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Research Article

Robust Adaptive Control for Coordinated Constrained Multiple Flexible Joint Manipulators with Hysteresis Loop

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This paper focuses on the position/force tracking control problem for constrained multiple flexible joint manipulators system with nonlinear input of hysteresis loop. Firstly, the dynamic model is given in the task space and the input of hysteresis loop model is approximated by a differential equation. Secondly, considering the disturbance with unknown bounds, a robust adaptive control strategy based on the sliding mode which consists of constraint force error and position error is designed. The proposed approach can not only compensate the model error, external disturbance, and flexible parameter uncertainties, but also drive the closed-loop system variables to reach the sliding model surface. Then it can be proved that both position and constraint force errors can be guaranteed to converge to zero. Finally, the simulation results can verify the effectiveness of the proposed method.

1. Introduction

The position and force control of the multiple manipulators system have been widely concerned in recent years [1, 2]. Although the multiple manipulators system, adapted to the complex industrial environment, has great advantages for grasping the heavier object, it also has many complex control problems such as parametric uncertainty and gear clearance.

For the position/force control problem, many control strategies have been proposed for single manipulator system. A sliding mode variable structure control was designed to drive the closed-loop system states to reach the adopted sliding model surface in [3]. To deal with the force tracking problem, a new adaptive sliding mode control based on a sliding mode consisting of force error was proposed in [4], in which the controller can guarantee the asymptotic convergence of the position and force errors. To discuss the time delay, an adaptive position/force control based on the backstepping for flexible joint manipulator with time delay was investigated in [5]. Many robot control strategies have been proposed for the weak flexible joint robots system [6, 7]. A singular perturbation (SP) analysis was proposed to guarantee the stability of system in [8], but it cannot ensure the stability for the strong flexible joint manipulators system. Based on the backstepping method, a robust adaptive control strategy was proposed for the unknown joint stiffness and unknown motor inertia in [9]. A new method for flexible joint robots with model uncertainties in both robot dynamics and actuator dynamics was presented in [10], in which the uniformly ultimate boundedness in a closed-loop adaptive system was proved. Considering the position and force control issues of constrained flexible joint robots with parametric uncertainty, a new adaptive and sliding mode control scheme was developed in [11]. In order to compensate the uncertainty caused by the model error, robust control, adaptive control, and neural network control were adopted in [12–14]. But only the single flexible joint manipulator system was considered in the above researches, while the multimanipulators system with object was not considered.

For the multiple manipulators system, the main control approaches can be classified into two directions: the master-slave control [15] and the hybrid position/force control [16]. Compared with the master-slave control, the task space can be decomposed into two orthogonal subspaces, where the position of the object was controlled in a certain direction of the workspace and the force was controlled in the other direction. A decentralized adaptive fuzzy control scheme was presented in paper [17], and position error and internal forces

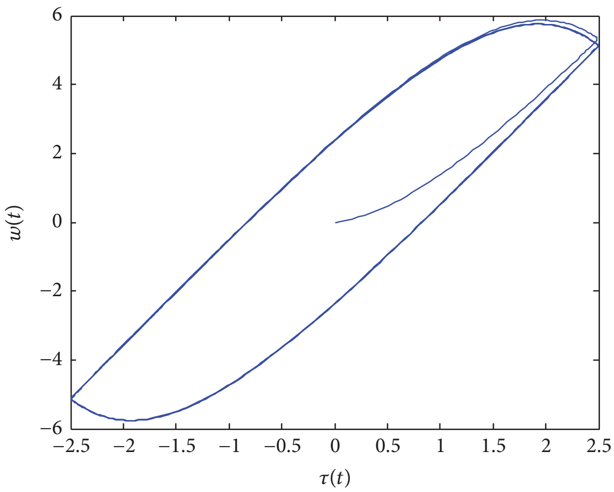

FIGURE 1: Nonlinear hysteresis loop.

error can be can be guaranteed to converge zero. An adaptive position/force control of coordinated multiple manipulators based on a new sliding mode was presented in the paper [18]. Considering the time delay, a robust adaptive control strategy based on a sliding mode is designed for the motion synchronization of multiple flexible joint manipulators with time delay in [1].

Due to the running for a long time in the actual system, the gear clearance of joint manipulators will occur which will influence system tracking performance. The performance of the gear clearance can be described by a hysteresis loop as in Figure 1.

It is well known that the stability of the system cannot be guaranteed and the system will have a poor tracing performance as a result of the existence of gear clearance.

Considering the existence of gear clearance in the multiple manipulators system, we proposed a robust adaptive control strategy based on a sliding mode to overcome the gear clearance. The performance of the gear clearance can be described by a hysteresis loop whose model is approximated by a differential equation. Then, the dynamic model including a hysteresis loop is obtained. It is proved that the controller can not only guarantee the stability of system, but also guarantee that the position and constraint force errors asymptotically converge to zero. Finally, the simulation results illustrate the validity and the feasibility of the proposed method in this paper.

2. Dynamical Model

Consider γ flexible joint manipulators holding a rigid object moving along m constraint surface. To facilitate the study, the following assumptions are made.

Assumption 1. The number of degrees of freedom and the number of joints are equal for each manipulator, the objects are rigid, and the joints are flexible.

Assumption 2. The system for each manipulator is nonredundant.

Assumption 3. All transformation matrices are of full rank.

2.1. Dynamic Modeling of Multiple Flexible Joint Manipulators. The dynamic equations of flexible joint manipulators with the disturbance can be described as follows [19]:

$$\begin{aligned} &D_i(q_i)\ddot{q}_i + C_i(q_i,\dot{q}_i)\dot{q}_i + G_i(q_i) + J_{ei}^{\mathrm{T}}(q_i)F_{ei} + f_i(t) \\ &\quad = w_i, \quad i = 1, 2, \ldots, \gamma, \end{aligned} \tag{1}$$

$$w_i(t) = c_i\tau_i(t) + \delta_i(\tau_i), \tag{2}$$

$$J_{mi}\ddot{q}_{mi} + \tau_i = u_i, \tag{3}$$

$$\tau_i = K_{mi}(q_{mi} - q_i), \tag{4}$$

where $i = 1, 2, \ldots, m$ is the manipulator number, $q_i, \dot{q}_i, \ddot{q}_i \in R^n$ are the joint position, joint velocity, and joint acceleration of the ith manipulator, $D_i \in R^{n\times n}$ is the inertia matrix, $C_i\dot{q}_i \in R^{n\times n}$ are the centrifugal and Coriolis forces, $G_i(q_i) \in R^n$ is the gravity matrix, $J_{ei} \in R^{n\times n}$ is the Jacobian matrix from the end-effector to joint space, $F_{ei} \in R^n$ is the force exerted on the object by the ith end-effector, $w_i \in R^n$ is the input control torque for ith manipulator, $q_{mi} \in R^n$ is the motor shafts angle, $J_{mi} \in R^{n\times n}$ is the symmetric positive definite inertia matrix of the actuator, $K_{mi} \in R^{n\times n}$ is the joint stiffness matrix, w_i is the input torque of the manipulator, τ_i are the transmission torques, and u_i is the vector of the motor torques.

