Research Article

On Leaderless and Leader-Following Consensus for Heterogeneous Nonlinear Multiagent Systems via Discontinuous Distributed Control Protocol

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This paper is concerned with consensus of heterogeneous nonlinear multiagent systems via distributed control. Both the cases of leaderless and leader-following are systematically investigated. Different from some existing results, completed consensus can be reached in this paper among heterogeneous multiagent network instead of bounded-consensus. First, a novel distributed control protocol is proposed, and some general consensus criterions are derived for multiagent systems without leader. Second, a leader with unknown but bounded input for the heterogeneous multiagent network is considered; aperiodically intermittent communications among followers are considered to avoid channel blocking in this case. Finally, two simulation examples are presented to verify the effectiveness of the main results.

1. Introduction

In recent years, distributed coordinated control of multiagent systems has been widely studied due to its easy implementation, strong robustness, and high self-organizability. There were many applications of multiagent systems in the field of robotic systems, UAVs (unmanned air vehicles), wireless sensor networks, and so on [1–4]. As a hot topic of multiagent systems, consensus has attracted great attention in systems and control theory. Many papers have been concerned with consensus problem of multiagent systems, in which, consensus can be reached among multiagent systems via sufficient local information exchanges between agents and their neighbors; one can see [5–11] and references therein.

There were two kinds of consensus named leaderless consensus and leader-following consensus, respectively. It is called leaderless consensus problem if there is no specified leader in the multiagent systems; it is called leader-following consensus problem otherwise. Both leaderless consensus and leader-following consensus have gotten many results recently. For example, distributed leaderless consensus algorithms for networked Euler-Lagrange systems have been investigated

in [12]. Reference [13] has studied leaderless consensus problem of a group of mobile agents interconnected by a star-like topology. On the other hand, leader-following consensus of multiagent systems has been considered in [14], in which both fixed and switching topologies have been considered. Based on M -matrix strategies, pinning-controlled leader-following consensus in multiagent systems has been discussed in [15]. More results about leaderless consensus or leader-following consensus can be seen in [16–20] and references therein. In general, the leader-following consensus problem could be translated into the stability problem of error systems. There were many theoretical tools and results about the analysis of the stability of dynamical systems [21–24]. To deal with the leaderless consensus problem, a virtual leader is often introduced or some matrix analysis techniques are used.

Note that the majority of above publications were concerned with identical systems. However, heterogeneous dynamic networks widely existed in real world, in which, each agent may have different parameters. A heterogeneous multiagent network cannot force consensus by static linear controllers. Thus, more results about heterogeneous dynamic networks were concerned with quasi-consensus (also named

bounded-consensus). For example, [25] investigates the bounded-consensus problem for cooperative heterogeneous agents with nonlinear dynamics in a directed communication network. Quasi-synchronization of heterogeneous dynamic networks via distributed impulsive control have been studied in [26]. More recently, quasi-synchronization is investigated for heterogeneous complex networks with switching sequentially disconnected topology in [27]. To the best of our knowledge, few literature works have appeared concerning the problems of complete consensus among heterogeneous multiagent systems, which provides the motivation of the current study. Note that nonlinearity is inescapable in real world systems [28–30]. The dynamics analysis of nonlinear systems has gotten many results [31–36], even to the nonlinear fractional order systems [37, 38]. Thus, nonlinear multiagent systems would be considered in this paper.

In this paper, we investigated both leaderless and leader-following consensus in a heterogenous network environment. The contributions of this paper are as follows: First, based a novel discontinuous distributed control protocol, the leaderless consensus has been reached under some simple conditions. Then, under the analysis method, the leader-following consensus also has been studied, in which case, the communication among followers is aperiodically intermittent. Most of the existing results about consensus for the heterogeneous multiagent systems have adopted the adaptive strategy; one can see [39–42] and references therein. Compared with some existing results, the control protocol in this paper is static, which may be easier to implement.

The rest of the paper is organized as follows: In Section 2, we introduce some definitions and some lemmas which are necessary for presenting our results in the following. The main results about leaderless consensus and leader-following consensus of heterogenous multiagents are presented in Section 3. Then, some examples are given to demonstrate the effectiveness of our results in Section 4. Conclusions are finally drawn in Section 5.

