Research Article

Dynamic Instability in Shallow Arches under Transversal Forces and Plane Frames with Semirigid Connections

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Structural engineering demands increasingly lighter systems, which can cause instability problems and compromise performance. A high slenderness index of a structural element makes it susceptible to instability. It is important to understand the problem, the limits of stability, and its postcritical behavior. An example that can occur in collapsed arches under a cross load is the dynamic snap-through behavior, where the structure in a given equilibrium condition jumps to a new remote equilibrium setting, causing usually sudden curvature. The semirigid connections are a source of physical nonlinearity and can influence the overall stability of the structural system, in addition to the distribution of stresses in the same system. Conventional approaches make use of static considerations. However, instability problems are inherently evolutionary processes, so a transient analysis is necessary for a complete description of structural behavior. The present work evaluates the geometrically nonlinear dynamic behavior of collapsed arches subjected to transverse force and plane frames with semirigid connections. The time domain responses, via Newmark's Method and positional formulation of the Finite Element Method, were obtained in terms of displacements, velocities, acceleration, and phase diagrams.

1. Introduction

Arches are widely used in mechanical, civil, or aerospace engineering. Because they present considerable slenderness, they may present a dynamic instability called snap-through behavior. For a critical loading level below the ultimate load, the arch jumps from the potential prebuckling valley and the response converges to a stable postcritical solution with the arch assuming an inverted configuration [2]. This process leads to large stress inversions and frequencies that significantly increase the possibility of failure of the structure. Therefore, it is necessary to identify the dynamic stability limits that separate small amplitude responses from those occurring in the postcritical region. The dynamic snap-through behavior is a strongly nonlinear phenomenon that generally has no analytical solution [3].

In recent years, research has been carried out on the subject. Zhou et al. [3] proposed a scaling approach to highlight similarities between dynamic snap-through behavior in different thin-curved structures using variations between geometric and/or boundary conditions for fast dynamic stability limits approximations. Rosas [4] conducted studies on a sine-shaped arch subjected to uniformly distributed loading, considering support conditions as discrete rotational springs. This same model was also evaluated by Galvão [2] and Silva [5]. Zhou et al. [6] demonstrated an analytical method to study the nonlinear stability and the disconnected equilibrium of shallow arches with asymmetric geometric imperfections. The solution proposed by them can be applied to arbitrary arches with arbitrary geometric imperfections. Ha et al. [7] analyzed the asymptotic stability of shallow arches, achieving the analytical response when the

FIGURE 1: (a) shallow-pinned arch; (b) typical load-displacement path in static analysis.

initial shape of the arch and the acting load, assumed to be independent of time, are linear combinations of sinusoidal functions. Virgin et al. [8] verified the response to a shallow arch subjected to loading and the influence of changes in the thermal environment. Chandra et al. [9] have obtained experimental and computational results of the transient behavior of arches culled under harmonic distributed load, establishing the ranges of parameters that lead to snap-through and chaotic responses.

With respect to semirigid connections, recent studies can be found in Turkalj et al. [10], presenting a geometrically nonlinear formulation for large displacements in composite beams with semirigid connections. Zhang et al. [11] propose a multisprings component (MSC) to represent the rotational stiffness of semirigid beam-column connections. Ozel et al. [12] present a finite element model that considers shear deformation effects, semirigid connection, and rotational inertia effects to obtain consistent stiffness and mass matrices.

The proposed work presents the dynamic response of shallow arches and frames with semirigid connections using the traditional Newmark algorithm for nonlinear transient problems and a comparison with literature results. The geometric nonlinear analysis was obtained using a positional formulation of the Finite Element Method in which, instead of the linear displacements of the nodes, the nodal positions are considered as unknowns of the problem.

