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Research Article

Adaptive Learning Based Tracking Control of Marine Vessels with Prescribed Performance

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A novel adaptive tracking controller of fully actuated marine vessels is proposed with completely unknown dynamics and external disturbances. The model of dominant dynamic behaviors and unknown disturbances of the vessel are learned by a neural network in real time. The controller is designed and it unifies backstepping and adaptive neural network techniques with predefined tracking performance constraints on the tracking convergence rate and the transient and steady-state tracking error. The stability of the proposed adaptive tracking controller of the vessel is proven with a uniformly bounded tracking error. The proposed adaptive tracking controller is shown to be effective in the tracking control of marine vessels by simulations.

1. Introduction

Tracking control of marine vessels has been widely used in civil and military applications such as shipping transportation, sea investigation, search and rescue, and security surveillance. Previous works on tracking control can be briefly classified into model-based and approximation-based approaches. Model-based methods require accurate dynamic models of the systems, which makes them difficult to be applied in reality due to the following reasons: (1) the dynamics of a vessel are generally time-varying and highly nonlinear and thus their actual mathematical model is hard to be obtained precisely using current modeling techniques; (2) vessels always suffer from unknown stochastic environmental disturbances.

Recently, adaptive NN control design techniques were presented for controlling marine vehicles with model uncertainties [1, 2], by which the dynamics of the vessels can be partially known or even completely unknown. In [3], a simple yet computationally efficient NN tracking control approach is presented for control of the fully actuated vessels. Reference [4] extends the control strategy in [3] to the underactuated vessels. Other approximation methods such as fuzzy tracking control schemes are also proposed for marine vehicles. In [5], a fuzzy tracking controller is proposed against the model uncertainty of the vessel dynamics which can obtain with a uniformly bounded tracking error. However, one problem with the existing approximation-based control schemes is that it is of great challenge to guarantee the transient response of the vessel dynamics; for example, unsuitable initialization of the weight of NN may lead to poor transient tracking performance.

For the purpose of solving this problem, an efficient prescribed performance control method was proposed in [6, 7] to regularize the response of strict-feedback systems in advance. The main idea is to formulate the tracking control as a constrained optimization problem with predefined constraints on the transient and steady-state tracking errors. It is then transformed into a relatively easier unconstrained optimization problem by defining a transformation function that incorporates the errors and their constraints into a single function analytically. This technique has been applied in several applications [8–11]. In [12], an adaptive NN control method was proposed with guaranteed transient and steady-state tracking performances for marine surface vessels with model uncertainties. Radial basis function (RBF) NNs are applied to approximate unknown vessel dynamics before control action applies with the assumptions of persistent excitation and recurrent orbits of the desired trajectory. However, the vessels always suffer from unknown environmental disturbances, and the estimated and stored model in memory may be violated during operating and the control performance cannot be guaranteed. Moreover, the persistent excitation condition was very restrictive making the method difficult to be applied in practice.

This paper addresses the tracking control of surface vessels with unknown system dynamics and stochastic disturbances. An adaptive NN controller is proposed to obtain desired tracking performances. First, the method can work with predefined convergence rate, maximum overshoot, and steady-state error as the desired tracking performance indices. Second, the adaptive NN control combined with backstepping technique is applied to generate control commands for tracking control with unknown model dynamics and external disturbances. Third, the stability of the control method is analyzed and the tracking error is proven to be uniformly bounded.

2. Problem Formulation

2.1. Dynamics of Surface Vessels. We consider the surge, sway, and yaw motions of the marine vessel. Let $\eta = [\eta_x, \eta_y, \eta_\psi]^T$ represent position (η_x, η_y) and heading (η_ψ) of the vessel. The vector $\nu = [\nu_x, \nu_y, r_\psi]^T$ represents the alongship, athwartship, and rotational velocity, respectively, in the body-fixed frame. The vessel dynamics is considered as follows:

$$\dot{\eta} = J(\eta)\nu \tag{1}$$

$$M\dot{\nu} + C(\nu)\nu + D(\nu)\nu + g(\eta) = \tau + \tau_d(\eta, \nu, t), \tag{2}$$

where

$$J(\eta) = \begin{bmatrix} \cos\psi_n & -\sin\psi_n & 0 \\ \sin\psi_n & \cos\psi_n & 0 \\ 0 & 0 & 1 \end{bmatrix}. \tag{3}$$

M is the diagonal inertia matrix, $C(\nu)$ is the matrix of Coriolis and centripetal, $D(\nu)$ is the damping matrix, $g(\eta)$ is the restoring force, τ is the vector of input signals, and τ_d is the time-varying unknown external disturbance and unmodeled dynamics. In this paper, we assume that M, $C(\nu)$, and $D(\nu)$ are completely unknown.

2.2. Prescribed Performance Control Preliminaries. The aim of this paper is to let the vessel position and heading in η track a smooth, bounded desired trajectory η_r with a uniformly bounded tracking error $\varepsilon(t) = \eta(t) - \eta_r(t)$. The predefined constraints of the tracking error $\varepsilon(t) = [\varepsilon_1(t), \varepsilon_2(t), \varepsilon_3(t)]^T$ $\forall t \geq 0$ are given as follows:

$$\begin{aligned} -\kappa\varrho_i(t) < \varepsilon_i(t) < \varrho_i(t) \quad &\text{if } \varepsilon_i(0) \geq 0 \\ -\varrho_i(t) < \varepsilon_i(t) < \kappa\varrho_i(t) \quad &\text{if } \varepsilon_i(0) \leq 0, \end{aligned} \tag{4}$$

where $0 \leq \kappa \leq 1$ and $\varrho_i(t)$, $i = 1, 2, 3$, are called performance functions [13] which are smooth, bounded, and strictly positive and decreasing.

