Hindawi
Mathematical Problems in Engineering
Volume 2018, Article ID 2947643, 13 pages
https://doi.org/10.1155/2018/2947643

Research Article

Couple-Group Consensus for Multiagent Systems via Time-Dependent Event-Triggered Control

Huiyang Liu [1] and Xiaoshuang Wang [2]

[1] *School of Mathematics and Physics, University of Science and Technology Beijing, Beijing, 100083, China*
[2] *School of Control and Computer Engineering, North China Electric Power University, Beijing 102206, China*

Correspondence should be addressed to Huiyang Liu; huiyangliu@163.com

Received 17 May 2018; Accepted 13 August 2018; Published 9 September 2018

Academic Editor: Xian-Ming Zhang

This paper investigates couple-group consensus problems for multiagent first-order and second-order systems. Several consensus protocols are proposed based on the time-dependent distributed event-triggered control. For the case of no communication delays, the time-dependent event-triggered strategies are applied to couple-group consensus problems. Based on the matrix theory, algebraic conditions for couple-group consensus are established. For the system with communication delays, based on event-triggered strategies, a Lyapunov-Krasovskii functional is constructed to prove the input-to-state stability of the systems. Moreover, Zeno behavior is excluded. Finally, numeral examples are given to illustrate the effectiveness of these results.

1. Introduction

In recent years, many researchers have paid attention to cooperative control of multiagent systems. This is because of the wide applications, such as formation control of satellite clusters, unmanned air vehicles, flocking, and swarming [1–5]. The objective in coordination control is to design appropriate control protocols such that the agents in a multiagent team can achieve a coordination objective asymptotically by using exchanged information between each other. The consensus problem is one type of the critical coordination control problems in the field of multiagent systems. It is simple in mathematics but represents a lot of real engineering systems, e.g., the attitude consensus and velocity consensus. Therefore, many researchers have focused on this topic and there emerge in large numbers of literatures [6–13].

In practical engineering applications, every autonomous agent is equipped with a microprocesser, which has limited resources and energy, to collect and process information. To reduce data transmission and save energy, event-triggered control strategies have been employed. A proper designed event-triggered strategy can also serve the desired properties of the ideal system, such as stability and convergence. Up to now, many theoretical results have been derived on event-triggered consensus problems by combining algebraic graph theory with matrix theory. The authors of [14] proposed event-triggered control strategies for first-order dynamics. A centralized formulation was considered, and then the results were extended to the decentralized counterparts. Some further results on event-based control for first-order dynamics were obtained in [15], including the analysis of discrete systems and continuous systems with high dimensions. The event-triggered tracking control protocol for first-order dynamics was proposed in [16]. The input-to-state stability (ISS) of the closed-loop multiagent system was analyzed by employing an ISS Lyapunov function. The consensus problem for second-order dynamics via event-triggered control was addressed in [17], where a centralized event-triggered strategy was designed, and the bound of interevent times was ensured. To overcome the limitation of the state-dependent event-triggered control to its practical implementation, a modified version of the event-based consensus scheme was proposed in [18]. A combinational measurement approach to designing the event-triggered scheme was developed in [19]. Among these literatures mentioned above, the data transmission only occurs when the measurement error exceeds the state-dependent threshold. However, the authors of [20, 21] designed event-triggered protocols where the event occurs when measurement error exceeds the time-dependent threshold. Moreover, in [22], flocking of multiagent systems with multiple groups was studied in fixed and switching heterogeneous networks. In [23], an event-triggered based protocol was designed for first-order discrete-time multiagent systems with time-varying topology. In [24], a distributed sampled-data based event-triggered consensus protocol was proposed. In [25], based on weighted average consensus, distributed event-triggered cubature information filtering was studied. There are a lot of results on the event-triggered consensus scheme reported in [26–32]. For more literatures, please refer to the survey paper [33].

To our best knowledge, all the aforementioned literatures have focused on consensus problems where all the agents can reach an agreement, even a dynamic agreement. In practical engineering applications, there is a phenomenon that the agreements are changed with the varying of environment and cooperative tasks. A novel consensus control strategy is introduced to multiagent systems, where agents in the same subnetwork reach a consistent value while no agreement between any two different subnetworks. It is called a couple-group consensus if there are only two subnetworks. In [34], a double-tree-form transformation was introduced and the first-order couple-group consensus problem was studied with finite switching topologies and bounded communication delays. In multiagent systems, communication delays are a kind of nonnegligible factors when they exist in the communication channel between any two agents. The authors of [35, 36] investigated this problem with undirected and directed topology. Moreover, the convergence analysis was discussed, and some criteria were established. The extension of this work to the second-order dynamics was addressed in [37], where two different kinds of consensus protocols were proposed for networks with fixed topology. In [38], the group consensus problem of second-order multiagent systems with time-delays was studied. In [39], the authors studied time-varying group formation analysis and design for general linear multiagent systems with directed topologies. In [40], group tracking control of second-order multiagent systems with fixed and Markovian switching topologies was studied.

In this paper, we want to concern the following problems. (1) The state-dependent threshold and the time-dependent threshold are usually used in the event-triggered control systems [33]. Since the time-dependent threshold has an advantage to avoid the continuous communication and also has a simpler form, we will consider the time-dependent threshold in this paper. (2) Although the authors of [20] adopted the time-dependent event-triggered strategy, single consensus was considered. It makes great sense to investigate the couple-group consensus in engineering applications. So far, however, few works have focused on the couple-group control problem via event-triggered control, especially the time-dependent event-triggered control. Motivated by the above discussion, the couple-group consensus problem via time-dependent event-triggered is considered in this paper. The contributions of this work are three-fold: (1) both first-order and second-order dynamics in the framework of event-triggered based control are considered in this paper. In addition, an important factor that may exist in the communication channel is considered; i.e., communication delays are considered when the event-triggered based protocols are proposed; (2) the time-dependent event-triggered protocol is introduced to deal with energy consumption and communication constraints; (3) we discuss the distributed event-based couple-group consensus in presence of both positive and negative adjacent weights. Zeno behavior is excluded for all cases.

The rest of this paper is organized as follows. Section 2 gives some preliminaries on graph theory, assumptions, and lemmas. Event-based couple-group consensus problems for first-order dynamics and second-order dynamics are presented in Sections 3 and 4, respectively. Numerical examples are provided in Section 5. Section 6 draws the conclusions and develops the vision for future work.

Notations. In this paper, standard notations are used to illustrate the problem. $\mathbb{R}^n$ represents the set of $n-$dimensional vectors. $\Re(\cdot)$ and $\Im(\cdot)$ are the real part and the imaginary part of a complex number, respectively. The vector $\mathbf{1}_n = [1, 1, \ldots, 1]^T \in \mathbb{R}^n$ has all elements equal to 1. $I_{n\times n}$ and $0_{n\times n}$ are $n-$dimensional identity matrix and $n-$dimensional zero matrix, respectively. $\|\cdot\|$ denotes Euclidean norm, and $*$ in this paper stands for a term of block that is induced by symmetry.

2. Preliminaries

Let $\mathscr{G} = (\mathscr{V}, \mathscr{E}, \mathscr{A})$ be a weighted graph to model the information communication among $n + m$ agents with the node set $\mathscr{V} = \{v_1, v_2, \ldots, v_{n+m}\}$, the edge set $\mathscr{E} \subseteq \mathscr{V} \times \mathscr{V}$, and the weighted adjacency matrix $\mathscr{A}$. The node indexes belong to a finite index set $\mathscr{P} = \{1, 2, \ldots, n+m\}$. An edge $e_{ij} \in \mathscr{E}$ means that node v_i can receive information from node v_j. $\mathscr{A} = [a_{ij}] \in \mathbb{R}^{(n+m)\times(n+m)}$ is defined as $a_{ij} \neq 0$ if $e_{ij} \in \mathscr{E}$ and $a_{ij} = 0$ if $e_{ij} \notin \mathscr{E}$. Moreover, in this paper, there is no self-loop, i.e., $a_{ii} = 0$. Let N_i be the neighbor set of agent v_i, denoted by $N_i = \{v_j \mid e_{ij} = (v_j, v_i) \in \varepsilon\}$. An undirected graph $\mathscr{G}$ is said to be connected if there is a path between any two nodes of $\mathscr{G}$.

The following definitions, notations, and lemmas are borrowed from [36]. Suppose that the network can be separated into two subnetworks $\mathscr{G}_1$ and $\mathscr{G}_2$. Denote $\mathscr{P}_1 = \{1, 2, \ldots, n\}$, $\mathscr{P}_2 = \{n+1, n+2, \ldots, n+m\}$, $\mathscr{V}_1 = \{v_1, v_2, \ldots, v_n\}$, and $\mathscr{V}_2 = \{v_{n+1}, v_{n+2}, \ldots, v_{n+m}\}$. Then $\mathscr{V} = \mathscr{V}_1 \cup \mathscr{V}_2$. Furthermore, letting $N_{1i} = \{v_j \in \mathscr{V}_1 \mid (v_i, v_j) \in \mathscr{E}\}$ and $N_{2i} = \{v_j \in \mathscr{V}_2 \mid (v_i, v_j) \in \mathscr{E}\}$, we have $N_i = \{v_j \in \mathscr{V} \mid (v_i, v_j) \in \mathscr{E}\} = N_{1i} \cup N_{2i}$. The Laplacian matrix $L = [l_{ij}]$ is defined as

$$l_{ij} = \begin{cases} -a_{ij}, & j \neq i, \\ \displaystyle\sum_{k=1,k\neq i}^{n+m} a_{ik}, & j = i. \end{cases} \tag{1}$$

For the two subgroups, we propose two assumptions as follows:

$$\sum_{j=n+1}^{n+m} a_{ij} = 0; \quad \forall i \in \mathscr{P}_1; \tag{A1}$$

$$\sum_{j=1}^{n} a_{ij} = 0; \quad \forall i \in \mathcal{P}_2, \tag{A2}$$

which means $\mathcal{G}_1$ and $\mathcal{G}_2$ are in-degree balanced to each other.

