Hindawi
Mathematical Problems in Engineering
Volume 2018, Article ID 2474150, 12 pages
https://doi.org/10.1155/2018/2474150

Research Article

Adaptive Finite-Time Synchronization for Complex Dynamical Network with Different Dimensions of Nodes and Time-Varying Outer Coupling Structures

Lihong Yan [1,2] and Junmin Li [1]

[1] *School of Mathematics and Statistics, Xidian University, Xi'an 710071, China*
[2] *College of Mathematics and Information Science, Xianyang Normal University, Xianyang 712000, China*

Correspondence should be addressed to Lihong Yan; 092212@163.com

Received 24 October 2018; Accepted 3 December 2018; Published 20 December 2018

Academic Editor: Tomasz Kapitaniak

In this paper, we investigate finite-time synchronization problems of complex dynamical networks with different dimensions of nodes, which contain unknown periodically coupling structures and bounded time-varying delay. Based on finite-time stability theory, the inequality techniques, and the properties of Kronecker production of matrices, some useful finite-time synchronization criteria for complex dynamical network with unknown periodical couplings have been obtained. In addition, with proper adaptive periodical learning law designed, the unknown periodical couplings have been estimated successfully. Finally, some simulation examples are performed to verify the theoretical findings.

1. Introduction

Complex dynamical networks, which are composed of a large set of nonlinear dynamical systems, wherein the isolated nodes are interdependent and interact with each other, have been applied to scientific research, engineering application and even our daily life [1–3]. The researchers mainly focused on network modeling, collective dynamical behavior analysis and evolution by adopting the knowledge of graph theory, statistical physics, computer simulation methods, and so on. The research of complex network is booming, and many important achievements have been obtained [4–10].

Up to now, complex dynamical network is still an active and promising scientific research area, especially synchronization controlling of complex dynamical networks. Many results are mainly focusing on asymptotical or exponential synchronization. However, from the perspective of practical application, finite-time synchronization has greater practical application value; for example, the system convergence time is very important in industrial control. In addition, the existing conclusion shows that the finite-time controller has better robustness and interference suppression performance [9]. Finite-time control methods mainly include sliding mode control [11], optimal control [12], learning control [13, 14], quantized control [15], and pure finite-time control [16, 17]. In recent years, many meaningful advances about finite-time synchronization have been reported in recent years.

As one knows, in the process of long-distance information transmission, time delay emerged ubiquitously as for the influence on finite signal transmission and channel bandwidth [10]. In addition, time delays always bring instability or other unpredictable negative effects to the controlled system. Thus, the influence of time delays for synchronization of complex dynamical networks is worthy of consideration. Lu and Chen [8] put forward complete synchronization analyses of networks of the coupled dynamical systems with time-varying couplings based on algebraic graph theory and dynamic system method. By adaptive learning control method, Li et al. [9] presented an intermittent control approach with multiple switched periods for the finite-time synchronization in complex networks with nondelay and time-varying delay couplings. In [12], He et al. presented the guaranteed cost synchronization of complex networks with Markovian jump and mode-dependent mixed time delay.

Guo and Li [13] presented a new synchronization algorithm for delayed complex dynamical networks. In 2015, Hao and Li [14] discussed the stochastic synchronization problem of complex dynamical networks with time-varying couplings. In [15], the author considered finite-time synchronization of dynamical networks by designing aperiodically intermittent pinning controllers with logarithmic quantization by using multiple Lyapunov functions and convex combination techniques. Chen et al. [16] presented conclusions about finite-time synchronization of multiweighted complex dynamical networks with and without coupling delay. In [17], the authors investigated adaptive lag synchronization for uncertain complex dynamical network with delay coupling.

In practice, the complex dynamical networks such as mobile communication networks and social networks are always time-varying networks. The weights of links are time varying, which results in variations of the network topology and coupling configuration over time [18, 19]. In [7], the author put forward passivity of directed and undirected complex dynamical networks with adaptive coupling weights. Mei et al. [20] investigated finite-time synchronization and parameter identification problem for drive-response dynamical networks with unknown network topological structure and system parameters based on the finite-time stability theory and the adaptive control method. By making use of finite-time stability theory and properties of Wiener process, finite-time topological identification and stochastic synchronization for two complex networks with multiple time delays were obtained in [21]; in addition, Zhao et al. studied finite-time topology identification and stochastic synchronization of complex network with multiple time delays. In 2016, the finite-time synchronization and identification for the uncertain system parameters and topological structure of complex delayed networks with Markovian jumping parameters and stochastic perturbations is studied in [22]. In [23], the author studied finite-time synchronization of multiweighted complex dynamical networks with and without coupling delay on the basis of Dini derivative and some inequality techniques.

Stated thus, we can see that the nodes of complex dynamical networks are assumed to have the same state dimension in the above literatures. However, synchronization for the dynamical networks with different dimensions of nodes deserves the same consideration. It is found that the dynamic characteristics of what the complex networks presented are particularly important for the study of networks with different nodes [24, 25], the network may exhibit different dynamical behaviors with nonidentical nodes couplings. In [24], Zhao et al. generalize the master stability function method for local synchronization of networks with identical nodes to the case of nonidentical nodes; a framework for global synchronization of dynamical networks with nonidentical nodes was presented. Wang and Fan [25, 26] discussed stabilization and synchronization of complex dynamical networks with similar nodes. Dai et al. [27] investigated generalized function matrix projective lag synchronization of uncertain complex dynamical networks with different dimension of nodes via adaptive control method combining with time-varying scaling matrix. In 2015, Tan and Tian [28] proposed a class of complex dynamical network modeling in which the nodes have different state dimensions, then with the help of finite-time theory, results of finite-time stabilization and synchronization are derived.

Motivated by the above discussion, in this paper, on the basis of adaptive controlling technique, finite-time stability theory, and the inequality techniques, an available control scheme is presented to accomplish the synchronization and estimation of the unknown periodical time-varying outer coupling of complex delayed dynamical networks with different dimensions of nodes and time delay. The rest of this paper is organized as follows: Section 2 gives the network model and some relative useful mathematical preliminaries. In Section 3, the finite-time control theory is employed to be addressed the synchronization and estimation of complex networks in detail, and some sufficient conditions are obtained. In Section 4, some representative examples are simulated to demonstrate the effectiveness of the proposed approach. Section 5 gives the conclusions of the paper.

2. Problem Formulation and Preliminaries

Consider the following complex dynamical network with different dimensions of nodes with unknown periodical outer coupling matrix and time-varying delay, the differential equation is

$$\begin{aligned}\dot{x}_i(t) &= f_i(x_i(t)) \\ &\quad + \sum_{\substack{j=1 \\ j\neq i}}^{N} c_{ij}(t)\left(\Gamma_{ij}x_j\left(t-\tau_j(t)\right) - \Gamma_{ii}x_i\left(t-\tau_i(t)\right)\right) \\ &\quad + u_i(t), \quad i = 1, 2, \ldots, N,\end{aligned} \tag{1}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{in_i}(t))^T \in R^{n_i}$ is the state variable of node i, $f_i : R^{n_i} \longrightarrow R^{n_i}, i = 1, 2, \ldots, N$ is a smooth nonlinear function, describes the local dynamics of each node for the network (1), $\tau_i(t)$ is the unknown bounded time-varying delay, $\Gamma_{ij} \in R^{n_i \times n_j}$, $i, j = 1, 2, \ldots, N$ is the inner coupling matrices, and $C(t) = (c_{ij}(t))_{N\times N}$ is the unknown periodical outer coupling matrix, which describes the topology structure of the whole network. The matrix $C(t)$ is defined as follows: if there is a connection from node j to node i ($i \neq j$), then $c_{ij}(t) \neq 0$; otherwise $c_{ij}(t) = 0$. The diagonal elements of matrix $C(t)$ satisfy

$$c_{ii}(t) = -\sum_{\substack{j=1 \\ j\neq i}}^{N} c_{ij}(t), \quad i = 1, 2, \ldots, N. \tag{2}$$

Here we need not assume that the coupling matrix $C(t)$ was symmetric.

Assume that $C([-\tau_0, 0], R^{n_i})$ is a Banach space of continuous functions mapping the intervals $[\tau_0, 0]$ into R^{n_i} with the norm $\|\Phi\| = \sup_{-t_0\le s\le 0}\|\Phi_i(s)\|$, where $0 \le t \le t_0$. For the network model (1), its initial conditions are given by $x_i(t) = \Phi_i(t) \in ([\tau_0, 0], R^{n_i})$.

The solution $s_i(t) \in R^{n_i}, i = 1, 2, \ldots, N$, of the following equations is the finite-time synchronization goal orbit:

$$\dot{s}_i(t) = f_i(s_i(t)), \quad i = 1, 2, \ldots, N. \tag{3}$$

In the paper, we will design proper controllers $u_i(t), i = 1, 2, \ldots, N$, to realize finite-time synchronization between the complex dynamical networks (1) and (3).

For further discussion, the following assumptions and lemmas needed to be introduced firstly.