Utilizing block diagonal metrics, then we have the dynamic equation as follows:

$$\begin{aligned} &D(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) + J_e^{\mathrm{T}}F_e + f(t) = w, \\ &w(t) = c\tau(t) + \delta(\tau), \\ &J_m\ddot{q}_m + \tau = u, \end{aligned} \tag{5}$$

where

$$\begin{aligned} &q = \left[q_1^{\mathrm{T}}, \ldots, q_i^{\mathrm{T}}, \ldots, q_\gamma^{\mathrm{T}}\right] \in R^{n\gamma}, \\ &D(q) = \operatorname{blockdiag}\left[D_1(q_1), \ldots, D_i(q_i), \ldots, D_\gamma(q_\gamma)\right] \\ &\quad \in R^{n\gamma\times n\gamma}, \\ &C(q,\dot{q}) = \operatorname{blockdiag}\left[C_1(q_1,\dot{q}_1), \ldots, C_i(q_i,\dot{q}_i), \ldots,\right. \\ &\quad \left. C_\gamma(q_\gamma,\dot{q}_\gamma)\right] \in R^{n\gamma\times n\gamma}, \\ &G(q) = \left[G_1(q_1), \ldots, G_i(q_i), \ldots, G_\gamma(q_\gamma)\right] \in R^{n\gamma}, \\ &w = \left[w_1, \ldots, w_i, \ldots, w_\gamma\right] \in R^{n\gamma}, \\ &F_e = \left[F_{e1}^{\mathrm{T}}, \ldots, F_{ei}^{\mathrm{T}}, \ldots, F_{e\gamma}^{\mathrm{T}}\right] \in R^{n\gamma}, \\ &J_e = \operatorname{blockdiag}\left[J_{e1}, \ldots, J_{ei}, \ldots, J_{e\gamma}\right] \in R^{n\gamma\times n\gamma}. \end{aligned} \tag{6}$$

2.2. The Dynamic Model of the Object. The dynamic model of the object can be obtained by the Newton-Euler method:

$$D_o(x)\ddot{x} + C_o(x,\dot{x})\dot{x} + G_o(x) = F_o + F_c, \tag{7}$$

where $x = [x_1{}^{\mathrm{T}}, x_2{}^{\mathrm{T}}, \ldots, x_n{}^{\mathrm{T}}]^{\mathrm{T}} \in R^n$ is the position of object, $D_o(x) \in R^{n\times n}$ is the inertia matrix, $C_o(x,\dot{x})\dot{x} \in R^{n\times n}$ are the centrifugal and Coriolis forces, $G_o(x) \in R^n$ is the gravity matrix, $J_{oi}(x)$ is the Jacobian matrix from end-effectors i to the center of mass of object, $F_{ei} \in R^n$ is the force exerted on the object by ith end-effector, $F_o \in R^n$ is the external force which contributes to the motion of object, and $F_{ei} \in R^n$ is the constraint force exerted on the object by ith end-effector.

Based on Assumptions 1–3, the environmental constraint can be expressed as

$$\Phi(x) = 0, \tag{8}$$

where $n > m$, constraint surface is holonomic and frictionless, and $\Phi(x) \in R^m$ is twice continuously differentiable.

Differentiating (8) with respect to time:

$$J_c(x)\dot{x} = 0, \tag{9}$$

where $J_c(x) = \partial\Phi(x)/\partial x \in R^{m\times n}$ donate the Jacobian matrix from the object coordinates to the constraint surface contact coordinates.

Then the constraint force of the object can be expressed as follows:

$$F_c = J_c^{\mathrm{T}}(x)\lambda, \tag{10}$$

where $\lambda \in R^m$ donate the generalized Lagrange multiplexer.

The external force and the internal force are defined in [17]; then F_e can be decomposed into the external force $F_o \in R^{\gamma n}$ and the internal force $F_I \in R^{\gamma n}$:

$$F_e = \left(J_o^{\mathrm{T}}\right)^+ \left[D_o(x)\ddot{x} + C_o(x,\dot{x})\dot{x} + G_o(x) - F_c\right] + F_I, \tag{11}$$

where F_I does not affect the position of the object motion, and it can be cancel out by J_o^{T}, where $J_o^{\mathrm{T}}F_I = 0$.

2.3. Dynamic Modeling of Coordinated Multiple Manipulators. The position of end-effectors can be expressed as

$$x_e = \varphi(q), \tag{12}$$

where $\varphi(q)$ is the transformation matrix from end-effectors to joint space.

Differentiating (12) with respect to time yields we have

$$\dot{x}_e = J_e(q)\dot{q}, \tag{13}$$

where

$$J_e(q) = \frac{\partial\varphi(q)}{\partial q} \in R^{\gamma n\times\gamma n}. \tag{14}$$

Then the velocity of end-effectors can be expressed by joint space variable and task space variable, respectively:

$$\dot{x}_e = J_e\dot{q}, \tag{15}$$

$$\dot{x}_e = J_o\dot{x}. \tag{16}$$

From (15) and (16) and the fact that J_e is invertible matrix, then the joint velocity and joint acceleration of the ith manipulator can be described by

$$\begin{aligned}\dot{q} &= J_e{}^{-1}J_o\dot{x},\\ \ddot{q} &= J_e^{-1}J_o\ddot{x} + \frac{d}{dt}\left(J_e^{-1}J_o\right)\dot{x}\\ &= J_e^{-1}J_o\ddot{x} + \left(J_e^{-1}\dot{J}_o - J_e^{-1}\dot{J}_eJ_e^{-1}J_o\right)\dot{x}.\end{aligned} \tag{17}$$

Substituting (17) into subsystem (1), then the dynamic model can be expressed as follows:

$$\begin{aligned}&\overline{D}(x)\ddot{x} + \overline{C}(x,\dot{x})\dot{x} + \overline{G}(x) + J_e{}^{\mathrm{T}}F_I + f\\ &\quad = w + J_e^{\mathrm{T}}\left(J_o^{\mathrm{T}}\right)^+ J_c^{\mathrm{T}}\lambda,\end{aligned} \tag{18}$$

where

$$\begin{aligned}\overline{D} &= DJ_e{}^{-1}J_o + J_e{}^{\mathrm{T}}\left(J_o{}^{\mathrm{T}}\right)^+ D_o,\\ \overline{C} &= D\left(J_e^{-1}\dot{J}_o - J_e^{-1}\dot{J}_eJ_e^{-1}J_o\right) + CJ_e{}^{-1}J_o\\ &\quad + J_e{}^{\mathrm{T}}\left(J_o{}^{\mathrm{T}}\right)^+ C_o,\\ \overline{G} &= G + J_e{}^{\mathrm{T}}\left(J_o{}^{\mathrm{T}}\right)^+ G_o.\end{aligned} \tag{19}$$

Multiplying (18) by $J_o{}^{\mathrm{T}}J_e{}^{-\mathrm{T}}$, we can have

$$\begin{aligned}&D_a(x)\ddot{x} + C_a(x,\dot{x})\dot{x} + G_a(x) + J_o^{\mathrm{T}}J_e^{-\mathrm{T}}f(t)\\ &\quad = w_a + F_c,\end{aligned} \tag{20}$$

where

$$\begin{aligned}D_a &= J_o{}^{\mathrm{T}}J_e{}^{-\mathrm{T}}DJ_e{}^{-1}J_o + D_o,\\ C_a &= J_o{}^{\mathrm{T}}J_e{}^{-\mathrm{T}}DJ_e{}^{-1}\left(\dot{J}_o - \dot{J}_eJ_e{}^{-1}J_o\right) + J_o{}^{\mathrm{T}}J_e{}^{-\mathrm{T}}CJ_e{}^{-1}J_o\\ &\quad + C_o,\\ G_a &= J_o{}^{\mathrm{T}}J_e{}^{-\mathrm{T}}G + G_o,\\ w_a &= J_o{}^{\mathrm{T}}J_e{}^{-\mathrm{T}}w.\end{aligned} \tag{21}$$

In a similar way, utilizing block diagonal metrics, the flexible joint dynamic system can be expressed as

$$\overline{J}_m\ddot{\tau} + h_m = u, \tag{22}$$

where $\overline{J}_m = J_mK_m^{-1}$, $h_m = J_m\ddot{q} + \tau$.