In this case, the Laplacian matrix has the following partition:

$$L = \begin{bmatrix} L_{11} & L_{12} \\ L_{12}^T & L_{22} \end{bmatrix}, \tag{2}$$

where $L_{11} \in \mathbb{R}^{n\times n}$ and $L_{22} \in \mathbb{R}^{m\times m}$. Since $\mathcal{G}$ is undirected, L is symmetric. Therefore, all the eigenvalues of L have real values.

Lemma 1 (see [36]). *L has a zero eigenvalue whose geometric multiplicity is at least two. If L only has two-simple zero eigenvalues and the rest of the eigenvalues of L are positive, then*

$$R = \lim_{t\longrightarrow+\infty} e^{-Lt} = \omega_{1r}\omega_{1l}^T + \omega_{2r}\omega_{2l}^T, \tag{3}$$

where $\omega_{il}^T L = 0$, $L\omega_{ir} = 0$, and $\omega_{il}^T\omega_{ir} = 1$ for any $i = 1, 2$.

3. Event-Triggered Group Consensus for First-Order Multiagent Systems

In this section, event-triggered control strategy is employed for first-order multiagent system without and with communication delays.

Considering a network with $n + m$ agents, each agent has the following dynamics:

$$\dot{x}_i(t) = u_i(t), \quad i = 1, 2, \dots, n + m, \tag{4}$$

where $x_i, u_i \in \mathbb{R}$ represent the state and control input of the ith agent, respectively.

3.1. Without Communication Delays. In this subsection, we consider the distributed event-triggered consensus scheme for first-order multiagent systems.

The distributed event-triggered control law is obtained:

$$u_i(t) = \begin{cases} \sum_{j\in N_{1i}} a_{ij}\left(\hat{x}_j(t) - \hat{x}_i(t)\right) + \sum_{j\in N_{2i}} a_{ij}\hat{x}_j(t), & i \in \mathcal{P}_1; \\ \sum_{j\in N_{2i}} a_{ij}\left(\hat{x}_j(t) - \hat{x}_i(t)\right) + \sum_{j\in N_{1i}} a_{ij}\hat{x}_j(t), & i \in \mathcal{P}_2, \end{cases} \tag{5}$$

where $\hat{x}_j(t) = x_j(t^j_{k'(t)})$, $k'(t) = \operatorname{argmin}_{l\in N: t>t^j_l}\{t - t^j_l\}$. Define the measurement error $e_i(t) = \hat{x}_i(t) - x_i(t) = x_i(t^i_k) - x_i(t)$, $t \in [t^i_k, t^i_{k+1})$, where $x_i(t^i_k)$ represents the kth triggered instant of agent i.

Definition 2 (see [35]). Couple-group consensus is asymptotically solved under protocol (5) if the states of agents in $\mathcal{G}$ satisfy $\lim_{t\longrightarrow\infty}\|x_i(t) - x_j(t)\| = 0; \forall i, j \in \mathcal{P}_1$; $\lim_{t\longrightarrow\infty}\|x_i(t) - x_j(t)\| = 0; \forall i, j \in \mathcal{P}_2$.

Remark 3. It is worth noting that the weight a_{ij} between two groups is allowed to be negative in the couple-group consensus problems. The term $a_{ij}\hat{x}_j$ for $i \in \mathcal{P}_1$ and $j \in \mathcal{P}_2$ can be regarded as disturbances from the other subnetwork. In addition, the existence of negative factors complicates the dynamics of the multiagent systems. Particularly, the nonzero eigenvalues of Laplacian matrix L does not satisfy $\lambda_i(L) > 0$ any more when the undirected topology is connected.

Let $x^1(t) = [x_1(t), x_2(t), \dots, x_n(t)]^T$ and $x^2(t) = [x_{n+1}(t), x_{n+2}(t), \dots, x_{n+m}(t)]^T$. Correspondingly, $e(t) = ((e^1)^T(t), (e^2)^T(t))^T = (e_1(t), e_2(t), \dots, e_{n+m}(t))^T$. With the protocol (5), the matrix form of the closed-loop system (4) is given as follows:

$$\begin{aligned} \dot{x}^1(t) &= -L_{11}x^1(t) - L_{11}e^1(t) - L_{12}x^2(t) - L_{12}e^2(t), \\ \dot{x}^2(t) &= -L_{21}x^1(t) - L_{21}e^1(t) - L_{22}x^2(t) - L_{22}e^2(t). \end{aligned} \tag{6}$$

Defining the average value of $x^1(t)$ as $\overline{x}^1(t)$, then

$$\begin{aligned} \dot{\overline{x}}^1(t) &= \frac{1}{n}\mathbf{1}_n^T\dot{x}^1(t) \\ &= \frac{1}{n}\mathbf{1}_n^T\left[-L_{11}\left(x^1(t) + e^1(t)\right) - L_{12}\left(x^2(t) + e^2(t)\right)\right]. \end{aligned} \tag{7}$$

Under assumptions $(A1)$ and $(A2)$, $\mathcal{G}_1$ and $\mathcal{G}_2$ are a balanced couple. Note that the Laplacian matrix L is symmetric. It follows that $\mathbf{1}_{n+m}^T L = 0$, $\mathbf{1}_n^T L_{11} = 0$, $\mathbf{1}_n^T L_{12} = 0$, and $\dot{\overline{x}}^1(t) = 0$. Normally, $\overline{x}^1(t) = \overline{x}^1(0) = a_1$ is calculated. The state of agents can be described as $x^1(t) = a_1\mathbf{1}_n + \theta^1(t)$, where $\theta^1(t)$ is the disagreement vector, satisfying $\mathbf{1}_n^T\theta^1(t) = 0$. Similarly, $x^2(t) = a_2\mathbf{1}_m + \theta^2(t)$, where $\mathbf{1}_m^T\theta^2(t) = 0$. Therefore

$$\dot{\theta}(t) = \dot{x}(t) = -L(\theta(t) + e(t)), \tag{8}$$

where $\mathbf{1}_{n+m}^T\theta(t) = 0$ and $\theta(t) = [\theta_1^T(t), \theta_2^T(t)]^T$. Obviously, the couple-group consensus problem of multiagent system is equivalent to the asymptotic stability of system (8).

Adopt the following event-triggered function [20]:

$$f(e_i(t)) = |e_i(t)| - c_1 e^{-\varrho_1 t}, \tag{9}$$

where $c_1 > 0$.

Theorem 4. *Under assumptions $(A1)$ and $(A2)$, multiagent system (4) with protocol (5) and distributed event-triggered function (9) asymptotically reaches couple-group consensus if L has two-simple zero eigenvalues and all the other nonzero eigenvalues are positive, where the parameter ϱ_1 in the event-triggered function satisfies $0 < \varrho_1 < \lambda_3(L)$, $0 = \lambda_{1,2}(L) < \lambda_3(L) \le \cdots \le \lambda_{n+m}(L)$. Furthermore, the system does not exhibit Zeno behavior for all initial conditions $x(0) \in \mathbb{R}^{n+m}$.*

Proof. From (8), it follows that

$$\theta(t) = e^{-Lt}\theta(0) - \int_0^t e^{-L(t-s)}Le(s)\,ds. \tag{10}$$

Therefore, $\theta(t)$ is bounded by

$$\|\theta(t)\| \le \left\|e^{-Lt}\theta(0)\right\| + \left\|\int_0^t e^{-L(t-s)}Le(s)\,ds\right\|. \tag{11}$$

From the exponential estimation of solutions, we have the following inequality:

$$\|\theta(t)\| \le c_\theta e^{-\lambda_3(L)t}\|\theta(0)\| + c_\theta \int_0^t e^{-\lambda_3(L)(t-s)}\|Le(s)\|\,ds, \tag{12}$$

where c_θ is a constant.

According to event-triggered function (9), it follows that

$$\|e(t)\| \le \sqrt{n+m}\,c_1 e^{-\varrho_1 t}. \tag{13}$$

Therefore,

$$\begin{aligned}\|\theta(t)\| &\le c_\theta e^{-\lambda_3(L)t}\|\theta(0)\| + \sqrt{n+m}\,c_1 c_\theta \|L\| \int_0^t e^{-\lambda_3(L)(t-s)} e^{-\varrho_1 s} ds, \\ &\le c_\theta e^{-\lambda_3(L)t}\|\theta(0)\| + \frac{\sqrt{n+m}\,c_1 c_\theta \|L\|\left(e^{-\varrho_1 t} - e^{-\lambda_3(L)t}\right)}{\lambda_3(L)-\varrho_1}.\end{aligned} \tag{14}$$

We obtain $\lim_{t\to\infty}\theta(t) = 0$ if $0 < \varrho_1 < \lambda_3(L)$.

Thus, the group consensus problem is solved. The verification of no Zeno behavior is similar to that of Sections 3.2 and 4 and is thus omitted here. When the absolute value of the measurement error of agent i exceeds a certain value, the event occurs and the corresponding controller updates, which keeps constant value until the next event occurs. □

Remark 5. The results can be extended to the more general directed graphs. Since the eigenvalues of the Laplacian matrix associated a directed graph may be complex values. For the directed graph case, $\lambda_3(L)$ is replaced with $\mathfrak{R}(\lambda_3(L))$, where $0 = \lambda_{1,2}(L) < \mathfrak{R}(\lambda_3(L)) \le \cdots \le \mathfrak{R}(\lambda_{n+m}(L))$.