Assumption 1. There exists a positive definite diagonal matrix $\mathscr{L} = \operatorname{diag}\{l_1, l_2, \ldots, l_N\}$, such that

$$\begin{aligned} &(x_i(t) - s_i(t))^T (f_i(x_i(t)) - f_i(s_i(t))) \\ &\quad \leq l_i (x_i(t) - s_i(t))^T (x_i(t) - s_i(t)) \end{aligned} \tag{4}$$

holds for all $x_i(t), s_i(t) \in R^{n_i}$, where $x_i(t) \neq s_i(t), i = 1, 2, \ldots, N$.

Assumption 2. In network (1), the unknown time-varying coupling elements $c_{ij}(t)$ are periodical parameters, that is, $c_{ij}(t + T) = c_{ij}(t)$ for $t \in [0, +\infty)$, in which T is the known common period of $c_{ij}(t)$. Moreover, suppose that there exist a positive constant B_c, such that

$$|c_{ij}(t)| \leq B_c. \tag{5}$$

As one knows that the outer coupling elements $c_{ij}(t)$ is periodical, so it must be bounded. Also we suppose the time delay $\tau(t)$ and its derivative $\dot{\tau}(t)$ are both bounded.

Assumption 3. Suppose there exist a constant τ, τ_d, such that

$$\begin{aligned} &0 \leq \tau_i(t) \leq \tau_0, \\ &0 \leq \dot{\tau}_i(t) \leq \tau_d \leq 1. \end{aligned} \tag{6}$$

Lemma 4 (see [27]). *Assume that there exist a continuous, positive definite function $V(t)$ which satisfies the following inequality:*

$$\dot{V}(t) \leq -\alpha V^{\eta}(t) + \beta V(t), \quad \forall t \geq t_0, \; V^{1-\eta}(t_0) \leq \frac{\alpha}{\beta}, \tag{7}$$

where $0 < \eta < 1$ and $\alpha > 0$, $\beta > 0$, then the setting time T_f satisfies

$$T_f \leq \frac{\ln\left(1 - (\beta/\alpha) V^{1-\eta}(t_0)\right)}{\beta(\eta - 1)}. \tag{8}$$

From the inequality (7), we can easily find that when $t_0 = 0$, $e^{-\beta(1-\eta)t} V^{1-\eta}(t) \leq V^{1-\eta}(0) - \alpha/\beta + (\alpha/\beta) e^{-(1-\eta)\beta t}$.

Especially, for the time-delay system, we have the following finite-time theorem.

Lemma 5 (see [9]). *Suppose that function $V(t)$ is continuous and nonnegative for $t \in [-\tau, +\infty]$ and satisfies the following conditions:*

$$\dot{V}(t) \leq -\alpha V^{\eta}(t) + \beta V(t), \tag{9}$$

where $0 < \eta < 1$ and $\alpha > 0$, $\beta > 0$, then the following inequality holds for $0 \leq t \leq T_f$,

$$e^{-\beta(1-\eta)t} V^{1-\eta}(t) \leq \overline{V}^{1-\eta}(0) - \frac{\alpha}{\beta} + \frac{\alpha}{\beta} e^{-(1-\eta)\beta t} \tag{10}$$

for the constant T_f which denotes the settling time, where $\overline{V}(0) = \sup_{-\tau \leq s \leq 0} V(s)$.

Lemma 6 (see [28]). *Let $\lambda_i, i = 1, 2, \ldots, m$, be eigenvalues of matrix $A \in C_{m \times m}$ and $\eta_i, i = 1, 2, \ldots, n$, be eigenvalues of matrix $B \in C_{n \times n}$, then eigenvalues of matrix $A \otimes B$ are $\lambda_i \eta_j, i = 1, 2, \ldots, m; j = 1, 2, \ldots, n$.*

For convenience, some necessary notations are needed to be introduced. T denotes the transpose of a matrix or a vector. I_{n_i} is an identity matrix of size n_i. I_N are some appropriate identity matrices. The notation $A > 0 (A < 0)$ means that the matrix A is positive (negative) definite. $\lambda_{max}(\cdot)$ and $\lambda_{min}(\cdot)$ mean the maximum and minimum eigenvalues of a matrix separately. And the inner coupling matrix Γ is denoted as

$$\Gamma = \begin{pmatrix} \Gamma_{n_1 \times n_1} & \Gamma_{n_1 \times n_2} & \cdots & \Gamma_{n_1 \times n_N} \\ \Gamma_{n_2 \times n_1} & \Gamma_{n_2 \times n_2} & \cdots & \Gamma_{n_2 \times n_N} \\ \cdots & \cdots & \cdots & \cdots \\ \Gamma_{n_N \times n_1} & \Gamma_{n_N \times n_2} & \cdots & \Gamma_{n_N \times n_N} \end{pmatrix}. \tag{11}$$

Let $e_i(t) = x_i(t) - s_i(t)$, then the dynamical error system is $\dot{e}_i(t) = \dot{x}_i(t) - \dot{s}_i(t)$; substitute (1) and (3) to it, we have

$$\begin{aligned} \dot{e}_i(t) &= f_i(x_i(t)) - f_i(s_i(t)) \\ &\quad + \sum_{\substack{j=1 \\ j \neq i}}^{N} c_{ij}(t) \left(\Gamma_{ij} e_j(t - \tau_j(t)) - \Gamma_{ii} e_i(t - \tau_i(t))\right) \\ &\quad + u_i. \end{aligned} \tag{12}$$

Obviously, we can see that finite-time synchronization of complex dynamical network (1) with controller u_i equals the finite-time stability of dynamical error system (12). So next we will design proper outer controller and adaptive periodical learning law to promise finite-time stability and unknown coupling structure estimation of complex dynamical network (1).

3. Finite-Time Synchronization for Complex Dynamical Networks

In this section, we consider the finite-time synchronization of time-varying delay coupled complex dynamical network

(1). In order to achieve the synchronization objective (3), the adaptive controller $u_i(t)$ for the ith node is designed as follows:

$$
\begin{aligned}
u_i(t) = &-\frac{\varepsilon}{\sqrt{2^{1+\sigma}}}\Bigg(\operatorname{sign}(e_i(t))\left|e_i(t)\right|^{\sigma} \\
&+\Bigg(\left(\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{\varphi_{ij}^{*}}\int_{t-T}^{t}\left(\left|\widehat{c}_{ij}(\nu)\right|+B_c\right)^2 d\nu\right)^{(1+\sigma)/2} \\
&+\left(\sum_{i=1}^{N}e_i^T(t)\,e_i(t)\right)^{(1+\sigma)/2}\Bigg)\frac{e_i(t)}{\left\|(e_i(t))\right\|^2}\Bigg)-k_i(t) \\
&\cdot e_i(t)-\left(\sum_{i=1}^{N}\int_{t-\tau}^{t}e_i^T(s)\,Q_i e_i(s)\,ds\right)^{(1+\sigma)/2} \\
&\cdot\frac{e_i(t)}{\left\|(e_i(t))\right\|^2},\quad \text{if } \left\|(e_i(t)\right\|\neq 0, \\
u_i(t) = &\ 0,\quad \text{if } \left\|(e_i(t)\right\| = 0.
\end{aligned}
\tag{13}
$$

$e_i(t)^{\sigma} = (e_{i1}(t)^{\sigma}, e_{i2}(t)^{\sigma}, \ldots, e_{in_i}(t)^{\sigma})^T$, $\operatorname{sign}(e_i(t)) = (\operatorname{sign}(e_{i1}(t)), \operatorname{sign}(e_{i2}(t)), \ldots, \operatorname{sign}(e_{in_i}(t)))^T$, $0 < \sigma < 1$, and ε is an adjusted constant. With the help of the update law design in [20], the adaptive parameter $k_i(t), i = 1, 2, \ldots, N$, is as follows:

$$
\begin{aligned}
\dot{k}_i(t) = &-\frac{\varepsilon}{\sqrt{2^{1+\sigma}}}r_i^{(1-\sigma)/2}\left|k_i\right|^{\sigma}\operatorname{sign}(k_i) \\
&+r_i\left(1-\frac{\eta_i}{k_i}\right)e_i^T(t)\,e_i(t).
\end{aligned}
\tag{14}
$$

The parameters r_i, η_i are some positive constants. The time-varying periodical couplings are designed by

$$
\widehat{c}_{ij}(t) = \begin{cases}
\widehat{c}_{ij}(t-T)+\varphi_{ij}^{*}e_i^T(t)\,\Gamma_{ij}e_j\left(t-\tau_j(t)\right), & t\in[kT,(k+1)T),\ k=1,2,\ldots, \\
\varphi_{ij}(t)\,e_i^T(t)\,\Gamma_{ij}e_j\left(t-\tau_j(t)\right), & t\in[0,T), \\
0, & t\in[-T,0).
\end{cases}
\tag{15}
$$

$\widehat{c}_{ij}(t)$ is the estimation to the coupling element $c_{ij}(t)$ and $\widehat{C} = (\widehat{c}_{ij}(t))$ is the estimated outer coupling matrix. Denote $\widetilde{c}_{ij}(t) = c_{ij}(t) - \widehat{c}_{ij}(t)$ to be the estimation error. φ_{ij}^{*} are some positive constants; $\varphi_{ij}(t)$ is a continuous and strictly increasing function for $t \in [0, T]$ and satisfies $\varphi_{ij}(0) = 0, \varphi_{ij}(T) = \varphi_{ij}^{*}$.