2. Dynamic Instability in Shallow Arches

Instability phenomena can be detected numerically in several problems in the context of solid mechanics. For these cases, numerical analyses are possible only within certain critical conditions when the occurrence of instability is not expected. The postcritical behavior of structures is an application of the chaos theory [13], in which a reorganization of the system in distant positions from precritical equilibrium positions occurs. The chaos is considered as a consequence of instability factors, far away from initial equilibrium position. However, the chaos reveals a certain degree of order, and so it would be incorrect to consider it by methods of random dynamics [14].

Shallow arches have small counter-deflections when compared to their spans (Figure 1(a)). This kind of structural

system is of interest to engineering because its geometry is typically used as a primary element in extensive structural systems. It is an interesting challenge for engineers because of the simplest structural configuration that presents the known mechanisms of buckling: symmetric snap-through behavior (limit point instability), antisymmetric snap-through behavior (unstable bifurcation), and classical stable bifurcation [15]. In addition, shallow arches may exhibit loss of stability oscillating relation to the critical load. This phenomenon is called flutter or oscillatory instability, and the critical load is called flutter load [14].

The typical representation of the snap-through behavior in static analysis can be visualized in Figure 1(b). When it occurs, the load-displacement diagram shows a jump, under load control, from point 1 to point 3. In some cases it is also possible to observe, under displacement control, a snap-back effect: the structure returns to its original configuration in a different equilibrium path (5 to 7). Points 2, 4, and 6 are critical points, associated with signal changes of the stiffness and mechanical behavior inflexion.

The structural failure is a dynamic process, so one must evaluate the buckling phenomenon and the stability from this point of view. A structural system is stable if a small change in the initial conditions of the problem leads to a small change in the response [14]. Nonlinear structural behavior is due to geometric nonlinearity or to physical nonlinearity.

Since the snap-through behavior is a highly nonlinear problem, numerical simulations of such a phenomenon require the use of numerically stable time-running algorithms. Chandra et al. [9] demonstrates numerical difficulties in usual time integration algorithms in this type of analysis. The effects of nonconservative forces acting on circular arches are analyzed in Detinko [16].

3. Frames with Kelvin-Voigt Rheological Connections

The Kelvin-Voigt connection was modeled by a spring k associated in parallel with a damper as showed in Figure 2. The parameters k and c were taken into account in the assembly of the Hessian matrix for the dynamical problem and in the assembly of the nodal force vector, with regard to both nodes i and j . For consideration of elastic connections,

FIGURE 2: (a) Kelvin-Voigt rheological model; (b) adopted graphical representation of the connection.

the variable c assumes zero. This scheme is extended for rotational movements.

Computationally, the nodes i and j were labeled differently but have the same position. The strategy adopted to manipulate degrees of freedom (DOF) of articulated joints was the same as that indicated by Greco and Coda [1]. According to Figure 3, the translational unknown variables are coupled while the rotational variables continue to be distinct after the union of two elements.

Here, the proposed scheme to deal with elastic and viscoelastic connections is considerably simpler than the usual approach presented in Galvão et al. [17] and Kassimali [18], based on variational principle of virtual work for static nonlinear analysis. Moreover, as presented in Greco et al. [19], at critical points the positional formulation presents better reliability for the numerical calculation. Numerical evaluation performed by the positional formulation can be potentially faster than the numerical evaluation performed by the corotational formulation, due to its simplicity, being very appropriate for complex structural analysis.

4. Finite Element Method: Positional Formulation

Greco [20] presents the equation of the Finite Element Method based on positional formulation, associating it with the Newmark Method for geometrically nonlinear dynamic analysis. Starting from the energy functional of the system, one has

$$\Pi = U_t - FX + K_c + K_a \quad (1)$$

where K_c (kinetic energy), K_a (damping), and U_t (potential energy) are given by

$$K_c = \int_V \frac{\rho}{2} \dot{X}^2 dV \quad (2)$$

$$K_a = \int_V c_m \rho X \dot{X} dV - \int_V \int_{X_k} c_m \rho \frac{X \ddot{X}}{\dot{X}} dX_k dV \quad (3)$$

$$U_t = \int_V \frac{1}{2} E \varepsilon^2 dV \quad (4)$$

where c_m is the coefficient of damping, ρ is the specific mass, F are the external forces, and X represents the set

of independent coordinates (positions and rotations) that a given nodal point can present. The strain energy function of the considered body (framed elements) is considered to be stored at the reference volume of the body (V) and E represents the material longitudinal elasticity modulus. An engineering strain measure (ε) can be used to calculate the strain energy, as presented in Rabelo et al. [21].