Consider the m constraints surface, the dimension of the object will be reduced to $n-m$; then the object position can be decomposed into $x = [x_1^{\mathrm{T}}\ \ x_2^{\mathrm{T}}]^{\mathrm{T}}$, where $x_1 \in R^{n-m}$, $x_2 \in R^m$, and there exists a function $\Omega(\cdot)$, where x_2 can be expressed as

$$x_2 = \Omega(x_1). \tag{23}$$

Defining

$$E_1 = \left[I_{n-m} \quad 0\right],$$
$$H = \begin{bmatrix} I_{n-m} & 0 \\ \dfrac{\partial\Omega(x_1)}{\partial x_1} & I_m \end{bmatrix}, \tag{24}$$

where $E_1 \in R^{(n-m)\times n}$, $H \in R^{n\times n}$, then position velocity and acceleration of the object can be described as

$$x = \left[{x_1}^{\mathrm{T}}, \Omega^{\mathrm{T}}(x_1)\right]^{\mathrm{T}}, \tag{25}$$

$$\dot{x} = HE_1^{\mathrm{T}}\dot{x}_1, \tag{26}$$

$$\ddot{x} = HE_1^{\mathrm{T}}\ddot{x}_1 + \dot{H}E_1^{\mathrm{T}}\dot{x}_1. \tag{27}$$

Substituting (26) and (27) into (20), then we can have

$$\begin{aligned} &D_aHE_1^{\mathrm{T}}\ddot{x}_1 + \left(D_a\dot{H}E_1^{\mathrm{T}} + C_aHE_1^{\mathrm{T}}\right)\dot{x}_1 + G_a \\ &\quad = w_a + J_c\lambda. \end{aligned} \tag{28}$$

Multiplying (28) by H^{T}, we can have

$$\begin{aligned} &H^{\mathrm{T}}D_aHE_1^{\mathrm{T}}\ddot{x}_1 + H^{\mathrm{T}}\left(D_a\dot{H} + C_aH\right)E_1^{\mathrm{T}}\dot{x}_1 + H^{\mathrm{T}}G_a \\ &\quad = H^{\mathrm{T}}w_a + H^{\mathrm{T}}J_c\lambda. \end{aligned} \tag{29}$$

Consider the problems of the nonlinear input of hysteresis loop; the model can be expressed as [16]

$$w(t) = c\tau(t) + \delta(\tau), \tag{30}$$

where

$$\begin{gathered} \delta(\tau) = \begin{cases} -\dfrac{c-b}{a} + C_1e^{-a\tau}, & \dot{\tau} > 0 \\ \dfrac{c-b}{a} + C_2e^{a\tau}, & \dot{\tau} < 0, \end{cases} \\ \left\|\delta_1(\tau), \delta_2(\tau)\right\|^{\mathrm{T}} = \|\delta(\tau)\| \le \overline{\delta}, \end{gathered} \tag{31}$$

where $\overline{\delta}$ donate the positive.

The dynamic equations have the following properties.

Property 4. D_i and $\overline{D}_i$ are symmetric and positive definite matrices.

Property 5. $J_cHE_1^{\mathrm{T}} = E_1H^{\mathrm{T}}{J_c}^{\mathrm{T}} = 0$.

Some basic knowledge should be noted.

Lemma 6 (see [17]). *If* $x : [0,\infty) \to R$ *is square integrable, that is,* $\lim_{t\to\infty}\int_0^t x^2(\tau)d\tau < \infty$, *and if* $\dot{x}(t)$, $t \in [0,\infty)$, *exists and is bounded, then* $\lim_{t\to\infty}x(t) = 0$.

3. Controller Design

In this section, an adaptive controller is designed based on neural network by using sliding mode which consists of position error and force error for the multiple flexible joint manipulators.

Consider the model parameter uncertainties; then we can have $\overline{D}_l = \overline{D}_{l0} + \Delta\overline{D}_l$, $\overline{C}_l = \overline{C}_{l0} + \Delta\overline{C}_l$, $\overline{G}_l = \overline{G}_{l0} + \Delta\overline{G}_l$, where $\overline{D}_{l0}$, $\overline{C}_{l0}$, $\overline{G}_{l0}$ donate the nominal part of the model; ΔD_l, ΔC_l, ΔG_l donate the norm-bounded uncertainties of the model; the dynamic equations can be expressed as

$$\overline{D}_{l0}\ddot{x}_1 + \overline{C}_{l0}\dot{x}_1 + \overline{G}_{l0} = w - J_e^{\mathrm{T}}F_I + J_e^{\mathrm{T}}\left(J_o^{\mathrm{T}}\right)^+ J_c^{\mathrm{T}}\lambda + \rho, \tag{32}$$

$$\begin{aligned} &D_{a0}HE_1^{\mathrm{T}}\ddot{x}_1 + \left(D_{a0}\dot{H} + C_{a0}H\right)E_1^{\mathrm{T}}\dot{x}_1 + G_{a0} \\ &\quad = w_a + J_c\lambda + \rho_a, \end{aligned} \tag{33}$$

where $\rho = -\Delta\overline{D}_{l0}\ddot{x}_1 - \Delta\overline{C}_{l0}\dot{x}_1 - \Delta\overline{G}_{l0} - J_o^{\mathrm{T}}J_e^{-\mathrm{T}}f$, $\rho_a = J_o^{\mathrm{T}}J_e^{-\mathrm{T}}\rho$.

Multiplying (33) by H^{T}, we can have

$$\begin{aligned} &H^{\mathrm{T}}D_{a0}HE_1^{\mathrm{T}}\ddot{x}_1 + H^{\mathrm{T}}\left(D_{a0}\dot{H} + C_{a0}H\right)E_1^{\mathrm{T}}\dot{x}_1 + H^{\mathrm{T}}G_{a0} \\ &\quad = H^{\mathrm{T}}w_a + H^{\mathrm{T}}J_c\lambda + H^{\mathrm{T}}\rho_a. \end{aligned} \tag{34}$$

By the assumption we can have $\|H\|\|\rho_a\| \le a_0 + a_1\|x\| + a_2\|\dot{x}\|^2$.

Multiplying (34) by E_1, we can have

$$\begin{aligned} &D_{al0}\ddot{x}_1 + C_{al0}\dot{x}_1 + G_{al0} \\ &\quad = E_1H^{\mathrm{T}}w_a + E_1H^{\mathrm{T}}J_c\lambda + E_1H^{\mathrm{T}}\rho_a, \end{aligned} \tag{35}$$

where $D_{al0} = E_1H^{\mathrm{T}}D_{a0}HE_1^{\mathrm{T}}$, $C_{al0} = E_1H^{\mathrm{T}}(D_{a0}\dot{H}+C_{a0}H)E_1^{\mathrm{T}}$, $G_{al0} = E_1H^{\mathrm{T}}G_{a0}$.

