Research Article

Robust Impulsive Stabilization of Uncertain Nonlinear Singular Systems with Application to Transportation Systems

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We consider the robust asymptotical stabilization problem for uncertain singular systems. We design a new impulsive control technique to ensure that the controlled singular system is robustly asymptotically stable and hence derive the corresponding stability criteria. These sufficient conditions are expressed in the form of algebra matrix inequalities and can be implemented numerically. We finally provide a numerical example of a transportation system to illustrate the effectiveness and usefulness of the proposed criteria.

1. Introduction

System stability is a fundamental issue for a nonlinear system. It not only relates to its system structure but also has relations with the exterior disturbance of the system. Singularity and parameter's uncertainty of the system can seriously affect its stability performance. For an unstable system, how to design a controller which stabilizes the system becomes a critical problem, which motivates the current research. The stabilization methods can be applied to a range of practical applications such as transportation systems [1, 2].

As for general nonlinear systems, various design methods to control these dynamical systems have been proposed. Among them, impulsive control methods have attracted considerable attention because impulsive control laws have a fast response time, strong robustness and resistance to disturbances, and low energy consumption. They have been applied to many disciplines [3–5]. The stabilization issue for the nonsingular certain system by use of the impulsive control method has been studied and many sufficient conditions of its asymptotical stability, such as [6–9], have been provided. Recent research work can be found in [10–15] and the references therein.

In this paper, we will develop the impulsive control method, the Lyapunov functional method, and the matrix inequality technology to solve the stabilizing issue of a class of singular systems with uncertainty, and design an impulsive controller of parts of state variables. Finally, the sufficient conditions of asymptotical stability will be given under our designed controller.

This paper's notations are quite standard. Let R^n denote the n -dimensional Euclidean space, R_+ the set of nonnegative real numbers, and $R^{m \times n}$ the set of all real $m \times n$ matrices. The superscript "T" represents the matrix transposition operation and $X \geq Y$ (respectively, $X > Y$), where X and Y are symmetric matrices, indicates that $X - Y$ is positive semidefinite (respectively, positive definite). The symbol I_n is $n \times n$ the identity matrix and $\|\cdot\|$ is the Euclidean norm in R^n . The $\lambda_M(P)$ and $\lambda_m(P)$ represent the largest and smallest eigenvalues of P , respectively. Let $K : R_+ \rightarrow R$ denote the set of all continuous real-valued functions. We say that the function $p : R_+ \rightarrow R_+$ is piecewise continuous if it is continuous on R_+ , except at the time points in the set $\{\tau_k\}$, is left-continuous, and has the right limit at τ_k for all k . The $PC(R_+, R_+)$ is the set of all such piecewise continuous functions p .

2. Preliminaries

2.1. Uncertain Singular System. Consider the following singular system with disturbances:

$$\begin{aligned} E\dot{x}(t) &= Ax(t) + w(t), \\ y(t) &= Cx(t), \\ x(t_0) &= x_0, \end{aligned} \quad (1)$$

where $x(t) \in R^n$ and $y(t) \in R^m$ are the system state and output vectors, respectively. The parameter $w(t, x(t))$ is the norm-bounded external uncertainty described by a continuous vector-valued function. The matrices $A \in R^{n \times n}$ and $C \in R^{m \times n}$ are constant ones of appropriate dimensions. In the situation that $0 < \text{rank}(E) < n$, we say that system (1) is an uncertain singular system. Without loss of generality, we assume that

$$E = \begin{pmatrix} I_u & 0 \\ 0 & 0 \end{pmatrix}, \quad (2)$$

where $I_u \in R^{u \times u}$ ($u \in N^+$, $u < n$) is an $u \times u$ identity matrix. We need the following assumptions for our later use.

Assumption 1. The uncertainty $w(t)$ is a norm-bounded nonlinear function and satisfies the following Lipschitz condition:

$$\|w(t_1) - w(t_2)\| \leq l \|x_u(t_1) - x_u(t_2)\|, \quad \forall t_1, t_2 \in R_+, \quad (3)$$

where $l > 0$ is a known constant scalar.

