Research Article

Simple Method to Predict Ground Displacements Caused by Installing Horizontal Jet-Grouting Columns

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During the horizontal jet grouting in soft ground, injection of large volumes of water and grout into the soil can lead to significant ground displacements. A simple method is proposed in this paper to predict the ground displacements caused by installing horizontal jet-grouting columns. The process of installing a horizontal column is simplified as the expansion of a cylindrical cavity with a uniform radial stress applied at plastic-elastic interface in a half plane. In this study, the analytical solution is adopted to calculate the deformation induced by the expansion of a cylindrical cavity. Considering the main jetting parameters (jetting pressure of the fluid, flow rate of the fluid, and withdrawal rate of the rod) and the soil properties (stiffness of the surrounding soil), an empirical equation to estimate the radius of plastic zone is developed. Two field tests are carried out in Shanghai, China, to verify the correctness and applicability of the proposed method. Comparisons between the predicted and measured values indicate that the proposed method can provide a reasonable prediction. The proposed simple method can be recommended as a useful tool for the design of ground improvement by means of horizontal jet grouting.

1. Introduction

Jet grouting has been deemed as one of the most widely used ground improvement techniques for strengthening soft soil deposits and preventing groundwater seepage from leaking fractures or fissures [1–15]. Jet grouting is frequently used for the launching of shield machines or tunnel construction in complex geological conditions. Jet grouting technology is based on the injection of high velocity fluids through small-diameter nozzles to erode the soil and mix it with injected grout to form a soil-cement column [16–21]. Based on the different methods of fluid injection, jet-grouting technology can be classified as (a) single fluid system (grout only), (b) double fluid system (grout + air), or (c) triple fluid system (water + grout + air) [22–25]. Generally, according to the injection direction of the fluid, the jet-grouting technology can be divided into (i) vertical jet-grouting technology and (ii) horizontal jet-grouting technology.

Since there is a risk of intrusion of an overlying layer, horizontal jet-grouting technology has been increasingly used to create jet-grouting umbrellas for protecting excavations and reducing surface settlements (Xu et al., 2016) [4, 26–31]. As jet grouting involves the injection of large volumes of water or grout into the soil, inevitable disturbance of the surrounding soil will occur, which may cause significant displacement of the ground [4, 19, 32–37]. When jet-grouting columns are installed horizontally, the impact on the surrounding soil is mainly due to the vertical orientation of the jet, which is of great concern for geotechnical engineers. The ground displacement induced by the jet-grouting process may result in additional earth pressures being imposed on existing building foundations or utilities, causing possible adverse effects. Thus, it is necessary to determine the magnitude of such additional loads and the anticipated subsoil movement and ground surface heave.

Recent advances in jet-grouting technology have mainly focused on the strength of hardened columns and the evaluation of achievable column size (Wang et al., 2012) [38–41]. There is, however, very little literature discussing the ground displacement induced by horizontal jet-grouting...
column installation. Based on the cavity expansion theory, Chai et al. [42–44] proposed a semitheoretical method to estimate the lateral displacements induced by installing soil-cement columns. However, Chai’s method was developed for the installation of vertical soil-cement columns and cannot be directly applied to the installation of horizontal jet-grouting columns.

In this paper, a method was proposed to predict the ground displacement caused by installing a horizontal jet-grouting column by considering the jetting parameters and soil properties. This was done by simplifying the process of installing a horizontal column as the expansion of a cylindrical cavity and with uniform radial stress in a half plane. The ground displacements from two field tests conducted in Shanghai, China, were compared with the predicted values to verify the correctness and applicability of the proposed method.

2. Brief Review of Existing Methods

There are three available methods for estimating ground displacement due to horizontal jet-grouting column installation: (i) the empirical method, (ii) the numerical simulation method, and (iii) the semitheoretical and semiempirical method.

Empirical methods are based upon field observations. However, the parameters used in the empirical methods due to a lack of the physical meaning were not defined clearly. Additionally, the parameters are usually derived for specific ground conditions and they are difficult to be applied to other jet-grouting projects that possess different ground conditions.