The desired position trajectory, desired constraint force multiplier, and desired internal force are denoted as x_d, F_{cd}, F_{Id}. The objective of the controller is to make the position errors and force errors converge to zero, and the internal force error can also be bounded.

Considering the influence of flexible joint and input of the hysteresis loop, the controller design is divided into three steps. First, a desired input torque of the manipulator w_d is designed to make the position errors and force errors converge to zero. Second, a desired transmission torque τ_d is designed to achieve the boundedness of input torque errors. Third, the motor torque u is obtained to make the transmission torque track the desired expected values. Therefore, multimanipulator system stability can be guaranteed.

The object position error, generalized force multiplier error, internal force error, and control error are defined as follows:

$$\begin{gathered} e = x_1 - x_{1d}, \\ e_{F_I} = F_I - F_{Id}, \\ e_\lambda = \lambda - \lambda_d, \\ e_w = w - w_d, \\ e_\tau = \tau - \tau_d. \end{gathered} \tag{36}$$

The auxiliary variable is designed as

$$\begin{gathered} v = E_1^{\mathrm{T}}\left(\dot{x}_{1d} - \Lambda_1 e\right) - J_c^{\mathrm{T}}s_2, \\ v_\tau = \tau_d - \Lambda_\tau e_\tau. \end{gathered} \tag{37}$$

Then the sliding mode is designed as

$$s = E_1^{\mathrm{T}} \dot{x}_1 - \nu = E_1^{\mathrm{T}} s_1 + J_c^{\mathrm{T}} s_2, \quad s_\tau = \dot{e}_\tau + \Lambda_\tau e_\tau, \tag{38}$$

where $s_1 = e + \Lambda_1 \dot{e}$, $s_2 = e_\lambda + \Lambda_2 \dot{e}_\lambda$, $\Lambda_1, \Lambda_2, \Lambda_\tau$ donate the positive definite diagonal matrix.

Then the desired input torque of the manipulator, desired transmission torque, and the motor torques are proposed as follows:

$$\begin{aligned} H^{\mathrm{T}} J_o^{\mathrm{T}} J_e^{-\mathrm{T}} w_d &= -H^{\mathrm{T}} J_c^{\mathrm{T}} \lambda - H^{\mathrm{T}} \widehat{\rho} + H^{\mathrm{T}} G_{a0} \\ &\quad + H^{\mathrm{T}} \left(D_{a0} \dot{H} + C_{a0} H \right) E_1^{\mathrm{T}} \dot{x}_1 \\ &\quad + H^{\mathrm{T}} J_o^{\mathrm{T}} \left(F_{Id} + K_{FI} e_{FI} \right) \\ &\quad - H^{\mathrm{T}} J_o^{\mathrm{T}} J_e^{-\mathrm{T}} K J_e^{-1} J_o D_{a0}^{-\mathrm{T}} H^{-\mathrm{T}} s \\ &\quad + \left(H^{\mathrm{T}} D_{a0} H \right) \left(E_1^{\mathrm{T}} \ddot{x}_{1d} - E_1^{\mathrm{T}} \Lambda_1 \dot{e} - J_c^{\mathrm{T}} \dot{s}_2 - \dot{J}_c^{\mathrm{T}} s_2 \right), \end{aligned} \tag{39}$$

$$\tau_d = \frac{1}{c} w_d - \Gamma \operatorname{sgn} \left(J_e^{-1} J_o D_{a0}^{-\mathrm{T}} H^{-\mathrm{T}} s \right), \tag{40}$$

$$u = \widehat{\overline{J}}_m \dot{v}_\tau + \widehat{h}_m - K_\tau s_\tau - P_\tau \operatorname{sgn}(s_\tau), \tag{41}$$

where K, K_{FI}, Γ, K_τ donate the positive definite diagonal matrix, and adaptive law is given by

$$\begin{aligned} \widehat{\rho} &= \left(\widehat{a}_0 + \widehat{a}_1 \|x\| + \widehat{a}_2 \|\dot{x}\|^2 \right) \operatorname{sgn} \left[H \left(H^{\mathrm{T}} D_{a0} H \right)^{-\mathrm{T}} s \right], \\ \dot{\widehat{a}}_0 &= \kappa \left\| \left(H^{\mathrm{T}} D_{a0} H \right)^{-1} H^{\mathrm{T}} \right\| \|s\|, \\ \dot{\widehat{a}}_1 &= \kappa \left\| \left(H^{\mathrm{T}} D_{a0} H \right)^{-1} H^{\mathrm{T}} \right\| \|x\| \|s\|, \\ \dot{\widehat{a}}_2 &= \kappa \left\| \left(H^{\mathrm{T}} D_{a0} H \right)^{-1} H^{\mathrm{T}} \right\| \|\dot{x}\|^2 \|s\|. \end{aligned} \tag{42}$$

$P_\tau = \delta_{\overline{J}_m} \|\dot{v}_\tau\| + \|\widehat{h}_m - h_m\|$ and $\delta_{\overline{J}_m}$ donate the error upper bound for the $(\widehat{\overline{J}}_m - \overline{J}_m)$.

From the proposed control scheme, we have the following theorem.

Theorem 7. *Consider the multiple flexible joint robotic manipulator system with input of the hysteresis loop described by (34), (30), and (22), based on Assumptions 1–3, the asymptotic convergence of object position errors and constraint force errors will be ensured by controls (39), (40), and (41) with the adaptive law (42), and internal force errors can be bounded.*

Proof. Consider the following Lyapunov function:

$$\begin{aligned} V &= \frac{1}{2} s^{\mathrm{T}} s + \frac{1}{2} s_\tau^{\mathrm{T}} \overline{J}_m s_\tau + e_\tau^{\mathrm{T}} \Lambda_\tau^{\mathrm{T}} K_\tau e_\tau + \frac{1}{2} \kappa^{-1} \widetilde{a}_0^2 \\ &\quad + \frac{1}{2} \kappa^{-1} \widetilde{a}_1^2 + \frac{1}{2} \kappa^{-1} \widetilde{a}_2^2. \end{aligned} \tag{43}$$

Differentiating (43) with respect to time t,

$$\begin{aligned} \dot{V} &= s^{\mathrm{T}} \dot{s} + \kappa^{-1} \widetilde{a}_0 \dot{\widetilde{a}}_0 + \kappa^{-1} \widetilde{a}_1 \dot{\widetilde{a}}_1 + \kappa^{-1} \widetilde{a}_2 \dot{\widetilde{a}}_2 + s_\tau^{\mathrm{T}} \overline{J}_m \dot{s}_\tau \\ &\quad + 2 e_\tau^{\mathrm{T}} \Lambda_\tau^{\mathrm{T}} K_\tau \dot{e}_\tau. \end{aligned} \tag{44}$$

