

## Research Article

# Complexity-Entropy Causality Plane Based on Return Intervals: A Useful Approach to Quantify the Aeroengine Gas Path Parameters

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The complexity-entropy causality plane, as a powerful tool for discriminating Gaussian from non-Gaussian process, has been recently introduced to describe the complexity among time series. We propose to use this method to distinguish the stage of climb-cruise-decline of aeroengine. Our empirical results demonstrate that this statistical physics approach is useful. Further, the return intervals based complexity-entropy causality plane is introduced to describe the complexity of aeroengine fuel flow time series. The results can infer that the cruise process has lowest complexity and the decline process has highest complexity.

## 1. Introduction

The understanding and analysis of aeroengine time series, especially the evolution of gas path system sequences, has been attracting the attention of mathematicians and physicists for many years. Doel and Urban modeled the gas path system by linear approaches [1, 2], such as weighted-least-squares [3], filtering approaches [4, 5], where the accuracy and reliability were limited. For improving calculation accuracy and reliability, nonlinear techniques, such as adaptive modeling [6, 7], neural networks [8–11], and genetic algorithms [12–15], were introduced to investigate the gas path system. The existence of autocorrelation between gas path system variables reveals the aeroengine efficiency because past observations can help to predict future variables. This question motivates the research on the subject, especially by aeroengine managers and analysts, trying to save aeroengine maintenance costs. For bearing fault detection, Liu has applied the detrended fluctuation analysis to analyze gas path system correlation [16]. Dong et al. have found that the exhaust gas temperature (EGT), the low-spool rotor speed (N1), the high-spool rotor speed (N2), and the fuel flow (FF) have greater correlation and cross-correlation than other observations, suggesting more predictability [17]. It was also

shown that the EGT, N1, N2, and FF play an important role in understanding gas path system. Thus, the analysis of EGT, N1, N2, and FF seems to be more efficient than other observations.

Nowadays, it is clear that the design of aeroengine gas path system is a typical complex system, which involves many subcomponents. The computing ranges of aeroengine were chosen from the process of take-off-climb-cruise-decline-approach-land of aeroengine [18, 19]. To distinguish the process of take-off-climb-cruise-decline-approach-land of aeroengine depends on the airplane's altitude in previous analysis. In this paper, we propose to test the gas path system of aeroengine time series by employing a recently introduced statistical tool: the complexity-entropy causality plane. It is shown that this plane allows distinguishing cruise and decline process.

To make a distinction between climb, cruise, and decline process, the return intervals of aeroengine time series are proposed. The central quantities here are the time interval between successive events above (or below) some threshold  $Q$ . By studying the statistics of the return intervals for gas path system time series, one aims to find out the laws distinguishing the climb, cruise, and decline process throughout the flight.

The organization of this paper is as follows. In the next section, we simply present the aeroengine gas path performance parameters employed in this paper. We show the complexity-entropy causality plane and empirical results in Section. In Section, we introduce a technique named return intervals based complexity-entropy causality plane enabling us to estimate the complexity characteristic for aeroengine gas path system. In particular, the ability to identify the complexity in gas path system is demonstrated. Finally, we draw some conclusions in Section.

## 2. The Dataset

In this paper, the complexity-entropy causality plane will be used to examine flight course of aeroengine. Here we make a brief description for gas path performance parameters, offered by Aircraft Maintenance and Engineering Corporation. In civil aviation flight management system, flight data are acquired from on-board flight data recorders, which are part of the Aircraft Condition Monitoring System (ACMS) such as Smart ACMS Recorder (SAR) and Quick Access Recorder (QAR) [20, 21]. The QAR data is more comprehensive where data includes an extensive list of flight parameters recorded at specific sampling intervals which are set by the manufacturer. Therefore, the QAR data is applied in this paper.

Previously, researches demonstrate that the parameters including EGT, N1, N2, and FF play an important role in understanding aeroengine system [17, 22]. For this reason, the parameters EGT, N1, N2, and FF are usually selected to amplify the study of aeroengine gas path system. Here, the parameter FF is selected as an example in this paper.

