

Research Article

A New Robust Nonfragile Controller Design Scheme for a Class of Hybrid Systems through Piecewise Affine Models

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This paper investigates the robust \mathcal{H}_∞ nonfragile control problem for a class of discrete-time hybrid systems based on piecewise affine models. The objective is to develop an admissible piecewise affine nonfragile controller such that the resulting closed-loop system is asymptotically stable with robust \mathcal{H}_∞ performance γ . By employing a state-control augmentation methodology, some new sufficient conditions for the controller synthesis are formulated based on piecewise Lyapunov functions (PLFs). The controller gains can be obtained via solving a set of linear matrix inequalities. Simulation examples are finally presented to demonstrate the feasibility and effectiveness of the proposed approaches.

1. Introduction

Over the past few decades, hybrid systems have drawn tremendous attention from the control community, as they contain the competence to model the interaction between logic components and continuous dynamics [1–6]. Piecewise affine (PWA) systems are a rich class of hybrid systems, which can provide a useful modeling approach for the stability analysis and controller synthesis of hybrid systems [7–9]. In addition, the PWA models can approximate smooth nonlinear plants with arbitrary degrees of accuracy [10, 11].

In recent years, many valuable references on the systematic analysis and synthesis for the PWA systems have been published [12–22]. In [12], the stability of continuous-time PWA systems was analyzed by utilizing discontinuous piecewise Lyapunov functions (PLFs). On the basis of the idea in [12], some algorithms for stability and performance analysis of discrete-time PWA systems were proposed in [14] and the continuity of the PLFs was not required for the discrete-time case. The authors in [20] designed a static output-feedback controller for discrete-time PWA systems such that the resulting closed-loop system was asymptotically stable with

robust \mathcal{H}_∞ performance γ . The authors in [21] developed a reconfigurable control approach for continuous-time PWA systems with actuator and sensor faults, and the output tracking problem was investigated simultaneously. In [23], the problem of delay-dependent fixed-order memory piecewise affine \mathcal{H}_∞ output-feedback control for a class of nonlinear systems with time-varying delay was considered. The authors in [24] have proposed a novel system-augmentation approach to the delay-dependent reliable piecewise affine \mathcal{H}_∞ static output-feedback control for nonlinear systems with time-varying delay and sensor faults in the piecewise-Markovian-Lyapunov-functional-based framework.

On the other hand, since 1980s, the robust \mathcal{H}_∞ control problem has drawn much attention as it is robust against the system parameter uncertainties, unmodeled dynamics, and external disturbances [25, 26]. It is implicitly assumed in this control approach that the controllers are implemented precisely. In fact, controllers gains inevitably possess some degrees of errors or parameter variations due to physical limitations such as the inherent inaccuracy in analog systems, finite word length in arbitrary digital systems, and round-off errors in process of numerical computations [27].

Numbers of literatures have already shown that relatively small perturbations in the controller parameters can even result in instability of the closed-loop system [28–30]. For this case, it is realistic to synthesize a controller for a given uncertain system such that the controller is insensitive to some predefined admissible variations with respect to its gains. More recently, many efforts have been devoted to the nonfragile robust \mathcal{H}_∞ controller design problem [31–40]. For instance, a nonfragile \mathcal{H}_∞ controller with multiplicative gain perturbations was proposed for linear time invariant (LTI) systems in [31]. The author in [32] developed a decentralized robust state-feedback nonfragile control method for uncertain discrete-time large-scale systems with time-delays and controller gain variations. In [34], a nonfragile robust controller with parameter uncertainties was constructed for a class of nonlinear systems based on T-S fuzzy neural models. The authors in [40] considered the nonfragile robust \mathcal{H}_∞ control problem for fuzzy systems based on output information. In spite of the attractive features in piecewise affine systems, the authors in [41] proposed a state-feedback nonfragile robust controller design method for continuous-time PWA systems. The results in [41] were derived based on a common Lyapunov function (CLF), and the matrices B_i are not empowered to have uncertainties. To the best of the authors' knowledge, few attempts have been devoted to the robust nonfragile controller design for discrete-time piecewise affine systems based on PLFs, which inspires us for this study.

In this paper, we will consider the piecewise affine nonfragile control problem for a class of discrete-time hybrid systems based on piecewise affine models. Via employing a state-control augmentation approach, a new nonfragile controller synthesis method is proposed based on piecewise Lyapunov functions (PLFs), which removes the restrictive condition that the input matrices do not have uncertainties. This feature enables one to design the nonfragile controller for a broader class of industrial systems. It is shown that the resulting closed-loop PWA system is asymptotically stable with a prescribed disturbance attenuation level γ , and the controller gains can be acquired via solving a set of linear matrix inequalities (LMIs). Finally, two simulation examples are presented to show the effectiveness and amenability of the proposed approaches. The main contributions of this paper are listed as follows: (1) The controller is designed based on PLFs and piecewise affine models and the conservatism can be reduced. (2) A new robust piecewise affine nonfragile controller is designed to deal with the situations that perturbations exist in the controller gains. (3) The input matrices are empowered to have uncertainties, which is more applicable for some practical requirements.

The rest of this paper is organized as follows. The piecewise affine model description and nonfragile controller design are given in Section 2. The asymptotic stability analysis for resulting closed-loop system is shown in Section 3. Simulation examples are shown in Section 4 to verify the effectiveness of the proposed methods. Section 5 gives the conclusions.

Notations. The notations employed in this paper are standard. A real symmetric matrix $A > 0$ (≥ 0) indicates A being positive-definite (positive-semi-definite). $\text{sym}\{X\}$ is the shorthand notation for $X^T + X$.

2. Problem Formulation

2.1. Discrete-Time Piecewise Affine Systems. A discrete-time piecewise affine system is shown as

$$\begin{aligned} x(t+1) &= (A_i + \Delta A_i)x(t) + a_i + \Delta a_i \\ &\quad + (B_i + \Delta B_i)u(t) + H_i w(t) \\ z(t) &= L_i x(t) + N_i u(t), \\ x(t) &\in \Sigma_i, \quad i \in \Pi := \{1, 2, \dots, \phi\} \end{aligned} \quad (1)$$

where $x(t) \in \mathfrak{R}^n$ is the system state; $u(t) \in \mathfrak{R}^m$ is the control input; $z(t) \in \mathfrak{R}^p$ is the regulated output; $w(t)$ is the external nonlinear disturbance belonging to $l_2[0, \infty)$; Σ_i is a polyhedral partition from the state-space and Π is the index set of these polyhedral regions; ΔA_i , Δa_i , and ΔB_i are the uncertainty terms of the i -th local affine model satisfying

$$[\Delta A_i \quad \Delta a_i \quad \Delta B_i] = W_{i1} \Delta_i(t) [M_{i1} \quad M_{i2} \quad M_{i3}], \quad i \in \Pi \quad (2)$$

with W_{i1} , M_{i1} , M_{i2} , and M_{i3} being known real constant matrices. $\Delta_i(t)$ is unknown time-varying matrix satisfying

$$\Delta_i^T(t) \Delta_i(t) \leq \mathbf{I}. \quad (3)$$

Remark 1. The system model in (1) is in fact affine systems rather than linear systems as the offset term ($a_i + \Delta a_i$) is involved. One can easily conclude that this type of models is more accurate for approximation to nonlinear systems [41].