Thus, the constraints on the tracking error $\varepsilon_i(t)$ are defined by $\varrho_i(t)$ and κ. Given the following exponential performance function

$$\varrho_i(t) = (\varrho_{i0} - \varrho_{i\infty})\,\varepsilon^{-l_i t} + \varrho_{i\infty}, \quad i = 1, 2, 3, \tag{5}$$

where $\varrho_{i0} = \varrho_i(0)$ and $\varrho_{i\infty} = \lim_{t\to\infty}\varrho_i(t)$, the constant ϱ_{i0} in (5) should be set such that (4) is satisfied at $t = 0$. l_i determines how fast $\varrho_i(t)$ decreases. Moreover, it is implied that the overshoot is required to be no larger than $\kappa\varrho_i(0)$.

The idea in [6, 7] of transforming the tracking error into a new function is borrowed here, by which the error constraints can be incorporated into the function analytically so that the controller optimization can be solved more efficiently. The error transformation is defined by

$$\xi_i = T_i\left(\frac{\varepsilon_i}{\varrho_i}\right), \quad i = 1, 2, 3, \tag{6}$$

where $T_i(\cdot)$, $i = 1, 2, 3$ are smooth, strictly increasing functions which define objective mappings:

$$\begin{aligned} T_i : (-\kappa, 1) \longrightarrow (-\infty, \infty) \quad &\text{for } \varepsilon_i(0) \geq 0 \\ T_i : (-1, \kappa) \longrightarrow (-\infty, \infty) \quad &\text{for } \varepsilon_i(0) \leq 0 \end{aligned} \tag{7}$$

for $i = 1, 2, 3$. Defining $\xi(t) = [\xi_1(t), \xi_2(t), \xi_3(t)]^T$, after differentiation we have

$$\dot{\xi} = T_\partial\left(\dot{\varepsilon} - \dot{\varrho}\frac{\varepsilon}{\varrho}\right), \tag{8}$$

where $T_\partial = \operatorname{diag}([(dT_1/d(\varepsilon_1/\varrho_1))(1/\varrho_1)\cdots(dT_3/d(\varepsilon_3/\varrho_3))(1/\varrho_3)])$ and $\dot{\varrho}(\varepsilon/\varrho) = [\dot{\varrho}_1(\varepsilon_1/\varrho_1)\cdots\dot{\varrho}_3(\varepsilon_3/\varrho_3)]^T$.

According to [6], if $\xi_i(t)$ is able to be kept bounded $\forall t \geq 0$ with the transformation function $T_i(\cdot)$, $i = 1, 2, 3$, then (4) is guaranteed.

3. Adaptive Tracking Control Design

In this section, we will develop an adaptive tracking controller by combining the backstepping design technique and a RBF NN. The main advantage of RBF is local approximation and fast learning, which has been widely used in researches [14, 15].

Assume that full state information ν and η of the vessel is available. Given the reference trajectory $\eta_r = [\eta_{xr}, \eta_{yr}, \eta_{\psi r}]^T$ and the tracking error

$$\varepsilon(t) = \eta(t) - \eta_r(t), \tag{9}$$

we can get

$$\dot{\varepsilon}(t) = J(\eta)\nu - \dot{\eta}_r(t). \tag{10}$$

The error is transformed based on (6) and (8) as follows:

$$\dot{\xi} = T_\partial \left(\dot{\varepsilon} - \dot{\varrho}\frac{\varepsilon}{\varrho} \right) = T_\partial \left(\dot{\eta}(t) - \dot{\eta}_r(t) - \dot{\varrho}\frac{\varepsilon}{\varrho} \right) = T_\partial \dot{\eta}(t) - T_\partial r = T_\partial J v - T_\partial r, \quad (11)$$

where $r = \dot{\eta}_r(t) + \dot{\varrho}(\varepsilon/\varrho)$.

According to [6, 13], the boundedness of the solution of (11) guarantees the predefined tracking performance of $\varepsilon(t)$ introduced via (4) for all $t \geq 0$. We introduce a virtual control based on the transformed error

$$\phi_1 = J^T r - K_1 (T_\partial J)^T \xi \quad (12)$$

with $K_1 = K_1^T > 0$ and $\dot{\phi}_1 = (\partial\phi_1/\partial\eta)\dot{\eta} + (\partial\phi_1/\partial\dot{\eta}_r)\ddot{\eta}_r + (\partial\phi_1/\partial\xi)\dot{\xi}_1$. Define a second error variable in the transformed space as

$$\xi_a = v(t) - \phi_1(t). \quad (13)$$

Assume that exogenous effects and uncertainties have finite energy; we have $|\tau_{di}(\eta, v, t)| \leq m_i(\eta, v) + n_i(t)$, $i = 1, 2, 3$, where $m_i(\eta, v) : R^3 \times R^3 \to R^+$ is a positive, smooth, and nondecreasing function and $n_i(t)$ is a positive scalar. Consider the following desired control law:

$$\bar{\tau} = -(T_\partial J)^T \xi - K_2 \xi_a + C(v) v + D(v) v + g(\eta) + M\dot{\phi}_1 - \mathrm{SGN}(\xi_a)(m(\eta, v) + n_i), \quad (14)$$

where $\mathrm{SGN}(\xi_a)$ is the diagonal matrix of signum functions $\mathrm{sgn}(\xi_{a,i})$, $i = 1, 2, 3$, $K_2 = K_2^T > 0$, and $K_2 = \mathrm{diag}(k_{ii}) \in R^{3\times 3}$.

Since the dynamics and disturbance of the vessel are completely unknown, a RBF NN $\widehat{W}^T S(\xi)$ is used to approximate the vessel model and effect of the disturbance given by

$$\widehat{W}^T S(\xi) = M\dot{\phi}_1 + C(v) v + D(v) v + g(\eta) - \mathrm{SGN}(\xi_a)(m(\eta, v) + n_i) - \epsilon(\xi), \quad (15)$$

where $\widehat{W} := \mathrm{blockdiag}[\widehat{W}_1^T, \widehat{W}_2^T, \widehat{W}_3^T]$ are the estimation of the desired weight W^*, $S(\xi) = [S_1^T(\xi), S_2^T(\xi), S_3^T(\xi)]^T$ are the basis functions, $\xi = [\eta^T, v^T, \phi_1^T, \dot{\phi}_1^T]$ are the inputs of the NN, and $\epsilon(\xi)$ is the approximation error. Thus, the control law and weight update law are defined as follows:

$$\tau = -(T_\partial J)^T \xi - K_2 \xi_a + \widehat{W}^T S(Z) \quad (16)$$

$$\dot{\widehat{W}}_i = -\Upsilon_i \left(S_i(\xi)\xi_{a,i} + \mu_i \widehat{W}_i \right), \quad (17)$$

where Υ_i are constant matrix and $\mu_i > 0$, $i = 1, 2, 3$, and $Z = [\eta^T, v^T, \xi^T, \dot{\xi}^T]$ are the inputs of NN.

