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Research Article

Sliding Mode Control with Adaptive Fuzzy Compensation for Uncertain Nonlinear System

Lina Wang and Haihui Zhang

College of Mechanical and Electronic Engineering, Northwest A&F University, Key Laboratory of Agricultural Internet of Things, Ministry of Agriculture, Key Laboratory of Agricultural Internet of Things, Ministry of Agriculture and Rural Affairs, Yangling, China

Correspondence should be addressed to Lina Wang; lina_wang@nwafu.edu.cn

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Fuzzy sliding mode control as a robust and intelligent nonlinear control technique is proposed to control processes with severe nonlinearity and unknown models. This paper proposes a new adaptive tracking fuzzy sliding mode controller for nonlinear systems in the presence of fuzzy compensation. The main contribution of the proposed method is that the fuzzy system is used to realize the adaptive approximation of the unknown part of the model, and the fuzzy gain can be reduced effectively. The fuzzy self-adaptive rate is derived through the Lyapunov method, and the stability and convergence of the whole closed-loop system are guaranteed by adjusting the adaptive weight value. The performance of the proposed approach is evaluated for double joint rigid manipulator problems. The simulation results illustrate the effectiveness of our proposed controller.

1. Introduction

Fuzzy control method is widely used in control theory and control engineering systems due to its great theoretical value and successful applications in complex practical systems. Adaptive fuzzy control is efficient to control uncertain nonlinear system [1]. Since the adaptive fuzzy approaches can approximate the complex nonlinear functions through fuzzy rules, the application of the fuzzy logic to deal with the chattering problem is proven as an effective way; it has been used in many engineering applications, such as robot systems and active suspension system [2, 3]. Adaptive fuzzy control has two types: indirect adaptive fuzzy control and direct adaptive fuzzy control [4, 5]. In indirect adaptive fuzzy control, there is a common problem that controls compensation. In [6], introduce unwanted high gain at the control input. In direct adaptive fuzzy control, the fuzzy controller parameters can be modified by the adaptive mechanism. We need to decrease the high gain through choosing one limiter for control input. The compensation control was proposed for the direct adaptive fuzzy control [7]. The advantage of fuzzy logic is that it can easily integrate human control experience into the controller through fuzzy rules and realize high level controller by designing fuzzy rules [8]. Some researchers have made significant advances about adaptive fuzzy control schemes [9, 10]. The most direct adaptive fuzzy control with a robust control term is based on Lyapunov stability.

The Sliding Mode Control (SMC) was originally introduced for variable structure systems in continuous domain by Utkin [11, 12]. It is known to have robust to model uncertainty, parameter variations, and good disturbance rejection properties [13–15]. But its major drawback is the chattering problem in practical applications, such as robotics, aerospace, and chaos control [16–19]. Traditional methods that replace the relay control by a saturating approximation, integral sliding control, and boundary layer technique have been proposed to eliminate this chattering problem [20–22]. However, if systems uncertainties are too large, the SMC would require a high switching gain with a thicker boundary layer. In [23], we can see SMC techniques for direct adaptive fuzzy control. The adaptive fuzzy control method considered a discontinuous law and a robust control law, but the control bandwidth of the controller is so wide. It was effective to eliminate chattering, however, leading to the inferior tracking performance. It is well known that we can reduce the error by introducing a compensator into the system. In order to enhance the tracking performance, we can use some adaptive strategy for compensation. The universal approximation theorem shows that the fuzzy system is a new universal approximator besides the polynomial function approximator and neural network approximation [24–26]. The adaptive fuzzy SMC has simple control structure, online learning ability, and model-free feature [27]. Here, we developed an adaptive fuzzy compensator SMC to control double joint rigid manipulator system. It has the advantage of representing system's nonlinear functions without the dynamic model requirement.

In this paper, during the control of the nonlinear system, the fuzzy system is used to realize the adaptive approximation of the unknown part of the model, and the fuzzy gain can be reduced effectively. The fuzzy self-adaptive rate is derived through the Lyapunov method; the convergence properties of the tracking error are analytically proven. The fuzzy self-adaptive rate is derived through the Lyapunov method, and the stability and convergence of the whole closed-loop system are guaranteed by adjusting the adaptive weight value. Combining the SMC with adaptive fuzzy compensation method, the stability and convergence of the whole closed-loop system are guaranteed by adjusting the adaptive weight value.