With the controller (13) and adaptive update laws (14) and (15), a sufficient condition for the controlled complex network which can realize finite-time synchronization will be presented.

Definite 1. Complex dynamical networks (1) and (3) are said to be synchronized in finite time, if for suitable designed feedback controller $u_i(t)$, there exist a constant $t_f > 0$, any initial time t_0, such that each state $x_i(t)$ of complex dynamical network (1) will satisfy

$$
\begin{gathered}
\left\|x_i(t)-s_i(t)\right\|\longrightarrow 0,\quad t\longrightarrow t_f, \\
\left\|x_i(t)-s_i(t)\right\|\equiv 0,\quad t\geq t_f, \\
i=1,2,\ldots,N.
\end{gathered}
\tag{16}
$$

t_f depends on the initial state vector value $e_i(t) = \phi_i(t)$ for $t \in [-\max_{k=0,1,\ldots,N,\theta\in R}\{\tau_k(\theta), T\}, 0]$.

Remark 7. For continence, let us suppose that the coupling node delay $\tau_i(t), i = 1, 2, \ldots, N$, is less than the period of time-varying coupling structure T.

Theorem 8. *Under Assumptions 1–3, if there exist positive constants r_i, η_i and positive definite matrices Q_i, $i = 1, 2, \ldots, N$, such that the following conditions hold:*

$$
\begin{gathered}
\mathscr{L}-\mathscr{H}+Q_i<0, \\
\frac{1}{2\delta}\left(\widehat{C}^T\widehat{C}\right)\otimes\left(\Gamma^T\Gamma\right)-\mathscr{Q}<0, \\
i=1,2,\ldots,N.
\end{gathered}
\tag{17}
$$

$\mathscr{Q} = \operatorname{diag}(Q_1 \otimes I_{n_1}, Q_2 \otimes I_{n_2}, \ldots, Q_N \otimes I_{n_N})$, $\mathscr{H} = \operatorname{diag}(\eta_1, \eta_2, \ldots, \eta_N)$, and $i, j = n_1, n_2, \ldots, n_N$. Then the controlled network (1) with adaptive update laws (12), (13), and (14) warrants finite-time synchronized in a finite time for any given positive initial time t_0:

$$
t_f = \frac{4\ln\left(1-(\delta/2\varepsilon)\,\overline{V}^{(1-\sigma)/2}(0)\right)}{\delta(\sigma-1)},
\tag{18}
$$

where $\overline{V}(0) = (1/2)\|\zeta\|^2 + (1/2)\sum_{i=1}^{N}\sum_{j=1}^{N}(1/\varphi_{ij}^{*})\int_{-T}^{0}\widetilde{c}_{ij}^{2}(\nu)d\nu + \sum_{i=1}^{N}\int_{-\tau}^{0}\zeta_i^T(s)Q_i\zeta_i(s)ds + (1/2)\sum_{i=1}^{N}(1/r_i(0))k_i(0,\zeta)^2$. $\zeta = (\zeta_1, \zeta_2, \ldots, \zeta_N)^T$ is the initial value of $e(t)$ and satisfies the condition $\{\zeta \mid \overline{V}^{(1-\sigma)/2}(0) < 2\varepsilon/\delta\}$.

Proof. Choose a Lyapunov-Krasovskii like function candidate as follows:

$$
V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t),
\tag{19}
$$

where

$$V_1(t) = \frac{1}{2}\sum_{i=1}^{N} e_i^T(t) e_i(t),$$

$$V_2(t) = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{\varphi_{ij}^*}\int_{t-T}^{t}\tilde{c}_{ij}^2(\tau)\,d\tau,$$

$$V_3(t) = \sum_{i=1}^{N}\int_{t-\tau(t)}^{t} e_i^T(s) Q_i e_i(s)\,ds, \tag{20}$$

$$V_4(t) = \frac{1}{2}\sum_{i=1}^{N}\frac{1}{r_i}k_i^2.$$

Obviously, we have

$$\dot{V}(t) = \sum_{i=1}^{4}\dot{V}_i(t) \tag{21}$$

If $e_i(t) \neq 0$, under the controller (13), the error system of coupling complex dynamical network (1) is reformulated as

$$\begin{aligned}
\dot{e}_i(t) &= f_i(x_i(t)) - f_i(s_i(t)) \\
&+ \sum_{\substack{j=1\\ j\neq i}}^{N} c_{ij}(t)\left(\Gamma_{ij}e_j\left(t-\tau_j(t)\right) - \Gamma_{ii}e_i\left(t-\tau_i(t)\right)\right) \\
&- \frac{\varepsilon}{\sqrt{2^{1+\sigma}}}\operatorname{sign}(e_i(t))\left|e_i(t)\right|^{\sigma} \\
&- \frac{\varepsilon}{\sqrt{2^{1+\sigma}}}\left(\left(\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{\varphi_{ij}^*}\int_{t-T}^{t}\left(\left|\left(\hat{c}_{ij}(\nu)\right| + B_c\right)^2 d\nu\right)^{(1+\sigma)/2}\right. \\
&\left. + \left(\sum_{i=1}^{N} e_i^T(t) e_i(t)\right)^{(1+\sigma)/2}\right)\frac{e_i(t)}{\left\|(e_i(t))\right\|^2} \\
&- \varepsilon\left(\sum_{i=1}^{N}\int_{t-\tau}^{t} e_i^T(s) Q_i e_i(s)\,ds\right)^{(1+\sigma)/2}\frac{e_i(t)}{\left\|(e_i(t))\right\|^2} \\
&- k_i(t) e_i(t),
\end{aligned} \tag{22}$$

Taking the derivative along the trajectories of the error system respectively, we get

$$\begin{aligned}
\dot{V}_1(t) &= \sum_{i=1}^{N} e_i^T(t)\dot{e}_i(t) = \sum_{i=1}^{N} e_i^T(t)\left(f_i(x_i(t)) - f_i(s_i(t))\right. \\
&- \sum_{j=1}^{N}\hat{c}_{ij}(t)\Gamma_{ij}e_j(t) \\
&\left. + \sum_{\substack{j=1\\ j\neq i}}^{N} c_{ij}(t)\left(\Gamma_{ij}e_j(t-\tau(t)) - \Gamma_{ii}e_i(t-\tau(t))\right)\right) \\
&- \sum_{i=1}^{N} e_i^T(t) \\
&\cdot\left(\frac{\varepsilon}{\sqrt{2^{1+\sigma}}}\left(\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{\varphi_{ij}^*}\int_{t-T}^{t}\left(\left|\hat{c}_{ij}(\nu)\right| + B_c\right)^2 d\nu\right)^{(1+\sigma)/2}\right. \\
&\left. + \varepsilon\left(\sum_{i=1}^{N}\int_{t-\tau}^{t} e_i^T(s) Q_i e_i(s)\,ds\right)^{(1+\sigma)/2}\right)\frac{e_i(t)}{\left\|e_i(t)\right\|^2} \\
&- \sum_{i=1}^{N} e_i^T(t)\left(\frac{\varepsilon}{\sqrt{2^{1+\sigma}}}\left(\operatorname{sign}(e_i(t))\left|e_i(t)\right|^{\sigma}\right.\right. \\
&\left.\left. + \left(\sum_{i=1}^{N} e_i^T(t) e_i(t)\right)^{(1+\sigma)/2}\right) + k_i e_i(t)\right) \le \sum_{i=1}^{N} e_i^T(t) \\
&\cdot\left((l_i - k_i) e_i(t) - \sum_{j=1}^{N}\hat{c}_{ij}(t)\Gamma_{ij}e_j\left(t-\tau_j(t)\right)\right. \\
&\left. + \sum_{j=1}^{N} c_{ij}(t)\Gamma_{ij}e_j\left(t-\tau_j(t)\right)\right) - \frac{\varepsilon}{\sqrt{2^{1+\sigma}}}\left|e_i(t)\right|^{\sigma+1} \\
&- \frac{\varepsilon}{\sqrt{2^{1+\sigma}}}\left(\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{\varphi_{ij}^*}\int_{t-T}^{t}\left(\left|\hat{c}_{ij}(\nu)\right| + B_c\right)^2 d\nu\right)^{(1+\sigma)/2} \\
&- \varepsilon\left(\sum_{i=1}^{N}\int_{t-\tau}^{t} e_i^T(s) Q_i e_i(s)\,ds\right)^{(1+\sigma)/2}
\end{aligned} \tag{23}$$

As to the second item, we obtain

$$\dot{V}_2(t) = \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{\varphi_{ij}^*}\left(\tilde{c}_{ij}^2(t) - \tilde{c}_{ij}^2(t-T)\right). \tag{24}$$

