Research Article

Wear Calculation-Based Degradation Analysis and Modeling for Remaining Useful Life Prediction of Ball Screw

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Ball screw is a kind of precise transmission element in drive system of machine tool. In this paper, the degradation model of ball screw is proposed based on wear calculation-based degradation analysis and experimental data-based validation. At first, fatigue wear is analyzed to be the predominant degradation mode of ball screw. The wear volume formula of ball screw is derived as the function of working load and stroke number. Secondly, the degradation rate of ball screw is analyzed to be affected by the total degradation and wear rate. Based on this finding, the degradation model of ball screw is theoretically derived as an exponential model by inputting wear volume formula. Thirdly, experimental data-based cross-validation method is proposed to validate the exponential degradation model. Determination coefficients are calculated to evaluate the fitting degree between the degradation model and real degradation path. Next, run-to-failure test of ball screw is carried out to collect experimental data in different working conditions. The averaged determination coefficient of different working conditions is calculated as 0.7848, which indicates that the proposed model can well fit the actual degradation path. In addition, the proposed model is applied to predict remaining useful life (RUL) of the tested ball screw by using collected data. RUL is estimated in a high and stable accuracy after 168000 strokes. For further validation, comparison with linear model is performed. All results show that the exponential degradation model is reasonable and correct in reflecting the degradation process of ball screw.

1. Introduction

Ball screw is a kind of precise transmission component in drive system of machine tool to convert rotary motion to translational motion [1, 2]. Its performance degradation and failure will directly reduce the machining accuracy and quality, which may lead to machine tool breaking down and even catastrophic events. To avoid this situation, condition-based maintenance (CBM) is a good solution [3, 4]. CBM is a kind of maintenance strategy which monitors the health condition of machinery and makes an optimal maintenance decision based on condition monitoring information, thus reducing unexpected failures and improving the safety of the system operation. Si et al. [5] researched a condition-based maintenance problem with observed degradation signals for the determination of the optimal replacement time of the system. Do et al. [6] promoted a proactive CBM strategy considering both perfect and imperfect maintenance actions for a deteriorating system with nonlinear Wiener degradation processes. The RUL estimation of machinery is one of the major tasks in CBM [7] and prognostics and health management (PHM) [8]. The main contribution of RUL prediction is to forecast the time left before the machinery loses its operation ability, which is crucial for making maintenance decision of CBM. RUL of ball screw is determined as the interval between current time and the first time to reach the predetermined failure threshold. Degradation-based modeling methods have been recognized as an essential and effective approach for RUL prediction [9]. To achieve an accurate RUL estimation, factors influencing the degradation processes of the systems are considered in degradation modeling. Many degradation models can be developed by considering different factors. Reasonable and effective degradation model usually contributes a lot to accurate RUL estimation. Therefore, studying the degradation model of ball screw is of great significance, which can help
to accurately predict the RUL, thus improving the machining stability and preventing machine tool from sudden failure.

There are many degradation modeling methods in previous studies: (1) physical mechanism-based method; (2) data driven method; and (3) hybrid method. The physical mechanism-based method attempts to build mathematical or physical models to describe the degradation process of system based on degradation mechanisms. Marble et al. [10] developed a physics-based model for bearing prognostics by computing the spall growth trajectory and time to failure based on operating conditions. Chen et al. [11] built the accelerated degradation model of aerospace electrical connector after researching its failure mechanism. Physical mechanism-based method is direct and convenience. It can provide accurate model if the degradation mechanisms are clear. For complex system like ball screw, however, it is difficult to completely understand the failure mechanisms and establish precise degradation model only based on physical mechanism.

The data driven method is an approach to derive the degradation model based on the available observed data. It can be categorized into machine learning-based method and model-based method. Data driven approach is becoming more and more appealing in recent years. It does not need to know the exact failure mechanism during modeling. Machine learning-based method attempts to derive the degradation model from measured data using machine learning techniques. Zhang et al. [12] developed a performance degradation model of screw using quantum genetic algorithm and dynamic fuzzy neural network based on measured vibration signals. Maio et al. [13] proposed a method based on relevance vector machine to estimate the RUL of bearing. Liu et al. [14] proposed an enhanced recurrent neural network to predict the RUL of lithium-ion battery. Zhang et al. [15] presented a degradation recognition method based on deep belief networks and multisensor data fusion to monitor the degradation of ball screw. Machine learning-based method could be beneficial for complex machine whose mechanical principles are not straightforward so that developing an accurate model is impossible. However, shortcomings still exist: the accuracy of machine learning-based method is highly dependent on the quantity and quality of the measured signals. For component with high reliability and long useful life like ball screw, it is prohibitively expensive to collect enough degradation data to establish a machine learning-based degradation model.