The rest of this paper is organized as follows. Section 2 presents the system descriptions and problem formulation method. Section 3 introduces our proposed controller scheme. Simulation results are included in Section 4. Section 5 provides the concluding remarks.

2. System Description and Problem Formulation

Consider the uncertain nonlinear systems as follows [28]:

$$\boldsymbol{D}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}}) + \mathrm{G}(\boldsymbol{q}) + \boldsymbol{F}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}}) = \tau \tag{1}$$

where $\boldsymbol{D}(\boldsymbol{q})$ is inertia moment, $\boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})$ is centripetal force and the goth moment, $\mathrm{G}(\boldsymbol{q})$ is gravity component, and $\boldsymbol{F}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}})$ is consisted by friction $\boldsymbol{F}$, disturbance $\tau_{\boldsymbol{d}}$, and the load change uncertainties.

Because the fuzzy system has universal approximation property[29–31], we can use $\hat{f}(x \mid \theta)$ to $f(x)$. Aiming to fuzzy system x_1 input and x_2, design five fuzzy sets, that is, $n = 2$, $i = 1, 2$, and $p_1 = p_2 = 25$; then we can obtain $p_1 \times p_2 = 25$ fuzzy rules. The following two steps are used to construct the fuzzy system $\hat{f}(x \mid \theta)$; the first, aiming to the variable $x_i (i = 1, 2)$, defines p_i fuzzy sets $A_i^{l_i} (l_i = 1, 2, 3, 4, 5)$ [10]. The second, using $\prod_{i=1}^{n} p_i = p_1 \times p_2 = 25$ fuzzy rules to construct fuzzy system $\hat{f}(x \mid \theta)$, then the j fuzzy rule is

$$R^{(j)}: \text{ IF } x_1 \text{ is } A_1^{l_1} \text{ and } x_2 \text{ is } A_1^{l_2} \text{ THEN } \hat{f} \text{ is } B^{l_1 l_2} \tag{2}$$

where $l_i = 1, 2, 3, 4, 5$, $i = 1, 2$, $j = 1, 2, \ldots, 25$, $B^{l_1 l_2}$ is the fuzzy set of the conclusion. Then the first step and the 25th fuzzy rule can be expressed as

$$R^{(1)}: \text{ IF } x_1 \text{ is } A_1^{1} \text{ and } x_2 \text{ is } A_2^{1} \text{ THEN } \hat{f} \text{ is } B^{1} \tag{3}$$

$$\vdots$$

$$R^{(25)}: \text{ IF } x_1 \text{ is } A_1^{5} \text{ and } x_2 \text{ is } A_1^{5} \text{ THEN } \hat{f} \text{ is } B^{25} \tag{4}$$

Fuzzy inference process takes the following four steps: (1) using the product inference engine to realize the premise inference of the rule, the inference result is $\prod_{i=1}^{2} \mu_{A_i^{l_i}}(x_i)$.

(2) Use singleton fuzzifier to calculate $\overline{y}_f^{l_1 l_2}$, that is, the function value $f(x_1, x_2)$ of the x-coordinate value (x_1, x_2) corresponding to the maximum value (1.0) of the membership function.

(3) Using product inference engine to realize the inference of rule premise and rule conclusion, then the inference result is $\overline{y}_f^{l_1 l_2}(\prod_{i=1}^{2} \mu_{A_i^{l_i}}(x_i))$. Calculate all the fuzzy rules; then the output of fuzzy system is $\sum_{l_1=1}^{5} \sum_{l_2=1}^{5} \overline{y}_f^{l_1 l_2}(\prod_{i=1}^{2} \mu_{A_i^{l_i}}(x_i))$.