Numerical simulation methods are not convenient for practical use because of the requirement of the input parameters. Determining the input parameters involves professional background knowledge and the calibration of the input parameters by performing laboratory experiments may be required. Additionally, the numerical simulation methods may not be capable of modelling the process of jet-grouting column installation.

Semitheoretical and semiempirical methods are based upon the cavity expansion theory in an infinite soil mass. The methods may be capable of describing the process of jet-grouting column installation. The semitheoretical approach to estimate the lateral displacement caused by the installation of soil-cement columns was proposed by Chai et al. [42–44], in which the radius of the cavity (\( R_u \)), a key parameter, was empirically determined by analyzing several field tests using the slurry double mixing (SDM) method, the dry jet mixing (DJM) method, and the wet jet mixing (WJM) method. Notwithstanding that, this method can only be applied to the prediction of lateral displacement caused by vertical jet-grouting column installation. Also, the effect of the jetting pressure which plays a leading role for horizontal jet grouting cannot be considered in this method.

3. General Consideration for Simplification of Horizontal Jet-Grouting Process

During the installation of a horizontal jet-grouting column at a depth \(( h )\), the high pressure fluid is injected through small-diameter nozzles installed on the monitor, which is continuously rotated at a constant rate and slowly withdrawn, as shown in Figure 1(a). Since the jet-grouting process
involves the injection of large volumes of water or grout into the soil, significant ground heave is expected. In this study, for simplicity purposes, it was assumed that the ground displacement due to the installation of a horizontal column was identical to the ground deformation induced by the expansion of a cylindrical cavity \((R)\) with uniform radial stress \((N)\) in a half plane, as shown in Figure 1(b). Due to the simplification, the following two questions are raised and discussed. (1) How can one calculate the ground deformation induced by the expansion of a cylindrical cavity with uniform radial stress in a half plane? (2) How can one describe the physical meaning of \(R\) and \(N\) and then determine their values?

Verruijt [45] developed an analytical solution for calculating the deformations caused by a uniform stress acted on the cavity boundary in a half plane. Considering the plastic zone \((r_p)\) and the stress at the interface of the plastic and elastic zones \((N_p)\), Figure 3 illustrates the parameters required for calculating the deformations, while the relevant equations are listed as follows:

\[
L_xA = \text{Re} \left( \frac{1 + \nu}{E} \left( (3 - 4\nu) f(Z) - Z f'(Z) - F(Z) \right) \right) \tag{1}
\]

\[
L_yA = \text{Im} \left( \frac{1 + \nu}{E} \left( (3 - 4\nu) f(Z) - Z f'(Z) - F(Z) \right) \right), \tag{2}
\]

where \(\text{Re}\) and \(\text{Im}\) represent the real and imaginary parameters respectively; \(L_{xA}\) is the displacement of point A in \(x\) direction; \(L_{yA}\) is the displacement of point A in \(y\) direction; \(\nu\) is Poisson’s ratio; \(E\) is Young’s modulus; \(Z = x + iy\); \(f(Z)\) and \(F(Z)\) are the analytic functions. Based on the solution of Verruijt [45], the equations to determine functions \(f(Z)\), \(F(Z)\), and \(f'(Z)\) are derived as follows:

\[
f(Z) = C_d \left( -2i \left( 1 + \eta^2 \right) Z \right) + 2i \frac{(1 + \eta^2) + i\hbar \left( 1 - \eta^2 \right)}{Z(1 + \eta^2) - i\hbar \left( 1 - \eta^2 \right)}
\]
where $h$ is the depth for installation of a horizontal column; $N_p$ is the stress at the interface of the plastic and elastic zones; $r_p$ is the radius of the plastic zone. In this study, calculations of the radius of plastic zone and the interface stress of the plastic and elastic zones based on the above equations have been implemented in the MATLAB environment.