3.2. With Communication Delays. Assume that the communication delays in all channels are uniform, bounded but time-varying, which satisfy $0 < \tau(t) < r$. The couple-group consensus protocol is written as

$$u_i(t) = \begin{cases} \sum_{j\in N_{1i}} a_{ij}\left(\hat{x}_j(t-\tau(t)) - \hat{x}_i(t-\tau(t))\right) + \sum_{j\in N_{2i}} a_{ij}\hat{x}_j(t-\tau(t)), & i \in \mathscr{P}_1; \\ \sum_{j\in N_{2i}} a_{ij}\left(\hat{x}_j(t-\tau(t)) - \hat{x}_i(t-\tau(t))\right) + \sum_{j\in N_{1i}} a_{ij}\hat{x}_j(t-\tau(t)), & i \in \mathscr{P}_2. \end{cases} \tag{15}$$

System (4) with protocol (15) and event error $e_i(t-\tau(t)) = \hat{x}_i(t-\tau(t)) - x_i(t-\tau(t))$ can be written in matrix form as

$$\dot{x}(t) = -Lx(t-\tau(t)) - Le(t-\tau(t)). \tag{16}$$

According to the analysis of system without communication delays, it is obvious that $\dot{\theta}(t) = \dot{x}(t)$. Then the dynamics of $\theta(t)$ is given as follows:

$$\dot{\theta}(t) = -L\theta(t-\tau(t)) - Le(t-\tau(t)). \tag{17}$$

Theorem 6. *Under assumptions (A1) and (A2), multiagent system (4) with protocol (15) and event-triggered function (9) reaches couple-group consensus asymptotically for bounded communication delay $0 < \tau(t) < r$ if L has two-simple zero eigenvalues while all the other nonzero eigenvalues are positive, and there exist matrices $\overline{P} > 0$, $\overline{R} > 0$, Q, and Y and constants ϵ, $\mu > 0$, $\eta > 0$, and $\mu+\eta > 2\varrho_1$, satisfying the following matrix inequality:*

$$\begin{bmatrix} \kappa_{11} & \kappa_{12} & rL\overline{R} & -L & 0_{(n+m)\times(n+m)} \\ * & -\epsilon Q^T - \epsilon Q & r\epsilon L\overline{R} & -\epsilon L & rQ^T \\ * & * & -r\overline{R}\left(1 - r(\mu+\eta)\right) & 0_{(n+m)\times(n+m)} & 0_{(n+m)\times(n+m)} \\ * & * & * & -\eta I_{(n+m)\times(n+m)} & 0_{(n+m)\times(n+m)} \\ * & * & * & * & -r\overline{R} \end{bmatrix} < 0, \tag{18}$$

where $\kappa_{11} = (\mu+\eta)\overline{P} - Y - Y^T$ and $\kappa_{12} = \overline{P} - Q - \epsilon Y^T$. Furthermore, the system does not exhibit Zeno behavior for all initial conditions $x(0) \in \mathbb{R}^{n+m}$.

Proof. Construct the following Lyapunov-Krasovskii functional:

$$V\left(\theta_t, \dot{\theta}_t\right) = \theta^T(t)P\theta(t) + \int_{t-r}^t \int_s^t \dot{\theta}^T(\xi) R\dot{\theta}(\xi)\,d\xi\,ds. \tag{19}$$

We verify that

$$\int_{t-r}^t \int_s^t \dot{\theta}^T(\xi) R\dot{\theta}(\xi)\,d\xi\,ds = \int_{t-r}^t (\xi - t + r)\,\dot{\theta}^T(\xi) R\dot{\theta}(\xi)\,d\xi$$

$$\leq r\int_{t-r}^{t}\dot{\theta}^{T}(\xi)R\dot{\theta}(\xi)\,d\xi. \tag{20}$$

For some constant $\mu > 0$, let $\dot{V}(\theta_t,\dot{\theta}_t)+\mu V(\theta_t,\dot{\theta}_t) < 0$, for $V(\theta_t,\dot{\theta}_t)\geq\|e(t-\tau(t))\|^2$. Denote $U=\dot{V}(\theta_t,\dot{\theta}_t)+\mu V(\theta_t,\dot{\theta}_t)+\eta(V(\theta_t,\dot{\theta}_t)-\|e(t-\tau(t))\|^2)$, where $\eta > 0$. Hence, taking the derivative of $V(\theta_t,\dot{\theta}_t)$, it can be obtained that

$$\begin{aligned}U&\leq\dot{\theta}^{T}P\theta+\theta^{T}P\dot{\theta}+r\dot{\theta}^{T}R\dot{\theta}-\int_{t-r}^{t}\dot{\theta}^{T}(\xi)R\dot{\theta}(\xi)\,d\xi\\&\quad+(\mu+\eta)\left[\theta^{T}P\theta+r\int_{t-r}^{t}\dot{\theta}^{T}(\xi)R\dot{\theta}(\xi)\,d\xi\right]\\&\quad-\eta e(t-\tau(t))^{T}e(t-\tau(t)).\end{aligned} \tag{21}$$

Denoting $\delta=-\dot{\theta}-L\theta(t-\tau(t))-Le(t-\tau(t))$, then $0=[\theta^{T}P_2^{T}+\dot{\theta}^{T}P_3^{T}]\delta+\delta^{T}[P_2\theta+P_3\dot{\theta}]$. We can get

$$\begin{aligned}U&\leq\dot{\theta}^{T}P\theta+\theta^{T}P\dot{\theta}+r\dot{\theta}^{T}R\dot{\theta}-\int_{t-r}^{t}\dot{\theta}^{T}(\xi)R\dot{\theta}(\xi)\,d\xi\\&\quad+(\mu+\eta)\left[\theta^{T}P\theta+r\int_{t-r}^{t}\dot{\theta}^{T}(\xi)R\dot{\theta}(\xi)\,d\xi\right]\\&\quad-\eta e(t-\tau(t))^{T}e(t-\tau(t))+\left[\theta^{T}P_2^{T}+\dot{\theta}^{T}P_3^{T}\right]\delta\\&\quad+\delta^{T}\left[P_2\theta+P_3\dot{\theta}\right].\end{aligned} \tag{22}$$

We apply the relation

$$\begin{aligned}&\theta(t-\tau(t))=\theta(t)-\int_{t-\tau(t)}^{t}\dot{\theta}(s)\,ds,\\&\int_{t-r}^{t}\dot{\theta}^{T}(s)R\dot{\theta}(s)\,ds\\&\quad\geq\left[\frac{1}{r}\int_{t-\tau(t)}^{t}\dot{\theta}(s)\,ds\right]^{T}rR\left[\frac{1}{r}\int_{t-\tau(t)}^{t}\dot{\theta}(s)\,ds\right]\end{aligned} \tag{23}$$

and let $\psi^{T}(t)=[\theta^{T}(t),\dot{\theta}^{T}(t),(1/r)\int_{t-\tau(t)}^{t}\dot{\theta}^{T}(s)ds,e^{T}(t)]$; then $U\leq\psi^{T}(t)\phi\psi(t)+r\dot{\theta}^{T}(t)R\dot{\theta}(t)$ holds, where

$$\Phi=\begin{bmatrix}\chi_{11}&\chi_{12}&rP_2^{T}L&-P_2^{T}L\\ *&-P_3^{T}-P_3&rP_3^{T}L&-P_3^{T}L\\ *&*&-rR(1-r(\mu+\eta))&0_{(n+m)\times(n+m)}\\ *&*&*&-\eta I_{(n+m)\times(n+m)}\end{bmatrix}, \tag{24}$$

with $\chi_{11}=(\mu+\eta)P-P_2^{T}L-L^{T}P_2$ and $\chi_{12}=P-P_2^{T}-L^{T}P_3$. Applying Schur Complements Lemma [41], $U < 0$ if the following linear matrix inequality holds:

$$\begin{bmatrix}\chi_{11}&\chi_{12}&rP_2^{T}L&-P_2^{T}L&0_{(n+m)\times(n+m)}\\ *&-P_3^{T}-P_3&rP_3^{T}L&-P_3^{T}L&rR\\ *&*&-rR(1-r(\mu+\eta))&0_{(n+m)\times(n+m)}&0_{(n+m)\times(n+m)}\\ *&*&*&-\eta I_{(n+m)\times(n+m)}&0_{(n+m)\times(n+m)}\\ *&*&*&*&-rR\end{bmatrix}<0. \tag{25}$$

We multiply (25) by $\operatorname{diag}\{P_2^{-1},P_2^{-1},R^{-1},I_p,R^{-1}\}$ and its transpose on the left and the right, respectively, and denote $P_3=\epsilon P_2$, $Q=P_2^{-1}$, $\overline{P}=Q^{T}PQ$, $\overline{R}=R^{-1}$, and $Y=LQ$; then inequality (25) can be rewritten as (18). Therefore, $\lim_{t\longrightarrow\infty}\theta(t)=0$, i.e., $\lim_{t\longrightarrow\infty}x(t)=[a_1\mathbf{1}_n^{T},a_2\mathbf{1}_m^{T}]^{T}$, which means that the couple-group consensus problem is solved. Furthermore, we will prove that the event-triggered function does not have Zeno behavior. Because

$$\dot{V}\left(\theta_t,\dot{\theta}_t\right)+(\mu+\eta)V\left(\theta_t,\dot{\theta}_t\right)-\eta\|e(t-\tau(t))\|^2<0, \tag{26}$$

we have

$$\begin{aligned}\theta^{T}(t)P\theta(t)&\leq V\left(\theta_t,\dot{\theta}_t\right)\\&\leq e^{-(\mu+\eta)(t-t_0)}V\left(\theta_{t_0},\dot{\theta}_{t_0}\right)\\&\quad+\int_{t_0}^{t}e^{-(\mu+\eta)(t-s)}\eta\|e(s-\tau(s))\|^2\,ds.\end{aligned} \tag{27}$$

Furthermore, $\lambda_{min}(P)\|\theta(t)\|^2\leq\theta^{T}(t)P\theta(t)\leq\lambda_{max}(P)\|\theta(t)\|^2$, and thus

$$\begin{aligned}&\|\theta(t)\|^2\\&\quad\leq\frac{\lambda_{max}(P)e^{-(\mu+\eta)t}}{\lambda_{min}(P)}\|\theta(0)\|^2\\&\qquad+\frac{1}{\lambda_{min}(P)}\int_{0}^{t}e^{-(\mu+\eta)(t-s)}\eta\|e(s-\tau(s))\|^2\,ds,\end{aligned} \tag{28}$$

that is,

$$\begin{aligned}&\|\theta(t)\|\\&\quad\leq\frac{\sqrt{\lambda_{max}(P)}e^{-(\mu+\eta)t/2}}{\sqrt{\lambda_{min}(P)}}\|\theta(0)\|\\&\qquad+\sqrt{\frac{1}{\lambda_{min}(P)}\int_{0}^{t}e^{-(\mu+\eta)(t-s)}\eta\,ds}\,\|e(t-\tau(t))\|_{\infty},\end{aligned}$$

$$\leq \frac{\sqrt{\lambda_{max}(P)}e^{-(\mu+\eta)t/2}}{\sqrt{\lambda_{min}(P)}} \|\theta(0)\| + \sqrt{\frac{1}{\lambda_{min}(P)}\frac{\eta}{\mu+\eta}} \|e(t-\tau(t))\|_{\infty}. \tag{29}$$

From the input-to-state stability (ISS) theory [42], the system (17) is ISS with the disturbance $e(t-\tau(t))$. It can reach couple-group consensus via event-triggered function (9) without Zeno behavior when $\mu+\eta > 2\varrho_1 > 0$. □

4. Event-Triggered Group Consensus for Second-Order Multiagent Systems

In this section, distributed event-triggered couple-group consensus problems for second-order multiagent systems are addressed.