(4) Use average defuzzizer to get the output of fuzzy system:

$$\hat{f}(x_1 \mid \theta) = \frac{\sum_{l_1=1}^{5} \sum_{l_2=1}^{5} \overline{y}_f^{l_1 l_2} \left(\prod_{i=1}^{2} \mu_{A_i^{l_i}}(x_i)\right)}{\sum_{l_1=1}^{5} \sum_{l_2=1}^{5} \left(\prod_{i=1}^{2} \mu_{A_i^{l_i}}(x_i)\right)} \tag{5}$$

where $\mu A_i^j(x_i)$ is the membership function of x_i. Make $\overline{y}_f^{l_1 l_2}$ free parameter and put it in set $\theta \in \boldsymbol{R}^{(25)}$. Introduce the fuzzy basis vectors $\xi(\boldsymbol{x})$, than (5) is as follows:

$$\hat{f}(\boldsymbol{x}_1 \mid \theta) = \hat{\theta}^{\boldsymbol{T}} \xi(\boldsymbol{x}) \tag{6}$$

where $\xi(\boldsymbol{x})$ is $\prod_{i=1}^{n} p_i = p_1 \times p_2 = 25$ fuzzy basis vectors; among the $l_1 l_2$ element is

$$\boldsymbol{\xi}_{l_1 l_2}(x) = \frac{\prod_{i=1}^{2} A_i^{l_i}(x_i)}{\sum_{l_1=1}^{5} \sum_{l_2=1}^{5} \left(\prod_{i=1}^{2} \mu_{A_i^{l_i}}(x_i)\right)} \tag{7}$$

Then using fuzzy system to approximate (1) $\boldsymbol{F}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}})$, it can be shown

$$\boldsymbol{F}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}}) = \hat{\boldsymbol{\theta}} \boldsymbol{\xi}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}}) \tag{8}$$

3. Controller Design

Assume that $\boldsymbol{D}(\boldsymbol{q})$, $\boldsymbol{C}(\boldsymbol{q}, \dot{\boldsymbol{q}})$, and $\boldsymbol{G}(\boldsymbol{q})$ are known and all state variables can be measured. Define SMC function [17, 32]

$$\boldsymbol{s} = \dot{\tilde{\boldsymbol{q}}} + \Lambda \tilde{\boldsymbol{q}} \tag{9}$$

where Λ is positively definite matrix and $\tilde{\boldsymbol{q}}(t)$ is tracking error, $\tilde{\boldsymbol{q}} = \boldsymbol{q} - \boldsymbol{q}_d$.

Define

$$\dot{\boldsymbol{q}}_r(t) = \dot{\boldsymbol{q}}_d(t) - \Lambda \tilde{\boldsymbol{q}}(t) \tag{10}$$

Define Lyapunov function [33, 34]

$$V(t) = \frac{1}{2}\left(\boldsymbol{s}^{\boldsymbol{T}} \boldsymbol{D} \boldsymbol{s} + \sum_{i=1}^{n} \widetilde{\boldsymbol{\Theta}}_{\boldsymbol{i}}^{\boldsymbol{T}} \boldsymbol{\Gamma}_{\boldsymbol{i}} \widetilde{\boldsymbol{\Theta}}_{\boldsymbol{i}}\right) \tag{11}$$

where $\widetilde{\boldsymbol{\Theta}}_{\boldsymbol{i}} = \widetilde{\boldsymbol{\Theta}}_{\boldsymbol{i}}^* - \widehat{\boldsymbol{\Theta}}_i$, $\boldsymbol{\Theta}_i^*$ is ideal parameter, and $\Gamma_i > 0$. The Gamma i matrix is symmetric and positive definite.

Due to $\boldsymbol{s} = \dot{\widetilde{\boldsymbol{q}}} + \Lambda\widetilde{\boldsymbol{q}} = \dot{\boldsymbol{q}} - \dot{\boldsymbol{q}}_d + \Lambda\widetilde{\boldsymbol{q}} = \dot{\boldsymbol{q}} - \dot{\boldsymbol{q}}_{\boldsymbol{r}}$, then

$$\boldsymbol{D}\dot{\boldsymbol{s}} = \boldsymbol{D}\ddot{\boldsymbol{q}} - \boldsymbol{D}\ddot{\boldsymbol{q}}_r = \boldsymbol{\tau} - \boldsymbol{C}\dot{\boldsymbol{q}} - \boldsymbol{G} - \boldsymbol{F} - \boldsymbol{D}\ddot{\boldsymbol{q}}_r \tag{12}$$