**4.2. Radius of Plastic Zone.** There are two key factors that significantly affect the radius of the plastic zone ($r_p$), that is, soil properties and jetting parameters. As the soil properties are basically the same, the radius of the plastic zone ($r_p$) will be larger if the higher values of the jetting parameters are introduced. Contrarily, in the event that the jetting parameters are basically the same, the radius of the plastic zone ($r_p$) will be smaller if the values of the soil properties are higher. In this study, two parameters, $E_n$ and $E$, were chosen to represent the influence of the jetting parameters and soil properties on the $r_p$ value, as depicted in Figure 4, and their relationship can be expressed by the following equation:

$$r_p = f(E_n, E),$$

where $E_n$ is the injected energy for the unit length of jet-grouting column; $E$ is Young’s modulus. Thus, the following empirical equation to determine the parameter $r_p$ was proposed for implementing the dimensional analysis:

$$r_p = \alpha_p \sqrt{\frac{E_n}{E}},$$

where $\alpha_p$ is a parameter that is related to the soil types. $E_n$ is related to the treatment energy for the unit length of jet-grouting column at the pump ($E_n$). Generally, the following equation is adopted to calculate the $E_n$ value:

$$E_n = \beta E_p = \beta \frac{pQ}{v_s},$$

where $E_p$ is the treatment energy for the unit length of jet-grouting column at the pump; $\beta$ is reduction coefficient that considers the energy losses in the transportation of jetting fluid to the nozzle; $p$ is the injection pressure at the pump; $Q$ is the flow rate; $v_s$ is the average withdrawal speed of the monitor. $\beta$ value of 0.8 suggested by Croce and Flora [22] has been adopted in this study.

**4.3. Stress at the Plastic-Elastic Zone Interface ($N_p$).** The factors that primarily affect the stress at the plastic-elastic zone interface ($N_p$) are related to the shear strength and stress state of ground soil. Vesic [46] proposed the following equations to determine the stress at the plastic-elastic zone interface ($N_p$):

$$N_p = \left( c \cdot F_c + p_0 \cdot F_q + c \cdot \cot \varphi \right) \sqrt{I_p \sec \varphi} \sin \varphi / (1 + \sin \varphi)$$

$$F_q = (1 + \sin \varphi) \left( \frac{I_p}{1 + I_p \cdot \Delta \sec \varphi} \right) \sin \varphi / (1 + \sin \varphi)$$

$$F_c = (F_q - 1) \cot \varphi$$

where $F_c$, $F_q$, and $p_0$ are the injection pressures of the grouting column; $c$ is the cohesion of soil; $\Delta$ is friction angle of soil; $\varphi$ is the inclination of grouting column; $I_p$ is the moment of inertia of the plastic zone obtained by the empirical equation to determine the parameter $r_p$.
\[ I_{rr} = \frac{I_r}{1 + I_r \cdot \Delta \cdot \sec \varphi} \]  
(10)

\[ I_r = \frac{E}{2 \left(1 + v\right) \left[c + p_0 \cdot \tan \varphi\right]} \]  
(11)

where \( c \) is cohesion of soil; \( F_c \) and \( F_q \) are coefficients for cylindrical cavity; \( \varphi \) is internal friction angle; \( E \) is Young’s modulus; \( \Delta \) is average volumetric strain; \( v \) is Poisson’s ratio; \( I_r \) is rigidity index; \( I_{rr} \) is reduced rigidity index; \( p_0 \) is initial mean normal stress, which can be calculated as \( p_0 = \frac{(2 + K_0)}{3} \sigma_{v0} \) in the case of horizontal jet grouting. For the case of \( \varphi = 0 \) and zero volumetric strain (\( \Delta = 0 \)), the following simple equation to calculate \( \sigma_p \) can be obtained:

\[ N_p = c_u + \frac{2 + K_0}{3} \sigma_{v0} \]  
(12)

where \( c_u \) is the undrained shear strength of the soil; \( K_0 \) is the at-rest horizontal earth pressure coefficient; \( \sigma_{v0} \) is the initial vertical stress.

4.4. Calculation Procedure for Ground Displacements in Elastic Zone. Figure 5 points out the parameters required for calculating the ground movement in the elastic zone \( (R_A \geq r_p) \). The procedure for calculating the ground displacements in the elastic zone is summarized as follows.