Theorem 1. *Consider the system consisting of vessel model (2), desired bounded trajectory η_d, the control law (16), and NN weight adaptation law (17). Given the performance functions $\varrho_i(t)$, $i = 1, 2, 3$, and constant κ satisfying $0 \leq \kappa \leq 1$, $i = 1, 2, 3$, which incorporate desired performance bounds on tracking errors $\varepsilon_i(t)$, $i = 1, 2, 3$, if initial conditions satisfy*

$$\begin{aligned} |\varepsilon_i(0)| < \varrho_i(0) \quad &\text{if } \varepsilon_i(0) \neq 0 \\ \kappa \neq 0, \quad &\text{if } \varepsilon_i(0) = 0 \end{aligned} \quad (18)$$

with transformation functions

$$T_i\left(\frac{\varepsilon_i(t)}{\varrho_i(t)}\right) = \begin{cases} a_i \ln\left(\dfrac{\kappa + \varepsilon_i(t)/\varrho_i(t)}{1 - \varepsilon_i(t)/\varrho_i(t)}\right), & \text{for } \varepsilon_i(0) \geq 0 \\ a_i \ln\left(\dfrac{1 + \varepsilon_i(t)/\varrho_i(t)}{\kappa - \varepsilon_i(t)/\varrho_i(t)}\right), & \text{for } \varepsilon_i(0) \leq 0, \end{cases} \quad (19)$$

where $a_i > 0$, $i = 1, 2, 3$, are positive design constants, then the following properties hold:

(1) *The tracking error of the vessel is uniformly bounded.*

(2) *The predefined tracking performance of the vessel in the sense of (4) and (5) is guaranteed.*

Proof. See the appendix. □

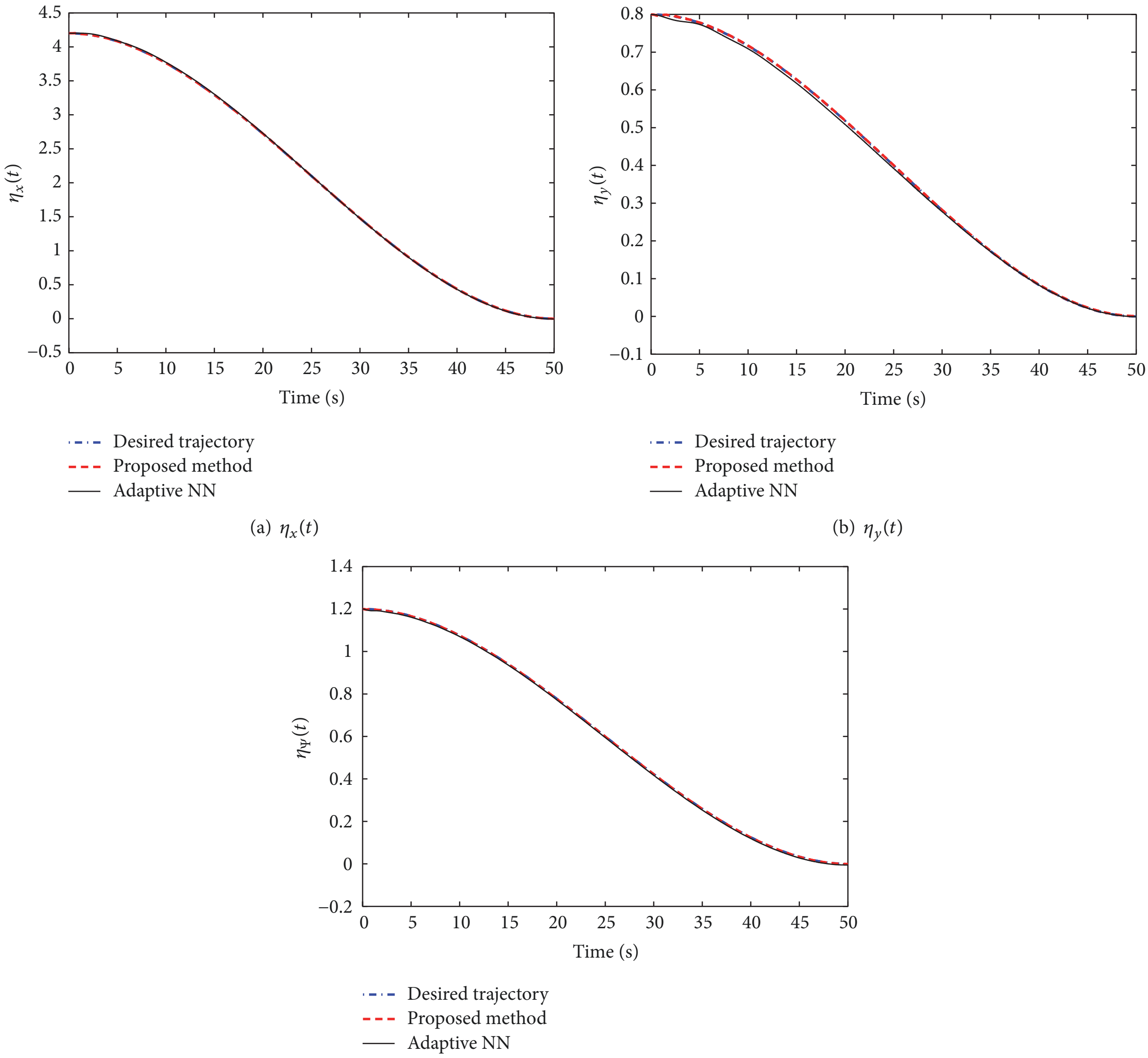
Table 1: Parameters of the target vessel.