The damping contribution (K_a) in the energy functional (Π) is implicit. Here, the algebraic development was performed to obtain a formulation consistent with the classical Rayleigh damping, proportional to mass. The mass matrix of the elements is considered discrete and the mass terms due to rotation were neglected, which explains the occurrence of zeros on the main diagonal of

$$M = \frac{\rho AL}{2} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (5)$$

The damping term K_a can be rewritten as

$$K_a = CX\dot{X} - C \int_{X_k} \frac{X\ddot{X}}{\dot{X}} dX_k \quad (6)$$

where the proportional damping matrix is given by

$$C = 2c_m M \quad (7)$$

The total energy functional described in (1) should be minimized from the Principle of Stationary Total Potential Energy for each finite element and its degrees of freedom.

$$\frac{\partial K_c}{\partial X_k} = \sum_{i=1}^{DOF} \frac{m_i}{2} 2\dot{X}_i \frac{\partial \dot{X}_i}{\partial X_k} \delta_{ik} \quad (8)$$

Applying the chain rule to obtain the derivative of velocity with respect to the nodal position X_k and returning to (8),

$$\frac{\partial K_c}{\partial X_k} = \sum_{i=1}^{DOF} c_{mi} \left(\frac{\partial X_i}{\partial X_k} \dot{X}_i + X_i \frac{\partial \dot{X}_i}{\partial X_k} - \frac{X_i \ddot{X}_i}{\dot{X}_i} \right) \delta_{ik} \quad (9a)$$

$$= \sum_{i=1}^{DOF} c_{mi} \left(1 \dot{X}_i + X_i \frac{\partial \dot{X}_i}{\partial t} \frac{\partial t}{\partial X_k} - \frac{X_i \ddot{X}_i}{\dot{X}_i} \right) \delta_{ik} \quad (9b)$$

$$\frac{\partial K_c}{\partial X_k} = \sum_{i=1}^{DOF} m_i \dot{X}_i \ddot{X}_i \frac{1}{\dot{X}_i} \delta_{ik} = \sum_{k=1}^{DOF} m_k \ddot{X}_k$$

where δ_{ik} is the Kronecker delta.

FIGURE 3: (a) Initial DOF; (b) DOF renumbered after assembly (extracted from Greco and Coda [1]).

It is important to note that to develop the last term of (6), the relation between a function and its primitive was considered; i.e.,

$$\frac{\partial}{\partial X_k} \left(\int_{X_k} \frac{X \ddot{X}}{\dot{X}} dX_k \right) = \frac{X \ddot{X}}{\dot{X}} \quad (9c)$$

For the portion due to the damping, one has

$$\begin{aligned} \frac{\partial K_a}{\partial X_k} &= \sum_{i=1}^{DOF} c_{mi} \left(\dot{X}_i + X_i \ddot{X}_i \frac{1}{\dot{X}_i} - \frac{X_i \ddot{X}_i}{\dot{X}_i} \right) \delta_{ik} \\ &= \sum_{k=1}^{DOF} c_{mk} \dot{X}_k \end{aligned} \quad (10)$$

The minimization of the functional can be represented as

$$\frac{\partial \Pi}{\partial X} = \frac{\partial U_t}{\partial X} - F + M \ddot{X} + C \dot{X} = 0 \quad (11)$$

that depends on the time t and on the nodal position vector X . It is necessary to realize a numerical integration in the time domain and to make the second derivative with respect to the position. By rewriting the same equation for the instant $t + \Delta t$, it is possible to obtain

$$\left. \frac{\partial \Pi}{\partial X} \right|_{t+\Delta t} = \left. \frac{\partial U_t}{\partial X} \right|_{t+\Delta t} - F_{t+\Delta t} + M \ddot{X}_{t+\Delta t} + C \dot{X}_{t+\Delta t} = 0 \quad (12)$$