2.2. Impulsive Control System Theory. Consider the following nonlinear system with impulses:

$$\dot{x}(t) = f(t, x(t)), \quad t \neq \tau_k, \quad (4a)$$

$$y = \varphi(x), \quad t \neq \tau_k, \quad (4b)$$

$$x(t_0^+) = x_0, \quad k = 1, 2, \dots, \quad (4c)$$

$$\Delta x(t) = u_k(y), \quad t = \tau_k, \quad (4d)$$

where $0 < \tau_1 < \dots < \tau_k < \dots$, with $\tau_k \rightarrow \infty$ as $k \rightarrow \infty$, and

$$\Delta x(\tau_k) = x(\tau_k^+) - x(\tau_k^-) = x(\tau_k^+) - x(\tau_k), \quad (5)$$

with $x(\tau_k^+) = \lim_{t \rightarrow \tau_k^+} x(t)$, and $x(\tau_k^-) = \lim_{t \rightarrow \tau_k^-} x(t)$. Let $\varphi \in C(R^n, R^m)$, $u_k \in C(R^m, R^n)$, $f \in C(R_+ \times R^n, R^n)$. The $C(R^n, R^m)$ is defined to be the set of all continuous functions mapping the value in R^n to the value in R^m and $C(R^m, R^n)$ and $C(R_+ \times R^n, R^n)$ can be defined similarly.

Definition 2. A sequence pair $\{\tau_k, u_k(y(\tau_k))\}$ is said to be an impulsive control pair for system (4a)-(4c). If system (4a)-(4c) is asymptotically stable after implementing an impulsive control, system (4a)-(4c) is said to be robustly impulsively stabilizable.

Given $\rho > 0$, let $S_\rho = \{x \in R^n : \|x(t)\| < \rho\}$. For $k = 1, 2, \dots$ and $(t, x) \in (\tau_{k-1}, \tau_k) \times R^n$, define

$$\begin{aligned} D^+V(t, x) &= \limsup_{h \rightarrow 0} \frac{1}{h} [V(t+h, x+hf(t, x)) - V(t, x)]. \end{aligned} \quad (6)$$

Furthermore, let $V(\cdot, \cdot) : R_+ \times S_\rho \rightarrow R_+$ be a continuous function on $R_+ \times S_\rho$, except possibly at the time points $\{\tau_k\}$ satisfying the following two conditions:

- (1) For each $x \in S_\rho$, $k = 1, 2, \dots$, $\lim_{(t,y) \rightarrow (\tau_k^-, x)} V(t, y) = V(\tau_k^-, x)$ exists;
- (2) $V(t, x)$ is locally Lipschitz in x .

To proceed, we need the following lemmas.

Lemma 3 (see [7]). Let $\varphi(0^-) = \varphi(0)$ and $\tau = \sup_k \{\tau_{k+1} - \tau_k\} < \infty$. Suppose that there exist $\alpha, \beta, c, \psi \in K$, $p \in PC(R_+, R_+)$, $V(t, x) \in V_0$, and $\sigma > 0$ such that

- (1) $x + u_k(x) \in S_\rho$ for all k ;
- (2) $\beta(\|x\|) \leq V(t, x) \leq \alpha(\|x\|)$, $\forall (t, x) \in R_+ \times S_\rho$;
- (3) $V(\tau_k, x + u_k(x)) \leq \psi_k(V(\tau_k, x))$, $k = 1, 2, \dots$;
- (4) $D^+V(t, x) \leq p(t)c(V(t, x))$, for $t \neq \tau_k$ and $x \in S_\rho$;
- (5) $\int_q^{\psi_k(q)} (ds/c(s)) + \int_{\tau_k}^{\tau_{k+1}} p(s)ds \leq -l_k$, $\forall q \in (0, \sigma)$, for all $l_k \in (0, +\infty)$, $k = 1, 2, \dots$, and $\sum_{k=1}^{\infty} l_k = \infty$.

Then, system (4a), (4b), (4c), and (4d) is asymptotically stable.

Lemma 4 (see [8]). Given a positive matrix S and a positive scalar γ , we have

$$MN + N^T M^T \leq \gamma^{-1} M S^{-1} M^T + \gamma N^T S N, \quad (7)$$

where M and N are real matrices with appropriate dimensions.