Parameter	Description	Value
m	Total mass	23.8 kg
L_v	Length of vessel	1.255 m
B_v	Breadth of vessel	0.290 m
z_g	Position along Z of the center of gravity	0.046 m
I_{vz}	Moments of inertia along Z	1.76 kg·m^2
ρ_{air}	Density of air	1.29 kg/m^3
ρ_{water}	Density of water	1025 kg/m^3
g_v	Gravitational acceleration	9.8 m/s^2

4. Simulations

In this section, the parameters of a supply vessel from Norwegian University of Science and Technology [16] are adopted for simulation with a 1 : 70 scaling to test the performance of the proposed adaptive NN controller. The vessel parameters used are given in Table 1.

The objective of this work is to make the vessel track the given smooth, bounded reference trajectory with predefined bounds for tracking errors. In this section, two simulation scenarios with different desired trajectories are tested. The tracking performance of the proposed controller is also evaluated under two conditions: with the predefined performance bounds and without the predefined performance bounds.



(a) $\eta_x(t)$

(b) $\eta_y(t)$

(c) $\eta_\Psi(t)$

Figure 1: Comparison of tracking performance.

4.1. Scenario I. In this scenario, the reference trajectory is given as follows:

$$\eta_r(t, t_r) = \eta_0 + \left(-2.0\frac{t^3}{t_r^3} + 3.0\frac{t^2}{t_r^2}\right)(\eta_f - \eta_0), \tag{20}$$

where $\eta_0 = [4.2, 0.8, 1.2]^T$ is the initial of the vessel, $\eta_f = [0, 0, 0]^T$ is the final position, and $t_r = 50$ s represents the stopping time.

The parameters κ, $i = 1, 2, 3$ appearing in (4) are set to be 0.8. The parameters in exponential performance functions (5) are $\varrho_{i0} = 0.1$, $\varrho_{i\infty} = 0.01$, and $l_i = 5$, $i = 1, 2, 3$. Furthermore, the parameters in error transformations function (19) are chosen as $a_i = 0.008$, $i = 1, 2, 3$.

The inputs of NNs are $Z = [\eta^T, \nu^T, \xi^T, \dot{\xi}^T]$. The number of nodes is $l = 512$. The centers of the nodes are evenly distributed in $[-2, 2]$, $[-2, 2]$, $[-\pi, \pi]$, $[-0.1, 0.1]$, $[-3, 3]$, $[-3, 3]$. The variance of center of Gaussian radial basis function is 5. The initial weights are zeros. The control gain matrices in (17) are $\Upsilon = \mathrm{diag}[0.001, 0.001, 0.0001]$ and $\mu = \mathrm{diag}[0.001, 0.001, 0.0001]$. The Lyapunov gain matrices are tuned as $K_1 = K_2 = \mathrm{diag}[20, 20, 20]$.

For comparative studies, we compare our method with the adaptive NN control without prescribed performance bound. Figure 1 shows the positions and heading evolution. The tracking errors are illustrated in Figure 2. It is observed that the proposed controller has much better transient and steady-state performance while the tracking errors are within the predefined bounds as compared to the one without considering the predefined tracking error bounds. The norm of the tracking error is shown in Figure 3(a). From Figure 3(b), we can see that, during transient stage, the control