From the estimation of unknown coupling structure items (15), we have

$$\begin{aligned}
&\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{\varphi_{ij}^*}\left(\tilde{c}_{ij}^2(t) - \tilde{c}_{ij}^2(t-T)\right) \\
&\quad = -\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{\varphi_{ij}^*}\left(\hat{c}_{ij}(t) - \hat{c}_{ij}(t-T)\right)^2 \\
&\quad\quad - 2\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{\varphi_{ij}^*}\left(c_{ij}(t) - \hat{c}_{ij}(t-T)\right) \\
&\quad\quad \cdot\left(\hat{c}_{ij}(t) - \hat{c}_{ij}(t-T)\right) \\
&\quad = -2\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{\varphi_{ij}^*}e_i^T(t)\Gamma_{ij}e_j(t) \\
&\quad\quad - \sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{\varphi_{ij}^*}\left(\hat{c}_{ij}(t) - \hat{c}_{ij}(t-T)\right)^2 \\
&\quad \le -2\sum_{i=1}^{N}\sum_{j=1}^{N}\tilde{c}_{ij}(t) e_i^T(t)\Gamma_{ij}e_j\left(t-\tau_j(t)\right).
\end{aligned} \tag{25}$$

So

$$\dot{V}_2(t) \le -\sum_{i=1}^{N}\sum_{j=1}^{N}\widetilde{c}_{ij}(t)\, e_i^T(t)\,\Gamma_{ij} e_j\left(t-\tau_j(t)\right). \qquad (26)$$

For the third item, according to the boundness of derivation of time-varying delay $\tau(t)$, we can yield

$$\begin{aligned}\dot{V}_3(t) &= \sum_{i=1}^{N}\left(e_i^T(t)\,Q_i e_i(t)\right.\\&\quad\left.-(1-\dot{\tau}(t))\,e_i^T(t-\tau(t))\,Q_i e_i(t-\tau(t))\right)\\&\le \sum_{i=1}^{N}\left(e_i^T(t)\,Q_i e_i(t)\right.\\&\quad\left.-(1-\tau_d)\,e_i^T(t-\tau(t))\,Q_i e_i(t-\tau(t))\right).\end{aligned} \qquad (27)$$

Combining with the adaptive update laws of the feedback gain (14), we have

$$\begin{aligned}\dot{V}_4(t) &= -\varepsilon\sum_{i=1}^{N}\frac{k_i}{r_i}\left(\frac{1}{\sqrt{2^{1+\sigma}}}r_i^{(1-\sigma)/2}\left|k_i\right|^{\sigma}\operatorname{sign}(k_i)\right)\\&\quad+\sum_{i=1}^{N}\frac{k_i}{r_i}r_i\left(1-\frac{\eta_i}{k_i}\right)e_i^T(t)\,e_i(t)\\&= -\varepsilon\sum_{i=1}^{N}\frac{1}{\sqrt{2^{1+\sigma}}}\frac{\left|k_i\right|^{\sigma+1}}{\sqrt{r_i}^{\sigma+1}}\\&\quad+\sum_{i=1}^{N}(k_i-\eta_i)\,e_i^T(t)\,e_i(t).\end{aligned} \qquad (28)$$

Based on Assumption 1, with the help of properties of Kronecker production, substitute (23), (26), (27), and (28) into the derivative term (21); we can further get

$$\begin{aligned}\dot{V}(t) &\le \sum_{i=1}^{N}e_i^T(t)\left(\left((l_i-\eta_i)\,I_{n_i}+Q_i\right)e_i(t)\right.\\&\quad\left.+\sum_{j=1}^{N}\widehat{c}_{ij}(t)\,\Gamma_{ij}e_j\left(t-\tau_j(t)\right)\right)-\frac{\varepsilon}{\sqrt{2^{1+\sigma}}}\left|e_i(t)\right|^{\sigma+1}\\&\quad-\frac{\varepsilon}{\sqrt{2^{1+\sigma}}}\left(\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{\varphi_{ij}^*}\int_{t-T}^{t}\left(\left|\widehat{c}_{ij}(\nu)\right|+B_c\right)^2 d\nu\right)^{(1+\sigma)/2}\\&\quad-\varepsilon\left(\sum_{i=1}^{N}\int_{t-\tau}^{t}e_i^T(s)\,Q_i e_i(s)\,ds\right)^{(1+\sigma)/2}\\&\quad-\sum_{i=1}^{N}(1-\tau_d)\,e_i^T(t-\tau(t))\,Q_i e_i(t-\tau(t))\\&\quad-\varepsilon\sum_{i=1}^{N}\frac{1}{\sqrt{2^{1+\sigma}}}\frac{\left|k_i\right|^{\sigma+1}}{\sqrt{r_i}^{\sigma+1}} = e^T(t)\left(\left(\left((l_i-\eta_i)\,I_{n_i}+Q_i\right)\right.\right.\\&\quad\left.\otimes I_N\right)e(t)+e^T(t)\left(\widehat{C}\otimes\Gamma\right)e(t-\tau(t))\\&\quad-\frac{\varepsilon}{\sqrt{2^{1+\sigma}}}\left|e_i(t)\right|^{\sigma+1}\\&\quad-\frac{\varepsilon}{\sqrt{2^{1+\sigma}}}\left(\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{\varphi_{ij}^*}\int_{t-T}^{t}\left(\left|\widehat{c}_{ij}(\nu)\right|+B_c\right)^2 d\nu\right)^{(1+\sigma)/2}\\&\quad-\varepsilon\left(\sum_{i=1}^{N}\int_{t-\tau}^{t}e_i^T(s)\,Q_i e_i(s)\,ds\right)^{(1+\sigma)/2}-(1-\tau_d)\\&\quad\cdot e^T(t-\tau(t))\,\mathcal{Q}e(t-\tau(t))-\varepsilon\sum_{i=1}^{N}\frac{1}{\sqrt{2^{1+\sigma}}}\frac{\left|k_i\right|^{\sigma+1}}{\sqrt{r_i}^{\sigma+1}}.\end{aligned} \qquad (29)$$

the vectors $e(t) = (e_1(t), e_2(t), \dots, e_N(t))^T$, $e(t-\tau(t)) = (e_1(t-\tau_1(t)), e_2(t-\tau_2(t)), \dots, e_N(t-\tau_N(t)))^T$. With Young's inequality, the item can be rewritten as

$$\begin{aligned}&e^T(t)\left(\widehat{C}\otimes\Gamma\right)e(t-\tau(t))\\&\quad\le\frac{\delta}{2}e^T(t)\,e(t)+\frac{1}{2\delta}e^T(t-\tau(t))\left(\widehat{C}\otimes\Gamma\right)^T\\&\qquad\cdot\left(\widehat{C}\otimes\Gamma\right)e(t-\tau(t)).\end{aligned} \qquad (30)$$

With Assumption 3, we can obtain

$$\begin{aligned}\dot{V}(t) &\le e^T(t)\left(\left(\left(l_i-\eta_i+\frac{\delta}{2}\right)I_{n_i}+Q_i\right)\otimes I_N\right)e(t)\\&\quad-\frac{1}{\sqrt{2^{1+\sigma}}}\left|e_i(t)\right|^{\sigma+1}-\varepsilon\sum_{i=1}^{N}\frac{1}{\sqrt{2^{1+\sigma}}}\frac{\left|k_i\right|^{\sigma+1}}{\sqrt{r_i}^{\sigma+1}}\\&\quad-\frac{\varepsilon}{\sqrt{2^{1+\sigma}}}\left(\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{\varphi_{ij}^*}\int_{t-T}^{t}\left(\left|\widehat{c}_{ij}(\nu)\right|+B_c\right)^2 d\nu\right)^{(1+\sigma)/2}\\&\quad-\varepsilon\left(\sum_{i=1}^{N}\int_{t-\tau}^{t}e_i^T(s)\,Q_i e_i(s)\,ds\right)^{(1+\sigma)/2}\\&\quad+e^T(t-\tau(t))\left(\frac{1}{2\delta}\left(\widehat{C}\otimes\Gamma\right)^T\cdot\left(\widehat{C}\otimes\Gamma\right)-\mathcal{Q}\right)\\&\quad\cdot e(t-\tau(t))\end{aligned} \qquad (31)$$

Base on the properties of Kronecker product of Lemma 6, one gets

$$\left(\widehat{C}\otimes\Gamma\right)^T\cdot\left(\widehat{C}\otimes\Gamma\right)=\widehat{C}^T\widehat{C}\otimes\Gamma^T\Gamma. \qquad (32)$$

According to Assumption 3, we can see that

$$\begin{aligned}&-\left(\sum_{i=1}^{N}\int_{t-\tau}^{t}e_i^T(s)\,Q_i e_i(s)\,ds\right)^{(1+\sigma)/2}\\&\quad\le-\left(\sum_{i=1}^{N}\int_{t-\tau(t)}^{t}e_i^T(s)\,Q_i e_i(s)\,ds\right)^{(1+\sigma)/2}.\end{aligned} \qquad (33)$$

As one can see, the estimation error of unknown coupling structure elements $\widetilde{c}_{ij}(t) = c_{ij}(t)-\widehat{c}_{ij}(t)$, so $\widetilde{c}_{ij}^2(t) \le (|\widehat{c}_{ij}(t)| +$