(1) Calculating the Stress at the Plastic-Elastic Zone Interface \( (N_p) \). Soil properties (unit weight, undrained shear strength, and at-rest earth pressure coefficient) can be first derived from ground investigations and/or laboratory experiments. For a given depth of construction, the stress at the plastic-elastic zone interface \( (N_p) \) can be calculated using (12).

(2) Calculating the Radius of the Plastic Zone \( (r_p) \). The jetting parameters (i.e., injection pressure at pump, flow rate, and average withdrawal speed of monitor) are collected, and Young’s modulus of soil is tested. Then the radius of the plastic zone \( (r_p) \) can be calculated using (5) and (6).

(3) Calculating the Ground Displacements in the Elastic Zone. By the calculated values of \( N_p \) and \( r_p \), the ground displacements in the elastic zone can thus be calculated using (1) and (2).

4.5. Calculation Procedure for Ground Displacements in Plastic Zone. Figure 6 points out the parameters required for calculating the ground movement in the plastic zone \( (r_c \leq R_A \leq r_p) \). Since Verruijt’s [45] solution cannot be utilized for calculating the ground displacements in a straightforward manner, by referring to an approximate formula from Chai et al. [42], two equations to calculate the ground displacements in plastic zone are proposed:

\[ L_{xA} = L_p \frac{2r_p + L_p}{2R_A + L_p + L_p^r_p/R_A R_A} x \]  
(13)

\[ L_{yA} = L_p \frac{2r_p + L_p}{2R_A + L_p + L_p^r_p/R_A} \frac{h + y}{R_A} \]  
(14)

where \( L_p \) is the soil displacement at the radius of plastic zone; \( R_A \) is the distance from the column center to point A, which can be calculated by \( R_A = \sqrt{x^2 + (h + y)^2} \). The calculation procedure for the ground displacements in plastic zone is described as follows.

(1) Calculating the Soil Displacement at the Radius of Plastic Zone \( (L_p) \). The radius of the plastic zone \( (r_p) \) and the stress at the plastic-elastic zone interface \( (N_p) \) can be calculated through the obtained soil parameters and jetting parameters. After that, the soil displacement at the radius of plastic zone.
Young's modulus of soil, the radius of the plastic zone horizontal jet-grouting column is described as follows for predicting the ground displacements induced by installing \( i \) of the column numbered \( y \) displacement of point A in \( L \) where \( i \) represents the number of the jet-grouting columns; \( L_{xi} \) is the displacement of point A in \( x \) direction due to the installation of the jet-grouting column numbered \( i \); \( L_{yi} \) is the displacement of point A in \( y \) direction due to the installation of the column numbered \( i \).

The procedure of utilizing the proposed simple method for predicting the ground displacements induced by installing horizontal jet-grouting columns is described as follows.

1. Based upon the energy imposed during jetting and Young's modulus of soil, the radius of the plastic zone \( r_p \) can be calculated using (5).

2. The stress at the plastic-elastic zone interface \( N_p \) can then be calculated by substituting the undrained shear strength of soil, \( c_u \), and the coefficient of lateral earth pressure at rest, \( k_0 \), into (12).

3. The associated ground displacements in the elastic zone and in the plastic zone induced by installing a horizontal jet-grouting column can be calculated using (1) and (2) and (13)- (14), respectively.

4. The ground displacements caused by installing a row of horizontal jet-grouting columns would be identical to the summation of the effects of each jet-grouting column installation and can be calculated using (15).

It should be noted that the method in the previous publication of the authors [19] is developed for analyzing the vertical jet grouting cases, but the authors found that Shen et al.'s method cannot be directly used to obtain a reasonable displacement prediction for the horizontal jet grouting cases in engineering practice. Thus, a new approach is developed in this paper to provide a reasonable displacement prediction for horizontal jet grouting cases. Additionally, two case histories (horizontal jet grouting cases), which are entirely different from the case histories (vertical jet-grouting cases) in Shen et al. [19], are analyzed to verify the applicability of the newly developed approach. An improved equation to calculate the parameter \( r_i \) is also provided considering \( E_n \) and \( E \) in this paper (see (5)), which is also different from the equation in Shen et al. [19] (see (9)). Moreover, because the boundary conditions for vertical and horizontal jet-grouting cases are different, the equation to calculate the value of \( N_p \) (see (12)) in this new manuscript is also different from the equation in Shen et al's [19] paper (see (16)).