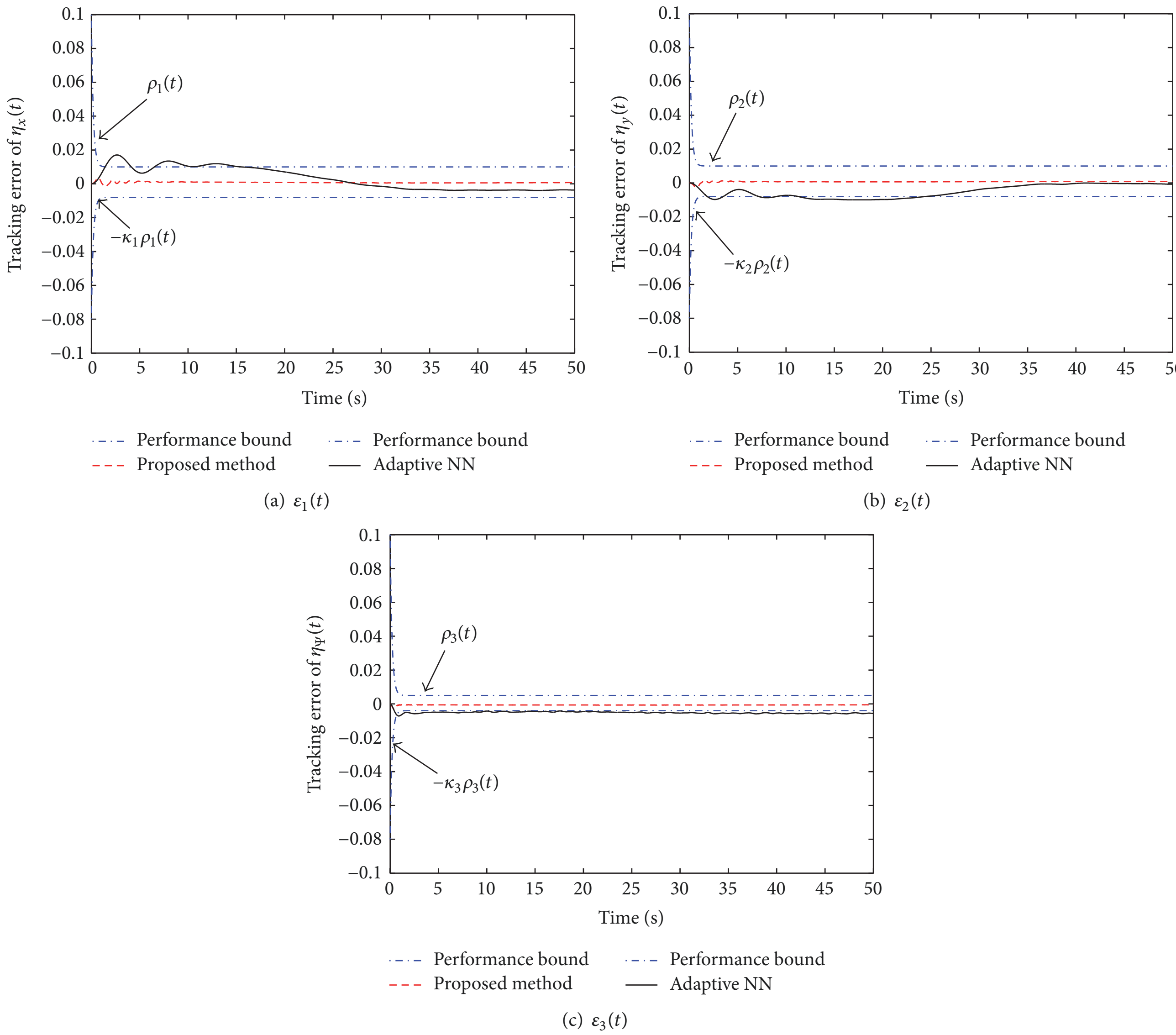


FIGURE 2: Tracking errors $\varepsilon(t)$ with respect to performance envelopes.

input of the controller without predefined error bounds oscillates seriously and requires a larger control effort, which results in larger overshoots and oscillations of the tracking performances as seen in Figure 2, whereas the control input of our proposed method is smooth. In addition, the proposed controller with predefined error bounds converges faster and the tracking errors stay strictly within the predefined bounds, while the one without considering the performance bounds cannot guarantee this.

4.2. Scenario II. In this scenario, the desired trajectory η_r is given by

$$\eta_r = \begin{bmatrix} 3\cos(0.06t + 0.63) \\ 2\sin(0.09t) \\ \pi + \pi\sin(0.13\pi t - 0.52) \end{bmatrix}, \tag{21}$$

where $\eta_0 = [2, 3, 2]^T$ is the payload initial position and $t_r = 200$ s represents the stopping time.

The parameters κ, $i = 1, 2, 3$ appearing in (4) are 0.7. The parameters in exponential performance functions (5) are $\varrho_0 = [2.2, 1.2, 0.63]^T$, $\varrho_{i\infty} = 0.01$, and $l_i = 0.45$, $i = 1, 2, 3$. Furthermore, the parameters in error transformations function (19) are chosen as $a_i = 0.01$, $i = 1, 2, 3$.

The inputs of NNs are $Z = [\eta^T, \nu^T, \xi^T, \dot{\xi}^T]$. The number of nodes is $l = 512$. The centers of the nodes are distributed in $[-2, 2]$, $[-2, 2]$, $[-\pi, \pi]$, $[-0.1, 0.1]$, $[-3, 3]$, $[-3, 3]$. The variance of center of Gaussian radial basis function is 5. The initial weights are zeros. The control gain matrices Υ and μ in (17) are chosen as $\Upsilon = \mathrm{diag}[0.001, 0.001, 0.0001]$ and $\mu = \mathrm{diag}[0.001, 0.001, 0.0001]$. The Lyapunov gain matrices are tuned as $K_1 = K_2 = \mathrm{diag}[50, 50, 50]$.

For comparative studies, we compare our controller to the one without considering the performance bounds.

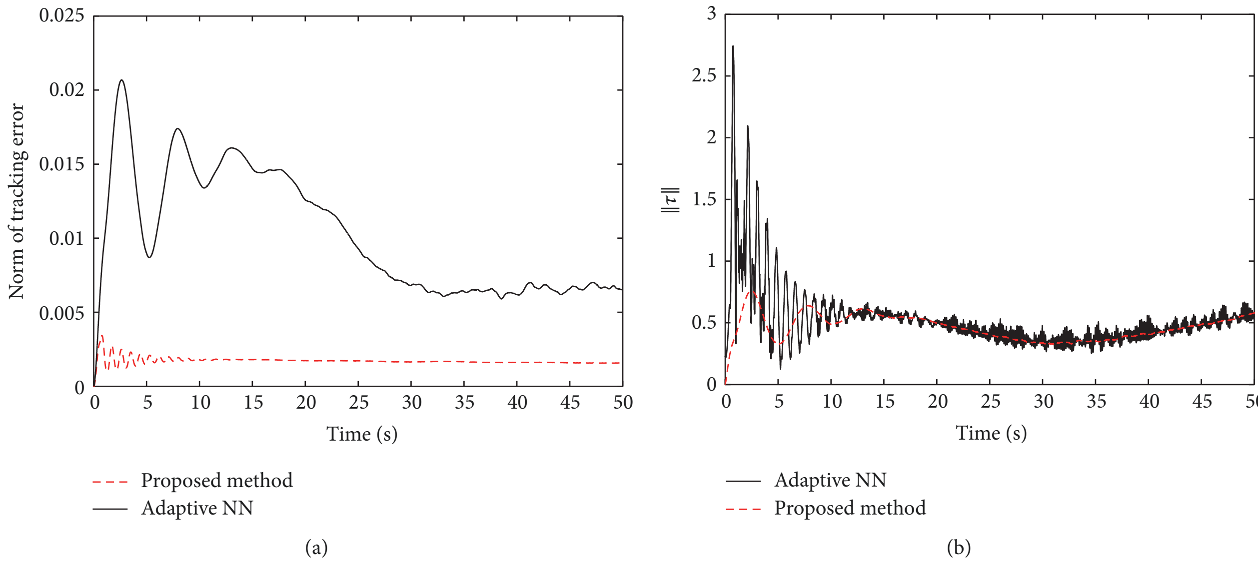


FIGURE 3: (a) $\|\varepsilon(t)\|$ and (b) $\|\tau\|$.