Following the idea given in [20], define the region index $\Pi = \Pi_0 \cup \Pi_1$, where Π_1 denotes the index set of subspaces without the origin and Π_0 refers to the index set of subspaces containing the origin.

For future use, a new set Ω is introduced to depict all possible region transitions

$$\Omega := \{(i, j) \mid x(t) \in \Sigma_i, x(t+1) \in \Sigma_j, i, j \in \Pi\}. \quad (4)$$

When $(i, j) \in \Omega$ and $i = j$, the state is involved in the same region Σ_i at the time t . Otherwise, the state trajectories will jump from the region Σ_i to Σ_j at that time.

In this paper, it is assumed that each polyhedral regions Σ_i can be outer approximated by an ellipsoid Ψ_i , that is to say, there exist matrices Q_i and q_i such that

$$\Sigma_i \subseteq \Psi_i, \quad \Psi_i := \{x \mid \|Q_i x + q_i\| \leq 1\}. \quad (5)$$

When Σ_i are slab regions, the above covering is useful, and then the parameters Q_i and q_i are guaranteed to exist, and $\Sigma_i \subseteq \Psi_i$ and $\Psi_i \subseteq \Sigma_i$. If the polyhedral regions Σ_i are slabs with

$$\Sigma_i = \{x \mid \alpha_i \leq x \leq \beta_i\}, \quad i \in \Pi, \quad (6)$$

where $\alpha_i, \beta_i \in \mathfrak{R}$, $\phi_i \in \mathfrak{R}^n$, then each slab can be represented by a degenerate ellipsoid as in (6) with

$$\begin{aligned} Q_i &= \frac{2\phi_i^T}{\beta_i - \alpha_i}, \\ q_i &= \frac{\beta_i + \alpha_i}{\alpha_i - \beta_i}. \end{aligned} \quad (7)$$

For each ellipsoid region, we have the following relationship:

$$\begin{bmatrix} 1 \\ x(t) \end{bmatrix}^T \begin{bmatrix} q_i^T q_i - 1 & q_i^T Q_i \\ * & Q_i^T Q_i \end{bmatrix} \begin{bmatrix} 1 \\ x(t) \end{bmatrix} \leq 0, \quad i \in \Pi. \quad (8)$$

2.2. Piecewise Affine Nonfragile Controller Design. Borrowing the idea from [42], define $\xi(t) = [x^T(t) \ u^T(t)]^T$. As the inaccuracies or uncertainties inevitably involved in the controllers in many practical situations, for the piecewise affine system (1), a new nonfragile piecewise affine controller in differential form is designed as

$$u(t+1) = (K_i + \Delta K_i)\xi(t) + k_i + \Delta k_i, \quad (9)$$

where $K_i \in \mathfrak{R}^{n_u \times (n_x + n_u)}$ and $k_i \in \mathfrak{R}^{n_u}$ are the controller gains to be determined, and $k_i = 0$ for $i \in \Pi_0$. ΔK_i and Δk_i are unknown matrices standing for the gain perturbations satisfying

$$[\Delta K_i \ \Delta k_i] = W_{i2} \Delta_{ui}(t) [U_{i1} \ U_{i2}], \quad i \in \Pi, \quad (10)$$

where W_{i2} , U_{i1} , and U_{i2} are known real constant matrices of appropriate dimensions. The real-valued matrix functions $\Delta_{ui}(t)$ satisfy

$$\Delta_{ui}^T(t) \Delta_{ui}(t) \leq \mathbf{I}. \quad (11)$$

The parameter uncertainties in (1) and (9) are recognized to be admissible if (2)-(3) and (10)-(11) hold.

By applying the piecewise affine nonfragile controller (9) into system (1), the closed-loop system can be formulated as

$$\begin{aligned} \xi(t+1) &= (\mathcal{A}_i + \Delta \mathcal{A}_i)\xi(t) + a_i + \Delta a_i + \overline{H}_i w(t) \\ z(t) &= \overline{L}_i \xi(t), \\ x(t) &\in \Sigma_i, \end{aligned} \quad (12)$$

$i \in \Pi,$

where $\mathcal{A}_i = \overline{A}_i + RK_i$, $\Delta \mathcal{A}_i = \Delta \overline{A}_i + R\Delta K_i$, $a_i = \overline{a}_i + Rk_i$, $\Delta a_i = \Delta \overline{a}_i + R\Delta k_i$, and

$$\begin{aligned} \overline{A}_i &= \begin{bmatrix} A_i & B_i \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \\ \Delta \overline{A}_i &= \begin{bmatrix} \Delta A_i & \Delta B_i \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \end{aligned}$$

$$\overline{a}_i = \begin{bmatrix} a_i \\ \mathbf{0} \end{bmatrix},$$

$$\Delta \overline{a}_i = \begin{bmatrix} \Delta a_i \\ \mathbf{0} \end{bmatrix},$$

$$R = \begin{bmatrix} \mathbf{0} \\ \mathbf{I}_{n_u} \end{bmatrix},$$

$$\overline{H}_i = \begin{bmatrix} H_i \\ \mathbf{0} \end{bmatrix},$$

$$\overline{L}_i = [L_i \ N_i].$$

(13)

Based on the augment vector $\xi(t)$, the following inequality implies (8) for each ellipsoid region,

$$\begin{bmatrix} 1 \\ \xi(t) \end{bmatrix}^T \begin{bmatrix} q_i^T q_i - 1 & q_i^T \overline{Q}_i \\ * & \overline{Q}_i^T \overline{Q}_i \end{bmatrix} \begin{bmatrix} 1 \\ \xi(t) \end{bmatrix} \leq 0, \quad i \in \Pi, \quad (14)$$

where $\overline{Q}_i = [Q_i \ \mathbf{0}_{n_u \times n_u}]$.

As practical control systems are always subject to external disturbance, thus, in this paper, we aim to synthesize an admissible piecewise affine nonfragile controller in the form of (9) such that the resulting closed-loop system is asymptotically stable with robust \mathcal{H}_∞ performance γ as

$$\|z\|_2 < \gamma \|w\|_2 \quad (15)$$

under zero initial conditions for all nonzero $w(t) \in l_2[0, \infty]$.

Before ending this section, the following lemmas will be employed to prove the main results in this paper.

Lemma 2 (see [22]). *Real matrices $Q > 0$, R , $\Delta(t)$, and S are regular matrices with appropriate dimensions. $\Delta(t)$ is time-varying satisfying $\Delta^T(t)\Delta(t) \leq \mathbf{I}$. The inequality*

$$Q + R\Delta(t)S + S^T\Delta^T(t)R^T < 0 \quad (16)$$

holds if and only if for some $\varepsilon > 0$

$$Q + \varepsilon R^T R + \varepsilon^{-1} S^T S < 0. \quad (17)$$

3. Piecewise Affine Nonfragile Controller Analysis and Design

On the basis of piecewise Lyapunov functions (PLFs), some new approaches to robust \mathcal{H}_∞ piecewise affine nonfragile controller synthesis will be proposed in this section.