Then

$$\begin{aligned}\dot{V}(t) &= s^T\boldsymbol{D}\dot{\boldsymbol{s}} + \frac{1}{2}s^T\dot{\boldsymbol{D}}\boldsymbol{s} + \sum_{i=1}^{n}\widetilde{\boldsymbol{\Theta}}_{\boldsymbol{i}}^{\boldsymbol{T}}\Gamma_i\dot{\widetilde{\boldsymbol{\Theta}}}_{\boldsymbol{i}} \\ &= -s^T\left(-\boldsymbol{\tau} + \boldsymbol{C}\dot{\boldsymbol{q}} + \boldsymbol{G} + \boldsymbol{F} + \boldsymbol{D}\ddot{\boldsymbol{q}}_r - \boldsymbol{C}\boldsymbol{s}\right) \\ &\quad + \sum_{i=1}^{n}\widetilde{\boldsymbol{\Theta}}_{\boldsymbol{i}}^{\boldsymbol{T}}\Gamma_i\dot{\widetilde{\boldsymbol{\Theta}}}_{\boldsymbol{i}} \\ &= -s^T\left(\boldsymbol{D}\ddot{\boldsymbol{q}}_r + \boldsymbol{C}\dot{\boldsymbol{q}}_r\boldsymbol{G} + \boldsymbol{F} - \boldsymbol{\tau}\right) + \sum_{i=1}^{n}\widetilde{\boldsymbol{\Theta}}_{\boldsymbol{i}}^{\boldsymbol{T}}\Gamma_i\dot{\widetilde{\boldsymbol{\Theta}}}_{\boldsymbol{i}}\end{aligned} \tag{13}$$

Among $\boldsymbol{F}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}})$ is unknown nonlinear function, using to MIMO fuzzy system to approximate $\widehat{\boldsymbol{F}}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}} \mid \boldsymbol{\Theta})$.

3.1. Adaptive Control Law Design. Design adaptive law

$$\begin{aligned}\boldsymbol{\tau} &= \boldsymbol{D}(\boldsymbol{q})\ddot{\boldsymbol{q}}_r + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}_r + \boldsymbol{G}(\boldsymbol{q}) + \widehat{\boldsymbol{F}}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}} \mid \boldsymbol{\Theta}) \\ &\quad - \boldsymbol{K}_D s\end{aligned} \tag{14}$$

where $\boldsymbol{K}_D = \operatorname{diag}(K_i)$, $K_i > 0$, i $= 1, 2, \ldots, n$, and

$$\begin{aligned}\widehat{\boldsymbol{F}}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}} \mid \boldsymbol{\Theta}) &= \begin{bmatrix}\widehat{F}_1(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}} \mid \boldsymbol{\Theta}_1) \\ \widehat{F}_2(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}} \mid \boldsymbol{\Theta}_2) \\ \vdots \\ \widehat{F}_n(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}} \mid \boldsymbol{\Theta}_n)\end{bmatrix} \\ &= \begin{bmatrix}\boldsymbol{\Theta}_1^T\boldsymbol{\xi}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}}) \\ \boldsymbol{\Theta}_2^T\boldsymbol{\xi}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}}) \\ \vdots \\ \boldsymbol{\Theta}_n^T\boldsymbol{\xi}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}})\end{bmatrix}\end{aligned} \tag{15}$$

Fuzzy approximation error is

$$\boldsymbol{w} = \boldsymbol{F}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}}) - \widehat{\boldsymbol{F}}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}} \mid \boldsymbol{\Theta}^*) \tag{16}$$