Figure 4 shows the positions and heading evolution; Figure 5 shows the tracking errors and Figure 6(a) shows the norm of the error vector. It can be seen that the proposed controller guarantees the tracking errors which are within the predefined bounds and produces better transient and steady-state performance as compared to the one without considering the performance bounds. From Figure 6(b), we can see that, during transient stage, the control input of the adaptive controller without considering the performance bounds oscillates seriously and requires a larger control effort compared with our proposed method, which results in larger overshoots and oscillations of the tracking performances as seen in Figure 5. Therefore, the proposed controller performs better than that without considering the performance bounds in both scenarios.

5. Conclusions

In this paper, the tracking control of the marine surface vessel is studied and a novel tracking controller with online modeling and predefined performance bounds was developed. The proposed control scheme can work with completely unknown vessel dynamic and unknown disturbances. Moreover, the tracking error was proven to be uniformly bounded. Simulation shows that the resulting tracking error stays strictly in the predefined bounds and the proposed controller performs better than that one without considering the predefined performance bounds.

Appendix

Proof of Theorem 1

Choose a Lyapunov function candidate with quadratic ξ_1,

$$V_1 = \frac{1}{2}\xi^T\xi. \tag{A.1}$$

According to (1), (11), (13), and $J(\eta)J^T(\eta) = I$, we obtain

$$\dot{\xi} = T_\partial J\left(\xi_a + \phi_1(t)\right) - T_\partial r, \tag{A.2}$$

$$\dot{V}_1 = \xi^T\left(T_\partial J\phi_1 - T_\partial r\right) + \xi^T T_\partial J\xi_a. \tag{A.3}$$

Consider the first term on the right-hand side of (A.3) and multiply both sides of (12) by J:

$$\begin{aligned}\xi^T\left(T_\partial J\phi_1 - T_\partial r\right) &= \xi^T T_\partial\left(J\phi_1 - r\right)\\ &= -\xi^T T_\partial JK_1\left(T_\partial J\right)^T\xi.\end{aligned} \tag{A.4}$$

Let $\omega = (T_\partial J)^T\xi$; the above equation can be written as $\xi^T(T_\partial J\phi_1 - T_\partial r) = -\omega^T K_1\omega \le 0$. And (A.3) can be rewritten as

$$\dot{V}_1 = -\omega^T K_1\omega + \xi^T T_\partial J\xi_a. \tag{A.5}$$

Since $T_\partial J$ is invertible, $\omega = (T_\partial J)^T\xi = 0$ implies that $\omega = 0$. Therefore, $\xi^T(T_\partial J\phi_1 - T_\partial r) = 0$ if and only if $\omega = 0$.

According to (13) and (2), differentiating ξ_a, we have

$$\begin{aligned}&\dot{\xi}_a\\ &= M^{-1}\left(-C(\nu)\nu - D(\nu)\nu - g(\eta) + \tau + \tau_d(\eta,\nu,t)\right)\\ &\quad - \dot{\phi}_1.\end{aligned} \tag{A.6}$$

Consider the following Lyapunov function candidate:

$$\overline{V} = V_1 + \frac{1}{2}\xi_a^T M\xi_a. \tag{A.7}$$

From (A.5) and (A.6), we have

$$\begin{aligned}\dot{\overline{V}} \le{}& -\omega^T k\omega + \xi^T T_\partial J\xi_a\\ &+ \xi_a^T\left(-C(\nu)\nu - D(\nu)\nu - g(\eta) - M\dot{\phi}_1 + \tau\right)\\ &+ \sum_{i=1}^{3}\left|\xi_{2,i}\right|\left(m_i(\eta,\nu) + n_i\right).\end{aligned} \tag{A.8}$$