Theorem 3. For a given constant scalar λ , the closed-loop system (12) is asymptotically stable with disturbance attenuation level γ if there exist matrices $0 < P_i = P_i^T \in \mathfrak{R}^{(n+m) \times (n+m)}$, $\bar{K}_i \in \mathfrak{R}^{m \times (n+m)}$, $\bar{k}_i \in \mathfrak{R}^m$, $V_{i1} \in \mathfrak{R}^{n \times n}$, $V_{i2} \in \mathfrak{R}^{m \times n}$, $V_{i3} \in \mathfrak{R}^{m \times m}$, and $i \in \Pi$ and two sets of scalars $\{r_{ij} < 0, i \in \Pi_1, (i, j) \in \Omega\}$, and $\{\varepsilon_i > 0, i \in \Pi\}$, such that the following LMIs hold:

$$\begin{bmatrix} \Gamma_{ij1} + \varepsilon_{ij} \Theta_{i1}^T \Theta_{i1} & \Lambda_{i1}^{(1)} & \Lambda_{i1}^{(2)} \\ * & -\varepsilon_{ij} \mathbf{I} & \mathbf{0} \\ * & * & -\varepsilon_{ij} \mathbf{I} \end{bmatrix} < 0, \quad (18)$$

$$i \in \Pi_0, j \in \Pi, (i, j) \in \Omega,$$

$$\begin{bmatrix} \Gamma_{ij2} + \varepsilon_{ij} \Theta_{i2}^T \Theta_{i2} & \Lambda_{i2}^{(1)} & \Lambda_{i2}^{(2)} \\ * & -\varepsilon_{ij} \mathbf{I} & \mathbf{0} \\ * & * & -\varepsilon_{ij} \mathbf{I} \end{bmatrix} < 0, \quad (19)$$

$$i \in \Pi_1, j \in \Pi, (i, j) \in \Omega,$$

where

$$\Gamma_{ij1} = \begin{bmatrix} P_j - V_i - V_i^T & V_i \bar{A}_i + E \bar{K}_i & V_i \bar{H}_i \\ * & -P_i + \bar{L}_i^T \bar{L}_i & \mathbf{0} \\ * & * & -\gamma^2 \mathbf{I} \end{bmatrix},$$

$$\Theta_{i1} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \bar{M}_{i1}^T & U_{i1}^T \\ \mathbf{0} & \mathbf{0} \end{bmatrix},$$

$$\Lambda_{i1}^{(1)} = \begin{bmatrix} V_i \bar{W}_{i1} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix},$$

$$\Lambda_{i1}^{(2)} = \begin{bmatrix} V_i R W_{i2} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix},$$

Γ_{ij2}

$$= \begin{bmatrix} P_j - V_i - V_i^T & V_i \bar{a}_i + E \bar{k}_i & V_i \bar{A}_i + E \bar{K}_i & V_i \bar{H}_i \\ * & -r_{ij} (1 - q_i^T q_i) & r_{ij} q_i^T \bar{Q}_i & \mathbf{0} \\ * & * & -P_i + r_{ij} \bar{Q}_i^T \bar{Q}_i + \bar{L}_i^T \bar{L}_i & \mathbf{0} \\ * & * & * & -\gamma^2 \mathbf{I} \end{bmatrix},$$

$$\Theta_{i2} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ M_{i2}^T & U_{i2}^T \\ \bar{M}_{i1}^T & U_{i1}^T \\ \mathbf{0} & \mathbf{0} \end{bmatrix},$$

$$\Lambda_{i2}^{(1)} = \begin{bmatrix} V_i \bar{W}_{i1} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix},$$

$$\Lambda_{i2}^{(2)} = \begin{bmatrix} V_i R W_{i2} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix},$$

$$V_i = \begin{bmatrix} V_{i1} & \lambda S V_{i2} \\ V_{i3} & V_{i2} \end{bmatrix},$$

$$E = \begin{bmatrix} \lambda S \\ \mathbf{I}_m \end{bmatrix},$$

$$\bar{A}_i = \begin{bmatrix} A_i & B_i \\ 0 & 0 \end{bmatrix},$$

$$\bar{a}_i = \begin{bmatrix} a_i \\ 0 \end{bmatrix},$$

$$\bar{H}_i = \begin{bmatrix} H_i \\ 0 \end{bmatrix},$$

$$\bar{W}_{i1} = \begin{bmatrix} W_{i1} \\ \mathbf{0} \end{bmatrix},$$

$$\bar{M}_{i1} = [M_{i1} \ M_{i3}],$$

(20)

and $S \in \mathfrak{R}^{(n+m) \times m}$ is an arbitrary matrix.

Moreover, the controller gains for each subspace can be determined via

$$K_i = V_{i2}^{-1} \bar{K}_i, \quad (21)$$

$$k_i = V_{i2}^{-1} \bar{k}_i,$$

Proof. It is easy to see that condition (18) for $i \in \Pi_0$ is a special case of condition (19) for $i \in \Pi_1$. Without loss of generality, in the following, the proof of the more complex case $i \in \Pi_1$ is to be considered. Take the piecewise Lyapunov function as

$$V(t) = \xi^T(t) P_i \xi(t), \quad i \in \Pi. \quad (22)$$

It is also assumed that $x(t+1) \in \Sigma_j$. Then we have $V(t+1) = \xi^T(t+1) P_j \xi(t+1)$. The closed-loop system in (12) is asymptotically stable with robust \mathcal{H}_∞ performance γ , if the inequality

$$V(t+1) - V(t) + z^T(t) z(t) - \gamma^2 w^T(t) w(t) < 0 \quad (23)$$

holds.

By substituting (22) into (23), it yields that

$$\begin{aligned} & \xi^T(t+1) P_j \xi(t+1) - \xi^T(t) P_i \xi(t) + z^T(t) z(t) \\ & - \gamma^2 w^T(t) w(t) < 0, \quad (i, j) \in \Omega. \end{aligned} \quad (24)$$

Along the trajectories of closed-loop system (12), the following inequality implies (24) with $(i, j) \in \Omega$,

$$\begin{aligned} & \begin{bmatrix} 1 \\ \xi(t) \\ w(t) \end{bmatrix}^T \left\{ \begin{bmatrix} a_i^T + \Delta a_i^T \\ \mathcal{A}_i^T + \Delta \mathcal{A}_i^T \\ \overline{H}_i^T \end{bmatrix} P_j(\star) + \begin{bmatrix} \mathbf{0} \\ \overline{\mathcal{L}}_i^T \\ \mathbf{0} \end{bmatrix} (\star) \right. \\ & \left. + \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \star & -P_i & \mathbf{0} \\ \star & \star & -\gamma^2 \mathbf{I} \end{bmatrix} \right\} \begin{bmatrix} 1 \\ \xi(t) \\ w(t) \end{bmatrix} < 0. \end{aligned} \quad (25)$$

