Research Article

FTS and FTB of Conformable Fractional Order Linear Systems

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In this paper, an extension of some existing results related to finite-time stability (FTS) and finite-time boundedness (FTB) into the conformable fractional derivative is presented. Illustrative example is presented at the end of the paper to show the effectiveness of the proposed result.

1. Introduction

In regard to the control theory area, finite-time stability investigation is largely used in different areas, for example, stabilization [1], fault estimation [2], and observer-based control [3]. Several research works were done to solve FTS and FTB problem for integer-order linear and nonlinear systems [4–7]. Nevertheless, noninteger derivatives constitute a property of many dynamic systems, so fractional order equations are the best way to describe these systems. For example, in electromagnetic systems [8] fractional order calculus has been used in economy [9], dielectric polarization [10], and image processing [11]. In recent decades, and with the growth of complex engineering systems and the development of science, the use of fractional calculus in many contexts of control theory, such as stability, FTS, has increased significantly.

Until now, FTS and FTB for fractional order systems are not well tackled in the literature. Indeed, there are only few works related to this topic. Readers can refer to [12–15] as an example of finite-time stability and finite-time boundedness of fractional order linear systems using Caputo derivative. Nevertheless, there is no works in the literature which investigate FTS and FTB of the new class of fractional order systems using conformable derivative.

Indeed, in [16], R. Khalil et al. proposed a new effective derivative called “the conformable fractional derivative”.

Both Riemann Liouville and Caputo derivative share some disadvantages; for example, the property $D^\alpha(1) = 0$ is not satisfied for Riemann Liouville definition, the product of two functions as $D^\alpha(fg) = fD^\alpha(g) + gD^\alpha(f)$ and the chain rule as $D^\alpha(f \circ g)(t) = D^\alpha(f)(g(t))D^\alpha(g)(t)$ are not satisfied neither for Riemann Liouville derivative nor for Caputo one. In addition, the monotonicity of a function f is not deduced from the sign of $D^\alpha(f)$. After that, in [17], T. Abdeljawad developed more properties for the conformable derivative. A lot of investigations on it are currently conducted. In addition, in regard to the physical meaning of such derivative, the work in [18] gave physical and geometrical interpretations of the fractional conformable derivative which thus indicate potential applications in physics and engineering.

In this paper, FTS and FTB, which are well tackled for integer-order LTI systems, are extended to conformable fractional order LTI systems. In addition, the design of a feedback controller for the same class of conformable fractional order systems is described. Note that the solution of LTI fractional conformable systems is given by T. Abdeljawad in [17].

The rest of the paper is organized as follows. In Section 2, preliminaries and useful results with respect to the conformable fractional order calculus, FTS, and FTB are introduced. In Section 3 the main results of the work are introduced detailing the FTS and FTB of a certain class of conformable LTI systems. To show the efficiency of

the proposed approach, simulation results are presented in Section 4. Finally, some conclusions are given in Section 5.

2. Preliminaries

In this section, we recall some definitions, notations, and traditional results.

Definition 1 ([16, 17] (conformable fractional derivative)). Consider a function $f : [0, \infty) \mapsto \mathbb{R}$. Thus, the conformable fractional derivative of f of order α is defined by

$${}^a T^\alpha f(t) = \lim_{\varepsilon \rightarrow 0} \frac{f(t + \varepsilon t^{1-\alpha}) - f(t)}{\varepsilon} \quad (1)$$

for all $t > 0$, $\alpha \in (0, 1]$. If f is α -differentiable in some $(0, a)$, $a > 0$, and $\lim_{t \rightarrow 0^+} T^\alpha f(t)$ exists, then by definition

$$T^\alpha f(0) = \lim_{t \rightarrow 0^+} T^\alpha f(t). \quad (2)$$

Definition 2 ([16, 17]). Let $\alpha \in (0, 1]$. The conformable fractional integral starting from a point 0 of a function $f : [0, \infty) \mapsto \mathbb{R}$ of order α is defined as

$$I^\alpha f(t) = \int_0^t s^{\alpha-1} f(s) ds. \quad (3)$$

Lemma 3 ([16, 17]). Assume that $f : [0, \infty) \mapsto \mathbb{R}$ is continuous and $\alpha \in (0, 1]$, so

$$T^\alpha I^\alpha f(t) = f(t), \quad \text{for all } t > 0. \quad (4)$$

Lemma 4. Let $\alpha \in (0, 1]$. Consider a continuous function $f : [0, \infty) \mapsto \mathbb{R}$ such that $T^\alpha f(t)$ exist on $(0, \infty)$. If $T^\alpha f(t) \geq 0$ (respectively, $T^\alpha f(t) \leq 0$), for all $t > 0$, so f is increasing (respectively, decreasing).