Eq. (12) represents the geometric nonlinear dynamic equilibrium condition for the problem under analysis, i.e., the movement equation. The nodal loading vector $F_{t+\Delta t}$ is known at each time point and is defined as

$$\begin{aligned} F_{t+\Delta t} &= F_0 [c_1 + c_2 t + c_3 t^2 + c_4 t^3 + c_5 \sin(c_6 t) \\ &\quad + c_7 \cos(c_8 t) + c_9 e^{c_{10} t}] \end{aligned} \quad (13)$$

where $c_1, c_2, c_3, \dots, c_{10}$ are coefficients of the load function applied to the system under study.

Using the Newmark Method for the numerical integration in the time domain, one can get the equations of this as a function of the positions:

$$X_{t+\Delta t} = X_t + \Delta t \dot{X}_t + \Delta t^2 \left[\left(\frac{1}{2} - \beta \right) \ddot{X}_t \right] + \beta \ddot{X}_{t+\Delta t} \quad (14)$$

$$\dot{X}_{t+\Delta t} = \dot{X}_t + \Delta t (1 - \gamma) \ddot{X}_t + \gamma \Delta t \ddot{X}_{t+\Delta t} \quad (15)$$

where the initial nodal acceleration is determined, based on (12):

$$\ddot{X}_0 = M^{-1} \left[F_0 - C \dot{X}_0 - \left. \frac{\partial U_t}{\partial X} \right|_0 \right] \quad (16)$$

As the proposed formulation is based on the description of the positions, the displacements are unused throughout the process. However, recognizing the positions, one can do the following:

$$X = X_0 + u \implies u = X - X_0 \quad (17)$$

The acceleration and velocity are given by

$$\ddot{u} = \ddot{X} \quad (18)$$

$$\dot{u} = \dot{X} \quad (19)$$

If (17) to (19) are replaced with (14) and (15), the expressions for the Newmark Method will be obtained as a function of the displacements.

Taking (14) and isolating the acceleration at time $t + \Delta t$, it is possible to obtain

$$\ddot{X}_{t+\Delta t} = \frac{X_{t+\Delta t} - X_t}{\beta \Delta t^2} - \frac{\dot{X}_t}{\beta \Delta t} - \left(\frac{1}{2\beta} - 1 \right) \ddot{X}_t \quad (20)$$

Taking (15) and (20) in (12):

$$\begin{aligned} \left. \frac{\partial \Pi}{\partial X} \right|_{t+\Delta t} &= \left. \frac{\partial U_t}{\partial X} \right|_{t+\Delta t} - F_{t+\Delta t} + \frac{M}{\beta \Delta t^2} X_{t+\Delta t} - M \dot{D}_t \\ &\quad + C E_t + \frac{\gamma C}{\beta \Delta t} X_{t+\Delta t} - \gamma \Delta t C Q_t = 0 \end{aligned} \quad (21)$$

FIGURE 4: (a) L-frame and input data; (b) sudden load applied at plane frame.

FIGURE 5: Load application point's horizontal displacement $\times$ time for undamped and damped cases.

where the vectors D_t and E_t contain the contributions of the variables considered in the previous time instant t :

$$D_t = \frac{X_t}{\beta \Delta t^2} + \frac{\dot{X}_t}{\beta \Delta t} + \left(\frac{1}{2\beta} - 1 \right) \ddot{X}_t \quad (22)$$

$$E_t = \dot{X}_t + \Delta t (1 - \gamma) \ddot{X}_t \quad (23)$$

Returning to (21) and deriving it from the positions at the present instant,

$$\left. \frac{\partial^2 \Pi}{\partial X^2} \right|_{t+\Delta t} = \nabla g(X^0) = \left. \frac{\partial^2 U_t}{\partial X^2} \right|_{t+\Delta t} + \frac{M}{\beta \Delta t^2} + \frac{\gamma C}{\beta \Delta t} \quad (24)$$

Eq. (24) constitutes the expression of the Hessian matrix for the dynamic problem.