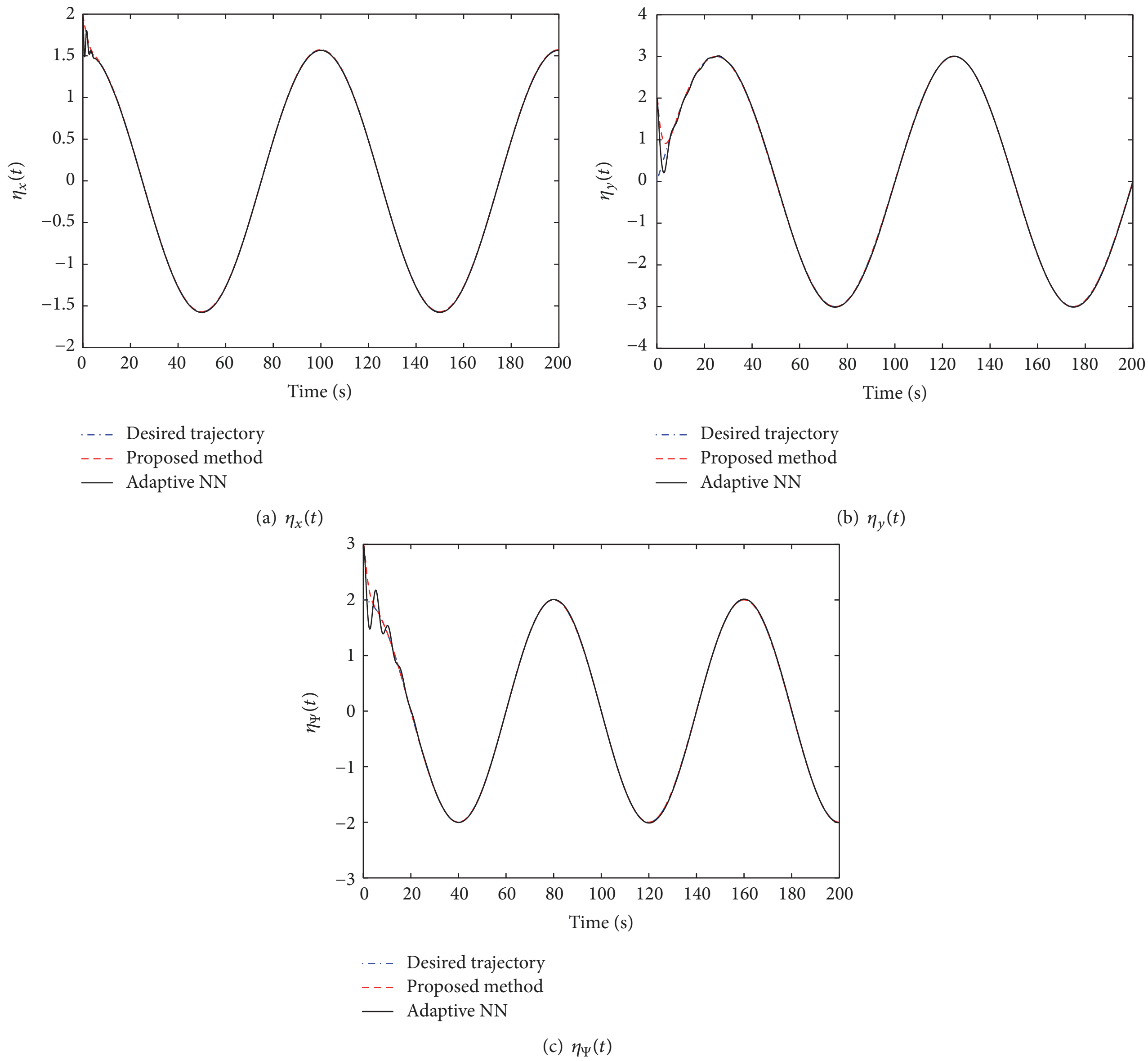


Figure 4: Comparison of tracking performance.

Substituting (14) into (A.8), the latter can be rewritten as $\dot{\overline{V}} \leq -\omega^T k\omega - \xi_a^T K_2 \xi_a$. Consider the augmented Lyapunov function candidate

$$V_2 = V_1 + \frac{1}{2}\xi_a^T M \xi_a + \sum_{i=1}^{3} \widetilde{W}_i^T \Upsilon_i^T \widetilde{W}_i^T, \tag{A.9}$$

where $\widetilde{W}_i = W_i - W_i^*$, W_i is NN estimated weight, W_i^* is the desired weight, and $\widetilde{W}_i$ is weight estimation error. Differentiating (A.9), we obtain

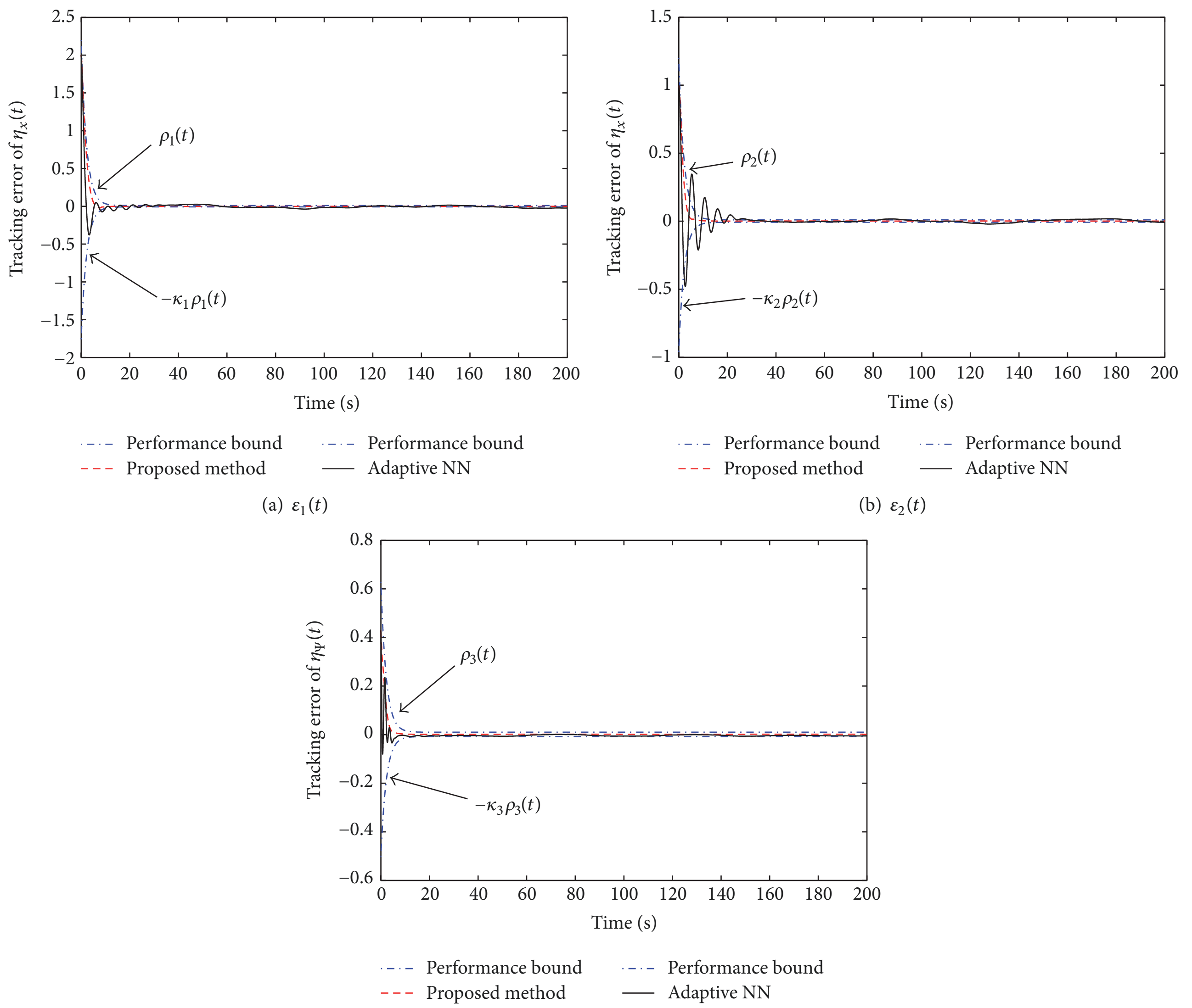
$$\begin{aligned}\dot{V}_2 \leq{}& -\omega^T K_1 \omega + \xi^T T_\partial J \xi_a + \xi_a^T \Bigg(-C(\nu)\nu - D(\nu)\nu \\ &+ \sum_{i=1}^{3} \widetilde{W}_i^T \Upsilon_i^T \widetilde{W}_i^T - g(\eta) - M\dot{\phi}_1 + \tau \Bigg) \\ &+ \sum_{i=1}^{3} \left|\xi_{2,i}\right| \left(p_i(\eta,\nu) + q_i\right) + \sum_{i=1}^{3} \mu_i \widetilde{W}_i^T \widehat{W}_i.\end{aligned} \tag{A.10}$$