Notations. In this paper, $\mathbb{R}^n$ denotes the set of all n -dimensional column vectors. Let I_n be n -dimensional identity matrix. For any $x \in \mathbb{R}^n$, $\|x\|_1 = \sum_{i=1}^n |x_i|$, $\|x\| = \sqrt{\sum_{i=1}^n x_i^2}$, $\text{sign}(x) = [\text{sign}(x_1), \text{sign}(x_2), \dots, \text{sign}(x_n)]^T \in \mathbb{R}^n$, where $\text{sign}(x_i) = 1$ when $x_i > 0$, $\text{sign}(x_i) = -1$ when $x_i < 0$; otherwise $\text{sign}(x_i) = 0$. For a square matrix $A \in \mathbb{R}^{m \times n}$, let $\|A\| = \sqrt{\sum_{i=1}^m \sum_{j=1}^n a_{ij}^2}$, A^T be its transpose, and $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ be the maximum and minimum eigenvalues of A , respectively. $\otimes$ denotes Kronecker product.

2. Preliminaries

Considering the following heterogeneous multiagent system consisting of N agents, the dynamical behavior of i th agent is described as

$$\dot{x}_i(t) = A_i x_i(t) + B_i f(x_i(t)) + u_i(t), \quad (1)$$

where $x_i(t) \in \mathbb{R}^n$ denotes the state of i th agent, $f(x_i(t)) = [f_1(x_i(t)), f_2(x_i(t)), \dots, f_n(x_i(t))]^T \in \mathbb{R}^n$ is a nonlinear

function, $u_i(t)$ is control input would be given later, and A_i and B_i are constant matrix.

Throughout this paper, the following assumption of nonlinear function $f(\cdot)$ should be satisfied for the system.

Assumption 1. For the nonlinear function $f(\cdot)$, there exist constants $l_{ij}^f \geq 0$, $i, j = 1, 2, \dots, n$, such that, for any $x, y \in \mathbb{R}^n$,

$$|f_i(x) - f_i(y)| \leq \sum_{j=1}^n l_{ij}^f |x_j - y_j|. \quad (2)$$

Remark 2. Let $L_f = (l_{ij}^f)_{n \times n}$. Then, for any diagonal matrices $\Lambda_f > 0$, this assumption implies that $(x - y)^T L_f^T \Lambda_f L_f (x - y) \geq (f(x) - f(y))^T \Lambda_f (f(x) - f(y))$. And note that a lot of systems can be satisfied, such as Chua's circuit and some chaotic neural networks.

To get our main results, some preliminaries of graph would be given. Let $\mathcal{G} = \{\Delta, E, W\}$ be a undirected graph of order N , where $\Delta = \{v_1, v_2, \dots, v_N\}$, $E \subseteq \Delta \times \Delta$ denote the set of nodes and edges, respectively. For any i , $a_{ii} = 0$. $e_{ij} = (v_i, v_j) \in E$ is an edge from i to j , where $i \neq j$, which means that v_j can receive message from v_i . v_i is a neighbor of v_j if $e_{ij} \in E$. The set of all neighbors of v_i can be denoted as $\mathcal{N}_i = \{v_j : e_{ij} \in E, j = 1, 2, \dots, N\}$. $A = \{a_{ij}\} \in \mathbb{R}^{N \times N}$ denotes weighted adjacency matrix, where a_{ij} is weight which is satisfied $a_{ij} = a_{ji} \neq 0$ if $e_{ij} \in E$ and $a_{ij} = 0$ otherwise. We assume $a_{ii} = 0$ for all $i = 1, 2, \dots, N$.

Assumption 3. The topological structure is undirected in this paper, and the undirected communication graph is connected.

3. Main Results

3.1. Leaderless Consensus of Heterogeneous Multiagent System.

This subsection considers the leaderless case; without loss of generality, define the state error as $e_i(t) = x_{i+1}(t) - x_1(t)$, for $i = 1, 2, \dots, N - 1$; then, one has

$$\begin{aligned} \dot{e}_i(t) &= A_{i+1} x_{i+1}(t) - A_1 x_1(t) + B_{i+1} f(x_{i+1}(t)) \\ &\quad - B_1 f(x_1(t)) + u_{i+1}(t) - u_1(t). \end{aligned} \quad (3)$$

In this case, $u_i(t)$ is designed as

$$u_i(t) = -c \sum_{j=1}^N l_{ij} x_j(t) - c \hat{d}_i \text{sign}(x_i(t) - x_1(t)), \quad (4)$$

where the Laplacian matrix $L = (l_{ij})_{N \times N}$ is associated with the adjacency matrix $\mathcal{A} = (a_{ij})_{N \times N}$ defined by the following.

$$l_{ij} = \begin{cases} -a_{ij} & \text{for } i \neq j \\ \sum_{j=1, j \neq i}^N a_{ij} & \text{for } i = j \end{cases} \quad (5)$$