## 3. Complexity-Entropy Causality Plane and Empirical Results

**3.1. Complexity-Entropy Causality Plane Method.** For measuring the information content of aeroengine system, a typical method is to evaluate probability distribution function, describing the distribution of some measurable or observable property. Therefore, the Shannon entropy, which can be of great help when analyzing aeroengine system data since it captures the uncertainty and disorder of the time series without imposing any limitations on the theoretical probability distribution [23–25], is used as a first natural approach. For a time series  $X = \{x_i: i = 1, 2, \dots, n\}$  with probability distribution  $P = \{p(x_i): i = 1, 2, \dots, n\}$ , the Shannon entropy is given by [26–28]

$$H(P) = - \sum_{x_i} p(x_i) \log p(x_i). \quad (1)$$

The Shannon entropy  $H(P) = 0$ , if the time series  $X$  is certain which of the possible outcomes  $x_i$  whose probabilities are given by  $p(x_i)$  will actually take place. The Shannon entropy should be maximal, if all the outcomes are equally likely (uncertainty is highest when all possible events are equiprobable).

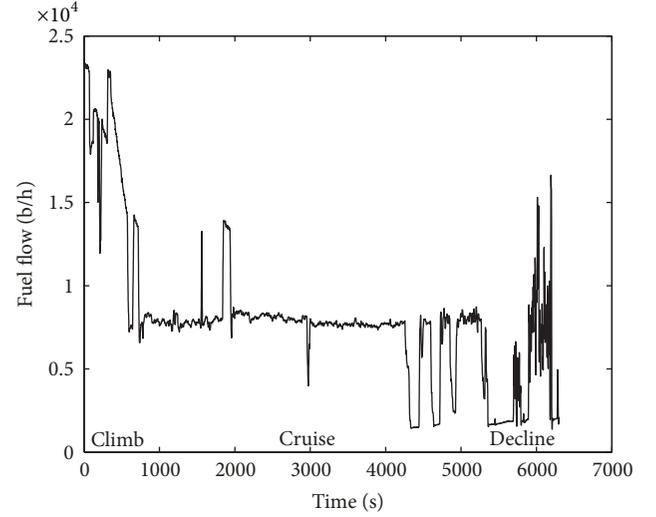


FIGURE 1: The process of climb-cruise-decline of aeroengine gas path system for FF time series.

For differentiating different degrees of periodicity and chaos, nevertheless, Lamberti et al. [29] proposed a statistical complexity measure (SCM) method. This method offers significant additional information regarding the peculiarities of the underlying probability distribution, not detected by the entropy. It is defined as follows:

$$C_{JS}(P) = Q_J(P, P_e) H_S(P), \quad (2)$$

where  $H_S(P) = H(P)/S_{\max}$  is the normalized Shannon entropy, with  $S_{\max} = H(P_e) = \ln N$ , ( $0 \leq H_S \leq 1$ ), and  $P_e = \{1/N, \dots, 1/N\}$ .

$Q_J(P, P_e) = Q_0 J(P, P_e)$  with  $J(P, P_e) = H(P + P_e)/2 - H(P)/2 - H(P_e)/2$  and  $Q_J$  is defined in terms of the extensive Jensen-Shannon divergence, and  $Q_0$  is a normalization constant, equal to the inverse of maximum possible value of  $J(P, P_e)$ .

The diagram of  $C_{JS}$  versus  $H_S(P)$  is the complexity-entropy causality plane defined in [23, 30]. Statistical complexity measure was recently shown to be necessary because it captures the property of organization [31]. SCM has been successfully used to research changes in system dynamics originated by modifications of some characteristic parameters [32–34].

**3.2. Complexity-Entropy Causality Plane Results.** In this section, we analyze the gas path system fuel flow (FF) parameters for 10 aeroengines. The data of the FF variables for climb-cruise-decline process are presented in Figure 1. We employ gas path parameters over climb-cruise-decline process, where the cruise process has lowest complexity (see Figure 1). In order to make comparisons, all aeroengines gas path parameters are studied for the same time length.