Model-based method builds mathematical model at first and then estimates model parameters based on collected data to describe the degradation path. Elwany et al. [16] presented a stochastic degradation modeling framework of partially degraded components to compute the RUL. The Paris-Erdogan (PE) model is one of the most widely used models in the RUL prediction of machinery. Lei et al. [17] transformed the PE model into an empirical model for RUL prediction of machinery. Liao [18] employed the Paris model combined with a genetic programming method to predict the RUL of bearing. Wiener process models are a kind of the most commonly used stochastic process models. Wang et al. [19] developed a linear Wiener process model for RUL prediction of machinery. Si et al. [20] present a relatively general degradation model based on a Wiener process for RUL estimation by considering three-source variability. Paroissin et al. [21] established a randomly delayed Wiener process model considering the degradation starting at a random time. In addition, Liu et al. [22] proposed a degradation modeling approach for a system with multiple degradation patterns based on inverse Gaussian process. Tian et al. [23] proposed a proportional hazard model-based method for the RUL prediction of the systems consisting of bearings. Gebrael et al. [24] established an exponential degradation model with random error terms and updated the model parameters using Bayesian approach and real-time condition monitoring information. The model-based method incorporates both expert knowledge and measured data, predicting the RUL of machinery with less data. Therefore, it has more advantages than the physical mechanism-based method and machine learning-based method for ball screw.

The exponential model is an empirical model representing the degradation process where the cumulative damage has effect on the degradation rate. It has become one of the most popular degradation models among all model-based studies since first proposed by Gebrael et al. in [24]. Si et al. [25] applied exponential-based degradation model to estimate the RUL of a general system with the combination of Bayesian updating and expectation maximization (EM) algorithm. Wang et al. [26] fitted the degradation path of LED light bar using a biexponential model to predict the RUL under operating conditions. Babel et al. [27] fitted the behavior of the change in insulation current to an exponential model to predict the RUL of insulation. In addition, exponential model is also one of the most widely used degradation models for RUL of bearings [28]. Many studies about bearings RUL prediction have achieved good results by treating exponential model as degradation model. Wang et al. [28] proposed two-stage strategy to predict the health status of bearing based on exponential model. Li et al. [29] proposed an improved exponential model to predict the RUL of rolling element bearings. The exponential model has achieved many good results in previous studies. As it can be learned from previous literature, the selection of exponential degradation model is usually based on experience. Despite the wide use of exponential model, no literature has applied it to research the degradation of ball screw. What is more, very little research has been reported to investigate the degradation model of ball screw so far, which is a big gap in present study for degradation research and RUL prediction of ball screw. To fill this gap, it is necessary to study the degradation model and to predict the RUL of ball screw.

Aimed at these problems, wear calculation-based degradation analysis is combined with experimental data to study the degradation model of ball screw in this paper. In the proposed method, degradation analysis based on wear volume calculation is performed to derive the degradation model at first, and then the derived model is further verified using degradation data collected in run-to-failure test. The RUL of ball screw is also predicted based on the proposed degradation model. The rest of the paper is organized as follows. Section 2 analyzes the wear type of ball screw.
Section 3 analyzes the normal contact force between ball and raceway and then calculates the wear volume of ball screw. The degradation model of ball screw based on wear volume formula and degradation analysis is derived in Section 4. Section 5 proposes a degradation data-based cross-validation method. Section 6 validates the derived degradation model using cross-validation method and RUL prediction based on measured run-to-failure degradation data. Section 7 provides a conclusion and a discussion of future research directions.

2. Wear Type Analysis of Ball Screw

Ball screw is composed of guide screw, nut, and ball. There are three main typical failure modes for ball screw: surface damage, deformation failure, and fracture failure [30]. The deformation failure is mainly caused by high static load or large impact load, and the fracture failure is mainly caused by excessive load or excessive instantaneous load. When the ball screw is running, the surface of its raceway will suffer from alternating normal stress because the balls do roll-slide movement in the raceway constantly. Balls and raceway contact with each other under external load and do relative sliding, suffering friction force and normal contact force, forming friction surface and wear. Therefore, the surface damage is considered as the main degradation mode for qualified ball screw when installation and working load both meet the requirements and no corrosion material or foreign matter contamination exists. It has been proposed by researchers about the degradation of bearing that if the bearing is properly loaded, lubricated, installed, and kept free of foreign contaminants, then the main mode of bearing failure is material fatigue [31]. However, literature about the wear types of ball screw is still lacking.