By some simple derivations, one can get the following error dynamic equation for $i = 1, 2, \dots, N-1$:

$$\begin{aligned} \dot{e}_i(t) = & A_{i+1}e_i(t) + B_{i+1}h(e_i(t)) + w_{i+1}(x_1(t)) \\ & - c \sum_{j=1}^N (l_{i+1,j} - l_{1j}) x_j(t) - c\hat{d}_{i+1} \text{sign}(e_i(t)), \end{aligned} \quad (6)$$

where $h(e_i(t)) = f(x_{i+1}(t)) - f(x_1(t))$, $w_i(x_1(t)) = (A_i - A_1)x_1(t) + (B_i - B_1)f(x_1(t))$. Let $\hat{L} = (\hat{l}_{ij})_{N \times N}$ and $\hat{l}_{ij} = l_{i+1,j+1} - l_{1,j+1}$ for $i, j = 1, 2, \dots, N-1$; one has $\sum_{j=1}^{N-1} \hat{l}_{ij} = l_{11} - l_{i+1,1}$ for $i = 1, 2, \dots, N-1$; then, we have $\sum_{j=1}^N (l_{i+1,j} - l_{1j})x_j(t) = \sum_{j=1}^{N-1} \hat{l}_{ij}e_j(t)$ and thus the error equation can be rewritten as

$$\begin{aligned} \dot{e}_i(t) = & A_{i+1}e_i(t) + B_{i+1}h(e_i(t)) + w_{i+1}(x_1(t)) \\ & - c \sum_{j=1}^{N-1} \hat{l}_{ij}e_j(t) - c\hat{d}_{i+1} \text{sign}(e_i(t)). \end{aligned} \quad (7)$$

The following assumption needs to be satisfied in this subsection.

Assumption 4. The $x_1(t)$ is bounded; i.e., for any initial $x_1(t_0)$, there exists T , such that $\|x_1(t)\| \leq \delta$, $t \geq T$, where δ is a positive constant. Furthermore, with Assumption 1, one has that $w_i(x_1(t))$ is also bounded; i.e., $\|w_i(x_1(t))\| \leq \varepsilon_i$, $t \geq T$, where ε_i is a positive constant.

Theorem 5. Under Assumptions 1, 3, and 4, the leaderless consensus of the heterogeneous multiagent system could be achieved if there exist positive definite diagonal matrix P and constants $\alpha > 0$, $\beta > 0$ such that

$$\hat{d}_{i+1} \geq \frac{\|P\|(\varepsilon_{i+1})}{c\lambda_{\min}(P)}; \quad (8)$$

$$\begin{bmatrix} PA_{i+1} + A_{i+1}^T P + L_f^T \Lambda_f L_f & PB_{i+1} \\ PB_{i+1} & -\Lambda_f \end{bmatrix} < \begin{bmatrix} \alpha P & 0 \\ 0 & 0 \end{bmatrix}; \quad (9)$$

$$(\alpha + \beta) I_{N-1} < 2c\hat{L}; \quad (10)$$

for $i = 1, 2, \dots, N-1$, where matrices Λ_f and L_f have been mentioned in Remark 2.

Proof. Choose a Lyapunov function as $V(t) = (1/2) \sum_{i=1}^{N-1} e_i^T(t) P e_i(t)$; taking the time derivative of $V(t)$ along the trajectories of (7), one obtains the following.

$$\begin{aligned} \dot{V}(t) = & \sum_{i=1}^{N-1} e_i^T(t) P \dot{e}_i(t) = \sum_{i=1}^{N-1} e_i^T(t) P \left\{ A_{i+1}e_i(t) \right. \\ & + B_{i+1}h(e_i(t)) + w_{i+1}(x_1(t)) - c \sum_{j=1}^{N-1} \hat{l}_{ij}e_j(t) \\ & \left. - c\hat{d}_{i+1} \text{sign}(e_i(t)) \right\} \leq \frac{1}{2} \end{aligned}$$

$$\cdot \sum_{i=1}^{N-1} e_i^T(t) (PA_{i+1} + A_{i+1}^T P) e_i(t)$$

$$\begin{aligned} & + \sum_{i=1}^{N-1} e_i^T(t) PB_{i+1}h(e_i(t)) \\ & + \sum_{i=1}^{N-1} e_i^T(t) Pw_{i+1}(x_1(t)) - c \sum_{i=1}^{N-1} e_i^T(t) P \sum_{j=1}^{N-1} \hat{l}_{ij}e_j(t) \\ & - c \sum_{i=1}^{N-1} \hat{d}_{i+1} e_i^T(t) P \text{sign}(e_i(t)) \end{aligned} \quad (11)$$