Noticing the state-space partition (14) and utilizing S-procedure, the following inequality implies (25) based on Schur complement:

$$\begin{aligned} & \begin{bmatrix} -P_j^{-1} & a_i + \Delta a_i & \mathcal{A}_i + \Delta \mathcal{A}_i & \overline{H}_i \\ \star & r_{ij}(q_i^T q_i - 1) & r_{ij} q_i^T \overline{Q}_i & \mathbf{0} \\ \star & \star & -P_i + r_{ij} \overline{Q}_i^T \overline{Q}_i + \overline{L}_i^T \overline{L}_i & \mathbf{0} \\ \star & \star & \star & -\gamma^2 \mathbf{I} \end{bmatrix} \\ & < 0, \quad i \in \Pi_1, (i, j) \in \Omega. \end{aligned} \quad (26)$$

It can be concluded that the Lyapunov matrices P_i and P_j are coupled with the system matrices. In order to facilitate

the controller design, we need to handle with the terms P_i and P_j^{-1} . To this end, for convenience in the controller synthesis, we make a congruence transformation to (26) by $\text{diag}\{V_i, \mathbf{I}, \mathbf{I}, \mathbf{I}\}$ as

$$\begin{aligned} & \begin{bmatrix} -V_i P_j^{-1} V_i^T & V_i(a_i + \Delta a_i) & V_i(\mathcal{A}_i + \Delta \mathcal{A}_i) & V_i \overline{H}_i \\ \star & r_{ij}(q_i^T q_i - 1) & r_{ij} q_i^T \overline{Q}_i & \mathbf{0} \\ \star & \star & -P_i(\mu) + r_{ij} \overline{Q}_i^T \overline{Q}_i + \overline{L}_i^T \overline{L}_i & \mathbf{0} \\ \star & \star & \star & -\gamma^2 \mathbf{I} \end{bmatrix} \\ & < 0, \quad i \in \Pi_1, (i, j) \in \Omega. \end{aligned} \quad (27)$$

Notice that

$$\begin{aligned} P_j - V_i - V_i^T + V_i P_j^{-1} V_i^T &= (P_j - V_i) P_j^{-1} (P_j - V_i)^T \\ &\geq 0, \end{aligned} \quad (28)$$

which indicates that

$$-V_i P_j^{-1} V_i^T \leq P_j - V_i - V_i^T. \quad (29)$$

Based on (29), the following inequality implies (27):

$$\begin{aligned} & \begin{bmatrix} P_j - V_i - V_i^T & V_i(a_i + \Delta a_i) & V_i(\mathcal{A}_i + \Delta \mathcal{A}_i) & V_i \overline{H}_i \\ \star & r_{ij}(q_i^T q_i - 1) & r_{ij} q_i^T \overline{Q}_i & \mathbf{0} \\ \star & \star & -P_i(\mu) + r_{ij} \overline{Q}_i^T \overline{Q}_i + \overline{L}_i^T \overline{L}_i & \mathbf{0} \\ \star & \star & \star & -\gamma^2 \mathbf{I} \end{bmatrix} < 0, \quad i \in \Pi_1, (i, j) \in \Omega. \end{aligned} \quad (30)$$

Notice that the controller gains are not involved in the first row of the matrices \mathcal{A}_i , $\Delta \mathcal{A}_i$, a_i , and Δa_i . For numerical tractability, we specify V_i as

$$V_i = \begin{bmatrix} V_{i1} & \lambda S V_{i2} \\ V_{i3} & V_{i2} \end{bmatrix}, \quad i \in \mathcal{S}_1, \quad (31)$$

where λ and S are defined as in (20).

Based on the matrices defined as in (31), one has that

$$V_i R K_i = \begin{bmatrix} V_{i1} & \lambda S V_{i2} \\ V_{i3} & V_{i2} \end{bmatrix} \begin{bmatrix} \mathbf{0} \\ \mathbf{I}_{n_u} \end{bmatrix} K_i = E \overline{K}_i, \quad (32)$$

$$V_i R k_i = \begin{bmatrix} V_{i1} & \lambda S V_{i2} \\ V_{i3} & V_{i2} \end{bmatrix} \begin{bmatrix} \mathbf{0} \\ \mathbf{I}_{n_u} \end{bmatrix} k_i = E \overline{k}_i,$$

where $R = [\lambda S^T \quad \mathbf{I}_{n_u}]^T$, $\overline{K}_i = V_{i2} K_i$, $\overline{k}_i = V_{i2} k_i$.

Substitute (32) into (30), and use Lemma 2 to handle the uncertainty terms. Then the following inequality implies (30) for $\varepsilon_i > 0$, $i \in \Pi$,

$$\begin{aligned} & \begin{bmatrix} P_j - V_i - V_i^T & V_i \overline{a}_i + E \overline{k}_i & V_i \overline{A}_i + R \overline{K}_i & V_i \overline{H}_i \\ \star & r_{ij}(q_i^T q_i - 1) & r_{ij} q_i^T \overline{Q}_i & \mathbf{0} \\ \star & \star & -P_i + r_{ij} \overline{Q}_i^T \overline{Q}_i + \overline{L}_i^T \overline{L}_i & \mathbf{0} \\ \star & \star & \star & -\gamma^2 \mathbf{I} \end{bmatrix} \\ & + \varepsilon_i^{-1} \begin{bmatrix} V_i \overline{W}_{i1} & V_i R W_{i2} \\ \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} (\star) + \varepsilon_i \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ M_{i2}^T & U_{i2}^T \\ \overline{M}_{i1}^T & U_{i1}^T \\ \mathbf{0} & \mathbf{0} \end{bmatrix} < 0 \end{aligned} \quad (33)$$

where

$$\overline{W}_{i1} = \begin{bmatrix} W_{i1} \\ \mathbf{0} \end{bmatrix}, \quad (34)$$

$$\overline{M}_{i1} = [M_{i1} \quad M_{i3}].$$

On the basis of Schur complement, it is easy to conclude that (19) implies (33). The proof is thus completed. \square

Remark 4. Note that in most existing nonfragile controller synthesis references for PWA systems, the control input matrices are not empowered to have uncertainties, which is restrictive for the control system design. In addition, it is also difficult to design a nonfragile piecewise affine controller via the approach proposed in [16], as one needs to develop a singular value decomposition to the control input matrices. However, by employing a state-control augmentation strategy for the nonfragile piecewise affine controller design, then the controller gains and the control input matrices can be decoupled from each other. As the state-control augmentation methodology is exploited, the uncertainty terms ΔK_i and Δk_i in the controller are decoupled from the system matrices. This feature makes it possible to design a piecewise affine nonfragile controller via convexification techniques in a uniform framework.

If the uncertainty terms, affine terms, and external disturbance do not exist, system (1) reduces to a nominal piecewise system as

$$x(t+1) = A_i x(t) + B_i u(t). \quad (35)$$

For system (35), a piecewise linear nonfragile controller in a differential form is designed as

$$u(t+1) = (K_i + \Delta K_i) \xi(t) \quad (36)$$

with $\xi(t)$ and ΔK_i defined as in (9).