Substituting (34) into (38), we have

$$\begin{aligned} \dot{s} &= E_1^{\mathrm{T}} \left(\ddot{e} + \Lambda_1 \dot{e} \right) + J_c^{\mathrm{T}} \dot{s}_2 + \dot{J}_c^{\mathrm{T}} s_2 \\ &= E_1^{\mathrm{T}} \left(\ddot{x}_1 - \ddot{x}_{1d} + \Lambda_1 \dot{e} \right) + J_c^{\mathrm{T}} \dot{s}_2 + \dot{J}_c^{\mathrm{T}} s_2 \\ &= E_1^{\mathrm{T}} \left(-\ddot{x}_{1d} + \Lambda_1 \dot{e} \right) + J_c^{\mathrm{T}} \dot{s}_2 + \dot{J}_c^{\mathrm{T}} s_2 + \left(D_{a0} H \right)^{-1} \\ &\quad \cdot J_o^{\mathrm{T}} J_e^{-\mathrm{T}} w + \left(D_{a0} H \right)^{-1} \\ &\quad \cdot \left(J_c^{\mathrm{T}} \lambda + \rho - G_{a0} - \left(D_{a0} \dot{H} + C_{a0} H \right) E_1^{\mathrm{T}} \dot{x}_1 \right). \end{aligned} \tag{45}$$

Substituting the desired input torque of the manipulator (39) into (45), we have

$$\begin{aligned} \dot{s} &= -H^{-1} D_{a0}^{-1} J_o^{\mathrm{T}} J_e^{-\mathrm{T}} K J_e^{-1} J_o D_{a0}^{-\mathrm{T}} H^{-\mathrm{T}} s \\ &\quad + \left(H^{\mathrm{T}} D_{a0} H \right)^{-1} \left(H^{\mathrm{T}} J_o^{\mathrm{T}} J_e^{-\mathrm{T}} e_w + H^{\mathrm{T}} \rho - H^{\mathrm{T}} \widehat{\rho} \right). \end{aligned} \tag{46}$$

Substituting the adaptive law (42) and (46) into (44), the following can be obtained:

$$\begin{aligned} \dot{V} &= \kappa^{-1} \widetilde{a}_0 \dot{\widetilde{a}}_0 + \kappa^{-1} \widetilde{a}_1 \dot{\widetilde{a}}_1 + \kappa^{-1} \widetilde{a}_2 \dot{\widetilde{a}}_2 + 2 e_\tau^{\mathrm{T}} \Lambda_\tau^{\mathrm{T}} K_\tau \dot{e}_\tau \\ &\quad + s^{\mathrm{T}} \left(H^{\mathrm{T}} D_{a0} H \right)^{-1} H^{\mathrm{T}} J_o^{\mathrm{T}} J_e^{-\mathrm{T}} e_w \\ &\quad - s^{\mathrm{T}} H^{-1} D_{a0}^{-1} J_o^{\mathrm{T}} J_e^{-\mathrm{T}} K J_e^{-1} J_o D_{a0}^{-\mathrm{T}} H^{-\mathrm{T}} s \\ &\quad + s^{\mathrm{T}} \left(H^{\mathrm{T}} D_{a0} H \right)^{-1} H^{-\mathrm{T}} \left(\rho - \widehat{\rho} \right) + s_\tau^{\mathrm{T}} \overline{J}_m \dot{s}_\tau \\ &\le s^{\mathrm{T}} \left(D_{a0} H \right)^{-1} J_o^{\mathrm{T}} J_e^{-\mathrm{T}} e_w \\ &\quad - s^{\mathrm{T}} H^{-1} D_{a0}^{-1} J_o^{\mathrm{T}} J_e^{-\mathrm{T}} K J_e^{-1} J_o D_{a0}^{-\mathrm{T}} H^{-\mathrm{T}} s + s_\tau^{\mathrm{T}} \overline{J}_m \dot{s}_\tau \\ &\quad + \|s\| \left\| \left(H^{\mathrm{T}} D_{a0} H \right)^{-1} \right\| \\ &\quad \cdot \left(\left\| H^{\mathrm{T}} \right\| \|\rho\| - a_0 - a_1 \|x\| - a_2 \|\dot{x}\|^2 \right) \\ &\quad + 2 e_\tau^{\mathrm{T}} \Lambda_\tau^{\mathrm{T}} K_\tau \dot{e}_\tau \le s^{\mathrm{T}} \left(D_{a0} H \right)^{-1} J_o^{\mathrm{T}} J_e^{-\mathrm{T}} e_w \\ &\quad - s^{\mathrm{T}} H^{-1} D_{a0}^{-1} J_o^{\mathrm{T}} J_e^{-\mathrm{T}} K J_e^{-1} J_o D_{a0}^{-\mathrm{T}} H^{-\mathrm{T}} s + s_\tau^{\mathrm{T}} \overline{J}_m \dot{s}_\tau \\ &\quad + 2 e_\tau^{\mathrm{T}} \Lambda_\tau^{\mathrm{T}} K_\tau \dot{e}_\tau. \end{aligned} \tag{47}$$

According to (30) and (40), we can have

$$e_w = c e_\tau - c\Gamma \operatorname{sgn} \left(J_e^{-1} J_o D_{a0}^{-\mathrm{T}} H^{-\mathrm{T}} s \right) + \delta(\tau). \tag{48}$$

Substituting (48) into (47) and choosing appropriate $c\Gamma$, the following can be obtained:

$$\begin{aligned} \dot{V} &\le s^{\mathrm{T}} \left(D_{a0} H \right)^{-1} J_o^{\mathrm{T}} J_e^{-\mathrm{T}} e_\tau \\ &\quad - s^{\mathrm{T}} H^{-1} D_{a0}^{-1} J_o^{\mathrm{T}} J_e^{-\mathrm{T}} K J_e^{-1} J_o D_{a0}^{-\mathrm{T}} H^{-\mathrm{T}} s + s_\tau^{\mathrm{T}} \overline{J}_m \dot{s}_\tau \\ &\quad + 2 e_\tau^{\mathrm{T}} \Lambda_\tau^{\mathrm{T}} K_\tau \dot{e}_\tau. \end{aligned} \tag{49}$$

Substituting the motor torques (41) into (49), we can have

$$\dot{V} = s^{\mathrm{T}}(D_{a0}H)^{-1} J_o^{\mathrm{T}} J_e^{-\mathrm{T}} e_\tau - s^{\mathrm{T}} H^{-1} D_{a0}^{-1} J_o^{\mathrm{T}} J_e^{-\mathrm{T}} K J_e^{-1} J_o D_{a0}^{-\mathrm{T}} H^{-\mathrm{T}} s - \dot{e}_\tau^{\mathrm{T}} K_\tau \dot{e}_\tau - e_\tau^{\mathrm{T}} \Lambda_\tau^{\mathrm{T}} K_\tau \Lambda_\tau e_\tau + s_\tau^{\mathrm{T}} \left[\left(\widehat{\overline{J}}_m - \overline{J}_m\right) \dot{v}_\tau + \widehat{h}_m - h_m - P_\tau \operatorname{sgn}(s_\tau) \right] \leq - \left[\left\| J_e^{-1} J_o D_{a0}^{-\mathrm{T}} H^{-\mathrm{T}} s \right\| \ \|e_\tau\| \ \|\dot{e}_\tau\| \right] Q \left[\left\| J_e^{-1} J_o D_{a0}^{-\mathrm{T}} H^{-\mathrm{T}} s \right\| \ \|e_\tau\| \ \|\dot{e}_\tau\| \right]^{\mathrm{T}} \leq 0, \quad (50)$$

where

$$Q = \begin{pmatrix} K & -\dfrac{I}{2} & 0 \\ -\dfrac{I}{2} & \Lambda_\tau^{\mathrm{T}} K_\tau \Lambda_\tau & 0 \\ 0 & 0 & K_\tau \end{pmatrix}. \quad (51)$$

Because of $V > 0$, $\dot{V} \leq 0$, the Lyapunov stability is guaranteed and $s, e_\tau, e_w, \dot{e}_\tau\ e, \dot{e}, e_\lambda, \dot{e}_\lambda$ are bounded.