5. Examples

The following examples demonstrate the effects of instability on plane frames with semirigid connections and shallow arches under dynamical snap-through behavior.

5.1. Example 1. This example is L-frame with semirigid connection subject to a dynamic load $P(t)$ as shown in Figure 4. The beam and the column are discretized with 10 finite elements for each one. A time step for temporal integration of 10^{-4} and a tolerance of 10^{-10} were considered. Two values for damping (c_m) were adopted: 2.0 (damped 1) and 4.0 (damped 2). The results are compared with those obtained by Silva [5].

Figure 5 shows the results obtained for the horizontal displacement of the application point of the load $P(t)$. One notes that the undamped case agrees well with that obtained by Silva [5]. Besides, the damped cases have the same frequency of undamped case and the amplitude relative to structure with damping 2 is smaller than the amplitude relative to structure with damping 1, even though both decrease throughout the analysis, as expected.

Figure 6 depicts the beam response for different values of the force P . One can note that as P increases, a nonperiodic and irregular behavior tends to be more visible. Furthermore, for $P = 800 \text{ kN}$ and $P = 900 \text{ kN}$ numerical instability is verified.

5.2. Example 2. This example is a beam with semirigid ends subject to a dynamic load $P(t)$ as shown in Figure 7. The

FIGURE 6: Load application point's horizontal displacement $\times$ time for different values of P .

FIGURE 7: (a) Beam with semirigid ends and input data; (b) sudden loads applied at beam.

structure is divided into 10 finite elements. A time step of 10^{-5} and a tolerance of 10^{-10} were chosen. Damping was not considered. A comparison is done with the works of Chan and Chui [22] and Silva [5].

Figure 8 shows the vertical displacement of the middle point of the beam with rigid connections obtained by Silva [5], by Chan and Chui [22], and by the proposed formulation using inertia I_1 and loading 1. Figure 9 depicts the case of the beam with semirigid connections using inertia I_1 and loading 1 as well. One concludes, then, that the results showed good agreement with those found in the literature.

The analysis was processed until $t = 10$ s as one can see in Figure 10. As expected, immediately after the load is over, the structure oscillates around the equilibrium response. It is highlighted that the algorithm showed being very stable throughout analysis.

An analysis of the beam response with elastic connections for different sudden loads values with time duration of 10^{-3} s reveals an unexpected behavior as one can observe from Figure 11. The response amplitude for $P = 180$ kN is smaller than for $P = 150$ kN, which is smaller than for $P = 100$ kN in free vibration. Unfollowing this pattern, for $P = 280$ kN the system oscillates with an amplitude greater than for $P = 100$ kN. Moreover, the frequency does not remain constant, which is another anomalous behavior. Then, it is possible that for P around 100 kN the beam suffers some kind of structural instability in view of qualitative analysis of its response.

Using inertia I_2 and loading 2, Figure 12 shows the response given by elastic connections and by viscoelastic connections. One observes that Kelvin-Voigt model decreases continuously the displacement amplitude in the free vibration, as predicted.

FIGURE 8: Vertical displacement $\times$ time for rigid connections. FIGURE 9: Vertical displacement $\times$ time for semirigid connections. FIGURE 10: Vertical displacement $\times$ time for semirigid connections (until $t=10$ s). FIGURE 11: Response of the beam with elastic connections for different values of P .

FIGURE 12: Response for elastic and viscoelastic connections.