Consider a multiagent network consisting of $n+m$ agents. The dynamics of a second-order agent is modeled by

$$\begin{aligned} \dot{\xi}_i(t) &= \zeta_i(t), \\ \dot{\zeta}_i(t) &= u_i(t), \\ & i = 1,2,\ldots,n+m, \end{aligned} \tag{30}$$

where $\xi_i, \zeta_i, u_i \in \mathbb{R}$.

Definition 7 (see [43]). Couple-group consensus for second-order multiagent system (30) is asymptotically solved if the states of agents in $\mathcal{G}$ satisfy $\lim_{t\longrightarrow\infty}\|\xi_i(t)-\xi_j(t)\| = 0$; $\lim_{t\longrightarrow\infty}\|\zeta_i(t)-\zeta_j(t)\| = 0$, $\forall i,j \in \mathcal{P}_1$, and $\lim_{t\longrightarrow\infty}\|\xi_i(t)-\xi_j(t)\| = 0$; $\lim_{t\longrightarrow\infty}\|\zeta_i(t)-\zeta_j(t)\| = 0$, $\forall i,j \in \mathcal{P}_2$.

4.1. Without Communication Delays. Similar to the first-order multiagent systems, the broadcast states are described by $\widehat{\xi}_i(t) = \xi_i(t_k^i)$ and $\widehat{\zeta}_i(t) = \zeta_i(t_k^i)$, $t \in [t_k^i, t_{k+1}^i)$. Then an event-based control strategy is proposed as follows:

$$u_i(t) = \begin{cases} \sum_{j\in N_{1i}} a_{ij}\left[\alpha\left(\widehat{\xi}_j(t)-\widehat{\xi}_i(t)\right)+\beta\left(\widehat{\zeta}_j(t)-\widehat{\zeta}_i(t)\right)\right] + \sum_{j\in N_{2i}} a_{ij}\left[\alpha\widehat{\xi}_j(t)+\beta\widehat{\zeta}_j(t)\right], & i\in\mathcal{P}_1; \\ \sum_{j\in N_{2i}} a_{ij}\left[\alpha\left(\widehat{\xi}_j(t)-\widehat{\xi}_i(t)\right)+\beta\left(\widehat{\zeta}_j(t)-\widehat{\zeta}_i(t)\right)\right] + \sum_{j\in N_{1i}} a_{ij}\left[\alpha\widehat{\xi}_j(t)+\beta\widehat{\zeta}_j(t)\right], & i\in\mathcal{P}_2, \end{cases} \tag{31}$$

where $\alpha > 0$, $\beta > 0$, $a_{ij} \geq 0$, $\forall i,j \in \mathcal{P}_1$, and $\forall i,j \in \mathcal{P}_2$; otherwise, $a_{ij} \in \mathbb{R}$.

We define the measurement errors $e_\xi(t) = \widehat{\xi}(t) - \xi(t)$ and $e_\zeta(t) = \widehat{\zeta}(t) - \zeta(t)$. This yields $u(t) = -\alpha L\xi(t) - \alpha L e_\xi(t) - \beta L\zeta(t) - \beta L e_\zeta(t)$. The matrix form of (30) with protocol (31) is given as

$$\begin{bmatrix} \dot{\xi}(t) \\ \dot{\zeta}(t) \end{bmatrix} = \Gamma \begin{bmatrix} \xi(t) \\ \zeta(t) \end{bmatrix} + \Omega \begin{bmatrix} e_\xi(t) \\ e_\zeta(t) \end{bmatrix}, \tag{32}$$

where

$$\begin{aligned} \Gamma &= \begin{bmatrix} 0_{(n+m)\times(n+m)} & I_{(n+m)\times(n+m)} \\ -\alpha L & -\beta L \end{bmatrix}, \\ \Omega &= \begin{bmatrix} 0_{(n+m)\times(n+m)} & 0_{(n+m)\times(n+m)} \\ -\alpha L & -\beta L \end{bmatrix}. \end{aligned} \tag{33}$$

Let $\xi(t) = \begin{bmatrix} \xi^1(t) \\ \xi^2(t) \end{bmatrix} = \begin{bmatrix} b_1\mathbf{1}_n \\ b_2\mathbf{1}_m \end{bmatrix} + \begin{bmatrix} c_1\mathbf{1}_n \\ c_2\mathbf{1}_m \end{bmatrix} t + \begin{bmatrix} \phi_{\xi^1}(t) \\ \phi_{\xi^2}(t) \end{bmatrix}$ and $\zeta(t) = \begin{bmatrix} c_1\mathbf{1}_n \\ c_2\mathbf{1}_m \end{bmatrix} + \begin{bmatrix} \phi_{\zeta^1}(t) \\ \phi_{\zeta^2}(t) \end{bmatrix}$. Denote $\phi_\xi(t) = [\phi_{\xi^1}^T(t), \phi_{\xi^2}^T(t)]^T$, $\phi_\zeta(t) = [\phi_{\zeta^1}^T(t), \phi_{\zeta^2}^T(t)]^T$, $e(t) = [e_\xi^T(t), e_\zeta^T(t)]^T$, and $\phi(t) = [\phi_\xi^T(t), \phi_\zeta^T(t)]^T$. The disagreement dynamics is given by

$$\dot{\phi}(t) = \Gamma\phi(t) + \Omega e(t). \tag{34}$$

In this section, we adopt the following event-triggered function [20]:

$$f(e_i(t)) = \|e_i(t)\| - c_2 e^{-\varrho_2 t}, \tag{35}$$

where $e_i(t) = \begin{bmatrix} e_{\xi_i}(t) \\ e_{\zeta_i}(t) \end{bmatrix}$, $c_2 > 0$.

Theorem 8. *Under assumptions (A1) and (A2), multiagent system (30) reaches couple-group consensus asymptotically with protocol (31) and distributed event-triggered function (35) if L has two-simple zero eigenvalues and the rest of nonzero eigenvalues are positive together with $\forall\alpha > 0$, $\beta > 0$, where the parameter ϱ_2 satisfies $0 < \varrho_2 < |\mathfrak{R}(\lambda_5(\Gamma))|$, $0 = \lambda_{1,2,3,4}(\Gamma) > \mathfrak{R}(\lambda_5(\Gamma)) \geq \cdots \mathfrak{R}(\lambda_{2(n+m)}(\Gamma))$. Furthermore, the system does not exhibit Zeno behavior for any initial conditions $\xi(0) \in \mathbb{R}^{n+m}$ and $\zeta(0) \in \mathbb{R}^{n+m}$.*

Proof. From (34), the following analytical solution is obtained:

$$\phi(t) = e^{\Gamma t}\phi(0) + \int_0^t e^{\Gamma(t-s)}\Omega e(s)\,ds. \tag{36}$$

According to the event-triggered function (35) and the exponential estimation of solutions, it follows that

$$\|\phi(t)\| \leq e^{\mathfrak{R}(\lambda_5(\Gamma))t} c_\phi \|\phi(0)\| + \int_0^t e^{\mathfrak{R}(\lambda_5(\Gamma))(t-s)} c_\phi \|\Omega e(s)\|\,ds,$$

$$\begin{aligned}&\leq e^{\Re(\lambda_5(\Gamma))t}c_\phi \|\phi(0)\| \\ &\quad + \sqrt{n+m}c_\phi c_2\sqrt{2}\|L\|\int_0^t e^{\Re(\lambda_5(\Gamma))(t-s)}e^{-\varrho_2 s}ds, \\ &\leq e^{\Re(\lambda_5(\Gamma))t}c_\phi \|\phi(0)\| \\ &\quad + \frac{\sqrt{n+m}c_\phi c_2\sqrt{2}\|L\|\left(e^{\Re(\lambda_5(\Gamma))t}-e^{-\varrho_2 t}\right)}{\Re(\lambda_5(\Gamma))+\varrho_2},\end{aligned} \tag{37}$$

where c_ϕ is a constant. Because $0 < \varrho_2 < |\Re(\lambda_5(\Gamma))|$, $\lim_{t\to\infty}\phi(t) = 0$. Then couple-group consensus of system (30) is asymptotically reached.