3. Robust Impulsive Stabilization

In this section, we shall design an impulsive control pair $\{\tau_k, u_k(x_u)\} = Bx_u(\tau_k)$ to stabilize system (1). System (1) under impulsive control can be rewritten as

$$\dot{x}_u(t) = A_{11}x_u(t) + A_{12}x_v(t) + w_u(t), \quad t \neq \tau_k, \quad (8a)$$

$$\Delta x_u(t) = x_u(t^+) - x_u(t^-) = Bx_u(t), \quad t = \tau_k, \quad (8b)$$

$$0 = A_{21}x_u(t) + A_{22}x_v(t) + w_v(t), \quad t \neq \tau_k, \quad (8c)$$

$$y(t) = Cx(t), \quad (8d)$$

where $x(t) = (x_u(t), x_v(t))^T$, $w(t) = (w_u(t), w_v(t))^T$ and $B \in R^{u \times u}$. The initial conditions are given as $x_u(t_0) = x_{u0}$, and $x_v(t_0) = x_{v0}$, $\tau_k \geq t_0$, $k = 1, 2, \dots$.

Theorem 5. Suppose Assumption 1 holds. The impulsive controlled system (8a), (8b), (8c), and (8d) is robustly asymptotically stable if the following conditions are satisfied:

- (1) $\lambda_M(Q_2) < 1$;
- (2) $\alpha = \lambda_m(A_{22}^T A_{22}) > 0$;
- (3) $\lambda_M(Q_1)\tau + \ln \lambda_M(Q_2) < 0$.

where

$$\begin{aligned} Q_1 &= A_{11} + A_{11}^T + A_{12}A_{12}^T + \frac{1}{\alpha}A_{21}^T A_{21} \\ &\quad + (1 + ml^2)I_p, \\ Q_2 &= (I_p + B)^T (I_p + B) \\ \text{and } m &= \max \left\{ 1, \frac{2}{\alpha} \right\}. \end{aligned} \quad (9)$$

Proof. Consider the following standard quadratic Lyapunov function candidate:

$$V(t, x) = x_u(t)^T x_u(t), \quad (10)$$

which satisfies condition (i) of Lemma 3. By virtue of the upper Dini derivative of Lyapunov function (10) along the solution of system (8a), (8b), (8c), and (8d), it follows that at $t \neq \tau_k$, $k = 1, 2, \dots$, it yields

$$\begin{aligned} D^+V(t, x) &= \dot{x}_u(t)^T x_u(t) + x_u(t)^T \dot{x}_u(t) \\ &= x_u(t)^T (A_{11} + A_{11}^T) x_u(t) \\ &\quad + 2x_u(t)^T A_{12}x_v + 2x_u(t)^T w_u(t). \end{aligned} \quad (11)$$

By Lemma 4, the following inequalities

$$2x_u(t)^T A_{12}x_v \leq x_u(t)^T A_{12}A_{12}^T x_u(t) + x_v^T x_v, \quad (12)$$

and

$$2x_u(t)^T w_u(t) \leq x_u(t)^T x_u(t) + w_u(t)^T w_u(t) \quad (13)$$

hold. Thus, it follows that (11) can be rewritten as

$$\begin{aligned} D^+V(t, x) &\leq x_v(t)^T x_v + w_u(t)^T w_u(t) \\ &\quad + x_u(t)^T (A_{11} + A_{11}^T) x_u(t) \\ &\quad + x_u(t)^T (I_u + A_{12}^T A_{12}) x_u(t). \end{aligned} \quad (14)$$

Moreover, we have

$$\begin{aligned} \lambda_m(A_{22}^T A_{22}) x_v^T x_v &\leq (A_{22}x_v)^T (A_{22}x_v) \\ &\leq 3x_u^T A_{21}^T A_{21} x_u + 3w_v^T w_v. \end{aligned} \quad (15)$$

Using condition (4a), (4b), (4c), and (4d) of Theorem 5, we obtain

$$x_v^T x_v \leq \frac{3}{\alpha} x_u^T A_{21}^T A_{21} x_u + \frac{3}{\alpha} w_v^T w_v, \quad (16)$$

and

$$\frac{3}{\alpha} w_v^T w_v + w_u^T w_u \leq ml^2 x_u^T x_u. \quad (17)$$

Combining (14), (16), and (17) together, we have

$$D^+V(t, x) \leq \lambda_M(Q_1) V(t). \quad (18)$$

Thus, condition (8a), (8b), (8c), (8d) is satisfied with $p(s) = \lambda_M(Q_1)$ and $c(s) = s$.