Substituting the controller (36) into system (35), then the resulting closed-loop system is

$$\xi(t+1) = (\bar{A}_i + RK_i + R\Delta K_i) \xi(t) \quad (37)$$

where \bar{A}_i is denoted as in (13).

Corollary 5. For a given constant scalar λ , the closed-loop system (37) is asymptotically stable if there exist matrices $0 < P_i = P_i^T \in \mathfrak{R}^{(n+m) \times (n+m)}$, $\bar{K}_i \in \mathfrak{R}^{m \times (n+m)}$, $V_{i1} \in \mathfrak{R}^{n \times n}$, $V_{i2} \in \mathfrak{R}^{m \times n}$, $V_{i3} \in \mathfrak{R}^{m \times m}$, and $i \in \Pi$ and a set of positive scalar $\varepsilon_i > 0$, $i \in \Pi$, such that

$$\begin{bmatrix} P_j - V_i - V_i^T & V_i \bar{A}_i + E \bar{K}_i & V_i R W_{i2} \\ * & -P_i + \varepsilon_i U_{i1}^T U_{i1} & \mathbf{0} \\ * & * & -\varepsilon_i \mathbf{I} \end{bmatrix} < 0, \quad (38)$$

$$i \in \Pi.$$

Moreover, the controller gain can be obtained by $K_i = V_{i2}^{-1} \bar{K}_i$.

The derivation procedures are similar to that of Theorem 3. The proof is omitted.

Remark 6. Note that when the uncertainty terms in the form of (10) in the controller gains do not exist, that is to say,

$$W_{i2} = U_{i1} = U_{i2} = 0, \quad i \in \Pi, \quad (39)$$

then the piecewise affine nonfragile controller reduces to a regular piecewise affine state-feedback controller as

$$u(t+1) = K_i \xi(t) + k_i, \quad i \in \Pi, \quad (40)$$

where $\xi(t) = [x^T(t) \ u^T(t)]^T$.

4. Simulation

In this section, two simulation examples are shown to illustrate the effectiveness of the proposed approaches.

Example 1. Consider a discrete-time piecewise affine system in the form of (1) as follows:

$$\begin{aligned} x(t+1) &= (A_i + \Delta A_i) x(t) + a_i + \Delta a_i \\ &\quad + (B_i + \Delta B_i) u(t) + H_i w(t) \\ z(t) &= L_i x(t) + N_i u(t), \end{aligned} \quad (41)$$

$$x(t) \in \Sigma_i, \quad i \in \{1, 2, 3\},$$

and the system matrices are given as

$$A_1 = \begin{bmatrix} 1.2 & 0 \\ -0.1 & 0.4 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0.5 & 0.1 \\ 0 & 1.1 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 1.1 & 0.3 \\ 0.1 & 0.2 \end{bmatrix},$$

$$a_1 = \begin{bmatrix} 0 \\ -0.45 \end{bmatrix},$$

$$a_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$

$$a_3 = \begin{bmatrix} 0 \\ 0.45 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 1 \\ 1.5 \end{bmatrix},$$

$$B_3 = \begin{bmatrix} 0.9 \\ 1.2 \end{bmatrix},$$

$$H_1 = \begin{bmatrix} 0.8 \\ 0.5 \end{bmatrix},$$

$$\begin{aligned}
 H_2 &= \begin{bmatrix} 0.7 \\ 0.6 \end{bmatrix}, \\
 H_3 &= \begin{bmatrix} 0.4 \\ 0.4 \end{bmatrix}, \\
 L_1 &= [1 \ 0.3], \\
 L_2 &= [1.2 \ 0.5], \\
 L_3 &= [0.5 \ 1], \\
 N_1 &= 1, \\
 N_2 &= 0.8, \\
 N_3 &= 1.2, \\
 W_{11} &= \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, \\
 W_{21} &= \begin{bmatrix} 0.3 \\ 0.1 \end{bmatrix}, \\
 W_{31} &= \begin{bmatrix} 0.4 \\ 0.1 \end{bmatrix}, \\
 M_{11} &= [0.01 \ 0.01], \\
 M_{21} &= [0.01 \ 0.03], \\
 M_{31} &= [0.03 \ 0.02], \\
 M_{12} &= 0.1, \\
 M_{22} &= 0.2, \\
 M_{32} &= 0.05, \\
 M_{i3} &= 0.01,
 \end{aligned} \tag{42}$$

and the parameter uncertainties in controllers in terms of (10) are

$$\begin{aligned}
 W_{i2} &= 0.8, \\
 U_{i1} &= [0.8 \ 0.8 \ 0.8], \\
 U_{i2} &= 0.8,
 \end{aligned} \tag{43}$$

$i \in \{1, 2, 3\}.$

The decomposed regions are given as

$$\begin{aligned}
 \Sigma_1 &= \{x(t) \mid -\beta < x_1(t) \leq -\alpha\}, \\
 \Sigma_2 &= \{x(t) \mid -\alpha < x_1(t) \leq \alpha\}, \\
 \Sigma_3 &= \{x(t) \mid \alpha < x_1(t) \leq \beta\},
 \end{aligned} \tag{44}$$

where $\alpha = 40$ and $\beta = 400$.

From the relationship in (7), we can obtain that $\phi_i^T = [1 \ 0]$, for $i \in \Pi$, and the parameters of the degenerate ellipsoids are

$$\begin{aligned}
 Q_1 &= \frac{2\phi_i^T}{\beta - \alpha}, \\
 q_1 &= \frac{\beta + \alpha}{\beta - \alpha}, \\
 Q_2 &= \frac{1}{d_1}, \\
 q_2 &= 0, \\
 Q_3 &= \frac{2\phi_i^T}{\beta - \alpha}, \\
 q_3 &= \frac{\beta + \alpha}{\alpha - \beta}.
 \end{aligned} \tag{45}$$

The objective is to design a piecewise affine nonfragile controller in the form of (9)-(10) such that the closed-loop system in (12) is asymptotically stable with a robust \mathcal{H}_∞ performance γ . Exploiting Theorem 3 with $S = [1 \ 0]^T$ and $\lambda = 1$, the controller gains can be obtained as

$$\begin{aligned}
 K_1 &= [-1.3002 \ -0.1249 \ -1.2656], \\
 k_1 &= 0.2580, \\
 K_2 &= [-1.5930 \ -0.1369 \ -1.5294], \\
 k_2 &= 0, \\
 K_3 &= [-1.5756 \ -0.1317 \ -1.5052], \\
 k_3 &= -0.1308,
 \end{aligned} \tag{46}$$

with robust \mathcal{H}_∞ performance $\gamma_{\min} = 1.4222$.

It is worth pointing out that when $k_i \equiv 0$, the piecewise affine nonfragile controller (9) is characterized as a piecewise linear nonfragile controller given in (36). With $\lambda = 1$, Table 1 compares the robust performance between piecewise affine nonfragile controllers with piecewise linear nonfragile controllers with different matrix S .

It can be seen from Table 1 that the piecewise affine nonfragile controllers have a better robust \mathcal{H}_∞ performance than the piecewise linear nonfragile controllers.