Using approximation (16), we obtain

$$\begin{aligned}\dot{V}_2 \leq{}& -\omega^T K_1 \omega + \xi^T T_\partial J \xi_a \\ &+ \xi_a^T \left[-W^{*T} S(Z) - \epsilon(Z) + \tau \right] \\ &+ \sum_{i=1}^{3} \widetilde{W}_i^T \Upsilon_i^{-1} \dot{\widehat{W}}_i.\end{aligned} \tag{A.11}$$

Substituting the control law (16) and the weight update law (17), (A.11) is rewritten as

$$\begin{aligned}\dot{V}_2 \leq{}& -\omega^T K_1 \omega - \xi_a^T K_2 \xi_a + \frac{1}{2}\xi_a^T \xi_a + \frac{1}{2}\|\epsilon\|^2 \\ &- \sum_{i=1}^{3} \mu_i \widetilde{W}_i^T \widehat{W}_i\end{aligned}$$



(a) $\varepsilon_1(t)$

(b) $\varepsilon_2(t)$

(c) $\varepsilon_3(t)$

FIGURE 5: Tracking errors $\varepsilon(t)$ with respect to performance envelopes.

$$\begin{aligned} &\leq -\xi^T (T_\partial J) K_1 (T_\partial J)^T \xi - \xi_a^T K_2 \xi_a + \frac{1}{2}\xi_a^T \xi_a \\ &\quad + \frac{1}{2}\|\epsilon\|^2 - \sum_{i=1}^{3}\frac{\mu_i}{2}\left\|\widetilde{W}_i\right\|^2 + \sum_{i=1}^{3}\frac{\mu_i}{2}\left\|W_i^*\right\|^2, \end{aligned} \tag{A.12}$$

$$\dot{V}_2 < -\beta V_2 + \delta, \tag{A.13}$$

where

$$\begin{aligned} \beta &= \Bigg\{ 2\lambda_{\min}\left((T_\partial J) K_1 (T_\partial J)^T\right), \\ &\quad \frac{2\lambda_{\min}(K_2 - (1/2) I_{3\times3})}{\lambda_{\max}(M)}, \min_{i=1,2,3}\left(\frac{\mu_i}{\lambda_{\max}(\Upsilon_i^{-1})}\right)\Bigg\} \end{aligned} \tag{A.14}$$

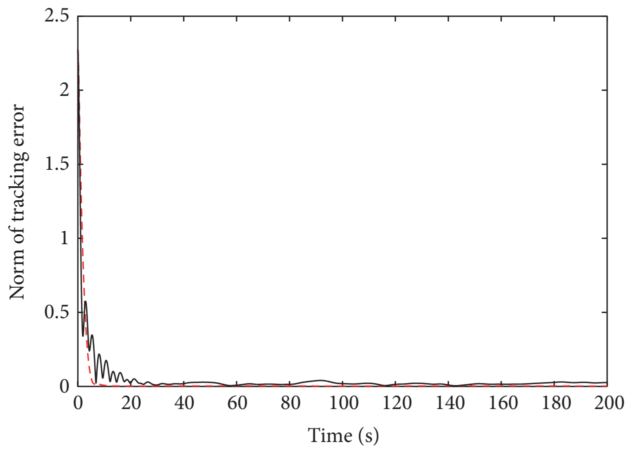
$$\delta = \frac{1}{2}\|\epsilon\|^2 + \sum_{i=1}^{3}\frac{\mu_i}{2}\left\|W_i^*\right\|^2,$$

where $\lambda_{\min}(\cdot)$ represents the minimum eigenvalue and $\lambda_{\max}(\cdot)$ is the maximum eigenvalue. The minimum eigenvalues of K_1 and $(K_2 - (1/2)I_{3\times3})$ should be positive in order to make $\beta > 0$.

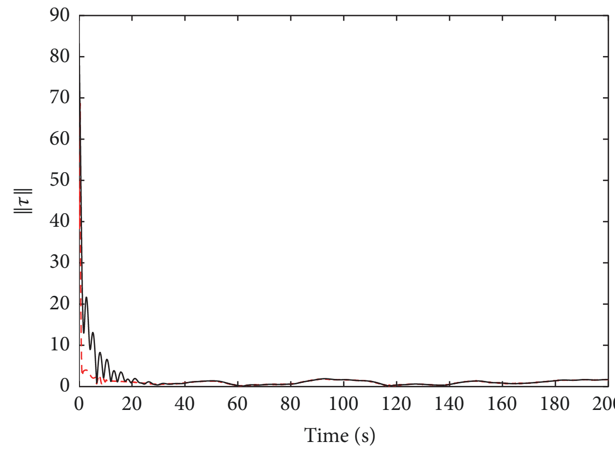
According to (A.13) and Lemma 1.2 in [17], it is straightforward to show that the signals ξ_1, ξ_2, $\widetilde{W}_1$, $\widetilde{W}_2$, and $\widetilde{W}_3$ are uniformly bounded. Since W_i^* is a constant, $\widehat{W}_i$ is also bounded, for $i = 1, 2, 3$. T_∂ is bounded by construction. Consequently, the control signal τ in (16) is bounded. The transformed error ξ_1 is proven to be bounded. The prescribed performance (4) of the tracking control system (2) is achieved. The boundedness of the solution of (8) guarantees the predefined performance bounds for the tracking error $e(t)$ introduced via (4) for all $t \geq 0$ [6, 13].

Conflicts of Interest

The authors declare that they have no conflicts of interest.



(a)



(b)

FIGURE 6: (a) $\|\varepsilon(t)\|$ and (b) $\|\tau\|$.

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