Substituting $H^{\mathrm{T}} J_o^{\mathrm{T}} J_e^{-\mathrm{T}} w_d$ into (34), we can have

$$\begin{aligned} H^{\mathrm{T}} D_{a0} H E_1^{\mathrm{T}} \ddot{e} &= H^{\mathrm{T}} J_o^{\mathrm{T}} J_e^{-\mathrm{T}} e_w - H^{\mathrm{T}} \widehat{\rho} + H^{\mathrm{T}} \rho \\ &\quad + \left(H^{\mathrm{T}} D_{a0} H\right) \left(-E_1^{\mathrm{T}} \Lambda_1 \dot{e} - J_c^{\mathrm{T}} \dot{s}_2 - \dot{J}_c^{\mathrm{T}} s_2 \right. \\ &\quad \left. - H^{-1} D_{a0}^{-1} J_o^{\mathrm{T}} J_e^{-\mathrm{T}} K J_e^{-1} J_o D_{a0}^{-\mathrm{T}} H^{-\mathrm{T}} s\right). \end{aligned} \quad (52)$$

Multiplying (52) by $E_1 H^{\mathrm{T}} (H^{\mathrm{T}} D_{a0} H)^{-1}$, it can be expressed as

$$E_1 H^{\mathrm{T}} E_1^{\mathrm{T}} \ddot{e} = \mu_1 (e, s, e_w, \widehat{\rho}, \rho), \quad (53)$$

where

$$\begin{aligned} &\mu_1 (e, s, e_w, \widehat{\rho}, \rho) \\ &\quad = E_1 H^{\mathrm{T}} \left(H^{\mathrm{T}} D_{a0} H\right)^{-1} H^{\mathrm{T}} J_o^{\mathrm{T}} J_e^{-\mathrm{T}} e_w \\ &\qquad + E_1 H^{\mathrm{T}} \left(H^{\mathrm{T}} D_{a0} H\right)^{-1} \left(H^{\mathrm{T}} \rho - H^{\mathrm{T}} \widehat{\rho}\right) \\ &\qquad - E_1 H^{\mathrm{T}} E_1^{\mathrm{T}} \Lambda_1 \dot{e} - E_1 H^{\mathrm{T}} \dot{J}_c^{\mathrm{T}} s_2 \\ &\qquad - E_1 H^{\mathrm{T}} H^{-1} D_{a0}^{-1} J_o^{\mathrm{T}} J_e^{-\mathrm{T}} K J_e^{-1} J_o D_{a0}^{-\mathrm{T}} H^{-\mathrm{T}} s. \end{aligned} \quad (54)$$

Because $\mu_1(e, s, e_w, \widehat{\rho}, \rho)$ is bounded, according to the analysis, we can obtain the fact that $E_1 H^{\mathrm{T}} E_1^{\mathrm{T}} \ddot{e}$ is bounded, and $\ddot{e}$ is also bounded. From (52), we can obtain the fact that $\dot{s}_2$ is bounded; then $\dot{s}$ is bounded. According to (50), $J_e^{-\mathrm{T}} J_o D_{a0}^{-\mathrm{T}} H^{-\mathrm{T}} s$ is square integrable, and $d(J_e^{-\mathrm{T}} J_o D_{a0}^{-\mathrm{T}} H^{-\mathrm{T}})/dt$ is bounded. According to Lemma 6, $J_e^{-\mathrm{T}} J_o D_{a0}^{-\mathrm{T}} H^{-\mathrm{T}} s$ can be guaranteed to converge to zero, because $J_e^{-\mathrm{T}} J_o D_{a0}^{-\mathrm{T}} H^{-\mathrm{T}}$ is bounded. Then s can be guaranteed to converge to zero. Then both s_1 and s_2 errors can be guaranteed to converge to zero. The asymptotic convergence of the position error and constraint force error can be ensured.

Substituting $w_d = J_e^{\mathrm{T}} (J_o^{\mathrm{T}})^{+} H^{-\mathrm{T}}$ into (32), we can have

$$J_e^{\mathrm{T}} \left(e_{FI} - K_{FI} e_{FI}\right) = \mu_2 (e, s, e_w, \widehat{\rho}), \quad (55)$$

where

$$\begin{aligned} \mu_2 (e, s, e_w, \widehat{\rho}, \rho) &= e_w + J_e^{\mathrm{T}} \left(J_o^{\mathrm{T}}\right)^{+} (\rho - \widehat{\rho}) - \overline{D}_{l0} \ddot{e} \\ &\quad + J_e^{\mathrm{T}} \left(J_o^{\mathrm{T}}\right)^{+} (D_{a0} H) \left(E_1^{\mathrm{T}} \Lambda_1 \dot{e} + J_c^{\mathrm{T}} \dot{s}_2 + \dot{J}_c^{\mathrm{T}} s_2 \right. \\ &\quad \left. - H^{-1} D_{a0}^{-1} J_o^{\mathrm{T}} J_e^{-\mathrm{T}} K J_e^{-1} J_o D_{a0}^{-\mathrm{T}} H^{-\mathrm{T}}\right). \end{aligned} \quad (56)$$

According to the analysis, we can obtain the fact that $\mu_2(e, s, e_w, \widehat{\rho}, \rho)$ is bounded, and J_e^{T} is full rank; then the convergence of internal force error e_{FI} can be ensured, and the internal force error can be reduced as small as required by changing the gain K_{Ip}. □

4. Simulation

A multiple flexible joint manipulators system with two manipulators is considered to verify the validity of the proposed controller, and each manipulator has two joints; the dynamic equation of the system can be shown as follows:

$$\begin{aligned} D_i(q_i) &= \begin{bmatrix} d_{i11}(q_i) & d_{i12}(q_i) \\ d_{i21}(q_i) & d_{i22}(q_i) \end{bmatrix}, \\ C_i(q_i, \dot{q}_i) &= \begin{bmatrix} c_{i11}(q_i, \dot{q}_i) & c_{i12}(q_i, \dot{q}_i) \\ c_{i21}(q_i, \dot{q}_i) & c_{i22}(q_i, \dot{q}_i) \end{bmatrix}, \\ G_i(q_i) &= \begin{bmatrix} g_{i1}(q_i) \\ g_{i2}(q_i) \end{bmatrix}, \\ J_{ei}(q_i) &= \begin{bmatrix} j_{ei11}(q_i) & j_{ei12}(q_i) \\ j_{ei21}(q_i) & j_{ei22}(q_i) \end{bmatrix}, \\ d_{i11}(q_i) &= (m_{i1} + m_{i2}) {l_{i1}}^2 + m_{i2} {l_{i2}}^2 \\ &\quad + 2 m_{i2} l_{i1} l_{i2} \cos q_{i2}, \\ d_{i12}(q_i) &= m_{i2} {l_{i2}}^2 + m_{i2} l_{i1} l_{i2} \cos q_{i2}, \\ d_{i21}(q_i) &= d_{i12}(q_i), \\ d_{i22}(q_i) &= m_{i2} {l_{i2}}^2, \\ c_{i11}(q_i, \dot{q}_i) &= -m_{i2} l_{i1} l_{i2} \dot{q}_{i2} \sin q_{i2}, \\ c_{i12}(q_i, \dot{q}_i) &= -m_{i2} l_{i1} l_{i2} \sin q_{i2} (\dot{q}_{i1} + \dot{q}_{i2}), \\ c_{i21}(q_i, \dot{q}_i) &= m_{i2} l_{i1} l_{i2} \dot{q}_{i1} \sin q_{i2}, \\ c_{i22}(q_i, \dot{q}_i) &= 0, \end{aligned}$$