Definition 5. Let $\alpha \in (0, 1]$; the conformable fractional exponential function is defined as

$$E_\alpha(\gamma, t) = \exp\left(\gamma \frac{t^\alpha}{\alpha}\right), \quad \forall t \geq 0 \quad (5)$$

with $\gamma \in \mathbb{R}$.

Remark 6. Let $x : [0, \infty) \mapsto \mathbb{R}^n$ be an α -differentiable on $(0, \infty)$ where $x(t) = (x_1(t), \dots, x_n(t))$ and $P = P^T > 0$. Then, $x^T P x$ is α -differentiable on $(0, \infty)$ and $T^\alpha x^T P x(t) = 2x(t)^T P T^\alpha x(t)$, $\forall t > 0$.

In what follows, some definitions and sufficient conditions of the FTS and the FTB for the integer-order linear time invariant system are introduced. Readers can refer to [7, 12].

Definition 7. The integer-order linear time invariant system

$$x'(t) = Ax(t), \quad x(0) = x_0, \quad t \in [0, T], \quad A \in \mathbb{R}^{n \times n} \quad (6)$$

is said to be FTS with respect to (c_1, c_2, T, S) with positive scalars $c_1, c_2, T, c_2 > c_1$ and matrix $S > 0$, if

$$x(0)^T S x(0) \leq c_1 \implies x(t)^T S x(t) < c_2, \quad t \in [0, T]. \quad (7)$$

Definition 8. Consider the integer-order linear time invariant system

$$x'(t) = Ax(t) + Dw(t), \quad x(0) = x_0, \quad t \in [0, T] \quad (8)$$

where $w(t) \in \mathbb{R}^m$ is the disturbance input and satisfies $w(t)^T w(t) \leq d$, $\forall t \in [0, T]$, $d \geq 0$, $A \in \mathbb{R}^{n \times n}$ and $D \in \mathbb{R}^{n \times m}$ are constant matrices, then, system (8) is said to be FTB with respect to (c_1, c_2, T, S, d) with positive scalars $c_1, c_2, T, c_2 > c_1$ and matrix $S > 0$, if

$$x(0)^T S x(0) \leq c_1 \implies x(t)^T S x(t) < c_2, \quad \forall t \in [0, T], \quad \forall w : w(t)^T w(t) \leq d, \quad \forall t \in [0, T]. \quad (9)$$

Remark 9. The FTS of integer-order systems was studied by F. Amato et al. in [7], after that Y. Ma et al. in [12] have generalized the work of Amato et al., 2001, to the Caputo fractional derivative.

3. Main Results

Consider the conformable fractional order LTI system

$$T^\alpha x(t) = Ax(t), \quad x(0) = x_0, \quad t \in (0, T], \quad (10)$$

where $x(t) = (x_1(t), \dots, x_n(t)) \in \mathbb{R}^n$ and $x_0 = (x_{01}, \dots, x_{0n}) \in \mathbb{R}^n$.

Sufficient conditions of the FTS for the conformable fractional order LTI system (10) are stated through the following theorem.

Theorem 10. For the conformable fractional order LTI system (10), assume that there exist a scalar β and a matrix $Q > 0$, $Q \in \mathbb{R}^{n \times n}$, verifying

$$PA + A^T P - \beta P < 0, \quad (11)$$

$$E_\alpha(\beta, T) \text{ cond}(Q) < \frac{c_2}{c_1}, \quad (12)$$

where $P = S^{1/2} Q S^{1/2}$. Thus, system (10) is FTS with respect to (c_1, c_2, T, S) .