Based on Cauchy inequality (i.e., $(\sum_{i=1}^n a_k b_k)^2 \leq \sum_{i=1}^n a_k^2 \sum_{i=1}^n b_k^2$) and the definition of the function $\text{sign}(\cdot)$, noting that P is a positive definite diagonal matrix, it is easy to get

$$\begin{aligned} -c\hat{d}_{i+1} e_i^T(t) P \text{sign}(e_i(t)) &= -c\hat{d}_{i+1} \sum_{j=1}^n p_j |e_{ij}(t)| \\ &\leq -c\hat{d}_{i+1} \lambda_{\min}(P) \|e_i(t)\|_1 \end{aligned} \quad (12)$$

$$\leq -c\hat{d}_{i+1} \lambda_{\min}(P) \|e_i(t)\|,$$

$$e_i^T(t) Pw_{i+1}(x_1(t)) \leq \varepsilon_{i+1} \|P\| \|e_i(t)\|.$$

According to Remark 2 and (12), we have

$$\begin{aligned} \dot{V}(t) \leq & \frac{1}{2} \\ & \cdot \sum_{i=1}^{N-1} e_i^T(t) (PA_{i+1} + A_{i+1}^T P + L_f^T \Lambda_f L_f) e_i(t) - \frac{1}{2} \\ & \cdot \sum_{i=1}^{N-1} h^T(e_i(t)) \Lambda_f h(e_i(t)) \\ & + \sum_{i=1}^{N-1} e_i^T(t) PB_{i+1}h(e_i(t)) \\ & + \sum_{i=1}^{N-1} \left(\varepsilon_{i+1} - c\hat{d}_{i+1} \frac{\lambda_{\min}(P)}{\|P\|} \right) \|P\| \|e_i(t)\| - ce^T(t) \quad (13) \\ & \cdot (\hat{L} \otimes P) e(t) = \frac{1}{2} \sum_{i=1}^{N-1} [e_i^T(t), h^T(e_i(t))] \\ & \cdot \begin{bmatrix} PA_{i+1} + A_{i+1}^T P + L_f^T \Lambda_f L_f & PB_{i+1} \\ PB_{i+1} & -\Lambda_f \end{bmatrix} \begin{bmatrix} e_i(t) \\ h(e_i(t)) \end{bmatrix} \\ & + \sum_{i=1}^{N-1} \left(\varepsilon_{i+1} - c\hat{d}_{i+1} \frac{\lambda_{\min}(P)}{\|P\|} \right) \|P\| \|e_i(t)\| - ce^T(t) \\ & \cdot (\hat{L} \otimes P) e(t), \end{aligned}$$

where $e(t) = [e_1^T(t), e_2^T(t), \dots, e_{N-1}^T(t)]^T$. Then, according to (8), (9), and (10), we have

$$\dot{V}(t) \leq e^T(t) \left(\left(\frac{\alpha}{2} I_{N-1} - c\hat{L} \right) \otimes P \right) e(t) \leq -\beta V(t). \quad (14)$$

This completes the proof. $\square$

3.2. Leader-Following Consensus with Aperiodically Intermittent Communication. The leader with unknown input is described as

$$\dot{x}_0(t) = A_0 x_0(t) + B_0 f(x_0(t)) + u_0(t), \quad (15)$$

where $u_0(t)$ is unknown input. $u_i(t)$ in this case is designed as

$$u_i(t) = \begin{cases} -c \sum_{j=1}^N l_{ij} x_j(t) - c d_i (x_i(t) - x_0(t)) - c \hat{d}_i \text{sign}(x_i(t) - x_0(t)) & \text{for } t \in [t_k, s_k] \\ -c \hat{d}_i \text{sign}(x_i(t) - x_0(t)) & \text{for } t \in (s_k, t_{k+1}) \end{cases} \quad (16)$$

where $0 = t_0 \leq s_0 \leq t_1 \leq s_1 \leq \dots \leq t_k \leq s_k \leq t_{k+1} \leq \dots$; time interval $[t_k, s_k]$ is called the communication time, i.e., every agent can interact with its neighbors in this time interval, while (s_k, t_{k+1}) is called rest time, in which, every agent can communicate only with

leader but not its neighbors. Denote by $c_k = s_k - t_k$ and $r_k = t_{k+1} - s_k$ the communication width and the rest width, respectively. Let error signal $e_i(t) = x_i(t) - s(t)$; by some simple derivation, one can get the following error dynamic equation:

$$\dot{e}_i(t) = \begin{cases} A_i e_i(t) + B_i g(e_i(t)) + w_i(s(t)) - u_0(t) - c \sum_{j=1}^N l_{ij} e_j(t) - c d_i (g_i(t)) - c \hat{d}_i \text{sign}(e_i(t)) & \text{for } t \in [t_k, s_k] \\ A_i e_i(t) + B_i g(e_i(t)) + w_i(s(t)) - u_0(t) - c \hat{d}_i \text{sign}(e_i(t)) & \text{for } t \in (s_k, t_{k+1}) \end{cases} \quad (17)$$

where $g(e_i(t)) = f(x_i(t)) - f(s(t))$, $w_i(s(t)) = (A_i - A_0)s(t) + (B_i - B_0)f(s(t))$.

Assumption 6. The $s(t)$ is bounded; i.e., for any initial $s(t_0)$, there exists T , such that $\|s(t)\| \leq \delta$, $t \geq T$, where δ is a positive constant. That is, $s(t)$ also could be an equilibrium point, a periodic orbit, or even a chaotic orbit throughout this paper. Furthermore, with Assumption 1, one has that $w_i(s(t))$ is also bounded; i.e., $\|w_i(s(t))\| \leq \varepsilon_i$, $t \geq T$, where ε_i is a positive constant.

Assumption 7. The unknown input of leader $u_0(t)$ in this paper is assumed bounded; let $\|u_0(t)\| \leq \varepsilon_0$.

Remark 8. There were many results about heterogeneous dynamic networks recently [1]; however, most of which have investigated quasi-synchronization or bounded-consensus of them. This paper would study the complete consensus for a heterogeneous multiagent network; furthermore, the leader in this paper has an unknown but bounded input, which has not been seen yet. The results in this paper can be applied for tracking an unknown target by a heterogeneous multiagent network. Note that communications among agents and their neighbors are aperiodic intermittent, which could prevent blocking the communication channel.

Theorem 9. Under Assumptions 1, 3, 6, and 7, consensus of the heterogeneous multiagent system could be achieved if there exist positive definite diagonal matrix P and constants $\alpha > 0$, $\beta > 0$ such that

$$\hat{d}_i \geq \frac{\|P\|(\varepsilon_0 + \varepsilon_i)}{c \lambda_{\min}(P)}; \quad (18)$$

$$\begin{bmatrix} PA_i + A_i^T P + L_f^T \Lambda_f L_f & PB_i \\ PB_i & -\Lambda_f \end{bmatrix} < \begin{bmatrix} \alpha P & 0 \\ 0 & 0 \end{bmatrix}; \quad (19)$$

$$(\alpha + \beta) I_N < 2c(L + D); \quad (20)$$

$$\lim_{k \rightarrow +\infty} \sum_{m=0}^{k-1} (\beta c_m - \alpha r_m) = +\infty; \quad (21)$$

where $D = \text{diag}\{d_1, d_2, \dots, d_N\}$ and $c_k = s_k - t_k$, $r_k = t_{k+1} - s_k$, $k = 0, 1, 2, \dots$, matrices Λ_f and L_f have been mentioned in Remark 2.

Proof. Choose a Lyapunov function as $V(t) = (1/2) \sum_{i=1}^N e_i^T(t) P e_i(t)$; for $t \in [t_k, s_k]$, $k = 0, 1, 2, \dots$, taking the time derivative of $V(t)$ along the trajectories of (17), one obtains the following.

$$\begin{aligned} \dot{V}(t) &= \sum_{i=1}^N e_i^T(t) P \dot{e}_i(t) = \sum_{i=1}^N e_i^T(t) P \left\{ A_i e_i(t) + B_i g(e_i(t)) + w_i(s(t)) - u_0(t) - c \sum_{j=1}^N l_{ij} e_j(t) - c d_i (g_i(t)) - c \hat{d}_i \text{sign}(e_i(t)) \right\} \\ &= \frac{1}{2} \cdot \sum_{i=1}^N e_i^T(t) (PA_i + A_i^T P) e_i(t) \\ &\quad + \sum_{i=1}^N e_i^T(t) PB_i g(e_i(t)) + \sum_{i=1}^N e_i^T(t) P w_i(s(t)) \\ &\quad + \sum_{i=1}^N e_i^T(t) P u_0(t) - c \sum_{i=1}^N \hat{d}_i e_i^T(t) P \text{sign}(e_i(t)) \\ &\quad - c \sum_{i=1}^N e_i^T(t) P \sum_{j=1}^N l_{ij} e_j(t) - c d_i \sum_{i=1}^N e_i^T(t) P e_i(t) \end{aligned} \quad (22)$$

