as follows:

\[
f = \frac{1}{1 + g} 
\]  

(15)

\[
g = k_{11}^2 + k_{12}^2 + k_{13}^2 + \cdots + k_{33}^2
\]  

(16)

where \( k_{11}, k_{12}, \ldots, k_{33} \) are elements of error matrix in (9) and \( f \) reaches the maximum when \( g \) is the minimum.

Thirdly, with a crossover or mutation operator according to the probability of crossover or mutation \( P_c \) and \( P_m \), a new population can be generated. Then, the above calculation process is repeated until the generation of evolutes is the largest. Finally, the output values of fitness functions are the maximum values of the error coefficients, and these results will be used to select the sensors (see Section 3.3).

3.3. Selection of Sensors. An INS and axial encoders can be chosen with maximum values of objective functions by giving the tolerance limits of three-dimensional coordinates, and these sensors with reasonable accuracy can guarantee errors of three-dimensional coordinates within tolerance limits in any mining area.

Suppose tolerance limits of three-dimensional coordinates are

\[
\begin{bmatrix}
\Delta X_i^g \\
\Delta Y_i^g \\
\Delta Z_i^g 
\end{bmatrix}
\leq
\begin{bmatrix}
\Delta P_i^g \\
\Delta Y_i^g \\
\Delta Z_i^g 
\end{bmatrix} 
\]

\[
(i = 1, 2).
\]

(17)

Combining with (8) and (10), we get INS and axial encoders with reasonable accuracy:

\[
K_i^{-1}
\begin{bmatrix}
\Delta X_i^g \\
\Delta Y_i^g \\
\Delta Z_i^g 
\end{bmatrix}
\leq
\begin{bmatrix}
\Delta \delta \\
\Delta \beta \\
\Delta \theta_1 
\end{bmatrix} 
\leq
K_i^{-1}
\begin{bmatrix}
\Delta X_i^g \\
\Delta Y_i^g \\
\Delta Z_i^g 
\end{bmatrix} 
\]

\[
(i = 1, 2).
\]

(18)

4. Experimental Results

Some experiments have been done with a MGI000/2660-WD shearer produced by Taiyuan Coal Mining Machinery Company, whose related parameters are \( B=1.8 \) m, \( l=9.82 \) m, \( L=3.54 \) m, \( 0^\circ \leq \theta_1 \leq 5^\circ \), and \( -18^\circ \leq \theta_2 \leq 0^\circ \). As illustrated in Figure 5, cutting paths of the shearer were achieved based on the model as follows.

From the geological survey report of the mining area, we can see \( \delta \in [89^\circ, 91^\circ] \), \( \beta \in [9^\circ, 11^\circ] \), and \( \gamma \in [-5^\circ, 5^\circ] \). Combined with the ranges of \( \delta, \beta, \gamma, \theta_1, \) and \( \theta_2 \), maximum values of objective functions can be achieved with GA firstly. The parameters of GA are shown in Table 1. The iteration process of proposed optimization algorithm is presented in Figure 6.

The mechanical parameter of longwall shear was determined by the thickness of coal seam for a shearer cuts the whole thickness of coal seam. So that, three typical longwall shearers working in thin seam, medium-thickness seam, and thick seam separately were selected for verifying the proposed optimization algorithm. Related parameters of the three type
### Table 1: Parameters of GA.

<table>
<thead>
<tr>
<th>The group number</th>
<th>The largest evolutes generations</th>
<th>The probability of crossover</th>
<th>The probability of mutation</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>100</td>
<td>0.5</td>
<td>0.01</td>
</tr>
</tbody>
</table>

### Table 2: Parameters of three kinds of shearers.

<table>
<thead>
<tr>
<th>Model</th>
<th>$R$ (m)</th>
<th>$L$ (m)</th>
<th>$l$ (m)</th>
<th>Angle of ranging arms (°)</th>
<th>Type of seam</th>
</tr>
</thead>
<tbody>
<tr>
<td>MG80/200-BW</td>
<td>1</td>
<td>1.41</td>
<td>3.8</td>
<td>[-13.2, 32.7]</td>
<td>Thin(0.76-1.4 m)</td>
</tr>
<tr>
<td>MG132/320-W</td>
<td>1.2</td>
<td>1.7</td>
<td>5.7</td>
<td>[-15.9, 19.6]</td>
<td>Medium-thick(1.3-3.0 m)</td>
</tr>
<tr>
<td>MG1000/2660-WD</td>
<td>1.8</td>
<td>3.54</td>
<td>9.82</td>
<td>[-18, 50]</td>
<td>Thick(4.1-6.2 m)</td>
</tr>
</tbody>
</table>

### Figure 6: Iteration process of GA.

### Figure 7: Desired error ranges of sensors for three kinds of shearers.

The group number, the largest evolutes generations, the probability of crossover, and the probability of mutation are presented in Table 1. Table 2 details the parameters of three kinds of shearers including their models, dimensions, and angles of ranging arms, along with the type of seam they are designed for. The iteration process of GA is depicted in Figure 6, while Figure 7 illustrates the desired error ranges of sensors for thin, medium-thick, and thick seams.

The angle between the arms and the shearer mainframe, $\theta$, is higher than the other two angles for a shearer working in thin seam. The geometric dimensioning of a shearer working in thin seam was relatively small. Accordingly, the changing in position of drums caused by the shear's attitude was smaller than that caused by the angle between the arms and the shearer mainframe. This is inverse for the case of medium-thickness seam and thick seam. The larger geometric dimensioning of a shearer working in medium-thickness seam and thick seam induced that the changing in position of drums was more sensitive to the shear's attitude.

So the desired accuracy of the pitch angle of a shearer mainframe, $\beta$, is higher than the angle between the arms and the shearer mainframe, $\theta$. According to the measurement principle (Figure 2), an axial encoder and INS can be selected with the desired accuracy exhibited in Figure 7.

### 5. Conclusions

(1) Based on the attitude and the angle between the arms and mainframe, a mathematical model of cutting path of a shearer was built relative to the local geographic coordinate frame.

(2) Considering variables in this mathematical model, errors of main-gate and tail-gate drum positions of a shearer were analyzed based on GA so that suitable and accurate sensors can be chosen according to the desired errors of cutting path.

(3) For the same desired errors of cutting path, the desired accuracy of the angle between the arms and the shearer mainframe, $\theta$, is higher than the other two angles for a shearer in thin seam. For the case of medium-thickness seam and thick seam, the desired accuracy of the pitch angle of a shearer mainframe, $\beta$, is higher.

### Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

### Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.
Acknowledgments

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References