Next, we will show that Zeno behavior is excluded. Assume that the agents trigger at time $t^* \geq 0$ and $e(t^*) = 0$, for $t \geq t^*$. Before the next trigger time, the following inequality holds:

$$\begin{aligned}\|e(t)\| &= \left\|\begin{bmatrix} e_\xi(t) \\ e_\zeta(t)\end{bmatrix}\right\| \leq \int_{t^*}^t \|\dot{e}(s)\|\,ds \\ &= \int_{t^*}^t \left\|\begin{bmatrix} e_\zeta(s) \\ -u(s)\end{bmatrix}\right\| ds \leq \int_{t^*}^t \left\|\begin{bmatrix} e(s) \\ u(s)\end{bmatrix}\right\| ds \\ &\leq \int_{t^*}^t (\|e(s)\| + \|u(s)\|)\,ds.\end{aligned} \tag{38}$$

From (31), (35), and (37), we obtain

$$\begin{aligned}\|u(t)\| &\leq \left\|-\alpha L\xi(t) - \beta L\zeta(t) - \alpha L e_\xi(t) - \beta L e_\zeta(t)\right\| \\ &= \left\|-\alpha L\phi_\xi(t) - \beta L\phi_\zeta(t) - \alpha L e_\xi(t) - \beta L e_\zeta(t)\right\| \\ &\leq \sqrt{\alpha^2+\beta^2}\|L\|\left(\|\phi(t)\| + \|e(t)\|\right), \\ \|e(t)\| &\leq \sqrt{n+m}c_2 e^{-\varrho_2 t}, \\ \|\phi(t)\| &\leq k_1 e^{\Re(\lambda_5(\Gamma))t} + k_2 e^{-\varrho_2 t},\end{aligned} \tag{39}$$

where $k_1 = c_\phi\|\phi(0)\| + \sqrt{n+m}c_\phi c_2\sqrt{2}\|L\|/|\Re(\lambda_5(\Gamma))+\varrho_2|$and $k_2 = \sqrt{n+m}c_\phi c_2\sqrt{2}\|L\|/|\Re(\lambda_5(\Gamma)) + \varrho_2|$. Then it can be deduced that

$$\|e(t)\| \leq \int_{t^*}^t \left(k_1^* e^{\Re(\lambda_5(\Gamma))s} + k_2^* e^{-\varrho_2 s}\right) ds, \tag{40}$$

where $k_1^* = \sqrt{\alpha^2+\beta^2}\|L\|k_1$ and $k_2^* = \sqrt{\alpha^2+\beta^2}\|L\|(k_2 + \sqrt{n+m}c_2) + 1$. The next event will not be triggered until $(t-t^*)(k_1^* e^{\Re(\lambda_5(\Gamma))t^*} + k_2^* e^{-\varrho_2 t^*}) = \sqrt{n+m}c_2 e^{-\varrho_2 t}$. Then we can calculate the trigger time interval $\varsigma = t - t^*$ from $(k_1^* e^{\Re(\lambda_5(\Gamma))+\varrho_2}t^*) + k_2^*)\varsigma = \sqrt{n+m}c_2 e^{-\varrho_2\varsigma}$, which implies that Zeno behavior is excluded for the lower bound of the interevent times ς. □

4.2. With Communication Delays. In this subsection, we analyze couple-group consensus for second-order multiagent systems with communication delays via event-triggered control.

Assume that the communication delay between any two neighbors is $d(t)$, satisfying $0 < d(t) < h$, $h > 0$, and $t > 0$. The event-based couple-group consensus protocol is given as

$$u_i(t) = \begin{cases}\sum_{j\in N_{1i}} a_{ij}\left[\alpha\left(\hat{\xi}_j(t-d(t)) - \hat{\xi}_i(t-d(t))\right) + \beta\left(\hat{\zeta}_j(t-d(t)) - \hat{\zeta}_i(t-d(t))\right)\right] + \sum_{j\in N_{2i}} a_{ij}\left[\alpha\hat{\xi}_j(t-d(t)) + \beta\hat{\zeta}_j(t-d(t))\right], & i\in\mathscr{P}_1; \\ \sum_{j\in N_{2i}} a_{ij}\left[\alpha\left(\hat{\xi}_j(t-d(t)) - \hat{\xi}_i(t-d(t))\right) + \beta\left(\hat{\zeta}_j(t-d(t)) - \hat{\zeta}_i(t-d(t))\right)\right] + \sum_{j\in N_{1i}} a_{ij}\left[\alpha\hat{\xi}_j(t-d(t)) + \beta\hat{\zeta}_j(t-d(t))\right], & i\in\mathscr{P}_2,\end{cases} \tag{41}$$

where $\alpha > 0$, $\beta > 0$, $a_{ij} \geq 0$, $\forall i,j \in \mathscr{P}_1$, and $\forall i,j\in\mathscr{P}_2$; otherwise, $a_{ij}\in\mathbb{R}$.

Defining $e_\xi(t)$ and $e_\zeta(t)$ as the measurement errors, system (30) with protocol (41) can be written as

$$\begin{aligned}\begin{bmatrix}\dot{\xi}(t) \\ \dot{\zeta}(t)\end{bmatrix} &= \Psi\begin{bmatrix}\xi(t) \\ \zeta(t)\end{bmatrix} + \Upsilon_1\begin{bmatrix}\xi(t-d(t)) \\ \zeta(t-d(t))\end{bmatrix} \\ &\quad + \Upsilon_1\begin{bmatrix}e_\xi(t-d(t)) \\ e_\zeta(t-d(t))\end{bmatrix}.\end{aligned} \tag{42}$$

With the definition of $\phi(t)$, the system (42) is further given as

$$\dot{\phi}(t) = \Psi\phi(t) + \Upsilon_1\phi(t-d(t)) + \Upsilon_1 e(t-d(t)), \tag{43}$$

where

$$\begin{aligned}\Psi &= \begin{bmatrix}0_{(n+m)\times(n+m)} & I_{(n+m)\times(n+m)} \\ 0_{(n+m)\times(n+m)} & 0_{(n+m)\times(n+m)}\end{bmatrix}, \\ \Upsilon_1 &= \begin{bmatrix}0_{(n+m)\times(n+m)} & 0_{(n+m)\times(n+m)} \\ -\alpha L & -\beta L\end{bmatrix}.\end{aligned} \tag{44}$$

Theorem 9. *Under assumptions (A1) and (A2), multiagent system (30) with protocol (41) and event-triggered function (35) asymptotically reaches couple-group consensus under bounded communication delay $0 < d(t) < h$ for $\forall\alpha > 0$, $\beta > 0$ if L has two-simple zero eigenvalues while all the other nonzero eigenvalues are positive and there exist matrices $\overline{P}_1 > 0$, $\overline{R}_1 > 0$, Q_1, and Y_1 and constants ϵ_1, $\mu_1 > 0$, $\eta_1 > 0$,*

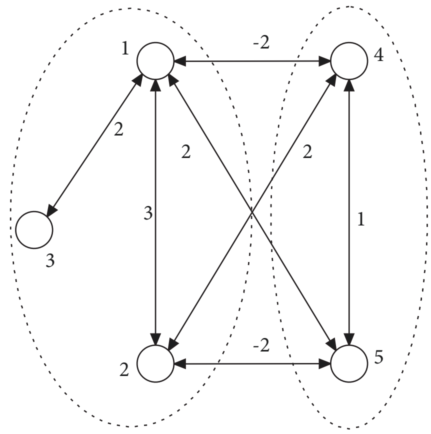

Figure 1: Topology 1.

and $\mu_1 + \eta_1 > 2\varrho_2$, *satisfying the following matrix inequality:*

$$\begin{bmatrix} \Xi_{11} & \Xi_{12} & h\Upsilon_1\overline{R} & \Upsilon_1 & 0_{(n+m)\times(n+m)} \\ * & -\epsilon_1 Q^T - \epsilon_1 Q & h\epsilon\Upsilon_1\overline{R} & \epsilon\Upsilon_1 & hQ_1^{\ T} \\ * & * & -h\overline{R}_1\left(1-h\left(\mu_1+\eta_1\right)\right) & 0_{(n+m)\times(n+m)} & 0_{(n+m)\times(n+m)} \\ * & * & * & -\eta_1 I_{(n+m)\times(n+m)} & 0_{(n+m)\times(n+m)} \\ * & * & * & * & -h\overline{R}_1 \end{bmatrix} < 0, \quad (45)$$

where $\Xi_{11} = (\mu_1+\eta_1)\overline{P}_1 + Y_1 + Y_1^{\ T}$ *and* $\Xi_{12} = \overline{P}_1 - Q_1 + \epsilon_1 Y_1^{\ T}$. *Furthermore, the system does not exhibit Zeno behavior for all initial conditions* $\xi(0) \in \mathbb{R}^{n+m}$ *and* $\zeta(0) \in \mathbb{R}^{n+m}$.

The proof is similar to that of Theorem 6 and thus is omitted here.

5. Simulations and Analysis

In this section, several examples are given to verify the effectiveness of the main results.

Example 1. Consider a multiagent team with five agents, and each agent is modeled by system (4). The communication topology is given by Figure 1 with initial state $x(0) = [-5, 2, 3, -8, 1]^T$. Agent 1, agent 2, and agent 3 are in one group while agent 4 and agent 5 are in another group. From the definition of Laplacian matrix, we can easily obtain the Laplacian matrix L. The eigenvalues of L are $\lambda_{1,2} = 0$, $\lambda_3 = 2$, $\lambda_4 = 3.55$, and $\lambda_5 = 8.45$, which implies that L has two-simple zero eigenvalues and all the other nonzero eigenvalues are positive. By tuning $c_1 = [0.12, 0.19, 0.2, 0.1, 0.2]$ and $\varrho_1 = [1.2, 1.35, 1.4, 1.3, 1.4]$, which are the parameters in event-triggered function (9), the trajectory and event times of every agent under event-triggered function (9) are shown by Figures 2 and 3, respectively. The evolution of $|e_i(t)|$ is plotted in Figure 4. It is clear that system (8) achieves couple-group consensus and the event-triggered scheme reduces the number of actuator updating to a great extent.