Similar to the proof of Theorem 5, one has

$$\begin{aligned} e_i^T(t) P w_i(s(t)) &\leq \varepsilon_i \|P\| \|e_i(t)\|, \\ e_i^T(t) P u_0(t) &\leq \varepsilon_0 \|P\| \|e_i(t)\| \\ -c\hat{d}_i e_i^T(t) P \operatorname{sign}(e_i(t)) &\leq -c\hat{d}_i \lambda_{\min}(P) \|e_i(t)\|. \end{aligned} \quad (23)$$

Then, from (23), (18), (19), and (20) and Remark 2, we have

$$\dot{V}(t) \leq -\beta V(t), \quad t \in [t_k, s_k]. \quad (24)$$

On the other hand, when $t \in (s_k, t_{k+1})$, $k = 0, 1, 2, \dots$, taking the time derivative of $V(t)$ along the trajectories of (17), similarly, one can obtain

$$\dot{V}(t) \leq \alpha V(t), \quad t \in (s_k, t_{k+1}). \quad (25)$$

Now, we estimate $V(t)$ based on the above two inequations.

For $t \in [t_0, s_0]$, it is easy to get $V(t) \leq V(t_0)e^{-\beta(t-t_0)}$ and $V(s_0) \leq V(t_0)e^{-\beta c_0}$.

For $t \in (s_0, t_1)$, it is easy to get $V(t) \leq V(s_0)e^{\alpha(t-s_0)} \leq V(t_0)e^{-\beta c_0 + \alpha(t-s_0)}$ and $V(t_1) \leq V(t_0)e^{-\beta c_0 + \alpha r_0}$.

For $t \in [t_1, s_1]$, one has $V(t) \leq V(t_1)e^{-\beta(t-t_1)} \leq V(t_0)e^{-\beta c_0 + \alpha r_0 - \beta(t-t_1)}$ and $V(s_1) \leq V(t_0)e^{-\beta(c_0+c_1)+\alpha r_0}$.

For $t \in (s_1, t_2)$, one has $V(t) \leq V(s_1)e^{\alpha(t-s_1)} \leq V(t_0)e^{-\beta(c_0+c_1)+\alpha r_0 + \alpha(t-s_1)}$ and $V(t_2) \leq V(t_0)e^{-\beta(c_0+c_1)+\alpha(r_0+r_1)}$.

Generally, when $t \in [t_k, s_k]$,

$$\begin{aligned} V(t) &\leq V(t_0) e^{-\beta \sum_{m=0}^{k-1} c_m + \alpha \sum_{m=0}^{k-1} r_m - \beta(t-t_k)} \\ &\leq V(t_0) e^{-\sum_{m=0}^{k-1} (\beta c_m - \alpha r_m)}, \end{aligned} \quad (26)$$

and when $t \in (s_k, t_{k+1})$,

$$\begin{aligned} V(t) &\leq V(t_0) e^{-\beta \sum_{m=0}^k c_m + \alpha \sum_{m=0}^{k-1} r_m - \alpha(t-s_k)} \\ &\leq V(t_0) e^{-\sum_{m=0}^{k-1} (\beta c_m - \alpha r_m) - (\alpha + \beta)c_k} \\ &\leq V(t_0) e^{-\sum_{m=0}^{k-1} (\beta c_m - \alpha r_m)}. \end{aligned} \quad (27)$$

Thus, for any $t > 0$, we have $V(t) \leq V(t_0)e^{-\sum_{m=0}^{k-1} (\beta c_m - \alpha r_m)}$, combined with (21), $\lim_{t \rightarrow +\infty} V(t) = 0$, which implies that $\lim_{t \rightarrow +\infty} \|e_i(t)\| = 0$; i.e., the consensus could be achieved; this completes our proof. $\square$

Note that condition (21) in Theorem 5 is difficult to check due to its infinity. The following results could be checked easily based on a higher request for the intermittent communication.

Corollary 10. *Under Assumptions 1, 3, 6, and 7, consensus of the heterogeneous multiagent system could be achieved if there exist positive definite diagonal matrix P and constants $\alpha > 0$, $\beta > 0$ such that (18)-(20) and the following condition holds:*

$$\frac{\check{c}}{\hat{T}} > \frac{\alpha}{\alpha + \beta}, \quad (28)$$

where $\check{c} = \min\{c_1, c_2, \dots\}$, $\hat{T} = \min\{t_1 - t_0, t_2 - t_1, \dots\}$.