Substitute (14) into (13), then

$$\begin{aligned}\dot{V}(t) &= -s^T\left(\boldsymbol{F}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}}) - \widehat{\boldsymbol{F}}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}} \mid \boldsymbol{\Theta}) + \boldsymbol{K}_D s\right) \\ &\quad + \sum_{i=1}^{n}\widetilde{\boldsymbol{\Theta}}_{\boldsymbol{i}}^{\boldsymbol{T}}\Gamma_i\dot{\widetilde{\boldsymbol{\Theta}}}_{\boldsymbol{i}} = -s^T\left(\boldsymbol{F}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}}) - \widehat{\boldsymbol{F}}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}} \mid \boldsymbol{\Theta})\right. \\ &\quad \left. + \widehat{\boldsymbol{F}}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}} \mid \boldsymbol{\Theta}^*) - \widehat{\boldsymbol{F}}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}} \mid \boldsymbol{\Theta}^*) + \boldsymbol{K}_D s\right) \\ &\quad + \sum_{i=1}^{n}\widetilde{\boldsymbol{\Theta}}_{\boldsymbol{i}}^{\boldsymbol{T}}\Gamma_i\dot{\widetilde{\boldsymbol{\Theta}}}_{\boldsymbol{i}} = -s^T\left(\widetilde{\boldsymbol{\Theta}}^T\boldsymbol{\xi}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}}) + \boldsymbol{w} + \boldsymbol{K}_D s\right) \\ &\quad + \sum_{i=1}^{n}\widetilde{\boldsymbol{\Theta}}_{\boldsymbol{i}}^{\boldsymbol{T}}\boldsymbol{\Gamma}_i\dot{\widetilde{\boldsymbol{\Theta}}}_{\boldsymbol{i}} = -s^T\boldsymbol{K}_D s - s^T\boldsymbol{w} \\ &\quad + \sum_{i=1}^{n}\left(\widetilde{\boldsymbol{\Theta}}_{\boldsymbol{i}}^{\boldsymbol{T}}\Gamma_i\dot{\widetilde{\boldsymbol{\Theta}}}_{\boldsymbol{i}} - s_i\widetilde{\boldsymbol{\Theta}}_{\boldsymbol{i}}^{\boldsymbol{T}}\boldsymbol{\xi}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}})\right)\end{aligned} \tag{17}$$

where $\widetilde{\boldsymbol{\Theta}} = \boldsymbol{\Theta}^* - \widehat{\boldsymbol{\Theta}}$, $\boldsymbol{\xi}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}})$ is the basic vector of fuzzy system, and $\widehat{\boldsymbol{F}}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}} \mid \boldsymbol{\Theta}^*) - \widehat{\boldsymbol{F}}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}} \mid \boldsymbol{\Theta}) = \widetilde{\boldsymbol{\Theta}}^T\boldsymbol{\xi}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}})$.

Adaptive control law is [35]

$$\dot{\widehat{\boldsymbol{\Theta}}}_i = -\Gamma_i^{-1}s_i\boldsymbol{\xi}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}}), \quad i = 1, 2, \ldots, n \tag{18}$$

Then

$$\dot{V}(t) = -\boldsymbol{s}^T\boldsymbol{K}_{\boldsymbol{D}}\boldsymbol{s} - \boldsymbol{s}^T\boldsymbol{w} \tag{19}$$

Because approximate error $\boldsymbol{w}$ is so small, when $\boldsymbol{K}_{\boldsymbol{D}}$ is big enough, we can guarantee $\dot{V}(t) \le 0$. When $\dot{V}(t) \equiv 0$ and $s \equiv 0$, according to LaSalle invariant theory, the closed-loop system is asymptotically stable, $t \longrightarrow \infty$, and $s \longrightarrow 0$.

3.2. Robust Adaptive Control Law. In order to eliminate the influence of the approximation error and ensure the stability of the system [36, 37], robust items should be adopted in the control law. Here we design robust adaptive law

$$\begin{aligned}\boldsymbol{\tau} &= \boldsymbol{D}(\boldsymbol{q})\ddot{\boldsymbol{q}}_{\boldsymbol{r}} + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}_{\boldsymbol{r}} + \boldsymbol{G}(\boldsymbol{q}) + \widehat{\boldsymbol{F}}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}} \mid \boldsymbol{\Theta}) \\ &\quad - \boldsymbol{K}_D s - \mathbf{W}\operatorname{sgn}(s)\end{aligned} \tag{20}$$

where $\mathbf{W} = \operatorname{diag}[\omega_{M_1}, \ldots, \omega_{M_n}]$ and $\omega_{M_i} \ge |\varpi_i|$, $i = 1, 2, \ldots, n$.

Add (20) into (17); then

$$\dot{V}(t) = -\boldsymbol{s}^{\mathrm{T}}\boldsymbol{K}_{\boldsymbol{D}}\boldsymbol{s} \le 0 \tag{21}$$

If we take MIMO fuzzy system $\widehat{\boldsymbol{F}}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}} \mid \boldsymbol{\Theta})$ to approximate $\boldsymbol{F}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}})$, then we aim to every input variable design k membership functions.