To verify the effectiveness of the designed controllers, simulations are carried out. With $S = [1 \ 0]^T$ and $\lambda = 1$, solving the LMIs in (18)-(19), we obtain the piecewise linear nonfragile controller gains as

$$\begin{aligned}
 K_1 &= [-1.3003 \ -0.1261 \ -1.2658], \\
 K_2 &= [-1.5903 \ -0.1367 \ -1.5266], \\
 K_3 &= [-1.5736 \ -0.1302 \ -1.5033].
 \end{aligned} \tag{47}$$

In this simulation example, the initial condition is $x_0 = [2 \ -1.5]^T$, and the external disturbance is $w(t) = 120\pi^{-3.6t}$.

TABLE 1: Comparison of robust \mathcal{H}_∞ performance for piecewise-affine/piecewise-linear nonfragile controllers.

Controller form	$S = \begin{bmatrix} 1 & 0 \end{bmatrix}^T$	$S = \begin{bmatrix} 1 & 1 \end{bmatrix}^T$	$S = \begin{bmatrix} 1 & -1 \end{bmatrix}^T$
piecewise affine controller	1.4222	1.6797	4.6871
piecewise linear controller	1.5924	1.7997	4.7362

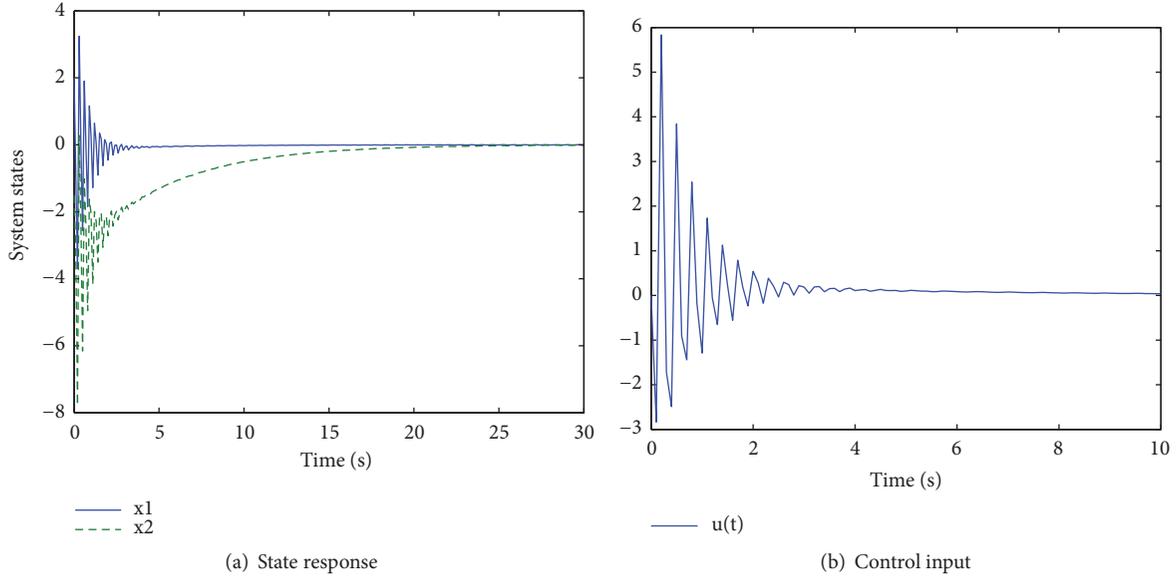


FIGURE 1: Simulation results in Example 1 by using piecewise affine nonfragile controller (9).

$\cos(4t)$. By using the piecewise affine nonfragile controller in (9), the state response of the closed-loop system is demonstrated in Figure 1(a), and the control input is shown in Figure 1(b). It can be seen from Figure 1(a) that the curves of system states converge to the zero after 20s.

With the same initial conditions and external disturbance, use the piecewise linear nonfragile controller as in (36), and the time responses of the closed-loop system are demonstrated in Figures 2(a) and 2(b). Figure 2(a) shows that the curves of the closed-loop system states converge to the zero after 28s.

Compare Figure 1(a) with Figure 2(a), and one can conclude that the convergence rate of the closed-loop system states with using piecewise affine nonfragile controller is faster than those with using piecewise linear nonfragile controller.

Under zero initial conditions, Figure 3 presents the \mathcal{H}_∞ performance. The ratio $\sqrt{\int_0^{t_f} z^T(t)z(t)dt} / \sqrt{\int_0^{t_f} w^T(t)w(t)dt}$ with using piecewise affine nonfragile controller is about 1.18, which is lower than $\gamma_{\min} = 1.4222$. The ratio $\sqrt{\int_0^{t_f} z^T(t)z(t)dt} / \sqrt{\int_0^{t_f} w^T(t)w(t)dt}$ with using piecewise linear nonfragile controller is about 1.3, which is lower than $\gamma_{\min} = 1.5924$. It can also be seen that a better disturbance attenuation level can be obtained with the piecewise affine nonfragile controller.

In order to verify the advantages of piecewise affine nonfragile controller (9) over the piecewise affine state-feedback controller (40), another simulation is conducted.

Solve the LMIs in (18)-(19) with $W_{i2} = 0$, $U_{i1} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$, and $U_{i2} = 0$, $i \in \{1, 2, 3\}$. Feasible solutions can be obtained with $\lambda = 1$ and $S = \begin{bmatrix} 1 & 0 \end{bmatrix}^T$, and the controller gains are

$$\begin{aligned}
 K_1 &= [-1.3202 \quad -0.1148 \quad -1.2689], \\
 k_1 &= 0.2038, \\
 K_2 &= [-1.4634 \quad -0.1183 \quad -1.3938], \\
 k_2 &= 0, \\
 K_3 &= [-1.4888 \quad -0.1168 \quad -1.4112], \\
 k_3 &= -0.1174.
 \end{aligned} \tag{48}$$

With the same initial conditions, disturbance, and parameter variations in the controller gains, apply controller (40) to system (41), and the trajectories of the closed-loop system states are shown in Figure 4. It can be seen from Figure 4 that the piecewise affine state-feedback controller (40) can not stabilize system (39) with parameter variations existing in the controller gains. From this case, we can see that the regular state-feedback controller can not deal with the situations that parameter variations exist in the controller gains.

This example clearly illustrates the advantages of the proposed piecewise affine nonfragile controller over the piecewise linear nonfragile controller and the piecewise affine state-feedback controller.