In Figure 2, we plot the cruise and decline of the different aeroengines for FF time series in the complexity-entropy causality plane. Observe that the cruise process has higher entropy (lower complexity) than decline ones. Therefore, the former is closer to the ideal point

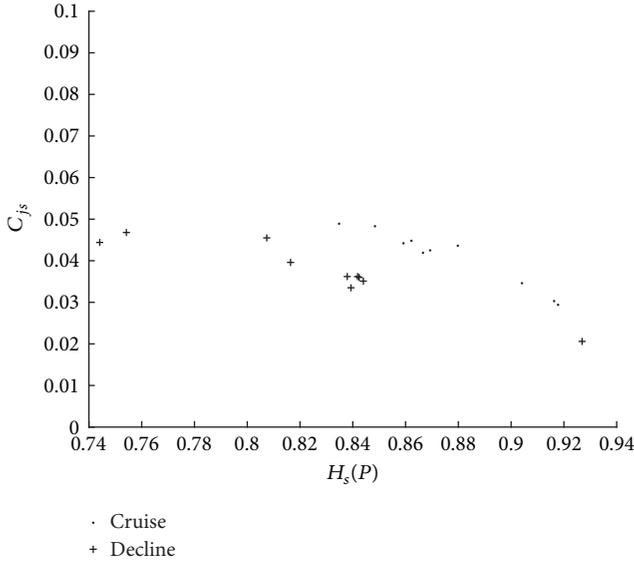


FIGURE 2: The complexity-entropy causality plane of the cruise and decline for the different aeroengines FF time series.

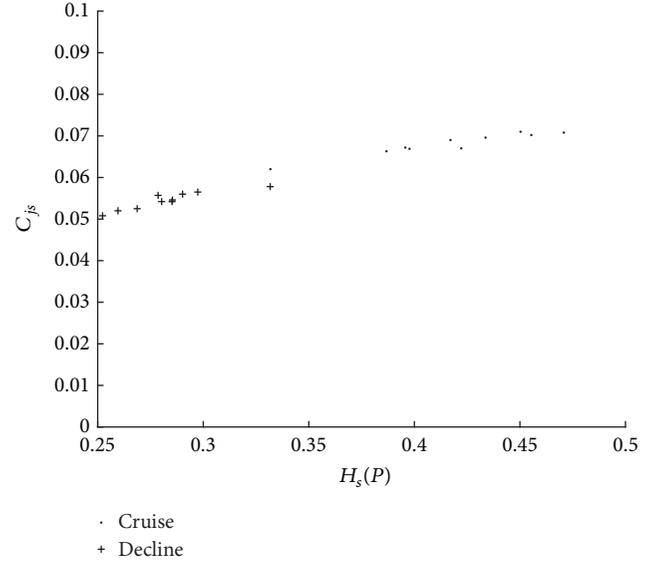


FIGURE 3: The complexity-entropy causality plane of the cruise and decline for the different aeroengines FF return intervals time series.

#### 4. Return Interval Based Complexity-Entropy Causality Plane and Empirical Results

**4.1. Return Interval Based Complexity-Entropy Causality Plane Method.** The understanding of the fluctuation of FF is of great importance in aeroengine fields. One of the central quantities here is the time interval between successive flows above (or below) some threshold  $Q$ , sometimes referred to as reoccurrence times or return intervals. Inspired by [24–26], we give the time interval for a time series  $X = \{x_i; i = 1, 2, \dots, n\}$  as follows.

*Step 1.* Describe the first-order difference  $\Delta^{(1)}X = \{\Delta x_i; i = 1, 2, \dots, n\}$

$$\Delta x_i = x_i - x_{i-1}. \quad (3)$$

*Step 2.* Calculate the time interval between successive sequences above threshold 0 for the first-order difference  $\Delta^{(1)}X$ , that is, the time interval between fuel flows that start increasing.