3.3. Fuzzy Compensation SMC for Friction. When $\boldsymbol{F}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}})$ only contain friction term, we can consider fuzzy compensation for approximate friction [26, 27, 38]. Due to the fact that is only related to the speed signal, the fuzzy system for approximate friction can be expressed as $\widehat{\boldsymbol{F}}(\dot{\boldsymbol{q}} \mid \boldsymbol{\theta})$, and we can design control law according to traditional fuzzy compensation controller design method like (14), (18), and (20) [39, 40].

Design fuzzy adaptive control law as follows:

$$\boldsymbol{\tau} = \boldsymbol{D}(\boldsymbol{q})\ddot{\boldsymbol{q}}_r + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}_r + \boldsymbol{G}(\boldsymbol{q}) + \widehat{\boldsymbol{F}}(\dot{\boldsymbol{q}} \mid \boldsymbol{\theta}) - \boldsymbol{K}_D s \tag{22}$$

Design robust fuzzy adaptive control law as follows:

$$\begin{aligned}\boldsymbol{\tau} = {} & \boldsymbol{D}(\boldsymbol{q})\ddot{\boldsymbol{q}}_r + \boldsymbol{C}(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}}_r + \boldsymbol{G}(\boldsymbol{q}) + \widehat{\boldsymbol{F}}(\dot{\boldsymbol{q}} \mid \boldsymbol{\theta}) - \boldsymbol{K}_D s \\ & - \mathbf{W}\operatorname{sgn}(s)\end{aligned} \tag{23}$$

Design adaptive law as follows:

$$\dot{\hat{\boldsymbol{\theta}}}_i = -\Gamma_i^{-1}\boldsymbol{\xi}(\dot{\boldsymbol{q}}), \quad i = 1, 2, \ldots, n \tag{24}$$

Fuzzy system can be designed

$$\widehat{\boldsymbol{F}}(\dot{\boldsymbol{q}} \mid \boldsymbol{\theta}) = \begin{bmatrix} \widehat{F}_1(\dot{\boldsymbol{q}}_1) \\ \widehat{F}_2(\dot{\boldsymbol{q}}_2) \\ \vdots \\ \widehat{F}_n(\dot{\boldsymbol{q}}_n) \end{bmatrix} = \begin{bmatrix} \boldsymbol{\theta}_1^T\boldsymbol{\xi}^1(\dot{\boldsymbol{q}}_1) \\ \boldsymbol{\theta}_2^T\boldsymbol{\xi}^2(\dot{\boldsymbol{q}}_2) \\ \vdots \\ \boldsymbol{\theta}_n^T\boldsymbol{\xi}^n(\dot{\boldsymbol{q}}_n) \end{bmatrix} \tag{25}$$

4. Illustrative Results and Discussion

Aiming to double joint rigid manipulator, the kinetic equation is [28]

$$\begin{aligned} & \begin{bmatrix} D_{11}(q_2) & D_{12}(q_2) \\ D_{21}(q_2) & D_{22}(q_2) \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} \\ & + \begin{bmatrix} -C_{12}(q_2)\dot{q}_2 & -C_{12}(q_2)(\dot{q}_1 + \dot{q}_2) \\ C_{12}(q_2)\dot{q}_1 & 0 \end{bmatrix} \begin{pmatrix} g_1(q_1+q_2)g \\ g_2(q_1+q_2)g \end{pmatrix} \\ & + \boldsymbol{F}(\boldsymbol{q},\dot{\boldsymbol{q}},\ddot{\boldsymbol{q}}) = \begin{pmatrix} \tau_1 \\ \tau_2 \end{pmatrix} \end{aligned} \tag{26}$$

among

$$\begin{aligned} D_{11}(q_2) &= (m_1+m_2)r_1^2 + m_2 r_2^2 + 2m_2 r_1 r_2 \cos(q_2) \\ D_{12}(q_2) &= D_{21}(q_2) = m_2 r_2^2 + m_2 r_1 r_2 \cos(q_2) \\ D_{22}(q_2) &= m_2 r_2^2 \\ C_{12}(q_2) &= m_2 r_1 r_2 \sin(q_2) \end{aligned} \tag{27}$$