Proof. Consider $V(x(t)) = x(t)^T P x(t)$; we have

$$T^\alpha V(x(t)) = x(t)^T [PA + A^T P] x(t). \quad (13)$$

From (11), we have

$$T^\alpha V(x(t)) \leq \beta V(x(t)). \quad (14)$$

Let $h(t) = E_\alpha(-\beta, t) V(x(t))$. We have

$$T^\alpha h(t) = -\beta E_\alpha(-\beta, t) V(x(t)) + T^\alpha V(x(t)) E_\alpha(-\beta, t). \quad (15)$$

From (14), we obtain

$$T^\alpha h(t) \leq 0. \quad (16)$$

Based on Lemma 4, we have

$$h(t) \leq h(0) \implies V(x(t)) \leq E_\alpha(\beta, t) V(x(0)). \quad (17)$$

It yields

$$x(t)^T P x(t) \leq E_\alpha(\beta, t) x(0)^T P x(0). \quad (18)$$

Knowing that $P = S^{1/2} Q S^{1/2}$, (18) can be written as

$$x(t)^T S^{1/2} Q S^{1/2} x(t) \leq E_\alpha(\beta, t) x(0)^T S^{1/2} Q S^{1/2} x(0). \quad (19)$$

Then,

$$\begin{aligned} \lambda_{\min}(Q) x(t)^T S x(t) \\ \leq \lambda_{\max}(Q) E_\alpha(\beta, t) x(0)^T S x(0). \end{aligned} \quad (20)$$

The two inequalities $x(0)^T S x(0) \leq c_1$ and (12) leads to $x(t)^T S x(t) < c_2, \forall t \in [0, T]$. This ends the proof. $\square$

For the system (10), we have also the following result.

Corollary 11. For the conformable fractional order linear time invariant system (10), assume that there exist a scalar β and a matrix $Q > 0$, $Q \in \mathbb{R}^{n \times n}$, verifying

$$AP + PA^T - \beta P < 0, \quad (21)$$

$$E_\alpha(\beta, T) \text{cond}(Q) < \frac{c_2}{c_1}, \quad (22)$$

where $P = S^{-1/2} Q^{-1} S^{-1/2}$. Then, system (10) is FTS with respect to (c_1, c_2, T, S) .

Based on the stability conditions presented in Theorem 10, the stabilizing controller design can be formulated. The main objective is to design a state controller $u(t) = Kx(t)$ for the conformable fractional order LTI system

$$T^\alpha x(t) = Ax(t) + Bu(t), \quad x(0) = x_0, \quad t \in (0, T], \quad (23)$$

where $x(t) = (x_1(t), \dots, x_n(t)) \in \mathbb{R}^n$ and $x_0 = (x_{01}, \dots, x_{0n}) \in \mathbb{R}^n$ such that the feedback system

$$T^\alpha x(t) = A_c x(t), \quad t \in (0, T], \quad (24)$$

where $A_c = A + BK$, $K \in \mathbb{R}^{l \times n}$, being FTS with respect to (c_1, c_2, T, S) .

We propose the following theorem.

Theorem 12. For the conformable fractional order linear time invariant system (23), assume that there exist a scalar β , a matrix $Q > 0$, $Q \in \mathbb{R}^{n \times n}$, and a matrix $L \in \mathbb{R}^{l \times n}$ verifying

$$AP + PA^T + BL + L^T B^T - \beta P < 0, \quad (25)$$

$$E_\alpha(\beta, T) \text{cond}(Q) < \frac{c_2}{c_1}, \quad (26)$$

where $P = S^{-1/2} Q^{-1} S^{-1/2}$. Thus, system (23) is finite-time stable with respect to (c_1, c_2, T, S) under the feedback control $u(t) = Kx(t) = LP^{-1}x(t)$.

Proof. Apply the state feedback controller $u(t) = LP^{-1}x(t)$ to (23) such that the conformable fractional closed-loop system is

$$T^\alpha x(t) = (A + BLP^{-1})x(t). \quad (27)$$

It is clear that (25) can be rewritten as

$$(A + BLP^{-1})P + P(A + BLP^{-1})^T - \beta P < 0. \quad (28)$$

Combining (26) and using Theorem 10, the FTS of the closed-loop system (23) can be given. $\square$

Now, consider the LTI conformal fractional order system presented by

$$T^\alpha x(t) = Ax(t) + Dw(t), \quad x(0) = x_0, \quad t \in (0, T], \quad (29)$$

where $x(t) = (x_1(t), \dots, x_n(t)) \in \mathbb{R}^n$ and $x_0 = (x_{01}, \dots, x_{0n}) \in \mathbb{R}^n$.