Example 2. Consider a multiagent system that has the same topology, initial state, and group division as that of Example 1. To solve the inequality in Theorem 6, we choose $\epsilon = 0.2$, $\eta = 3.2$, and $\mu = 0.0001$, satisfying $\mu + \eta > 2\varrho_1 > 0$, and then the feasible solution is given as

$$\begin{aligned} \overline{P} &= 10^8 \\ &\times \begin{bmatrix} 0.6773 & 0.4238 & 0.2510 & -0.2027 & 0.2013 \\ 0.4238 & 0.8662 & 0.0142 & 0.0881 & -0.1038 \\ 0.2510 & 0.0142 & 1.1345 & 0.0883 & -0.0800 \\ -0.2027 & 0.0881 & 0.0883 & 1.1274 & 0.2099 \\ 0.2013 & -0.1038 & -0.0800 & 0.2099 & 1.1454 \end{bmatrix}, \\ \overline{R} &= 10^8 \\ &\times \begin{bmatrix} 1.8567 & 1.8022 & 1.5618 & -0.1765 & 0.1634 \\ 1.8022 & 2.5641 & 0.9150 & 0.3275 & -0.3322 \\ 1.5618 & 0.9150 & 2.5323 & -0.1948 & 0.1877 \\ -0.1765 & 0.3275 & -0.1948 & 2.7320 & 1.8314 \\ 0.1634 & -0.3322 & 0.1877 & 1.8314 & 2.7118 \end{bmatrix}, \end{aligned} \quad (46)$$

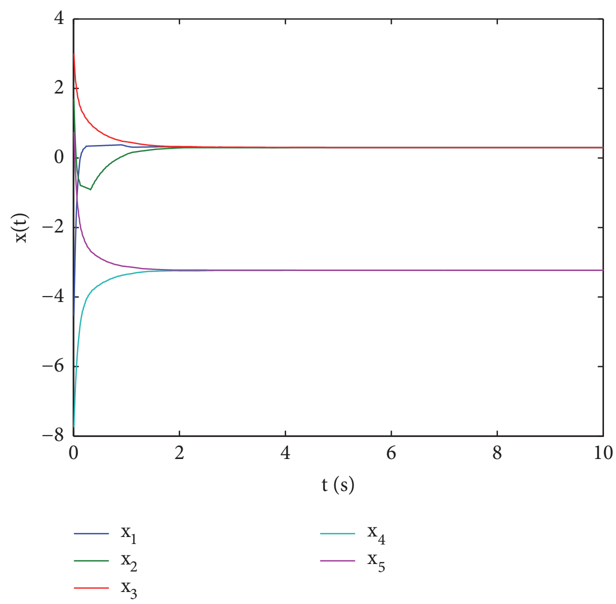

Figure 2: The state trajectories of five agents without communication delay.

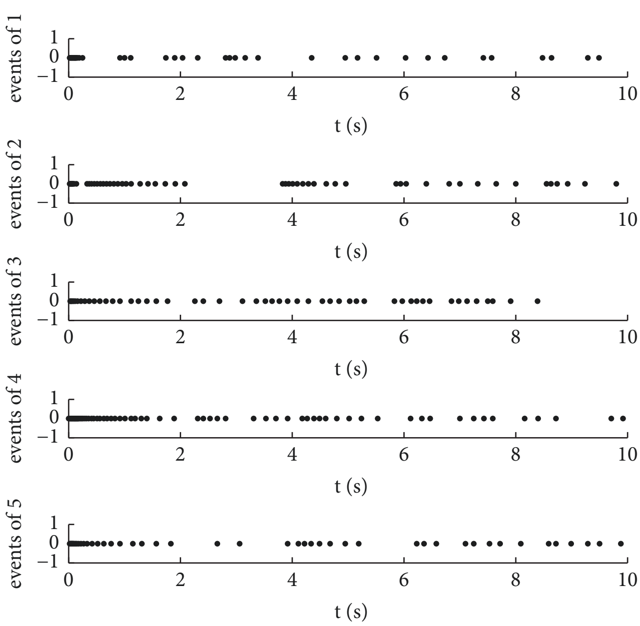

Figure 3: The event times of five agents without communication delay.

and $r_{max} = 0.18s$ is obtained. When $r = 0.1s < r_{max}$, the trajectories of all the agents are shown by Figure 5. It is clear that the couple-group consensus is achieved when $r < r_{max}$.

Example 3. Consider multiagent system (30) with five agents. The communication topology is given by Figure 6 with initial state $\xi(0) = [-5, 1, 2, -1, 6]^T$ and $\zeta(0) = [0, 0.3, 0.2, 0.8, 0.4]^T$. Agent 1, agent 2, and agent 3 are in one group while agent 4 and agent 5 are in another group. The eigenvalues of the Laplacian matrix are $\mu_{1,2} = 0$, $\mu_3 = 1.2984$, $\mu_4 = 3$, and $\mu_5 = 7.7016$, and the eigenvalues of Γ are $\lambda_{1,2,3,4} = 0$, $\lambda_5 = -6.5204$,

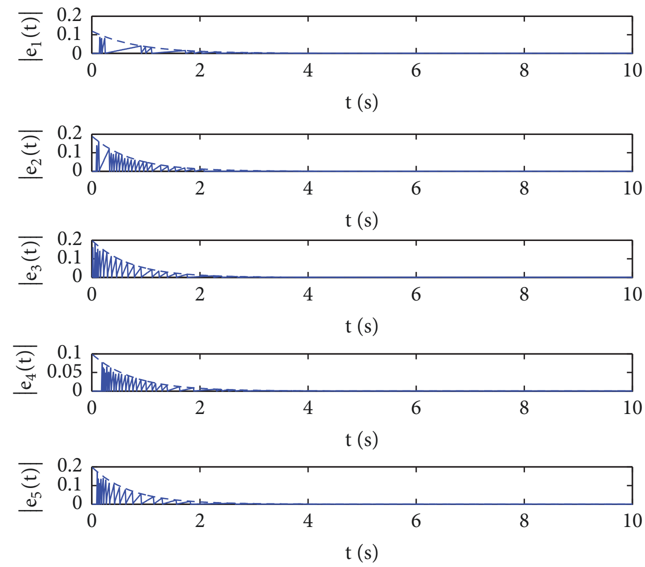

Figure 4: Evolution of the measurement error absolute $|e_i(t)|$ with event-triggered scheme (9).

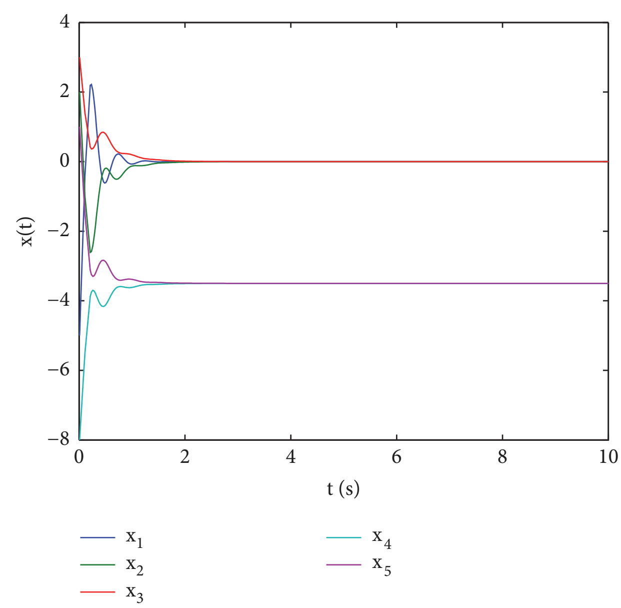

Figure 5: The state trajectories of five agents with communication delay $r = 0.1s$.

$\lambda_6 = -1.1811$, $\lambda_{7,8} = -1.5 \pm 0.866\boldsymbol{i}$, and $\lambda_{9,10} = -0.6492 \pm 0.9365\boldsymbol{i}$. It is obvious that Γ has a zero eigenvalue whose algebraic multiplicity is four and the real parts of all nonzero eigenvalues are negative. Choose $\alpha = 1$ and $\beta = 1$ as the parameter values of second-order couple-group consensus protocol (31). By tuning $c_2 = [0.28, 0.2, 0.19, 0.28, 0.3]$ and $\varrho_2 = [0.246, 0.241, 0.227, 0.246, 0.192]$, which are the parameter values of event-triggered function (35), the trajectories of position, velocity, and event times of all agents under event-triggered control protocol (31) are shown by Figures 7 and 8, respectively. The evolution of $\|e_i(t)\|$ is plotted by Figure 9. It is clear that system (34) achieves couple-group consensus and the event-triggered scheme reduces the

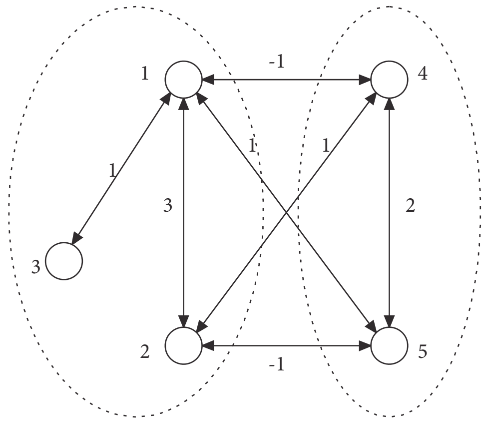

FIGURE 6: Topology 3.

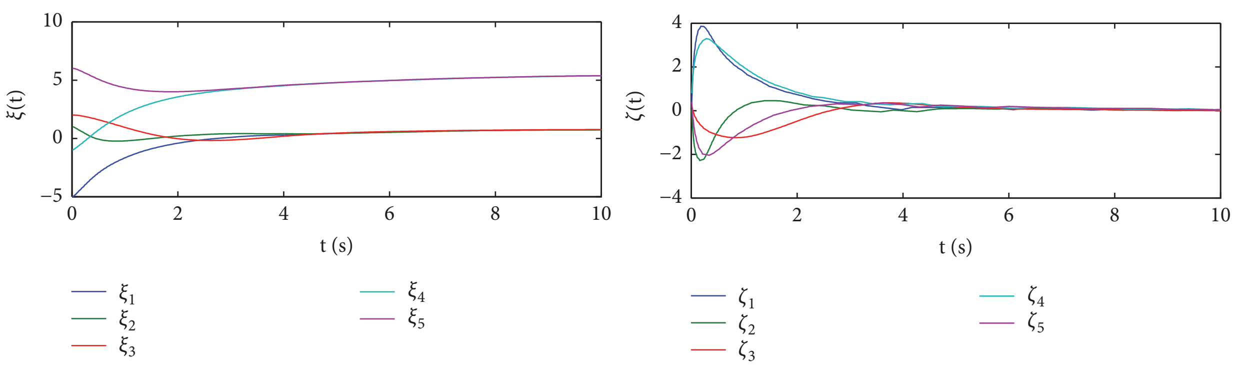

FIGURE 7: The position and velocity trajectories of five agents without communication delay.