At $t = \tau_k$, $k = 1, 2, \dots$, we have

$$\begin{aligned} V(x_u(t) + Bx_u(t)) &= x_u^T(I_u + B)^T(I_u + B)x_u \\ &\leq \lambda_M(Q_2) V(x_u(t)). \end{aligned} \quad (19)$$

Then condition (7) of Lemma 3 is satisfied with $\psi_k(s) = \lambda_M(Q_2)s$. It follows from condition (1) of Theorem 5 and (19) that

$$\|x_u(t) + Bx_u(t)\| \leq \|x_u\|, \quad (20)$$

which indicates that $x_u \in S_{\rho_0}$, $\rho_0 \in (0, \rho)$; i.e., $x_u + Bx_u \in S_{\rho_0}$ for all k . Thus, condition (ii) of Lemma 3 is also guaranteed. For $\tau < \infty$, the following two integrals are valid:

$$\int_q^{\psi_k(q)} \frac{ds}{c(s)} = \ln \lambda_M(Q_2) \quad (21)$$

and

$$\int_{\tau_k}^{\tau_{k+1}} p(s) ds \leq \lambda_M(Q_1) (\tau_{k+1} - \tau_k). \quad (22)$$

Hence, it follows from condition (7) of Theorem 5 that

$$\begin{aligned} \int_q^{\psi_k(q)} \frac{ds}{c(s)} + \int_{\tau_k}^{\tau_{k+1}} p(s) ds &\leq \lambda_M(Q_1) \tau + \ln \lambda_M(Q_2) \\ &= -r_k < 0 \end{aligned} \quad (23)$$

and

$$\sum_{k=1}^{\infty} l_k = \infty. \quad (24)$$

Condition (10) of Lemma 3 is also satisfied. Therefore, by Lemma 3 and Definition 2, system (8a)-(8b) is robustly asymptotically stable. Obviously, we can see that $\lim_{t \rightarrow +\infty} x_u(t) = 0$ as $\|w(t)\| \leq l\|x_u(t)\|$, which leads to $\lim_{t \rightarrow +\infty} x(t) = 0$. Therefore, system (8a), (8b), (8c), and (8d) is robustly asymptotically stable under the designed impulsive control. The proof is complete. $\square$

4. Application to Transportation Systems

In this section, we will consider a transportation application to illustrate our results obtained in Section 3. Consider a

transportation system which is modelled by (1) with the following specifications:

$$\begin{aligned} A_{11} &= 1, \\ A_{12} &= 0.52, \\ A_{21} &= -0.48, \\ A_{22} &= 2, \\ w(t) &= \left(\sqrt{\frac{x_1^2}{2} + x_2}, \sqrt{\frac{x_1^2}{2} - x_2} \right)^T, \\ E &= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \\ x(0) &= (-2, 1)^T. \end{aligned} \quad (25)$$

It is clear that $l = 1$. Then this transportation system is a singular 2-dimensional system with uncertainty. Now, we design an impulsive pair $\{\tau_k, Bx_1(\tau_k)\}$, where $B = -0.8$ and $\tau = \tau_{k+1} - \tau_k = 0.02$. Then, we have $\alpha = 1 > 0$, $Q_1 = 18$, $Q_2 = 0.04$, and $\lambda_M(Q_1)\tau + \ln \lambda_M(Q_2) = -3 < 0$.

Consequently, it follows from Theorem 5 that the transportation system is impulsively stabilizable under the following impulsive control pair:

$$\{\tau_k, Bx_1(\tau_k) = -0.8x_1(\tau_k)\}. \quad (26)$$

From the example, it is concluded that the impulsive control method can effectively stabilize the transportation singular systems.

5. Conclusion

In this paper, we have proposed a design method for robust impulsive stabilizing control for a singular transportation system with uncertainty. Sufficient conditions are derived to guarantee the global asymptotical stability of the system. An application to transportation systems shows that our designed impulsive stabilizing control is effective and strongly robust.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The author declares that they have no conflicts of interest.

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