6. Method Verification

6.1. Single Column Installation. Shen et al. [4] reported on a field test located near the Huangpu River in the Pudong New Development Area, China. In this field test, the single fluid system was used for horizontal jet grouting, where the metro tunnel stating shaft was protected by the excavation (Wu et al., 2015c; 2016; 2017a, b) [19, 47, 48]. Figure 8 shows the geotechnical profile and soil properties for this case history.
The subsoil profile consists mainly of three soil layers: backfill, silty clay, and soft clay. The water content for the soils on this test site ranged from 32% to 45%, while the undrained shear strengths were in the range of 21–39 kPa.

The ground surface heave and lateral displacement during the installation of a horizontal jet-grouting column were monitored. Six ground settlement gauges (S1 to S6) spaced at 3.0 m were installed, and one soil inclinometer was set at a depth of 15 m, as shown in Figures 9(a) and 9(b). The horizontal jet-grouting column was installed at a depth of 6.5 m. Jetting parameters for this field test were as follows: the jetting pressure \( p = 30 \) MPa; the flow rate \( Q = 90 \) L/min; and the withdrawal rate of the rod \( v_s = 15 \) cm/min. By (6), the \( E_n \) value was calculated to be 14.4 MJ/m. Then, by \( c_u = 30 \) kPa and \( K_0 = 0.5 \), the \( N_p \) value was calculated to be 127 kPa. For silty clay, the constrained modulus \( E \) of about 5.0 MPa reported by Huang and Gao [49], Shen and Xu [50], Tan and Wei [51], Ye et al. (2013), Du et al. [52], and Shen et al. [53] was used in the calculation. As discussed previously, the empirical parameter \( \alpha_p \) related to the soil types is an unknown parameter and is difficult to determine. It is noted from a trial calculation that the predicted lateral displacements caused by the installation of a single column were in good agreement with the measured lateral displacements, with the \( \alpha_p \) value set to 1.0.

Substituting the calculated values of \( E_n, E \), and \( \alpha_p \) into (5), the \( r_p \) value was calculated to be 1.70 m. Substituting the values of \( r_p, N_p \), and \( h \) into (1) to (3), which were coded in MATLAB, the ground surface heave and the lateral displacement caused by the installation of a horizontal column can be obtained. Figure 10 presents comparisons for the predicted and measured values of the ground surface heave, caused by the installation of a horizontal column. As evident, the proposed method can provide a reasonable prediction. Additionally, the comparisons of the predicted and measured values of the lateral displacement appear to be quite satisfactory (Figure 11).