$$
\begin{aligned}
g_{i1}(q_i) &= (m_1 + m_2)\, g l_{i1} \cos q_{i1} \\
&\quad + m_{i2} g l_{i2} \cos (q_{i1} + q_{i2}), \\
g_{i2}(q_i) &= m_{i2} g l_{i2} \cos (q_{i1} + q_{i2}), \\
j_{ei11}(q_i) &= -l_{i1} \sin q_{i1} - l_{i2} \sin (q_{i1} + q_{i2}), \\
j_{ei12}(q_i) &= -l_{i2} \sin (q_{i1} + q_{i2}), \\
j_{ei21}(q_i) &= l_{i1} \cos q_{i1} + l_{i2} \cos (q_{i1} + q_{i2}), \\
j_{ei22}(q_i) &= l_{i2} \cos (q_{i1} + q_{i2}), \\
I_{mi} &= \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}, \\
k_{mi} &= \begin{pmatrix} 100 & 0 \\ 0 & 100 \end{pmatrix}.
\end{aligned}
\tag{57}
$$

The dynamic equation of the object:

$$
\begin{aligned}
D_o(x) &= \begin{bmatrix} m_0 & 0 \\ 0 & m_0 \end{bmatrix}, \\
C_o(x, \dot{x}) &= 0, \\
G_o(x) &= \begin{bmatrix} 0 \\ m_0 g \end{bmatrix}, \\
J_o(x) &= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}^{\mathrm{T}}.
\end{aligned}
\tag{58}
$$

The manipulator i parameters: $l_{i1} = l_{i2} = 1.0$ m, $m_{i1} = m_{i2} = 1.0$ kg., and $m_{i1} = m_{i2} = 1.0$ kg.

The object parameters: $m_0 = 2.0$ kg, $g = 9.8\,\mathrm{m/s^2}$, $R = 0.2$ m., and $R = 0.2$ m.

Desired internal force: $F_{Id} = [10, 10, -10, -10]^{\mathrm{T}}$ N.

Desired position of object:

$$
\begin{aligned}
x_{1d} &= -0.1 \sin (t) \ \mathrm{m}, \\
x_2 &= R \cos \alpha - \tan \alpha \left[L - (x_1 + R \sin \alpha) \right] \ \mathrm{m}, \\
\lambda_d &= 10\,\mathrm{N}, \\
F_{Id} &= [10, 10, -10, -10]^{\mathrm{T}} \ \mathrm{N}.
\end{aligned}
\tag{59}
$$

Initial values of position of object: $x_1(0) = 0.1$ m, and $x_1(0) = -0.1$ m..

Controller parameters: $K = 10I$, $K_{FI} = 20I$, $\Gamma = 2I$, and $K_\tau = 10I$.

The time delay: $T = 0.07$ s.

The simulation results are shown in Figures 2–9. Figures 2, 4, and 6 show the position tracking performances of the object, the position errors, and the constraint force errors with no hysteresis loop compensation, respectively. From these figures, one can see that the convergence of tracking errors, both position and constraint force, can be guaranteed, but position and constraint force errors cannot converge to zero. Figures 3, 5, and 7 show the position tracking performances of the object, the position errors, and the constraint force errors with hysteresis loop compensation, respectively. We can see that the tracking errors, both position and constraint force, can converge to zero.

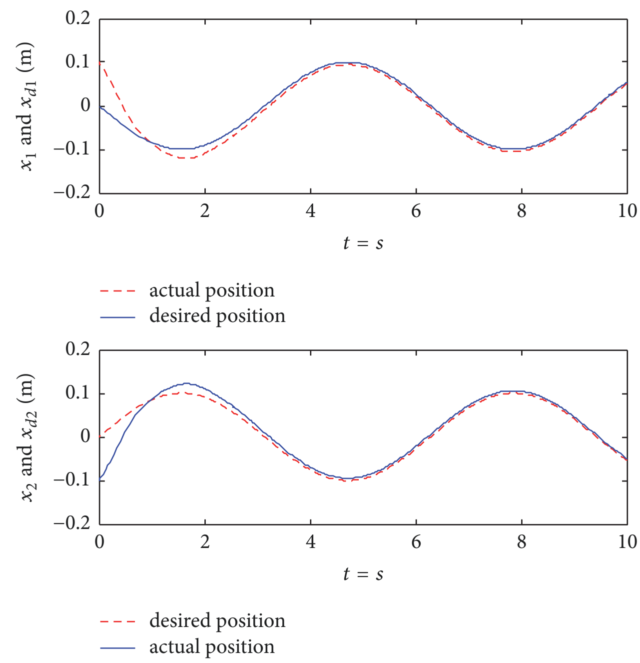

Figure 2: Position and velocity tracking of joint 1 (no hysteresis loop compensation).

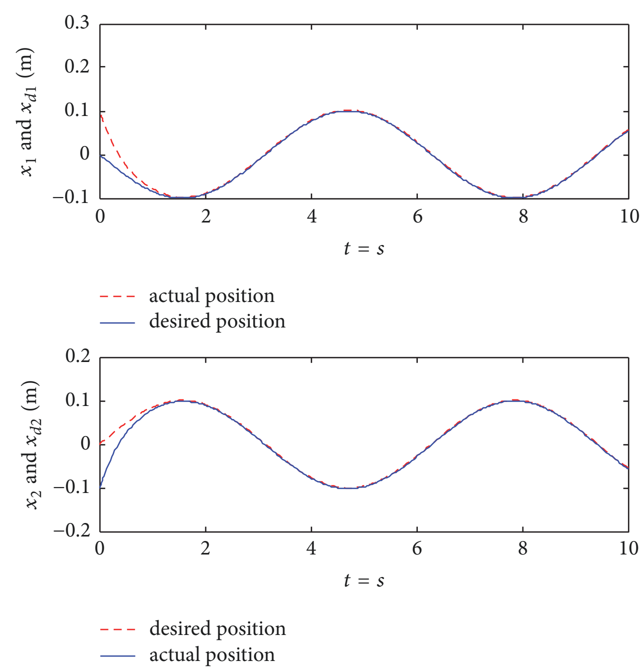

Figure 3: Position and velocity tracking of joint 1.