The following theorem provides a sufficient condition of the FTB for the conformable fractional order LTI system (29).

Theorem 13. Consider system (29) and assume that $\exists \beta \in \mathbb{R}$ and two matrices $P_1 > 0$, $P_1 \in \mathbb{R}^{n \times n}$, and $P_2 > 0$, $P_2 \in \mathbb{R}^{m \times m}$, verifying

$$\begin{pmatrix} AP + PA^T - \beta P & DP_2 \\ P_2 D^T & -\beta P_2 \end{pmatrix} < 0, \quad (30)$$

$$\lambda_{\max}(P_1) E_\alpha(\beta, T) \left(\frac{\beta d T^\alpha}{\lambda_{\min}(P_2) \alpha} + \frac{c_1}{\lambda_{\min}(P_1)} \right) < c_2, \quad (31)$$

where $P = S^{-1/2} P_1 S^{-1/2}$. Thus, system (29) is FTB with respect to (c_1, c_2, T, S, d) .

Proof. Suppose that $V(x(t)) = x(t)^T P^{-1} x(t)$. We have

$$T^\alpha V(x(t)) = (x(t)^T \quad w(t)^T) \begin{pmatrix} P^{-1}A + A^T P^{-1} & P^{-1}D \\ D^T P^{-1} & 0 \end{pmatrix} \begin{pmatrix} x(t) \\ w(t) \end{pmatrix}. \quad (32)$$

Taking into account assumption (30), the pre- and post-multiplying of (30) by $\begin{pmatrix} P^{-1} & 0 \\ 0 & P_2^{-1} \end{pmatrix}$ give

$$\begin{pmatrix} P^{-1}A + A^T P^{-1} - \beta P^{-1} & P^{-1}D \\ D^T P^{-1} & -\beta P_2^{-1} \end{pmatrix} \leq 0. \quad (33)$$

Combining (32) and (33), we have

$$\begin{aligned} T^\alpha V(x(t)) &\leq (x(t)^T \quad w(t)^T) \begin{pmatrix} \beta P^{-1} & 0 \\ 0 & \beta P_2^{-1} \end{pmatrix} \begin{pmatrix} x(t) \\ w(t) \end{pmatrix} \\ &= \beta V(x(t)) + \beta w(t)^T P_2^{-1} w(t). \end{aligned} \quad (34)$$

This together with $w(t)^T P_2^{-1} w(t) \leq \lambda_{\max}(P_2^{-1}) w(t)^T w(t) \leq d/\lambda_{\min}(P_2)$ gives

$$T^\alpha V(x(t)) \leq \beta V(x(t)) + r, \quad t \in (0, T], \quad (35)$$

where $r = \beta d/\lambda_{\min}(P_2)$.

Suppose that

$$g(t) = E_\alpha(-\beta, t) V(x(t)) - r \frac{t^\alpha}{\alpha}. \quad (36)$$

Then,

$$\begin{aligned} T^\alpha g(t) &= -\beta E_\alpha(-\beta, t) V(x(t)) \\ &\quad + E_\alpha(-\beta, t) T^\alpha V(x(t)) - r. \end{aligned} \quad (37)$$

From (35), we have

$$T^\alpha g(t) \leq r E_\alpha(-\beta, t) - r \leq 0, \quad t \in (0, T]. \quad (38)$$

Thus, based on Lemma 4 we have

$$g(t) \leq g(0) = V(x(0)), \quad t \in [0, T]. \quad (39)$$

Then,

$$\begin{aligned} V(x(t)) &\leq E_\alpha(\beta, t) \left[V(x(0)) + r \frac{t^\alpha}{\alpha} \right], \quad t \in [0, T] \\ &\leq E_\alpha(\beta, t) \left[V(x(0)) + r \frac{T^\alpha}{\alpha} \right]. \end{aligned} \quad (40)$$

Knowing that $P = S^{-1/2} P_1 S^{-1/2}$, the following inequality holds:

$$\begin{aligned} V(x(t)) &\geq \frac{x(t)^T S x(t)}{\lambda_{\max}(P_1)}, \\ V(x(0)) &\leq \frac{c_1}{\lambda_{\min}(P_1)}. \end{aligned} \quad (41)$$

From (40), we have

$$\begin{aligned} x(t)^T S x(t) &\leq \lambda_{\max}(P_1) E_\alpha(\beta, T) \left[r \frac{T^\alpha}{\alpha} + \frac{c_1}{\lambda_{\min}(P_1)} \right]. \end{aligned} \quad (42)$$

By using (31) and (42), we get

$$x(t)^T S x(t) < c_2, \quad \forall t \in [0, T]. \quad (43)$$

This ends the proof. $\square$

Remark 14. The obtained results of the FTS and the FTB for the conformable fractional order LTI systems are the generalization of the integer-order LTI systems.