Proof. It is obviously that (29) implies (21) based on the definition of notations $\check{c}$ and $\hat{T}$. Consequently, the result can be obtained. $\square$

The periodically intermittent communication is obviously a special case of Theorem 5; in periodical case, let $T \equiv t_{k+1} - t_k$, and $\hat{c} \equiv s_k - t_k$ for any $k = 0, 1, 2, \dots$; the following corollary could be obtained naturally.

Corollary 11. *Under Assumptions 1, 3, 6, and 7, consensus of the heterogeneous multiagent system could be achieved if there exist positive definite diagonal matrix P and constants $\alpha > 0$, $\beta > 0$ such that (18)-(20) and the following condition holds:*

$$\frac{\hat{c}}{T} > \frac{\alpha}{\alpha + \beta}. \quad (29)$$

4. Numerical Simulations

In this section, two examples are given to check our theorem results above.

4.1. Leaderless Consensus. In this subsection, we have a multiagent system consisting of four heterogeneous agents. The Laplacian matrix L is given as

$$L = \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ 0 & -1 & -1 & 2 \end{bmatrix}. \quad (30)$$

In this example, let $n = 3$, $x_i(t) = [x_{i1}(t), x_{i2}(t), x_{i3}(t)]^T$, $f_j(x_{ij}(t)) = 0.5(|x_{ij}(t) + 1| - |x_{ij}(t) - 1|)$, $i = 1, 2, 3, 4$. The matrices A_i and B_i are given as

$$A_1 = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -0.2 & 0 & 0 \\ 0 & -0.2 & 0 \\ 0 & 0 & -0.2 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} -0.1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix},$$

$$A_4 = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 1.25 & -3.2 & -3.2 \\ -3.2 & 1.1 & -4.4 \\ -3.2 & 4.4 & 1 \end{bmatrix},$$

FIGURE 1: Trajectory of the $[x_{11}(t), x_{12}(t), x_{13}(t)]$ with initial values $[0.1, 0.1, 0.1]$.

FIGURE 3: Trajectory of the $[x_{31}(t), x_{32}(t), x_{33}(t)]$ with initial values $[6, -2, 12]$.

FIGURE 2: Trajectory of the $[x_{21}(t), x_{22}(t), x_{23}(t)]$ with initial values $[4, -6, 8]$.

FIGURE 4: Trajectory of the $[x_{41}(t), x_{42}(t), x_{43}(t)]$ with initial values $[2, -4, 10]$.

$$B_2 = \begin{bmatrix} 1.25 & 3 & -3.2 \\ -3.2 & 1.1 & -4.4 \\ -3.2 & 4.4 & 1 \end{bmatrix},$$

$$B_3 = \begin{bmatrix} 1.5 & -2.2 & -3 \\ -3.2 & 1 & -4.4 \\ -3.2 & 4.4 & 1 \end{bmatrix},$$

$$B_4 = \begin{bmatrix} -1.25 & -3.2 & -3 \\ -3.2 & 1 & -4.4 \\ -1 & 4.4 & 1 \end{bmatrix}.$$

(31)

Without control, the trajectories of these four agents are shown as Figures 1–4 with corresponding initial values. It is obviously that they have different dynamics, the first agent has a chaotic behavior, the second agent may have a convergent trajectory, the third agent has a periodic orbit, etc. According to the simulation, we have $\varepsilon_2 = 5.7084$, $\varepsilon_3 = 2.5655$, $\varepsilon_4 = 7.6434$, and based on the analysis of Section 3.1, the $\hat{L}$ could be obtained as

$$\hat{L} = \begin{bmatrix} 4 & 0 & -1 \\ 0 & 4 & -1 \\ 0 & 0 & 2 \end{bmatrix}. \quad (32)$$

In order to reach consensus for the heterogeneous multiagent system, let $c = 10$. The L_f can be selected as I_3 . To solve (9) and (10), we set $\alpha = 20$; then, by using MATLAB LMI Toolbox one has $\beta = 9.8989$ and $P = 0.3136I_3$. By some simple computations about (8), $\hat{d}_i$ can be selected as $\hat{d}_1 = 0.6$, $\hat{d}_2 = 0.3$, $\hat{d}_3 = 0.8$. Then, all conditions of Theorem 5 can be satisfied. Figures 5 and 6 give the simulations for the state variables $x_{ij}(t)$ and error variables $e_{ij}(t) = x_{i+1,j}(t) - x_{1j}(t)$, respectively.