6.2. Group Column Installation. Wang et al. [32] reported on a case history related to a field trial involving the installation of five horizontal jet-grouting columns at the entrance shaft of Bailixincun Road Station of Shanghai Metro line 11. The jet-grouting columns in this field trial were formed using the double fluid method, known as the horizontal twin-jet-grouting method. Figure 12 shows the geotechnical profile and soil properties for this case history. The subsoil profile consisted of backfill (from 0.0 to 1.6 m), clayey silt (from 1.6 to 3.7 m deep), soft silty clay (from 3.7 to 13.0 m deep), and soft clay (from 13.0 to 27.0 m deep). Typical soil properties are summarized in Figure 12. The jet-grouting field test was conducted in the soft silty clay layer of low shear strength, which is subjected to time-dependent compressibility due to creep behavior (Wu et al. 2015d, e; Wu et al. 2017a, b) [54–56]. The soft soils were normal to slightly overconsolidated and were classified as low plasticity clays (CL), based on the Unified Soil Classification System (USCS). The water content of these soils varies from 30% to 50%, with unconfined compressive strengths \( q_u \) varying between 30 and 55 kPa.
In this field trial, due to the installation of five horizontal jet-grouting columns, the lateral displacement of subsoil and the ground surface heave were monitored. Figures 13 and 14 present the plan and sectional view of the layout of the ground settlement gauges and inclinometer for this case history. The instruments installed and monitored included one soil inclinometer and five ground settlement gauges (S1 to S5). The inclinometer was installed at a depth of 24 m, while the five horizontal columns were constructed at a depth range of 10.75–11.65 m. The construction sequence of the five horizontal columns was from C1 to C5, and their installations were completed in two days. In this case history, jetting parameters of the field test were as follows: the jetting pressure ($p$) = 12 MPa; the flow rate ($Q$) = 86 L/min; the withdrawal rate of the rod ($v_s$) = 60 cm/min. By (6), the $E_n$ value was calculated to be 1.4 MJ/m. Substituting the $c_u$ and $K_0$ values of 17.5 kPa and 0.5, respectively, into (7), the value of $N_p$ was calculated to be 171 kPa. For soft silty clay, the constrained modulus ($E$) reported by Huang and Gao [49] and Tan and Li [57] is about 2.1 MPa. Substituting $E_n = 1.4$ MJ/m, $E = 2.1$ MPa, and $\alpha_p = 1.0$ into (5), the value of $r_p$ was 0.81 m. Substituting the values of $r_p$, $N_p$, and $h$ into the MATLAB program that was written to solve (1) to (3) and due to the installation of five horizontal columns, the ground surface heave and the lateral displacements can be calculated. Figures 15 and 16 plot comparisons between the predicted and measured values of the ground surface heave and the lateral displacement caused by the installation of five horizontal columns, respectively. As evident, there were some discrepancies between the predicted and measured values, likely due to the omission of the influence of installation sequence and rheological properties of the grout in the calculation. Moreover, after the construction of horizontal columns, either their very low initial strengths or the dissipation of
4.0 m 4.0 m
S1 S2 S3 S4 S5
Settlement gauges
7.5 m 9.0 m
Retaining structure
Jet-grouting zone
Inclinometer
2.9 m 6.0 m
Figure 13: Plan view of the layout of ground settlement gauges and inclinometer (group column installation case).

Jet-grouting column
Inclinometer
Centerline of the column
Ground surface
0.6 m 1.5 m 4.4 m
Figure 14: Sectional view of the layout of ground settlement gauges and inclinometer (group column installation case).

10.75 m
4.0 m 0.5 m
C1 C2 C3 C4 C5
Figure 15: Comparisons of the predicted and measured values of the ground surface heave caused by the installation of five horizontal columns.

24.0 m
0 10 20 30 40 50 150 120 90 60 30
Distance to the center of horizontal column (m)
Ground surface upheaval (mm)
Centerline of the column (C5)
α_p = 1.0
α_p = 1.0
Construction depth
Depth (m)
−− Predicted curve
−−− Observed data
Lateral displacement (mm)
0 50 100 150 200 250 300 350 400 450 500
Figure 16: Comparisons of the predicted and measured values of the lateral displacement caused by the installation of five horizontal columns.

excess pore water pressure generated during construction could result in a reduction in the ground surface heave.

7. Conclusions
In this paper, a simple method to predict the ground displacements induced by installing horizontal jet-grouting columns was proposed. In this method, Verruijt [45] solution for calculating deformations based on the expansion theory of a cylindrical cavity with uniform radial stress in a half plane was adopted. An empirical equation to consider the total applied energy and soil properties to estimate the radius of plastic zone was developed. Based upon the comparisons between the measured ground displacements and those predicted through the proposed method, the following conclusions can be drawn:

1) The proposed method was applied to two case histories to verify its correctness and applicability, in which one involved the installation of a horizontal column and the other involved the installation of five horizontal columns.

2) The comparisons between the predicted and measured values of the lateral displacement and the ground surface heave showed that, in the single column installation case, the proposed method can yield a satisfactory prediction as the α_p value was set to 1.0. In the group column installation case, there were several discrepancies between the predicted and measured values, which were most likely due to the omission of the influence of installation sequence and rheological properties of the grout in the calculation procedure.

3) Despite the discrepancies, the proposed method is deemed appropriate in guiding and improving the design of ground improvement by horizontal jet grouting.

Conflicts of Interest
The authors declare that there are no conflicts of interest regarding the publication of this paper.
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References


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