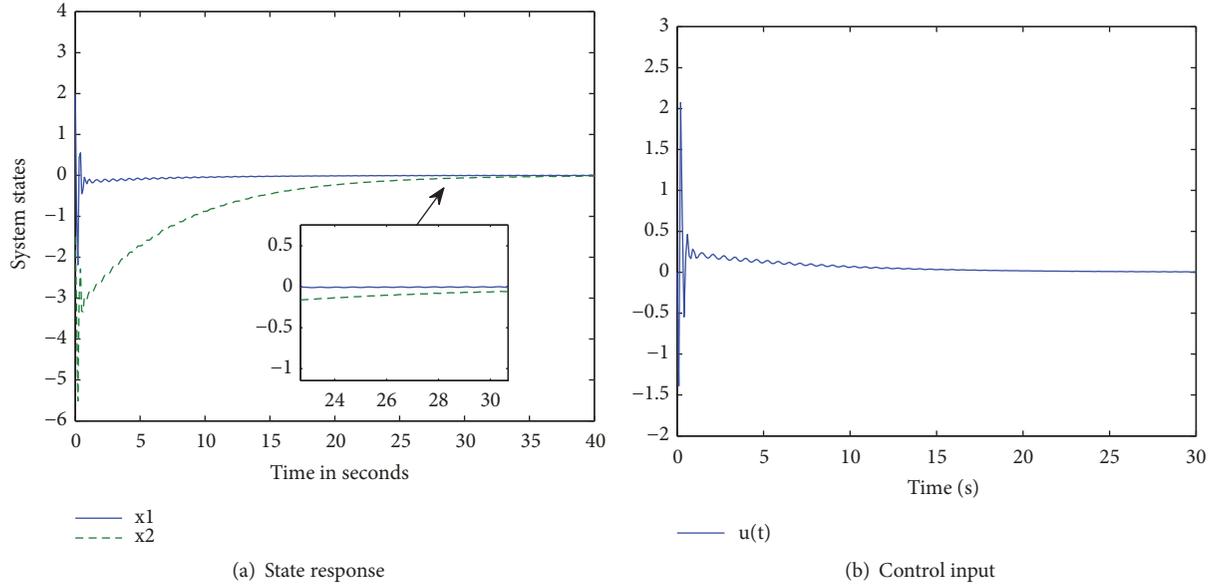


FIGURE 2: Simulation results in Example 1 by using piecewise linear nonfragile controller (36).

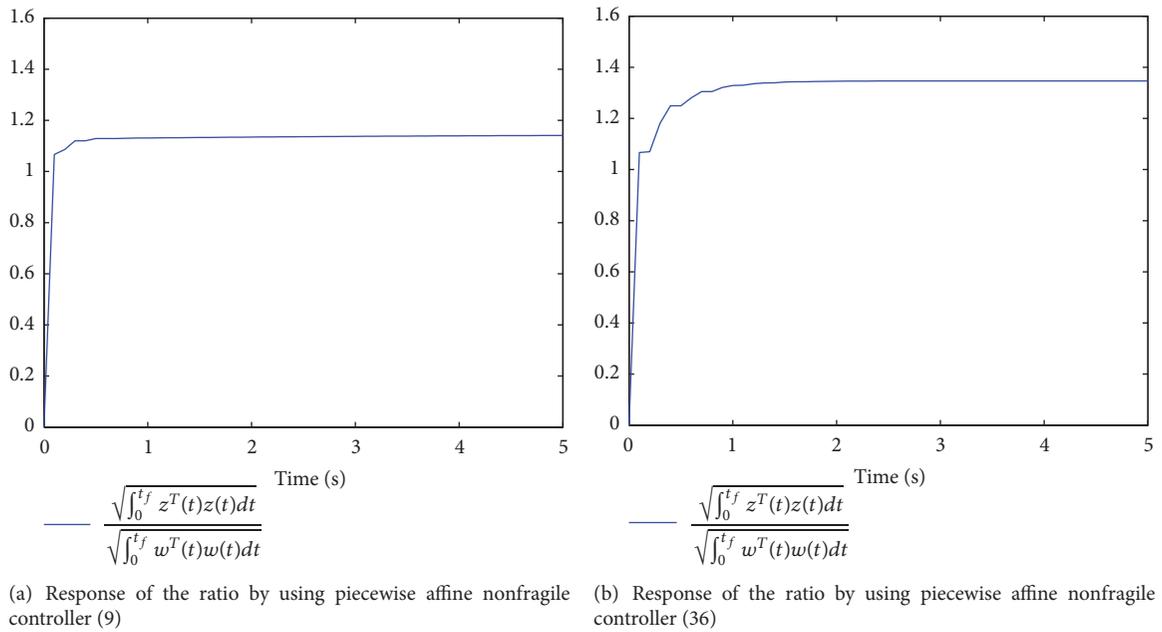


FIGURE 3: Response of the ratio $\sqrt{\int_0^{t_f} z^T(t)z(t)dt} / \sqrt{\int_0^{t_f} w^T(t)w(t)dt}$ in Example 1.

Example 2. Consider another discrete-time Chua's circuit system represented by the piecewise affine model as

$$\begin{aligned}
 x(t+1) &= A_i x(t) + a_i + B_i u(t) + H_i w(t) \\
 z(t) &= L_i x(t) + N_i u(t), \quad (49) \\
 x(t) &\in \Sigma_i, \quad i \in \{1, 2, 3\}
 \end{aligned}$$

where the system matrices are

$$A_1 = \begin{bmatrix} 0.9955 & 0.0070 & 0 \\ 0.0007 & 0.9993 & -0.0011 \\ 0 & 0.0074 & 1 \end{bmatrix}, \quad a_1 = \begin{bmatrix} 0.0028 \\ 0 \\ 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0.9983 & 0.0070 & 0 \\ 0.0007 & 0.9993 & -0.0011 \\ 0 & 0.0074 & 1 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 0.9955 & 0.0070 & 0 \\ 0.0007 & 0.9993 & -0.0011 \\ 0 & 0.0074 & 1 \end{bmatrix},$$

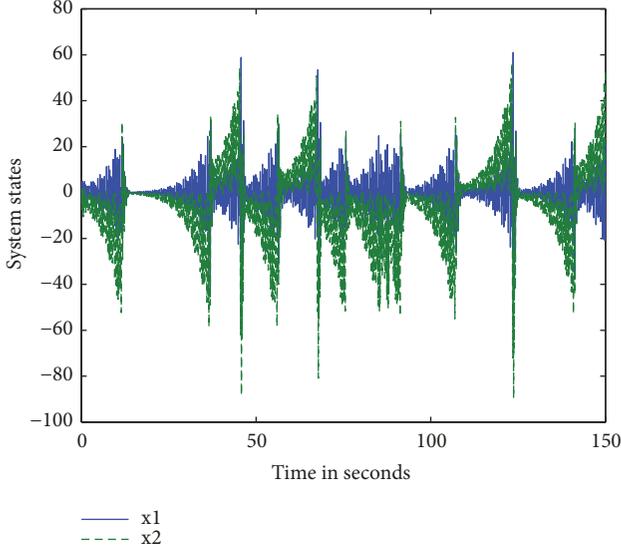


FIGURE 4: State responses in Example 1 by using piecewise affine state-feedback controller (40).

$$a_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix},$$

$$a_3 = \begin{bmatrix} -0.0028 \\ 0 \\ 0 \end{bmatrix},$$

$$B_1 = B_3 = \begin{bmatrix} -0.00998 \\ 0 \\ 0 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} -0.00999 \\ 0 \\ 0 \end{bmatrix},$$

$$H_1 = H_3 = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix},$$

$$H_2 = \begin{bmatrix} 0.5 \\ 0 \\ 0 \end{bmatrix},$$

$$L_1 = [1 \ 0 \ 0],$$

$$L_2 = [0.5 \ 0 \ 0],$$

$$L_3 = [1.5 \ 0 \ 0],$$

$$N_i = 0, \quad i \in \{1, 2, 3\}.$$

(50)

The decomposed regions are given as

$$\begin{aligned} \Sigma_1 &= \{x(t) \mid -\beta < x_1(t) \leq -\alpha\}, \\ \Sigma_2 &= \{x(t) \mid -\alpha < x_1(t) \leq \alpha\}, \\ \Sigma_3 &= \{x(t) \mid \alpha < x_1(t) \leq \beta\}, \end{aligned} \quad (51)$$

where $\alpha = 1, \beta = 5$.