5.3. Example 3. The third example consists of a shallow sinusoidal arch subjected to a sudden force according to Figure 13(a). This problem was analyzed by Galvão [2], Silva [5], and Rosas [4], with different approaches. The amplitude of the arch is $z_0 = 20$ mm. The specific mass considered for the analysis was $78 \times 10^{-9} \text{Ns}^2/\text{mm}^4$. The modulus of elasticity equals 210GPa, and the damping ratio (ξ) used was 0.10. The arch discretization was made with 18 elements and its fundamental frequency is equal to 250Hz. The relation between the damping coefficient (c_m) and the damping ratio (ξ) is given by $\xi\omega_n = c_m$. The time interval considered was $\Delta t = 0.0001$ s.

Figure 14 indicates the results of the central node displacement. A considerable agreement with the results presented by Galvão [2] is observed. In the relations $\lambda = P/h$ between 0.1 and 0.3 (Figures 14(a)–14(c)) the arch vibrates in its precritical configuration, around a constant value, equivalent to the static instability analysis. In the relation $\lambda = P/h = 0.35$ (Figure 14(d)) the occurrence of snap-through behavior is observed: vibration occurs around a constant

FIGURE 13: Shallow arch and loads considered in the analysis: (a) sudden load; (b) increasing triangular load.

FIGURE 14: Central node displacements $\times$ time for different load values.

FIGURE 15: Central node acceleration $\times$ time for different load values.

value equivalent to the static analysis considering the post-critical configuration. A small horizontal deviation of the solutions is observed, between 0.2s and 0.8s.

Regarding the acceleration (Figure 15), similar behavior is observed in situations involving λ ranging from 0.1 to 0.3. However, in the occurrence of the jump to the postcritical configuration, a time acceleration peak corresponding to 0.09s can be seen. Consequently, it tends to oscillate in small values close to zero, corresponding to the situation of static analysis, indicating that, from this moment, the forces of inertia do little to contribute to the response.

Regarding the velocities, similar behavior for λ between 0.1 and 0.3 (Figures 16(a)–16(c)) was observed, as occurring with the acceleration. At the time of the snap-through behavior (Figure 16(d)), speed spikes can be checked between 0.04s and 0.06s.

The phase diagrams are shown in Figure 17. Their shape is characteristic of geometrically nonlinear dynamic instability problems. The spiral shape tends to converge to a particular point in the diagram, called focus, where velocity is zero and the displacement is corresponding to the response in static analysis. Figures 17(a)–17(c) show similar behavior; however, Figure 17(d) shows a change in the novel trajectory, according to the time before the snap-through behavior and from this moment on, where the spiral behavior is again verified, which corresponds to the later period to the same phenomenon.

5.4. Example 4. The last numerical example consists of the same arch but is subjected to increasing triangular loading (Figure 13(b)). The specific mass considered was $78 \times 10^{-8} \text{Ns}^2/\text{mm}^4$. The modulus of elasticity equals 210GPa,

FIGURE 16: Velocity (central node) $\times$ time for different load values.

and the damping ratios used were 0.10, 0.5, 0.05, and 0.0. The arch discretization was the same as in the previous example. The time interval considered was $\Delta t = 0.0004$ s. The load factor considered was $\lambda = P/h = 0.2$.

Figure 18(a) shows the transient response of the vertical displacement (central node) for different damping ratio values. The time interval between 0s and 1.7s corresponds to the period before the snap-through behavior. After 1.7s, it is observed that the damped responses tend to a static value (18mm for $t = 4.5$ s). The undamped response oscillates around the static response. For $\xi = 0.10$ chaotic behavior occurs between 4.3s and 4.5s. The displacement of the central node presents a configuration change, oscillating between 4.37s and 4.5s, around the position 12mm (Figure 18(b)). This is a characteristic behavior of loss of stability with increased amplitudes due to the coupling of vibration modes

(flutter). This phenomenon also influences the acceleration and velocities for $\xi = 0.10$.