$B_c)^2$. Based on Assumption 1, with the help of the inequality $\|x_1\|^q + \|x_2\|^q + \cdots + \|x_n\|^q \le (\|x_1\|^2 + \|x_2\|^2 + \cdots + \|x_n\|^2)^{q/2}$, we can further get

$$\begin{aligned}\dot{V}(t) \le{}& e^T(t)\left((l_i - \eta_i) I_{n_i} + Q_i\right) \otimes I_N e(t) \\ &- \varepsilon V^{(1+\sigma)/2}(t) + e^T(t-\tau(t)) \\ &\cdot \left(\frac{1}{2\delta}\left(\widehat{C}^T\widehat{C}\right) \otimes \left(\Gamma^T\Gamma\right) - \mathcal{Q}\right) e(t-\tau(t)) + \frac{\delta}{2} \\ &\cdot e^T(t) e(t).\end{aligned} \tag{34}$$

$Q = \operatorname{diag}(Q_1 \otimes I_{n_1}, Q_2 \otimes I_{n_2}, \ldots, Q_i \otimes I_{n_N})$. Combine the conditions

$$\begin{aligned}&(l_i - \eta_i) I_{n_i} + Q_i < 0, \quad i = 1, 2, \ldots, N, \\ &\frac{1}{2\delta}\left(\widehat{C}^T\widehat{C}\right) \otimes \left(\Gamma^T\Gamma\right) - \mathcal{Q} < 0.\end{aligned} \tag{35}$$

So when condition (17) of Theorem 8 is satisfied, can we have

$$\begin{aligned}\dot{V}(t) &\le -\varepsilon V^{(1+\sigma)/2}(t) + \frac{\delta}{2} e^T(t) e(t) \\ &\le -\varepsilon V^{(1+\sigma)/2}(t) + \frac{\delta}{2} V(t).\end{aligned} \tag{36}$$

Based on finite-time stabilization theorems Lemmas 4 and 5 of the error system, can we obtain that the controlled complex dynamical network (1) can realize finite-time synchronization less than $t_f \le t_0 + 4\ln(1 - (\delta/2\varepsilon)\overline{V}^{(1-\sigma)/2}(t_0))/\delta(\sigma - 1)$ for any given initial time t_0. So based on the theory of finite-time synchronization, can we know that the synchronization of complex dynamical network (1) is realized. □

Remark 9. Theorem 8 gives a matrix dependent condition, according to the second item of condition (17), if we choose proper parameter δ, by calculation of $(1/2\delta)(\widehat{C}^T\widehat{C}) \otimes (\Gamma^T\Gamma)$, the matrix $\mathcal{Q}$ can be decided, that is, the blocking matrix Q_i is known, then substitute Q_i to the first item of condition (17), and the diagonal matrix $\mathcal{H}$ will be given. So we can see matrix calculation plays an important role in this process; in fact, the number of nodes for the dynamical network is usually large in real network, the calculation will be a little bit complex, and next we will present a simplified conclusion with the properties of eigenvalue of matrices showed in Corollary 11.

Remark 10. The adjustable parameters ε and σ can not only adjust the time of the synchronization time t_f itself, but also ensure that the periodic coupling structure matrix of the system $\widehat{C}(t)$ be estimated in the limited time $T < t_f$ by selecting the appropriate parameters. It means that at least one periodical coupling structure matrix has been estimated successfully when the finite-time synchronization of complex dynamical network (1) with controller (17) is realized. If the outer coupling structure of one period is known, the coupling structure of all periods is naturally determined.

Condition (17) of Theorem 8 is time-delay dependent, which has lower conservatism than the delay independent condition. Combining with Lemma 6, rewriting the theorem with the eigenvalue form of matrix, then Corollary 11 will be obtained.

Corollary 11. *Under Assumptions 1–3, if there exist positive constants r_i, η_i and positive definite matrices Q_i, $i = 1, 2, \ldots, N$, such that the following conditions hold:*

$$\begin{aligned}&\eta_{min} > l_{max} + \lambda_{max}(Q_i), \\ &\lambda_{min}(Q_i) > \frac{1}{2\delta}\lambda_{max}\left(\widehat{C}^T\widehat{C}\right)\lambda_{max}\left(\Gamma^T\Gamma\right), \\ &\qquad i = 1, 2, \ldots, N.\end{aligned} \tag{37}$$

$\eta_{min} = \min_{i=1,\ldots,N}\{\eta_i\}$ and $l_{max} = \max_{i=1,\ldots,N}\{l_i\}$. Then the controlled network (1) with the controller (13) and adaptive update laws (14) and (15) warrants finite-time synchronized in a finite time for any given positive initial time t_0:

$$t_f = \frac{4\ln\left(1 - (\delta/2\varepsilon)\overline{V}^{(1-\sigma)/2}(0)\right)}{\delta(\sigma - 1)}, \tag{38}$$

where $\overline{V}(0) = (1/2)\|\zeta\|^2 + (1/2)\sum_{i=1}^N\sum_{j=1}^N(1/\varphi_{ij}^*)\int_{-T}^0 \tilde{c}_{ij}^2(\nu)d\nu + \sum_{i=1}^N\int_{-\tau}^0 \zeta_i^T(s)Q_i\zeta_i(s)ds + (1/2)\sum_{i=1}^N(1/r_i(0))k_i(0,\zeta)^2$. $\zeta = (\zeta_1, \zeta_2, \ldots, \zeta_N)^T$ is the initial value of $e(t)$ and satisfies the condition $\{\zeta \mid \overline{V}^{(1-\sigma)/2}(0) < 2\varepsilon/\delta\}$.

In addition, if the feedback gain is constant, the adaptive finite-time controller becomes linear finite-time controller, so the following results will be presented.

Corollary 12. *Under Assumptions 1–3, if there exist large enough positive constant k_i and positive definite matrices Q_i, $i = 1, 2, \ldots, N$, the following conditions hold:*

$$\begin{aligned}&k_{min} > l_{max} + \lambda_{max}(Q_i), \\ &\lambda_{min}(Q_i) > \frac{1}{2\delta}\lambda_{max}\left(\widehat{C}^T\widehat{C}\right)\lambda_{max}\left(\Gamma^T\Gamma\right), \\ &\qquad i = 1, 2, \ldots, N.\end{aligned} \tag{39}$$

Then the controlled network (1) with the periodical adaptive update laws (15) and the controller

$$\begin{aligned}u_i(t) = -\frac{\varepsilon}{\sqrt{2^{1+\sigma}}}\Bigg(&\operatorname{sign}(e_i(t))\left|e_i(t)\right|^\sigma \\ &+ \left(\left(\sum_{i=1}^N\sum_{j=1}^N \frac{1}{\varphi_{ij}^*}\int_{t-T}^t \left(\left|\left(\hat{c}_{ij}(\nu)\right| + B_c\right)^2 d\nu\right)^{(1+\sigma)/2}\right. \\ &\left. + \left(\sum_{i=1}^N e_i^T(t) e_i(t)\right)^{(1+\sigma)/2}\right)\frac{e_i(t)}{\left\|(e_i(t))\right\|^2}\Bigg) - k_i e_i(t)\end{aligned}$$

$$-\left(\sum_{i=1}^{N}\int_{t-\tau}^{t} e_i^T(s)\, Q_i e_i(s)\, ds\right)^{(1+\sigma)/2} \frac{e_i(t)}{\left\|(e_i(t))\right\|^2},$$

$$\text{if } \left\|(e_i(t)\right\| \neq 0,$$

$$u_i(t) = 0, \quad \text{if } \left\|(e_i(t)\right\| = 0 \tag{40}$$

warrants finite-time synchronized in a finite time t_f:

$$t_f = \frac{4\ln\left(1-(\delta/2\varepsilon)\overline{V}^{(1-\sigma)/2}(0)\right)}{\delta(\sigma-1)}, \tag{41}$$

where $\overline{V}(0) = (1/2)\|\zeta\|^2 + (1/2)\sum_{i=1}^{N}\sum_{j=1}^{N}(1/\varphi_{ij}^*)\int_{-T}^{0}\tilde{c}_{ij}^2(\nu)d\nu + \sum_{i=1}^{N}\int_{-\tau}^{0}\zeta_i^T(s)Q_i\zeta_i(s)ds$. ζ *is the initial value of* $e(t)$ *and satisfies the condition* $\{\zeta \mid \overline{V}^{(1-\sigma)/2}(0) < 2\varepsilon/\delta\}$.

Proof. Choose the Lyapunov function candidate as follows:

$$\begin{aligned} V(t) &= \frac{1}{2}\sum_{i=1}^{N} e_i^T(t)\, e_i(t) + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\frac{1}{\varphi_{ij}^*}\int_{t-T}^{t}\tilde{c}_{ij}^2(\tau)\, d\tau \\ &\quad + \frac{1}{2}\sum_{i=1}^{N}\int_{t-\tau(t)}^{t} e_i^T(t)(s)\, Q_i e_i(s)\, ds. \end{aligned} \tag{42}$$

Q_i, $i = 1, 2, \ldots, N$, are any positive definite symmetric matrices.