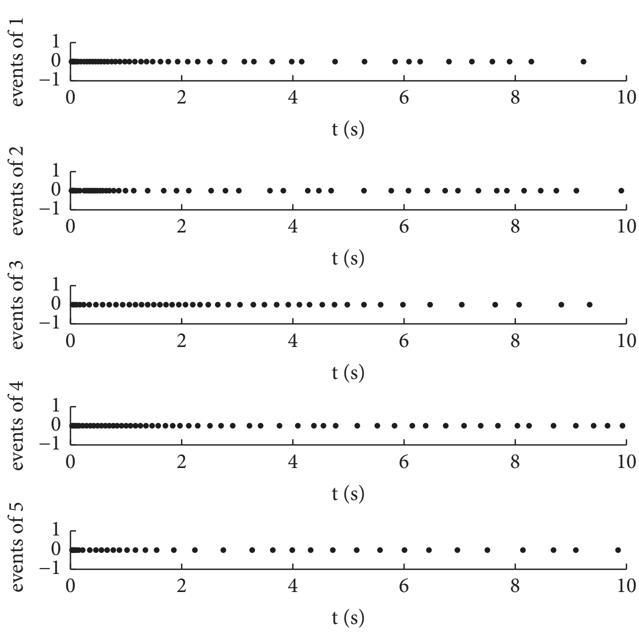

FIGURE 8: The event times of five agents without communication delays.

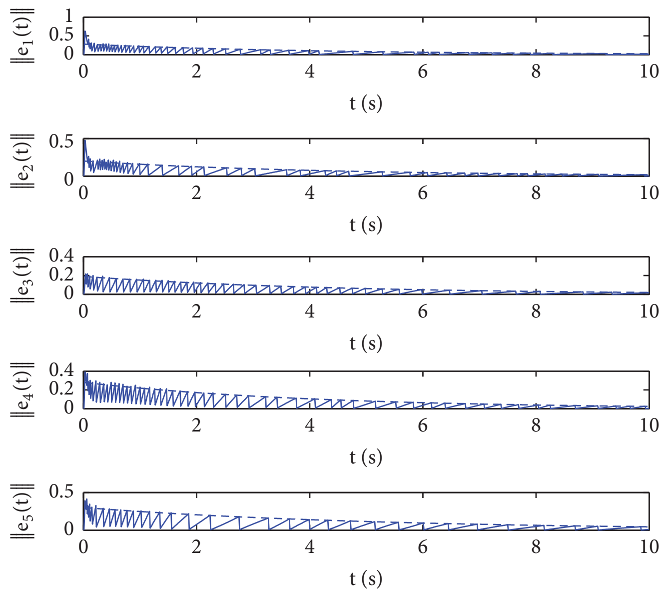

FIGURE 9: Evolution of the measurement error norm $\|e_i(t)\|$ with event-triggered scheme (35).

number of actuator updating to a great extent. This is consistent with the theoretical results.

Example 4. Consider a multiagent system that has the same topology, initial state, and group division as that of Example 3. To solve the inequality in Theorem 9, we choose $\epsilon_1 = 0.1$, $\eta_1 = 3.6$, and $\mu_1 = 0.0002$, satisfying $\mu_1 + \eta_1 > 2\varrho_2 > 0$. The feasible solution is given as

$$\overline{P} = 10^3 \times \begin{bmatrix} 1.7289 & 0.3463 & 0.1825 & -0.0399 & 0.0399 & -0.9328 & 0.3470 & 0.1832 & -0.0408 & 0.0408 \\ 0.3463 & 1.8318 & 0.0796 & 0.0609 & -0.0609 & 0.3470 & -0.8305 & 0.0809 & 0.0613 & -0.0613 \\ 0.1825 & 0.0796 & 1.9956 & -0.0210 & 0.0210 & 0.1832 & 0.0809 & -0.6667 & -0.0205 & 0.0205 \\ -0.0399 & 0.0609 & -0.0210 & 1.9010 & 0.3567 & -0.0408 & 0.0613 & -0.0205 & -0.7595 & 0.3569 \\ 0.0399 & -0.0609 & 0.0210 & 0.3567 & 1.9010 & 0.0408 & -0.0613 & 0.0205 & 0.3569 & -0.7595 \\ -0.9328 & 0.3470 & 0.1832 & -0.0408 & 0.0408 & 1.7328 & 0.3415 & 0.1834 & -0.0369 & 0.0369 \\ 0.3470 & -0.8305 & 0.0809 & 0.0613 & -0.0613 & 0.3415 & 1.8329 & 0.0833 & 0.0580 & -0.0580 \\ 0.1832 & 0.0809 & -0.6667 & -0.0205 & 0.0205 & 0.1834 & 0.0833 & 1.9909 & -0.0211 & 0.0211 \\ -0.0408 & 0.0613 & -0.0205 & -0.7595 & 0.3569 & -0.0369 & 0.0580 & -0.0211 & 1.9009 & 0.3568 \\ 0.0408 & -0.0613 & 0.0205 & 0.3569 & -0.7595 & 0.0369 & -0.0580 & 0.0211 & 0.3568 & 1.9009 \end{bmatrix},$$

$$\overline{R} = 10^3 \times \begin{bmatrix} 4.0263 & 0.4736 & 0.2916 & -0.0318 & 0.0318 & -3.0403 & 0.4688 & 0.2831 & -0.0326 & 0.0326 \\ 0.4736 & 4.1469 & 0.1709 & 0.0614 & -0.0614 & 0.4688 & -2.9173 & 0.1601 & 0.0627 & -0.0627 \\ 0.2916 & 0.1709 & 4.3290 & -0.0296 & 0.0296 & 0.2831 & 0.1601 & -2.7316 & -0.0301 & 0.0301 \\ -0.0318 & 0.0614 & -0.0296 & 4.2494 & 0.5421 & -0.0326 & 0.0627 & -0.0301 & -2.8198 & 0.5313 \\ 0.0318 & -0.0614 & 0.0296 & 0.5421 & 4.2494 & 0.0326 & -0.0627 & 0.0301 & 0.5313 & -2.8198 \\ -3.0403 & 0.4688 & 0.2831 & -0.0326 & 0.0326 & 4.0562 & 0.4338 & 0.2751 & -0.0251 & 0.0251 \\ 0.4688 & -2.9173 & 0.1601 & 0.0627 & -0.0627 & 0.4338 & 4.1627 & 0.1686 & 0.0522 & -0.0522 \\ 0.2831 & 0.1601 & -2.7316 & -0.0301 & 0.0301 & 0.2751 & 0.1686 & 4.3214 & -0.0271 & 0.0271 \\ -0.0326 & 0.0627 & -0.0301 & -2.8198 & 0.5313 & -0.0251 & 0.0522 & -0.0271 & 4.2585 & 0.5066 \\ 0.0326 & -0.0627 & 0.0301 & 0.5313 & -2.8198 & 0.0251 & -0.0522 & 0.0271 & 0.5066 & 4.2585 \end{bmatrix}, \tag{47}$$

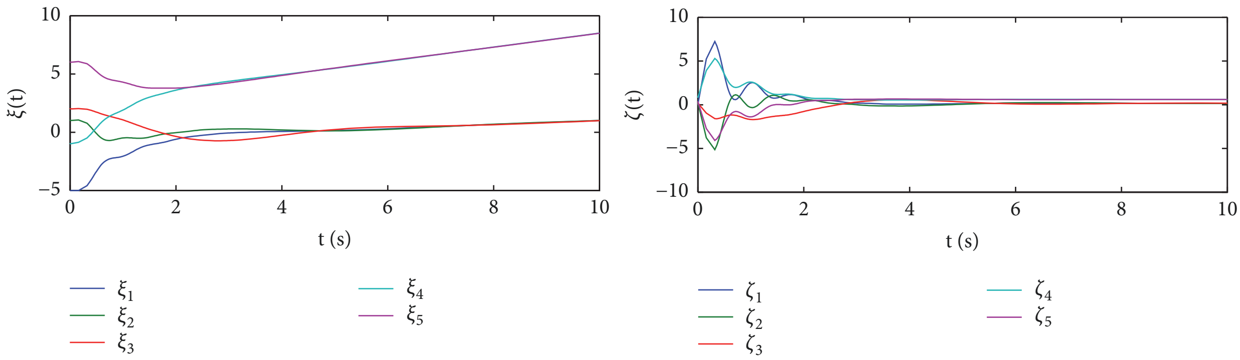

Figure 10: The state trajectories of five agents with communication delay $h = 0.135s$.

and $h_{max} = 0.21s$ is obtained. When $h = 0.135s < h_{max}$, the trajectories of all the agents are shown by Figure 10. It is clear that the couple-group consensus is achieved when $h < h_{max}$.

6. Conclusion

In this paper, the couple-group consensus problems of first-order and second-order multiagent systems with undirected topology via time-dependent event-triggered control protocols have been considered. Based on algebraic graph theory and matrix theory, the conditions for the cases without delays have been established to ensure couple-group consensus. For the cases with communication delays, the Lyapunov method has been used to guarantee the stability of the disagreement dynamics. The presence of time-dependent event-triggered function has saved energy consumption to a great extent. In addition, Zeno behavior has been excluded in all of those cases. In future work, we will focus on couple-group consensus of multiagent systems with directed topologies.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This work was supported by the National Natural Science Foundation of China (Grant no. 61304170) and the Fundamental Research Funds for the Central Universities (Grant no. FRF-TP-17-022A2).