The velocities (Figure 19(a)) are approximately zero before the snap-through behavior ($t = 1.7$ s) and present a sudden increase after this time. The damped responses ($\xi = 0.005$ and $\xi = 0.05$) present a reduction to the static condition. The undamped response ($\xi = 0$) increases gradually due to the rise of the load in the time. Figure 19(b) demonstrates the chaotic behavior of the velocity for $\xi = 0.10$. Figure 19(c) shows a detail of the sudden increase of velocity for this damping ratio at time interval 1.9s to 2.2s, and Figure 19(d) shows the effects of coupling vibration modes (flutter) between 4.3s and 4.5s.

The acceleration responses of central node are shown in Figure 20(a). The undamped response has the highest amplitudes due to the rise of the load in the time. It should

FIGURE 17: Phase diagrams (central node) for different loading values.

FIGURE 18: Displacements $\times$ time (central node) for different damping ratio ($\lambda = 0.2$): (a) nonchaotic responses; (b) chaotic response.

FIGURE 19: Velocities $\times$ time (central node) for different damping ratio ($\lambda = 0.2$): (a) nonchaotic responses; (b) chaotic response ($\xi=0.10$); (c) detail at interval of 1.5s-2.5s ($\xi=0.10$); (d) detail at interval of 4.3s-4.5s ($\xi=0.10$).

be noted that the chaotic response is equivalent to the damping ratio $\xi=0.10$ (Figure 20(b)), which shows an initial peak in the magnitude of the response between 1.9 and 2.2s (snap-through behavior) for the damping ratio considered (Figure 20(c)) and a behavior with large amplitudes in the range of 4.38 to 4.5s (Figure 20(d)).

Figure 21 shows the phase diagrams for the arch. The responses for $\xi=0.005$ and $\xi=0.05$ (except for damping) converge to a zero velocity focus and approximate displacement of 24mm, considering the inverted arch curvature. For the case without damping there is no convergence, since there is no energy dissipation in the structural system. Figure 22 presents the chaotic phase diagrams for the arch ($\xi=0.10$). However, the presence of a chaotic attractor corresponding to the displacement 24mm is observed. This behavior is typical in situations where the initial conditions undergo minimal changes, producing a trajectory that diverges exponentially from the original path. The system becomes unstable and the response in time is unpredictable.

6. Conclusions

This work presented the dynamic response of shallow arches under transverse force and plane frames with semirigid connections via numerical integration using the Newmark Method for nonlinear transient problems. A positional formulation of the Finite Element Method was used. The answers acquired possess consistency in relation to problems found in literature. The dynamic snap-through behavior phenomenon could be observed and commented on in each arch example.

Data Availability

The data used to write the article are public (accessible from scientific journals) and the obtained results are also public to be published in an open-access journal.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

(a)

(b)

(c)

(d)

FIGURE 20: Acceleration versus time (central node) for different damping ratio ($\lambda = 0.2$). FIGURE 21: Phase diagrams (central node) for different damping ratios ($\lambda = 0.2$) at interval of 0s-4.5s. FIGURE 22: Chaotic phase diagram (central node) for damping ratio $\xi=0.10$ ($\lambda = 0.2$) at interval of 0s-4.5s.