With similar methods, the derivative of $V(t)$ will be less than

$$\begin{aligned} \dot{V}(t) &\leq e^T(t)\left((l_i - k_i)\, I_{n_i} + Q_i\right) e(t) - \varepsilon V^{(1+\sigma)/2}(t) \\ &\quad + e^T(t-\tau(t))\left(\frac{1}{2\delta}\left(\widehat{C}^T\widehat{C}\right)\otimes\left(\Gamma^T\Gamma\right) - \mathcal{Q}\right) \\ &\quad \cdot e(t-\tau(t)) + \frac{\delta}{2}V(t). \end{aligned} \tag{43}$$

So when the following condition is satisfied,

$$\begin{gathered} (l_i - k_i)\, I_{n_i} + Q_i < 0, \quad i = 1, 2, \ldots, N, \\ \frac{1}{2\delta}\left(\widehat{C}^T\widehat{C}\right)\otimes\left(\Gamma^T\Gamma\right) - \mathcal{Q} < 0. \end{gathered} \tag{44}$$

That is, when condition (39) is satisfied, one can obtain

$$\dot{V}(t) \leq -\varepsilon V^{(1+\sigma)/2}(t) + \frac{\delta}{2}V(t). \tag{45}$$

That is, when the condition (39) is satisfied, the controlled complex dynamical network (1) will be finite-time synchronized in a finite-time t_f:

$$t_f = \frac{4\ln\left(1-(\delta/2\varepsilon)\overline{V}^{(1-\sigma)/2}(0)\right)}{\delta(\sigma-1)}. \tag{46}$$

□

Remark 13. Choosing appropriate parameters can help us to realize finite-time synchronization and unknown outer coupling estimation, so we put forward a calculation scheme to prepare the conditions of Corollary 11 as follows:

(1) Choose a proper parameter δ, for the given dynamical network model, calculate the eigenvalues of matrices $\Gamma^T\Gamma$ and $\widehat{C}^T\widehat{C}$, and then decide the values of matrices Q_i such that $\lambda_{min}(Q_i) > (1/2\delta)\lambda_{max}(\widehat{C}^T\widehat{C})\lambda_{max}(\Gamma^T\Gamma)$.

(2) Calculate the Lipschitz constant, decide the parameters l_i, and then combine with the values of Q_i, to decide the parameters η_i, such that $\eta_{min} > l_{max} + \lambda_{max}(Q_i)$.

(3) With the initial matrix Φ^*, determine the exponential parameter σ of the controller and the feedback parameters k and to calculate periodical coupling matrices $C_{ij}(t)$.

4. Numerical Simulations

In this section, we will illustrate the effectiveness of the proposed approaches to achieve finite-time synchronization of complex networks (1) with different chaotic models as the node dynamics with unknown time-varying coupling and time delay.

Without loss of generality, we choose five different chaotic dynamical as the nodes of complex dynamical networks (1). The system function is as follows:

$$\begin{aligned} \dot{x}_i(t) &= f_i(x_i(t)) \\ &\quad + \sum_{\substack{j=1 \\ j\neq i}}^{N} c_{ij}(t)\left(\Gamma_{ij}x_j\left(t-\tau_j(t)\right) - \Gamma_{ii}x_i\left(t-\tau_i(t)\right)\right) \\ &\quad + u_i(t), \quad i = 1, 2, \ldots, 5. \end{aligned} \tag{47}$$

The five nodes of the controlled network (1) separately are chaotic systems with different dimensions, including Sprott-O, hyperchaotic *lü*, Lorenz, duffing, and Chen systems. Also the synchronization orbits are the following five isolate dynamical systems.

The single Sprott-O system can be described as

$$\begin{pmatrix} \dot{x}_{11} \\ \dot{x}_{12} \\ \dot{x}_{13} \end{pmatrix} = \begin{pmatrix} x_{12} \\ x_{11} - x_{13} \\ x_{11} + x_{11}x_{13} + 2.7x_{12} \end{pmatrix}. \tag{48}$$

The single hyperchaotic *lü* system function is as follows:

$$\begin{pmatrix} \dot{x}_{21} \\ \dot{x}_{22} \\ \dot{x}_{23} \\ \dot{x}_{24} \end{pmatrix} = \begin{pmatrix} 36\left(x_{21} - x_{22}\right) + x_{24} \\ -x_{21}x_{23} + 20x_{22} \\ x_{21}x_{22} - 3x_{23} \\ x_{21}x_{23} + x_{24} \end{pmatrix}. \tag{49}$$

The single Lorenz system can be described as follows:

$$\begin{pmatrix} \dot{x}_{31} \\ \dot{x}_{32} \\ \dot{x}_{33} \end{pmatrix} = \begin{pmatrix} 10\left(x_{32} - x_{31}\right) \\ 28x_{31} - x_{31}x_{33} - x_{32} \\ x_{31}x_{32} - \dfrac{8}{3}x_{33} \end{pmatrix}. \quad (50)$$

The single Duffing system is

$$\begin{pmatrix} \dot{x}_{41} \\ \dot{x}_{42} \end{pmatrix} = \begin{pmatrix} x_{42} \\ -0.25x_{41} - x_{41}^{3} + 11\cos(t) \end{pmatrix}. \quad (51)$$

The Chen system of drive network can be described as follows:

$$\begin{pmatrix} \dot{x}_{51} \\ \dot{x}_{52} \\ \dot{x}_{53} \end{pmatrix} = \begin{pmatrix} 35\left(x_{52} - x_{51}\right) \\ -7x_{51} - x_{51}x_{53} + 28x_{52} \\ x_{51}x_{52} - 3x_{53} \end{pmatrix}. \quad (52)$$

The outer coupling matrix which depicts the structure of network (1) is a 5×5 matrix, where we choose $c_{11}(t) = -2.5 - 1.4\sin(\pi/3)t - 1.3\cos(2\pi/3)t$, $c_{12}(t) = 1 + 0.4\sin(\pi/3)t$, $c_{13}(t) = 0$, $c_{14}(t) = 0.5 + \sin(\pi/3)t$, $c_{15}(t) = 1 - 1.3\cos(2\pi/3)t$; $c_{21}(t) = 1$, $c_{22}(t) = -5.3 - \cos\pi t$, $c_{23}(t) = 1.3 + \cos\pi t$, $c_{24}(t) = 1$, $c_{25}(t) = 2$; $c_{31}(t) = 1 - \sin(\pi/4)t$, $c_{32}(t) = 1 + 0.5\cos(\pi/4)t$, $c_{33}(t) = -2.7 - 0.5\cos(\pi/4)t + \sin(\pi/4)t$, $c_{34}(t) = 0$, $c_{35}(t) = 0.7$; $c_{41}(t) = 1 + 0.5\cos(\pi/4)t$, $c_{42}(t) = \sin\pi t$, $c_{43}(t) = \cos\pi t$, $c_{44}(t) = -1 - \sin\pi t - \cos\pi t - 0.5\cos(\pi/4)t$, $c_{45}(t) = 0$; $c_{51}(t) = \cos(2\pi/3)t$, $c_{52}(t) = \cos(\pi/4)t$, $c_{53}(t) = 1$, $c_{54}(t) = 1$, $c_{55}(t) = -2 - \cos(2\pi/3)t - \cos(\pi/4)t$. we can calculate that the bound of $\|C_{ij}(t)\| \le B_c = 6.3$.

By calculation, we get the common period $T = 12$ of $c_{ij}(t)$; the parameters $\Phi^{*} = (\varphi_{ij}^{*})$ are designed as

$$\Phi^{*} = \left(\varphi_{ij}^{*}\right) = \begin{pmatrix} 0.5 & 0.3 & 0.7 & 2 & 1.1 \\ 0.1 & 1.2 & 0.4 & 0.5 & -1.9 \\ 0.9 & 0.8 & 0.5 & -0.7 & 1.1 \\ 0.5 & 1.5 & -0.5 & 2.1 & 1.8 \\ 1.2 & 1.4 & 1.8 & 2.0 & -2.2 \end{pmatrix}, \quad (53)$$

and $\varphi_{ij}(t) = (t/12)\varphi_{ij}^{*}$ for $i, j = 1, 2, 3, 4, 5$. The time delays are chosen as $\tau_j(t) = 0.1 + 0.2\sin(jt)$, $j = 1, 2, \ldots, 5$. And the bound of time delay is $\tau = 0.3$.