*Step 3.* Compute  $C_{JS}$  and  $H_S(P)$  for time interval sequences, and then plot  $C_{JS}$  versus  $H_S(P)$ .

**4.2. Return Interval Based Complexity-Entropy Causality Plane Results.** Next, we study the way the return intervals are arranged in time. Figure 3 shows the complexity-entropy causality plane of the cruise and decline for the different aeroengines FF time series. Obviously, the higher entropy in cruise process has been induced by the return interval based complexity-entropy causality plane, compared with Figure 2. Accordingly, the return interval time series are sensitive indicators for original series.

In order to gain a deeper insight, we analyze the complexity of climb-cruise-decline for return interval based

aeroengines FF time series in Figure 4. Figure 4(a) is the complexity-entropy causality plane of climb and cruise process for the different aeroengines FF time series and Figure 4(b) gives the climb and decline process for the different aeroengines FF time series.

Figure 1 shows that the cruise process has lowest complexity and the decline process has highest complexity. Thus we expect that the result of return intervals will show a similar behavior, too. This is shown in Figure 4, where, for climb, cruise, and decline values, the cruise process has highest entropy, and the decline process has lowest entropy. Furthermore, this return interval based method provides an effective method to distinguish the stage of climb-cruise-decline of aeroengine. The agreement between observed data and return interval based model is striking.

#### 5. Conclusion

In this paper, we consider complexity-entropy causality plane method to understand the complexity characteristics in aeroengine gas path system firstly. The technique has been implemented on the fuel flow time series. Our empirical results demonstrate that the cruise process has lower complexity than decline ones.

And then, by constructing the return intervals based complexity-entropy causality plane method to fuel flow parameters, we obtain the similar complexity characteristics exhibited by  $C_{JS}$  versus  $H_S(P)$  implying the higher entropy of fuel flow time series for the cruise and decline process.

To capture the degree of climb-cruise-decline of aeroengine in detail, we apply return intervals based complexity-entropy causality plane to research the complexity between climb-cruise-decline processes for aeroengine gas path systems. The results not only distinguish the stage of climb-cruise-decline of aeroengine, but also infer that the cruise

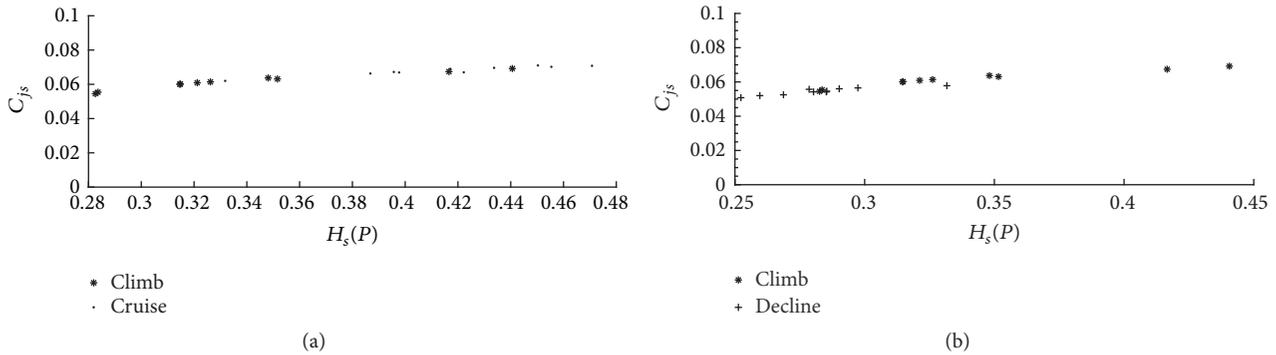


FIGURE 4: The complexity-entropy causality plane of (a) the climb and cruise and (b) climb and decline for the different aeroengines FF return intervals time series.

process has lowest complexity and the decline process has highest complexity.

### Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

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