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References

- [1] M. Greco and H. B. Coda, "Positional FEM formulation for flexible multi-body dynamic analysis," *Journal of Sound and Vibration*, vol. 290, no. 3, pp. 1141–1174, 2006.
- [2] Galvão A. S., *Static and dynamic instability of plane frames with semi-rigid connections*, Pontifícia Universidade Católica do Rio de Janeiro, 2004.
- [3] Y. Zhou, I. Stanciulescu, T. Eason, and M. Spottswood, "Fast approximations of dynamic stability boundaries of slender curved structures," *International Journal of Non-Linear Mechanics*, vol. 95, pp. 47–58, 2017.
- [4] L. R. B. Rosas, *Dynamic analysis of framed structures with contact restraints*. MSc dissertation, Universidade Federal de Ouro Preto, 2016.
- [5] A. R. D. Silva, *computational system for advanced static and dynamic analysis of steel structures*, Graduation program of civil engineering, 2009.
- [6] Y. Zhou, W. Chang, and I. Stanciulescu, "Non-linear stability and remote unconnected equilibria of shallow arches with asymmetric geometric imperfections," *International Journal of Non-Linear Mechanics*, vol. 77, pp. 1–11, 2015.
- [7] J. Ha, S. Gutman, S. Shon, and S. Lee, "Stability of shallow arches under constant load," *International Journal of Non-Linear Mechanics*, vol. 58, pp. 120–127, 2014.
- [8] L. N. Virgin, R. Wiebe, S. M. Spottswood, and T. G. Eason, "Sensitivity in the structural behavior of shallow arches," *International Journal of Non-Linear Mechanics*, vol. 58, pp. 212–221, 2014.
- [9] Y. Chandra, R. Wiebe, I. Stanciulescu, L. N. Virgin, S. M. Spottswood, and T. G. Eason, "Characterizing dynamic transitions associated with snap-through of clamped shallow arches," *Journal of Sound and Vibration*, vol. 332, no. 22, pp. 5837–5855, 2013.
- [10] G. Turkalj, D. Lanc, J. Brnic, and I. Pesic, "A beam formulation for large displacement analysis of composite frames with semi-rigid connections," *Composite Structures*, vol. 134, pp. 237–246, 2015.
- [11] J. Zhang, J. Jiang, S. Xu, and Z. Wang, "An Investigation of the Effect of Semi-rigid Connections on Sudden Column Removal in Steel Frames," *International Journal of Structures*, vol. 13, 2018.
- [12] H. F. Ozel, A. Saritas, and T. Tasbahji, "Consistent matrices for steel framed structures with semi-rigid connections accounting for shear deformation and rotary inertia effects," *Engineering Structures*, vol. 137, pp. 194–203, 2017.
- [13] I. Prigogine, *The end of certainty time, chaos and the new laws of nature*, The Free Press, NY, USA, 1997.
- [14] Z. P. Bažant and L. Cedolin, *Stability of Structures: Elastic, Inelastic, Fracture and Damage Theories*, World Scientific, 2010.
- [15] H. M. Caldwell, *Optimization of Shallow Arches against Snap-Through Buckling [Ph.D. thesis]*, Faculty of the Division of Graduate Studies and Research, Georgia Institute of Technology, 1977.
- [16] F. M. Detinko, "On the elastic stability of uniform beams and circular arches under nonconservative loading," *International Journal of Solids and Structures*, vol. 37, no. 39, pp. 5505–5515, 2000.
- [17] A. S. Galvão, A. R. D. Silva, R. A. M. Silveira, and P. B. Gonçalves, "Nonlinear dynamic behavior and instability of slender frames with semi-rigid connections," *International Journal of Mechanical Sciences*, vol. 52, no. 12, pp. 1547–1562, 2010.
- [18] A. Kassimali, *Matrix analysis of structures*, Cengage Learning, Cengage Learning, Stamford, USA, 1999.
- [19] M. Greco, R. C. G. Menin, I. P. Ferreira, and F. B. Barros, "Comparison between two geometrical nonlinear methods for truss analyses," *Structural Engineering and Mechanics*, vol. 41, no. 6, pp. 735–750, 2012.
- [20] M. Greco, *Nonlinear structural contact/impact analysis problems using the finite element method*, Universidade de São Paulo, 2004.
- [21] J. M. G. Rabelo, J. S. Becho, M. Greco, and C. A. Cimini Jr, "Modeling the creep behavior of GFRP truss structures with positional FEM," *Latin American Journal of Solids and Structures*, vol. 15, p. e17, 2018.
- [22] S. L. Chan and P. P. T. Chui, *Non-linear static and cyclic analysis of steel frames with semi-rigid connections*, Elsevier, Oxford, UK, 2000.

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