References

[1] W. Yu, G. Chen, and M. Cao, "Distributed leader-follower flocking control for multi-agent dynamical systems with time-varying velocities," *Systems & Control Letters*, vol. 59, no. 9, pp. 543–552, 2010.

[2] B. Fidan, V. Gazi, S. Zhai, N. Cen, and E. Karatas, "Single-view distance-estimation-based formation control of robotic swarms," *IEEE Transactions on Industrial Electronics*, vol. 60, no. 12, pp. 5781–5791, 2013.

[3] H. Su, X. Wang, and G. Chen, "Rendezvous of multiple mobile agents with preserved network connectivity," *Systems & Control Letters*, vol. 59, no. 5, pp. 313–322, 2010.

[4] L. Ding, Q. Han, L. Y. Wang, and E. Sindi, "Distributed cooperative optimal control of dc microgrids with communication delays," *IEEE Transactions on Industrial Informatics*, 2018.

[5] H. Liu and G. Xie, "Sampled-data based containment control of continuous-time multi-agent systems with switching topology and time-delays," *Neurocomputing*, vol. 286, pp. 19–30, 2018.

[6] W. Ren and E. Atkins, "Distributed multi-vehicle coordinated control via local information exchange," *International Journal of Robust and Nonlinear Control*, vol. 17, no. 10-11, pp. 1002–1033, 2007.

[7] K. Liu, Z. Ji, G. Xie, and L. Wang, "Consensus for heterogeneous multi-agent systems under fixed and switching topologies," *Journal of The Franklin Institute*, vol. 352, no. 9, pp. 3670–3683, 2015.

[8] Y. Gao, B. Liu, M. Zuo, T. Jiang, and J. Yu, "Consensus of continuous-time multiagent systems with general linear dynamics and nonuniform sampling," *Mathematical Problems in Engineering*, Article ID 718759, 7 pages, 2013.

[9] K. Liu, G. Xie, and L. Wang, "Consensus for multi-agent systems under double integrator dynamics with time-varying communication delays," *International Journal of Robust and Nonlinear Control*, vol. 22, no. 17, pp. 1881–1898, 2012.

[10] L. Ding, Q.-L. Han, and G. Guo, "Network-based leader-following consensus for distributed multi-agent systems," *Automatica*, vol. 49, no. 7, pp. 2281–2286, 2013.

[11] F. Jiang, D. Xie, and M. Cao, "Dynamic consensus of double-integrator multi-agent systems with aperiodic impulsive protocol and time-varying delays," *IET Control Theory & Applications*, vol. 11, no. 16, pp. 2879–2885, 2017.

[12] Z. Zhang, L. Zhang, F. Hao, and L. Wang, "Leader-following consensus for linear and Lipschitz nonlinear multiagent systems with quantized communication," *IEEE Transactions on Cybernetics*, vol. 47, no. 8, pp. 1970–1982, 2017.

[13] Y. Zheng, J. Ma, and L. Wang, "Consensus of hybrid multi-agent systems," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 29, no. 4, pp. 1359–1365, 2018.

[14] D. V. Dimarogonas, E. Frazzoli, and K. H. Johansson, "Distributed event-triggered control for multi-agent systems," *IEEE Transactions on Automatic Control*, vol. 57, no. 5, pp. 1291–1297, 2012.

[15] Z. Liu and Z. Chen, "Event-triggered average-consensus for multi-agent systems," in *Proceedings of the 29th Chinese Control Conference (CCC '10)*, pp. 4506–4511, Beijing, China, July 2010.

[16] J. Hu, G. Chen, and H. Li, "Distributed event-triggered tracking control of leader-follower multi-agent systems with communication delays," *Kybernetika*, vol. 47, no. 4, pp. 630–643, 2011.

[17] J. Hu, "Second-order event-triggered multi-agent consensus control," in *Proceedings of the 29th Chinese Control Conference*, pp. 4506–4511, Beijing, China, 2010.

[18] W. Zhu, Z.-P. Jiang, and G. Feng, "Event-based consensus of multi-agent systems with general linear models," *Automatica*, vol. 50, no. 2, pp. 552–558, 2014.

[19] Y. Fan, G. Feng, Y. Wang, and C. Song, "Distributed event-triggered control of multi-agent systems with combinational measurements," *Automatica*, vol. 49, no. 2, pp. 671–675, 2013.

[20] G. S. Seyboth, D. V. Dimarogonas, and K. H. Johansson, "Event-based broadcasting for multi-agent average consensus," *Automatica*, vol. 49, no. 1, pp. 245–252, 2013.

[21] N. Mu, X. Liao, and T. Huang, "Consensus of second-order multi-agent systems with random sampling via event-triggered control," *Journal of The Franklin Institute*, vol. 353, no. 6, pp. 1423–1435, 2016.

[22] G. Jing, Y. Zheng, and L. Wang, "Flocking of multi-agent systems with multiple groups," *International Journal of Control*, vol. 87, no. 12, pp. 2573–2582, 2014.

[23] J. Ma, J. Zhou, and Y. Gao, "Distributed event-triggered control of multiagent systems with time-varying topology," *Mathematical Problems in Engineering*, vol. 2014, Article ID 276373, 6 pages, 2014.

[24] G. Guo, L. Ding, and Q.-L. Han, "A distributed event-triggered transmission strategy for sampled-data consensus of multi-agent systems," *Automatica*, vol. 50, no. 5, pp. 1489–1496, 2014.

[25] Q. Tan, X. Dong, Q. Li, and Z. Ren, "Distributed event-triggered cubature information filtering based on weighted average concensus," *IET Control Theory & Applications*, vol. 12, no. 1, pp. 78–86, 2018.

[26] E. Garcia, Y. Cao, and D. W. Casbeer, "Decentralized event-triggered consensus with general linear dynamics," *Automatica*, vol. 50, no. 10, pp. 2633–2640, 2014.

[27] D. Yang, W. Ren, X. Liu, and W. Chen, "Decentralized event-triggered consensus for linear multi-agent systems under general directed graphs," *Automatica*, vol. 69, pp. 242–249, 2016.

[28] H. Zhang, G. Feng, H. Yan, and Q. Chen, "Observer-based output feedback event-triggered control for consensus of multi-agent systems," *IEEE Transactions on Industrial Electronics*, vol. 61, no. 9, pp. 4885–4894, 2014.

[29] F. Xiao and T. Chen, "Sampled-data consensus in multi-agent systems with asynchronous hybrid event-time driven interactions," *Systems & Control Letters*, vol. 89, pp. 24–34, 2016.

[30] X. Liu, C. Du, P. Lu, and D. Yang, "Distributed event-triggered feedback consensus control with state-dependent threshold for general linear multi-agent systems," *International Journal of Robust and Nonlinear Control*, vol. 27, no. 15, pp. 2589–2609, 2017.

[31] T.-H. Cheng, Z. Kan, J. R. Klotz, J. M. Shea, and W. E. Dixon, "Event-triggered control of multiagent systems for fixed and time-varying network topologies," *Institute of Electrical and Electronics Engineers Transactions on Automatic Control*, vol. 62, no. 10, pp. 5365–5371, 2017.

[32] W. Xu, D. W. Ho, L. Li, and J. Cao, "Event-triggered schemes on leader-following consensus of general linear multiagent systems under different topologies," *IEEE Transactions on Cybernetics*, vol. 47, no. 1, pp. 212–223, 2017.

[33] L. Ding, Q. Han, X. Ge, and X. Zhang, "An overview of recent advances in event-triggered consensus of multiagent systems," *IEEE Transactions on Cybernetics*, vol. 48, no. 4, pp. 1110–1123, 2018.

[34] J. Yu and L. Wang, "Group consensus in multi-agent systems with switching topologies and communication delays," *Systems & Control Letters*, vol. 59, no. 6, pp. 340–348, 2010.

[35] J. Yu and L. Wang, "Group consensus of multi-agent systems with directed information exchange," *International Journal of Systems Science*, vol. 43, no. 2, pp. 334–348, 2012.

[36] J. Yu and L. Wang, "Group consensus of multi-agent systems with undirected communication graphs," in *Proceedings of the 7th Asian Control Conference (ASCC '09)*, pp. 105–110, Hong Kong, China, August 2009.

[37] Y. Feng, S. Xu, and B. Zhang, "Group consensus control for double-integrator dynamic multiagent systems with fixed communication topology," *International Journal of Robust and Nonlinear Control*, vol. 24, no. 3, pp. 532–547, 2014.

[38] D. Xie and T. Liang, "Second-order group consensus for multi-agent systems with time delays," *Neurocomputing*, vol. 153, no. 4, pp. 133–139, 2015.

[39] X. Dong, Q. Li, Q. Zhao, and Z. Ren, "Time-varying group formation analysis and design for general linear multi-agent systems with directed topologies," *International Journal of Robust and Nonlinear Control*, vol. 27, no. 9, pp. 1640–1652, 2017.

[40] D. Xie, L. Shi, and F. Jiang, "Group tracking control of second-order multi-agent systems with fixed and Markovian switching topologies," *Neurocomputing*, vol. 281, no. 3, pp. 37–46, 2018.

[41] S. Boyd, L. El Ghaoui, E . Feron, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*, SIAM, Philadelphia, PA, USA, 1994.

[42] P. Pepe and Z.-P. Jiang, "A Lyapunov-Krasovskii methodology for ISS and iISS of time-delay systems," *Systems & Control Letters*, vol. 55, no. 12, pp. 1006–1014, 2006.

[43] Y. Feng, J. Lu, S. Xu, and Y. Zou, "Couple-group consensus for multi-agent networks of agents with discrete-time second-order dynamics," *Journal of The Franklin Institute*, vol. 350, no. 10, pp. 3277–3292, 2013.