Make $\boldsymbol{y} = [q_1, q_2]^{\mathrm{T}}$, $\boldsymbol{\tau} = [\tau_1, \tau_2]^{\mathrm{T}}$, and $\boldsymbol{x} = [q_1, \dot{q}_1, q_2, \dot{q}_2]^{\mathrm{T}}$. Take system parameter $r_1 = 1\mathrm{m}$, $r_2 = 0.8\mathrm{m}$, $m_1 = 1\mathrm{kg}$, and $m_2 = 1.5\mathrm{kg}$. The control objective is to make the output q_1, q_2 of double joint rigid manipulator track desired trajectory $y_{d_1} = 0.3\sin t$ and $y_{d_2} = 0.3\sin t$. Aiming for the fuzzy system inputs $\dot{q}_1, \dot{q}_2$ design five fuzzy sets separately, that is, $n = 2$, $i = 1, 2$, and $p_1 = p_2 = 25$, then has $p_1 \times p_2 = 25$ fuzzy rules. Define membership functions is

$$\mu_{A_i^l}(x_i) = \exp\left(-\left(\frac{x_i - \overline{x}_i^l}{\pi/24}\right)^2\right) \tag{28}$$

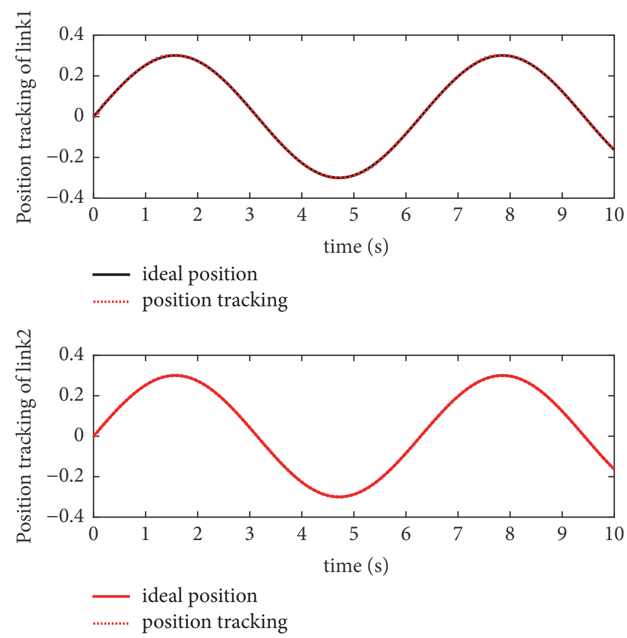

Figure 1: Double joint angle position tracking.

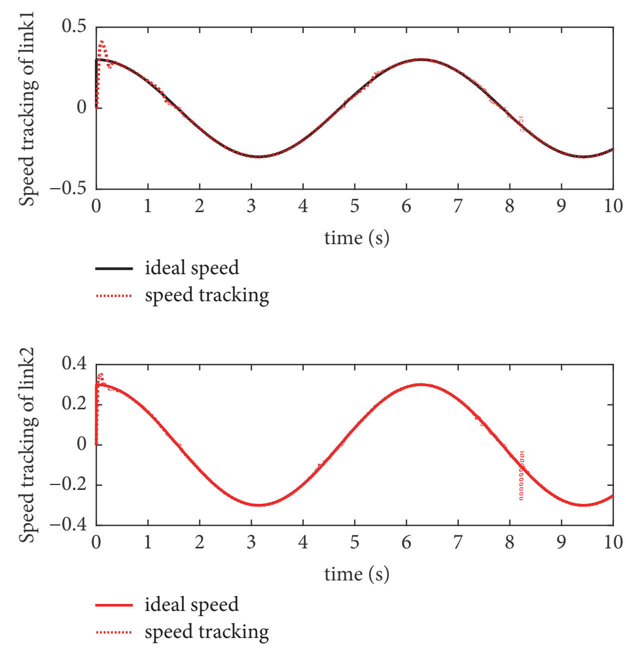

Figure 2: Double joint angular velocity tracking.

where $\overline{x}_i^l$ is $-\pi/6$, $-\pi/12$, 0, and $\pi/12$, $\pi/6$ and $i = 1, 2, 3, 4, 5$, A_i is separately NB, NS, ZO, PS, and PB.