4.2. Leader-Following Consensus. In this example, the coefficient matrices A , B , A_i , B_i are given as

$$A = \begin{bmatrix} -2.5 & 10 & 0 \\ 1 & -1 & 1 \\ 0 & -18 & 0 \end{bmatrix},$$

$$A_1 = \begin{bmatrix} -2.5 & 10 & 0 \\ 1 & -1 & 1 \\ 0 & -18 & -0.5 \end{bmatrix},$$

FIGURE 5: Time evolution of state variables of four heterogeneous agents with control.

FIGURE 6: Time evolution of error variables $e_i(t)$ with control.

$$A_2 = \begin{bmatrix} -2.5 & 10 & 0 \\ 1 & 1 & 1 \\ 0 & -18 & 0.3 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} -2.6 & 10 & 0 \\ 1 & -0.9 & 1 \\ 0 & -23 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} \frac{35}{6} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} \frac{35}{6} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

FIGURE 7: Trajectory of the $[s_1(t), s_2(t), s_3(t)]$ with initial values $[0.1, 0.1, 0.1]$.

FIGURE 8: Trajectory of the $[x_{11}(t), x_{12}(t), x_{13}(t)]$ with initial values $[4, -2, 1]$.

$$B_3 = \begin{bmatrix} \frac{29}{6} & 0 & 0 \\ 0 & 0 & 0 \\ 0.1 & 0 & 0 \end{bmatrix},$$

$$B_4 = \begin{bmatrix} \frac{35}{6} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

(33)

Meanwhile, let $n = 3$, $x_i(t) = [x_{i1}(t), x_{i2}(t), x_{i3}(t)]^T$, $f_j(x_{ij}(t)) = 0.5(|x_{ij}(t) + 1| - |x_{ij}(t) - 1|)$, $i = 1, 2, 3$, $j = 1, 2, 3$. Without control, the different trajectories of the leader and three followers are shown as Figures 7–10 with corresponding initial values. According to the simulation, we have $\varepsilon_1 = 3.1291$, $\varepsilon_2 = 2.0604$, $\varepsilon_3 = 4.0596$. Assume that the network is connected and the Laplacian matrix L is

$$L = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}. \quad (34)$$

Let $D = \text{diag}\{2, 4, 6\}$ and $u_0(t) = 1.1 \sin(t)$. Then, choosing $\alpha = 30$, $c = 20$ and solving (19) and (20) yield $\beta =$

FIGURE 9: Trajectory of the $[x_{21}(t), x_{22}(t), x_{23}(t)]$ with initial values $[3, 2, -1]$.

FIGURE 10: Trajectory of the $[x_{31}(t), x_{32}(t), x_{33}(t)]$ with initial values $[-1, 2, 5]$.

31.7071 and $P = 0.0048I_3$. From (18), one can let $\hat{d}_1 = 0.25$, $\hat{d}_2 = 0.36$, $\hat{d}_3 = 0.37$. For the sake of convenience, in this simulation, the period intermittent strategy will be considered. By Corollary 11, one can choose $\hat{c}/T = 0.7$. Thus, one can conclude that the leader-follower consensus can be achieved according to Corollary 11. Figures 11 and 12 give the simulations for the state variables $x_{ij}(t)$, $s_j(t)$ and error variables $e_{ij}(t) = x_{ij}(t) - s_j(t)$, respectively.

5. Conclusion

The leaderless consensus and leader-follower consensus of heterogeneous multiagent network have been studied in this paper. By utilizing a discontinuous communication protocol, leaderless consensus criterions formed as LMIs have been derived at first. Then, an unknown leader has been considered; under our discontinuous control protocol, the consensus could be obtained based on some conditions. The results in this paper could be applied to tracking control problem for an unknown target. In the leader-follower case, the communication among followers was aperiodically intermittent, which could prevent blocking the ways of signal transmission. Finally, simulation results have also been given to check the obtained theory results. Noting that this paper just considered an undirected topology of the network, future

FIGURE 11: Time evolution of state variables of leader and three followers with control.

FIGURE 12: Time evolution of error variables $e_i(t)$ with control.

works include the study of directed structure and even switching topologies, which are more suitable to the real world. Time-delays are also inescapable in real world systems [16, 43–45]; however, this paper has not considered the time-delay, which would be a significant topic in our future work.

Data Availability

No data have been used to support this study.

Disclosure

The numerical simulation part was performed using the MATLAB software.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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