Now, consider the design of state feedback controllers $u(t) = Kx(t)$ to stabilize the FTB problem for the conformable fractional order LTI system

$$\begin{aligned} T^\alpha x(t) &= Ax(t) + Bu(t) + Dw(t), \\ x(0) &= x_0, \quad \forall t \in (0, T], \end{aligned} \quad (44)$$

where $x(t) = (x_1(t), \dots, x_n(t)) \in \mathbb{R}^n$ and $x_0 = (x_{01}, \dots, x_{0n}) \in \mathbb{R}^n$.

Theorem 15. For the conformable fractional order linear time invariant system (44), assume that there exist a scalar, two matrices $P_1 > 0$, $P_1 \in \mathbb{R}^{n \times n}$, and $P_2 > 0$, $P_2 \in \mathbb{R}^{m \times m}$, and a matrix $L \in \mathbb{R}^{l \times n}$ such that the inequality (31) and

$$\begin{pmatrix} AP + PA^T + BL + L^T B^T - \beta P & DP_2 \\ P_2 D^T & -\beta P_2 \end{pmatrix} < 0 \quad (45)$$

hold, where $P = S^{-1/2} P_1 S^{-1/2}$.

Thus, system (44) is FTB with respect to (c_1, c_2, T, S, d) under the control law $u(t) = Kx(t) = LP^{-1}x(t)$.

Proof. Applying the state feedback controller $u(t) = Kx(t)$ to (44) results in the closed-loop system as

$$T^\alpha x(t) = (A + BL P^{-1}) x(t) + Dw(t), \quad \forall t \in (0, T]. \quad (46)$$

It is easy to see that (45) can be rewritten as

$$\begin{pmatrix} (A + BL P^{-1}) P + P (A + BL P^{-1})^T - \beta P & DP_2 \\ P_2 D^T & -\beta P_2 \end{pmatrix} < 0, \quad (47)$$

which, by Theorem 13, implies that the closed-loop system (44) is FTB with respect to (c_1, c_2, T, S, d) . $\square$

4. Numerical Example

Let us consider the conformable fractional order LTI system with a disturbance defined by

$$T^\alpha x(t) = Ax(t) + Dw(t), \quad t \in (0, T], \quad (48)$$

with $A = \begin{pmatrix} -1 & 1 \\ -1.5 & 0.5 \end{pmatrix}$, $D = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $w(t) = (0.2 \sin(t) \ 0.2 \cos(t))^T$. The parameters are given as $c_1 = 1$, $c_2 = 56$, $d = 0.04$, $T = 10$, $\beta = 0.04$, and the matrix $R = I$.

Applying Theorem 13, the feasible solution of the condition (30) is given by

$$P = \begin{pmatrix} 7.5243 & -3.8465 \\ -3.8465 & 5.7412 \end{pmatrix}, \quad (49)$$

$$P_2 = \begin{pmatrix} 0.1090 & -0.0244 \\ -0.0244 & 0.0726 \end{pmatrix}.$$

Therefore, system (48) is FTB with respect to $(1, 56, 20, R, 0.04)$.

The state trajectory over 0–10s with the initial state $x(0) = (0 \ 1)^T$ is shown in Figure 1. It is easy to see that system (48) is FTB with respect to $(1, 56, 10, R, 0.04)$ from Figure 2.

5. Conclusion

The FTS and FTB problem for integer-order systems have been subjected to several research works. However, much less interest is given to the new generalized mathematical representation, namely, conformable fractional order systems. In this paper, some existing results in the literature, related to FTS for linear integer-order systems, are extended into the new conformable fractional derivative. Finally, some simulation results are given to validate theoretical results.

FIGURE 1: The state $x(t)$ of system (48) versus time.

FIGURE 2: The trajectory of $x(t)^T R x(t)$ of system (48) versus time.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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