With the manipulator of friction fuzzy compensation, controller parameter is $\lambda_1 = 10$, $\lambda_2 = 10$, $\boldsymbol{K}_D = 20\boldsymbol{I}$, and $\Gamma_1 = \Gamma_2 = 0.0001$. Taking the initial state of the system $q_1(0) = q_2(0) = \dot{q}_1(0) = \dot{q}_2(0) = 0$, the friction item is $\boldsymbol{F}(\dot{\boldsymbol{q}}) = \begin{bmatrix} 15\dot{q}_1 + 6\operatorname{sgn}(\dot{q}_1) \\ 15\dot{q}_2 + 6\operatorname{sgn}(\dot{q}_2) \end{bmatrix}$ and the interference term is $\boldsymbol{\tau}_{\mathrm{d}} = \begin{bmatrix} 0.05\sin(20t) \\ 0.1\sin(20t) \end{bmatrix}$. During robust control law, $\mathbf{W} = \operatorname{diag}[2, 2]$.

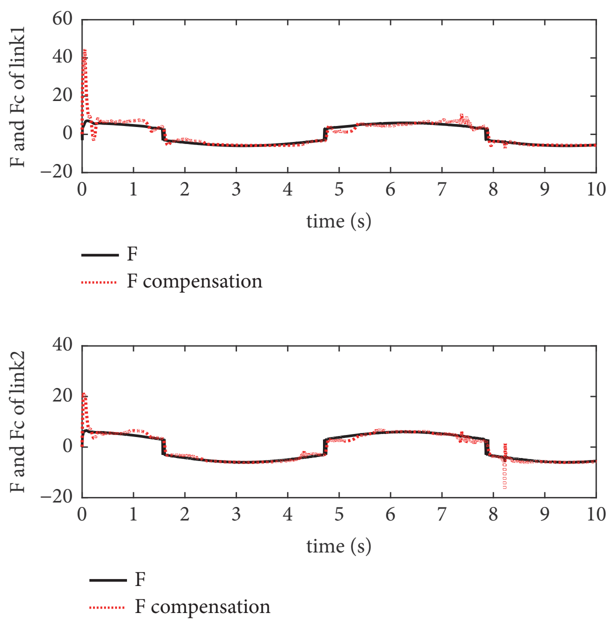

FIGURE 3: Double joint friction and compensation.

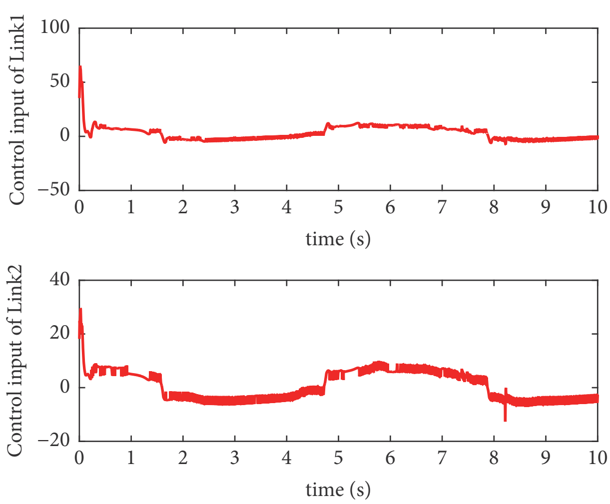

FIGURE 4: Double joint control input.

Taking robust control law (23), adaptive law is (24) and the simulation results are shown in Figures 1–4.

5. Conclusion

In this paper, a new adaptive fuzzy SMC scheme based on fuzzy compensation is proposed to control a nonlinear system. Use fuzzy system to realize adaptive approximation of the unknown part of the model. In order to show the ease of use and performance of the proposed method, it has been used to control the double joint rigid manipulator. The stability of the overall system was proved by the Lyapunov method and simulation results showed that the designed adaptive fuzzy SMC controller performs quite well in set-point tracking.

Data Availability

We use Matlab programming and build mathematical model and simulation. The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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