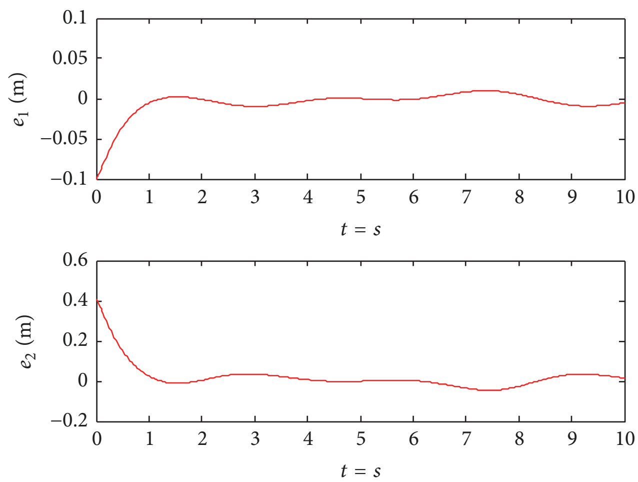

FIGURE 4: Position and velocity tracking errors of joint 1 (no hysteresis loop compensation).

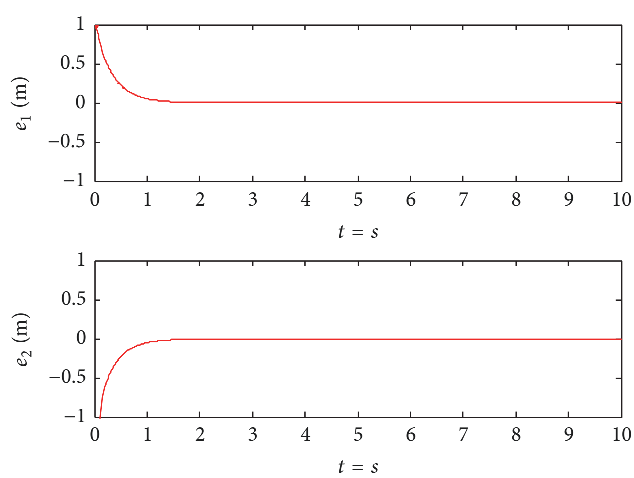

FIGURE 5: Position and velocity tracking errors of joint 1.

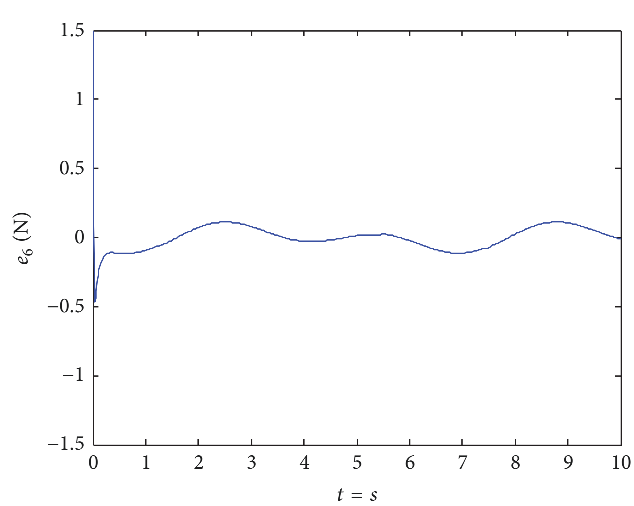

FIGURE 6: Contacting force tracking errors (no hysteresis loop compensation).

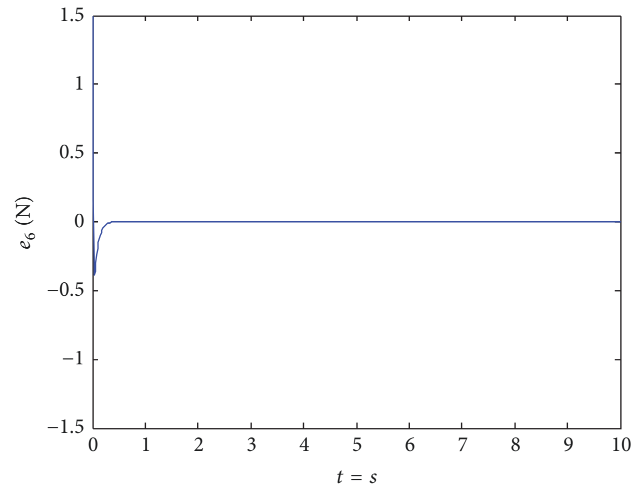

FIGURE 7: Contacting force tracking errors.

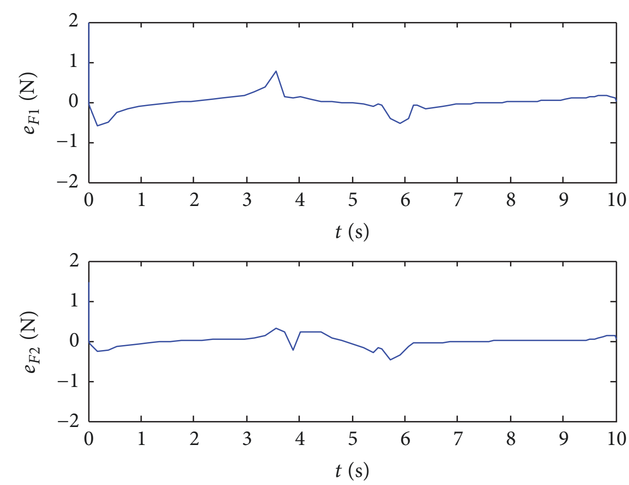

FIGURE 8: Internal force error of manipulator 1.

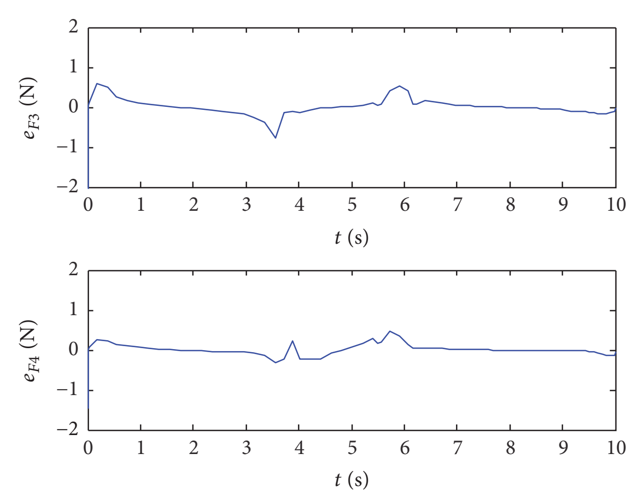

FIGURE 9: Internal force error of manipulator 2.

The constraint force errors for linear sliding mode are shown in Figure 8. We can see that the force errors are bounded in Figure 8 compared with Figure 7. The internal force errors are shown in Figure 8. We can see that the internal force errors are bounded. The simulation results show that the proposed method can make the position errors and force errors converge to zero.

5. Conclusions

In this paper, a robust adaptive position/force control strategy based on sliding mode is proposed for multiple flexible joint manipulators system. The major contribution of this paper is that the nonlinear input of hysteresis loop is considered, which is approximated as a differential equation. Then the modified sliding mode consisting of position error and constraint force error is adopted for coordinated multiple manipulators system. Considering the uncertainties consist of model parameter and distraction, a control strategy based on sliding mode is designed. By the Lyapunov theory, the control strategy can guarantee that both position and constraint force errors converge to zero. And the boundedness of parameters errors and internal force errors can be guaranteed. Finally, the simulation results illustrate the feasibility of the proposed method.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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