The inner coupling matrices are designed as

$$\Gamma_{12} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix},$$

$$\Gamma_{14} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & -1 \end{pmatrix},$$

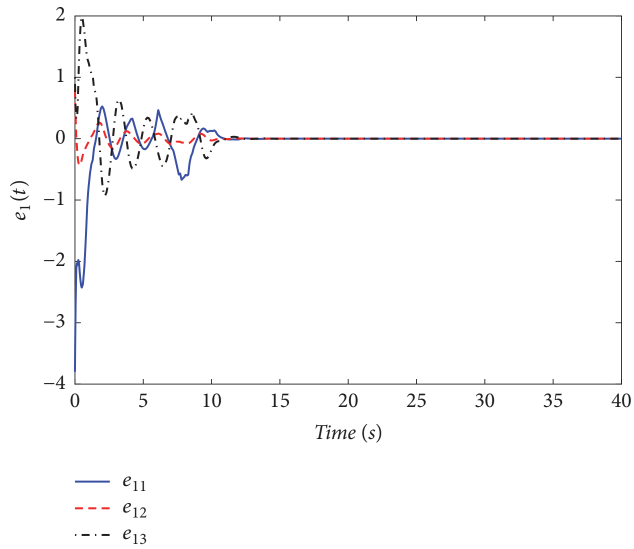

Figure 1: The error evolution of the first node.

$$\Gamma_{24} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad (54)$$

$\Gamma_{32} = \Gamma_{52} = \Gamma_{12}$, $\Gamma_{21} = \Gamma_{23} = \Gamma_{25} = \Gamma_{12}^{T}$, $\Gamma_{34} = \Gamma_{54} = \Gamma_{14}$, $\Gamma_{41} = \Gamma_{43} = \Gamma_{45} = \Gamma_{14}^{T}$, $\Gamma_{42} = \Gamma_{24}^{T}$, $\Gamma_{11} = \Gamma_{33} = \Gamma_{55} = I_3$, $\Gamma_{22} = I_4$, $\Gamma_{44} = I_2$, $\Gamma_{ij} = \Gamma_{ji} = I_3$, $i, j = 1, 3, 5$.

Choosing the initial values of complex dynamical networks (49) randomly in $[-5, 5]$, since $t_0 = 0$, the finite-time synchronization of the complex dynamical network (1) with controller (12) is achieved in a finite time $t_f = 4\ln(1 - (\delta/2\varepsilon)\overline{V}^{(1-\sigma)/2}(0))/\delta(\sigma - 1)$, where $\overline{V}(0) = (1/2)\|\zeta\|^2 + (1/2)\sum_{i=1}^{N}\sum_{j=1}^{N}(1/\varphi_{ij}^{*})\int_{-T}^{0}\tilde{c}_{ij}^{2}(\nu)d\nu + \sum_{i=1}^{N}\int_{-\tau}^{0}\zeta_i^{T}(s)Q_i\zeta_i(s)ds + (1/2)\sum_{i=1}^{N}(1/r_i(0))k_i(0,\zeta)^2$. The error curves of states of the complex dynamical network (1) are showed in Figures 1–5, which indicates that the finite-time synchronization of the complex dynamical network (49) is achieved with the controllers (13), (14) and the adaptive periodical undate law (15).

Simultaneously, Figure 6 displays the time evolution curves of the feedback gains $k_i(t)$, $i = 1, 2, \ldots, 5$, with the initial values $k_i(0) = 0.2$; as times goes on, the five adaptive feedback gains converge to the determined value $k_1 = 8.17$, $k_2 = 4.77$, $k_3 = 3.57$; $k_4 = 2.23$, $k_5 = 0.48$ gradually. And the unknown adaptive outer coupling structure $\hat{c}_{ij}$ are presented in Figure 7, from which we can see that the outer coupling connection matrix is periodical absolutely.

5. Conclusions

In past twenty years, thanks to the concerted efforts of physicists, mathematicians, and control scientists, complex networks have enjoyed unprecedented prosperity. As one

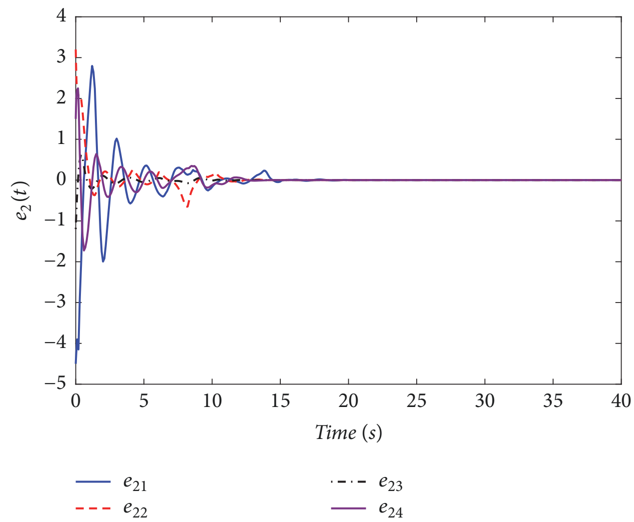

Figure 2: The error evolution of the second node.

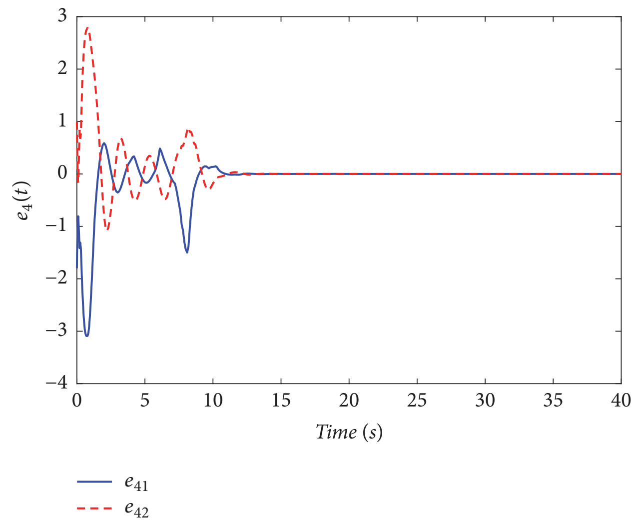

Figure 4: The error evolution of the fourth node.

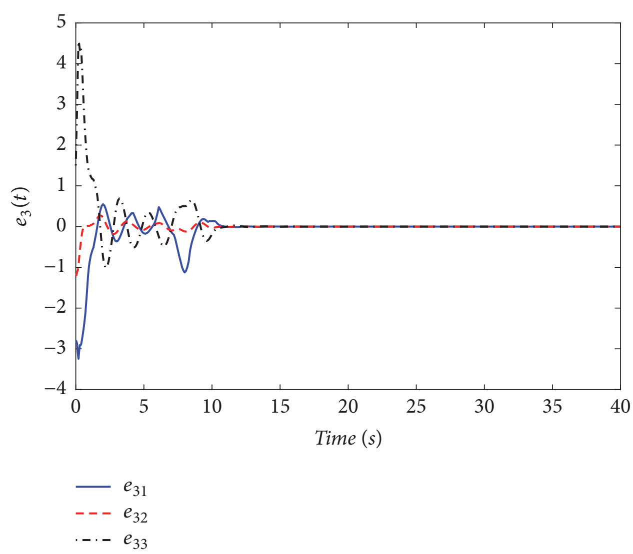

Figure 3: The error evolution of the third node.

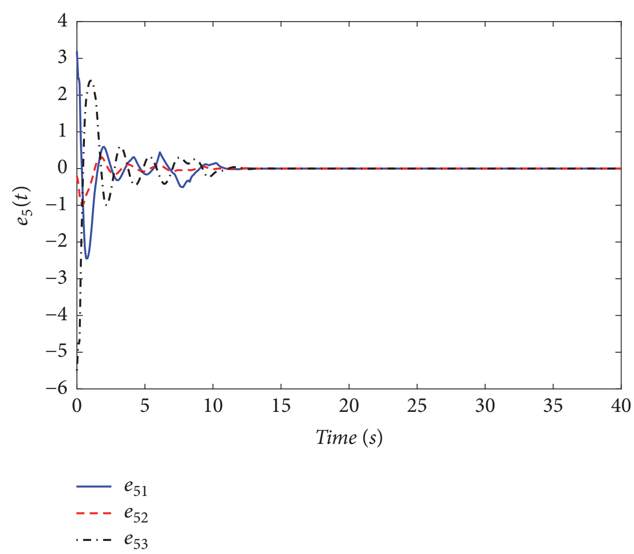

Figure 5: The error evolution of the fifth node.

can see, the synchronization results of complex dynamical networks have been applied to automatic control, secure communication, and other fields. For better understanding the principles and strategies of synchronization phenomena, in this paper, we investigate finite-time synchronization problems of complex dynamical networks with different number of nodes, which contains unknown periodically coupling structures and bounded time-varying delay. Based on finite-time stability theory and Lyapunov-Krasovskii method, some sufficient finite-time synchronization criteria for complex dynamical network with unknown periodical couplings have been presented.

Also some illustrative examples with their numerical simulations based on the different numbers of nodes are provided to demonstrate the effectiveness and feasibility of the proposed synchronization methods. In the simulations, the authors take heterogeneity and periodical connections of the dynamical nodes into account, which is much more correspondence with the actual complex network system greatly, also which shows that finite-time control techniques have better robustness and disturbance rejection properties. In the future, the author will take factors such as switching, uncertainties, and noise disturbance into account, to study the systems' finite-time synchronization.