From (7), we can obtain that $\phi_i^T = [1 \ 0 \ 0]$, for $i \in \Pi$, and the parameters of the degenerate ellipsoid are

$$\begin{aligned} Q_1 &= [0.5 \ 0 \ 0], \\ q_1 &= -1.5, \\ Q_2 &= [1 \ 0 \ 0], \\ q_2 &= 0, \\ Q_3 &= [0.5 \ 0 \ 0], \\ q_3 &= 1.5. \end{aligned} \quad (52)$$

It is assumed in this example that the perturbations existing in the controller gains are set as

$$\begin{aligned} M_{i2} &= 0.1, \\ U_{i1} &= [0.1 \ 0.1 \ 0.1 \ 0.1], \\ U_{i2} &= 0.3, \end{aligned} \quad (53)$$

$i \in \{1, 2, 3\}.$

As the controller gains contain parameter perturbations, we will design a piecewise affine nonfragile controller in (9) to guarantee that the closed-loop system is asymptotically stable with a robust \mathcal{H}_∞ performance γ . Exploiting Theorem 3 with $S = [1 \ 0]^T$ and $\lambda = 1$, the controller gains can be obtained as

$$\begin{aligned} K_1 &= [-0.1688 \ -0.0053 \ -0.1464 \ 0.0017], \\ k_1 &= -4.7949 \times 10^{-4}, \\ K_2 &= [-0.0147 \ -0.0005 \ -0.0127 \ 0.0001], \\ k_2 &= 0, \\ K_3 &= [-0.0204 \ -0.0006 \ -0.0176 \ 0.0002], \\ k_3 &= 5.7296 \times 10^{-5}, \end{aligned} \quad (54)$$

with robust \mathcal{H}_∞ performance $\gamma_{\min} = 0.2755$.

To present the effectiveness of the designed controllers, simulations are carried out. With initial conditions $x_0 = [3.5 \ 0 \ -3]^T$ and the external disturbance $w(t) = 120\pi^{-3.6t} \cdot \cos(4t)$, the state trajectories of the closed-loop system are demonstrated in Figure 5(a).

Solving the LMIs in (18)-(19) with $W_{i2} = 0, U_{i1} = [0 \ 0 \ 0], U_{i2} = 0, i \in \{1, 2, 3\}$, and $\lambda = 1, S = [2 \ 2 \ 1]^T$, then

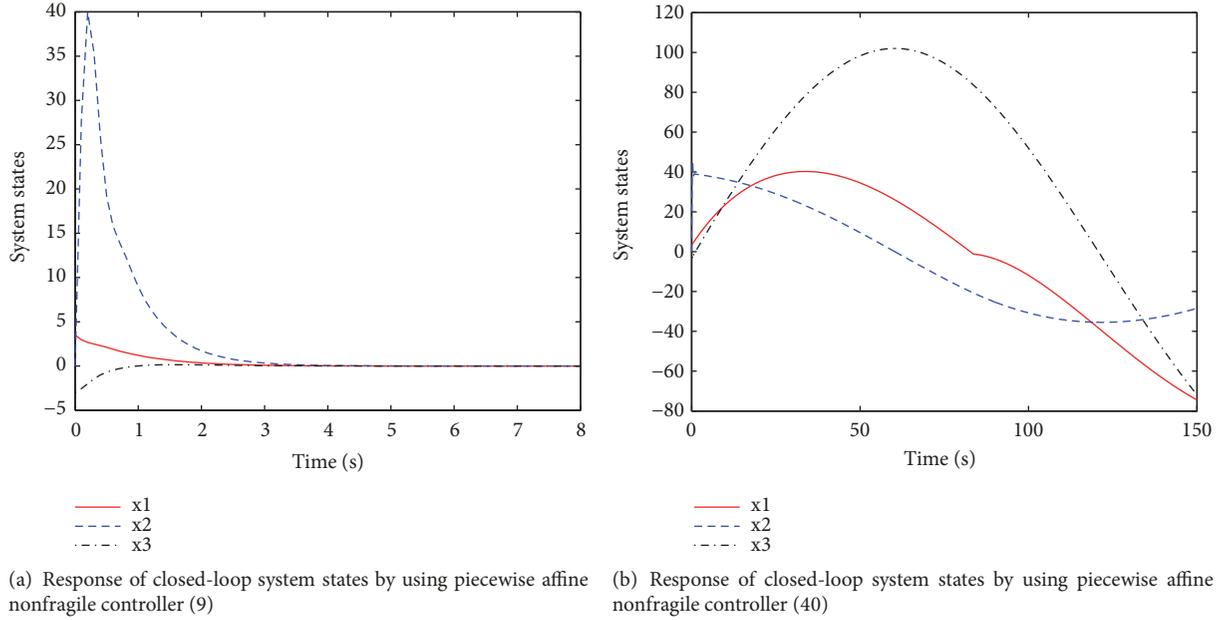


FIGURE 5: Response of the system states in Example 2.

the gains of the piecewise affine state-feedback controller (40) are

$$\begin{aligned}
 K_1 &= [-0.1407 \quad -0.0041 \quad -0.1471 \quad 0.0014], \\
 k_1 &= 1.3805 \times 10^{-6}, \\
 K_2 &= [-0.1688 \quad -0.0050 \quad -0.1772 \quad 0.0017], \\
 k_2 &= 0, \\
 K_3 &= [-0.1536 \quad -0.0045 \quad -0.1613 \quad 0.0015] \times 10^{-6}, \\
 k_3 &= 4.3196 \times 10^{-10}.
 \end{aligned} \tag{55}$$

With the same initial conditions and disturbance, we use controller (40) to system (49), and the closed-loop system states are demonstrated in Figure 5(b). It can be seen from Figure 5 that the piecewise affine state-feedback controller (40) can not stabilize system (49) with parameter variations existing in the controller gains.

5. Conclusion

In this paper, we have studied the nonfragile control problem for a class of discrete-time hybrid systems based on piecewise affine model. Based on PLFs, some new sufficient conditions for piecewise affine nonfragile controller synthesis are presented. By virtue of several convexification strategies, the controller gains can be attained via solving a set of linear matrix inequalities. Finally, simulation examples are carried out to demonstrate the advantages of the proposed methodologies.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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