Data Availability

The Matlab based models used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

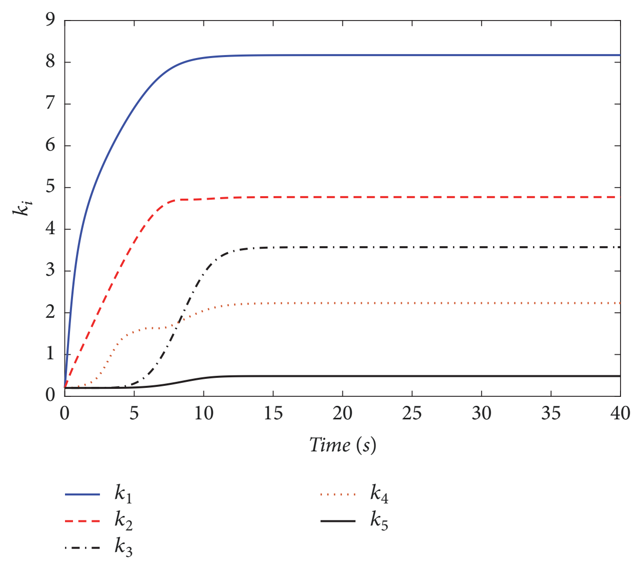

FIGURE 6: The adaptive update of feedback gains k_i, $i = 1, 2, 3, 4, 5$.

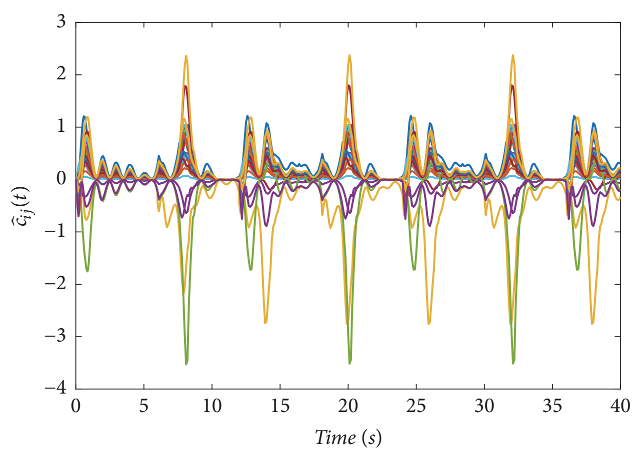

FIGURE 7: The adaptive undate of feedback gains $\hat{c}_{ij}(t)$, $i, j = 1, 2, 3, 4, 5$.

Authors' Contributions

Lihong Yan and Junmin Li formulated the problem and solved the problem. Lihong Yan computed and analyzed the results. All the authors equally contributed to writing of the paper.

Acknowledgments

The work is supported by the National Natural Science Foundation of China under Grants nos. 61573013 and 11705122, the Natural Science Funds of Science and Technology Department of Shaanxi Province under Grant no. 2014JM2-1002, the Nature Scientific Research Project of Shaanxi Education Department under Grant no. 18JK0829, and Natural Science Foundation of Xianyang Normal University under Grant no. 14XSYK004.

References

[1] W. Liu and Z. C. Yang, "Modeling the propagation of mobile metalware on complex networks," *Communications in Nonlinear Science and Numerical Simulation*, vol. 37, pp. 249–264, 2016.

[2] S. Green, M. Şerban, R. Scholl, N. Jones, I. Brigandt, and W. Bechtel, "Network analyses in systems biology: new strategies for dealing with biological complexity," *Synthese*, vol. 195, no. 4, pp. 1751–1777, 2018.

[3] A. Ghaffari and S. Arebi, "Pinning control for synchronization of nonlinear complex dynamical network with suboptimal SDRE controllers," *Nonlinear Dynamics*, vol. 83, no. 1-2, pp. 1003–1013, 2016.

[4] P. Selvaraj, R. Sakthivel, and C. K. Ahn, "Observer-based synchronization of complex dynamical networks under actuator saturation and probabilistic faults," *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, vol. 289, pp. 1–11, 2018.

[5] P. He, "Pinning control and adaptive control for synchronization of linearly coupled reaction-diffusion neural networks with mixed delays," *International Journal of Adaptive Control and Signal Processing*, vol. 32, no. 8, pp. 1103–1123, 2018.

[6] L. Yan, "Outer average synchronization between two coupled networks with different numbers of nodes," *IMA Journal of Mathematical Control and Information*, 2018, https://doi.org/10.1093/imamci/dny023.

[7] J.-L. Wang, H.-N. Wu, and T. Huang, "Passivity-based synchronization of a class of complex dynamical networks with time-varying delay," *Automatica*, vol. 56, pp. 105–112, 2015.

[8] X. Wang, G. Jiang, and X. Wu, "State Estimation for General Complex Dynamical Networks with Incompletely Measured Information," *Entropy*, vol. 20, no. 1, p. 5, 2018.

[9] L. Li, Z. Tu, J. Mei, and J. Jian, "Finite-time synchronization of complex delayed networks via intermittent control with multiple switched periods," *Nonlinear Dynamics*, vol. 85, no. 1, pp. 375–388, 2016.

[10] M. Kalpana, P. Balasubramaniam, and K. Ratnavelu, "Direct delay decomposition approach to synchronization of chaotic fuzzy cellular neural networks with discrete, unbounded distributed delays and Markovian jumping parameters," *Applied Mathematics and Computation*, vol. 254, pp. 291–304, 2015.

[11] T. Jing, F. Chen, and X. Zhang, "Finite-time lag synchronization of time-varying delayed complex networks via periodically intermittent control and sliding mode control," *Neurocomputing*, vol. 199, pp. 178–184, 2016.

[12] P. He and Y. Li, "Optimal guaranteed cost synchronization of coupled neural networks with Markovian jump and mode-dependent mixed time-delay," *Optimal Control Applications and Methods*, vol. 37, no. 5, pp. 922–947, 2016.

[13] X. Guo and J. Li, "A new synchronization algorithm for delayed complex dynamical networks via adaptive control approach," *Communications in Nonlinear Science and Numerical Simulation*, vol. 17, no. 11, pp. 4395–4403, 2012.

[14] X. Hao and J. Li, "Stochastic synchronization for complex dynamical networks with time-varying couplings," *Nonlinear Dynamics*, vol. 80, no. 3, pp. 1357–1363, 2015.

[15] C. Xu, X. Yang, J. Lu, J. Feng, F. E. Alsaadi, and T. Hayat, "Finite-Time Synchronization of Networks via Quantized Intermittent Pinning Control," *IEEE Transactions on Cybernetics*, vol. 48, no. 10, pp. 3021–3027, 2018.

[16] C. Chen, L. Li, and S. Ren, "Finite-time synchronization of multi-weighted complex dynamical networks with and without coupling delay," *Neurocomputing*, vol. 275, pp. 1250–1260, 2018.

[17] B. Liu, X. Wang, H. Su, H. Zhou, Y. Shi, and R. Li, "Adaptive synchronization of complex dynamical networks with time-varying delays," *Circuits, Systems and Signal Processing*, vol. 33, no. 4, pp. 1173–1188, 2014.

[18] X. Liu, D. W. Ho, Q. Song, and W. Xu, "Finite/Fixed-Time Pinning Synchronization of Complex Networks With Stochastic Disturbances," *IEEE Transactions on Cybernetics*, vol. 99, pp. 1–6, 2018.

[19] W. Lu and T. Chen, "Synchronization of networks with time-varying couplings," *Applied Mathematics-A Journal of Chinese Universities*, vol. 28, no. 4, pp. 438–454, 2013.

[20] J. Mei, M. Jiang, and J. Wang, "Finite-time structure identification and synchronization of drive-response systems with uncertain parameter," *Communications in Nonlinear Science and Numerical Simulation*, vol. 18, no. 4, pp. 999–1015, 2013.

[21] H. Zhao, L. Li, H. Peng, J. Xiao, Y. Yang, and M. Zheng, "Finite-time topology identification and stochastic synchronization of complex network with multiple time delays," *Neurocomputing*, vol. 219, pp. 39–49, 2017.

[22] Q. Xie, G. Si, Y. Zhang, Y. Yuan, and R. Yao, "Finite-time synchronization and identification of complex delayed networks with Markovian jumping parameters and stochastic perturbations," *Chaos, Solitons & Fractals*, vol. 86, pp. 35–49, 2016.

[23] S.-H. Qiu, Y.-L. Huang, and S.-Y. Ren, "Finite-time synchronization of multi-weighted complex dynamical networks with and without coupling delay," *Neurocomputing*, vol. 275, pp. 1250–1260, 2018.

[24] J. Zhao, D. J. Hill, and J. Zhao, "Synchronization of dynamical networks with nonidentical nodes: Criteria and control," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 58, no. 3, pp. 584–594, 2011.

[25] Y. H. Wang, Y. Q. Fan, Q. Y. Wang, and Y. Zhang, "Stabilization and synchronization of complex dynamical networks with different dynamics of nodes via decentralized controllers," *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 59, no. 8, pp. 1786–1795, 2012.

[26] Y.-Q. Fan, Y.-H. Wang, Y. Zhang, and Q.-R. Wang, "The synchronization of complex dynamical networks with similar nodes and coupling time-delay," *Applied Mathematics and Computation*, vol. 219, no. 12, pp. 6719–6728, 2013.

[27] H. Dai, G. Si, and Y. Zhang, "Adaptive generalized function matrix projective lag synchronization of uncertain complex dynamical networks with different dimensions," *Nonlinear Dynamics*, vol. 74, no. 3, pp. 629–648, 2013.

[28] M. Tan and W. Tian, "Finite-time stabilization and synchronization of complex dynamical networks with nonidentical nodes of different dimensions," *Nonlinear Dynamics*, vol. 79, no. 1, pp. 731–741, 2015.