In order to further study the wear type of ball screw, the microstructure of its raceway is observed using electron microscope. Properly loaded and lubricated ball screw that is installed on a test bench is selected. For comparison, the micrographs of new screw raceway and degraded screw raceway are both photographed as shown in Figure 1. It can be learned by comparing Figures 1(a) and 1(b) and Figures 1(c) and 1(d) that metallic particles flaking and spalled pit are generated on the raceway of degraded screw, which is judged to be fatigue spalls. Fatigue spalls typically occur at microstructural discontinuities such as inclusions and carbide clusters where the resultant stress exceeds the local microyield limit at that fatigue cycle.

Repeated cyclic stress is considered as the main cause of fatigue spalls. Cracks initiate at the surface stress concentrators and branch up toward the free surface when they reach a critical length or depth, removing a piece of surface material, and form a pit as shown in Figure 1(d) [31]. Fatigue crack growth will finally cause rolling contact fatigue and results in metallic particles flaking from the surface of the balls and raceway. This process that is caused by rolling contact fatigue is the fatigue wear, which is the predominant degradation mode for properly loaded, lubricated, and installed ball screw.
3. Fatigue Wear Volume Calculation

3.1. Normal Contact Force Analysis of Ball Screw. Single nut ball screw with single arc raceway is chosen to analyze the normal contact force in contact surface between ball and raceway. The force analysis diagram of ball is shown in Figure 2(a). Contact points between ball and guide screw, ball and nut are H and G, respectively. Normal contact forces of ball at points H and G are expressed as \( p_{n0} \) and \( p_{n1} \), respectively, when the axial working load is \( F_n \). Points H, G and ball center O are collinear. According to the equilibrium condition of two forces, normal contact forces in H and G are equal in magnitude but opposite in direction, which can be expressed as 

\[ p_{n0} = p_{n1}. \]

Coordinate system \( Ox y z \) is established to facilitate the normal contact force analysis and calculation as shown in Figure 2(b), where x axis is in the axial direction of screw, y axis is in the radial direction of screw, and z axis is in the tangent direction of screw in point O. According to the geometrical relationship shown in Figure 2(b), the included angle between y axis and the \( HG \) connecting line is the contact angle \( \alpha \), and mapping of included angle between \( x \) axis and the projection of \( HG \) connecting line in \( xOz \) surface is the helix angle \( \gamma \).

Hence the relationship between axial working load \( F_n \) and normal contact forces \( p_{n0}, p_{n1} \) is

\[ F_n = z_n p_{n0} \sin \alpha \cos y = z_n p_{n1} \sin \alpha \cos y \] (1)

where \( z_n \) is the number of working balls.

The normal contact forces \( p_{n0}, p_{n1} \) can be derived by formula (1).

\[ p_{n0} = p_{n1} = \frac{F_n}{z_n \sin \alpha \cos y} \] (2)

3.2. The Wear Volume Calculation. It is found that the guide screw wear is more serious than the nut in practice [32], and thus the wear volume of guide screw is calculated to indicate the performance degradation of ball screw in this paper. Researchers in International Business Machines Corporation (IBM) have put forward a model for calculating the wear, which is the function of two variables: the stroke number and energy [33, 34], and can be expressed as a differential equation

\[ d Q = \left( \frac{\partial Q}{\partial E} N \right) d E + \left( \frac{\partial Q}{\partial N} E \right) d N \] (3)

where \( Q \) is the measurable wear volume, \( E \) is the energy consumed during each stroke, and \( N \) is the number of strokes used to express the useful life of ball screw.

Formula (3) can be rewritten as a differential equation about fatigue wear volume \( W \) and stroke number \( N \) for fatigue wear [33, 35]

\[ d \left[ \frac{W}{(\tau_{\text{max}} S)^{9/2}} \right] = K_1 d N \] (4)

where \( K_1 \) is the fatigue wear constant of screw, \( \tau_{\text{max}} \) is the maximum shear stress that ball screw suffers from, and \( S \) is the sliding distance in each stroke.

In order to simplify the calculation, it is assumed that the axial working load of ball screw is constant. Hence the maximum shear stress in contact surface between ball and raceway is

\[ \tau_{\text{max}} = \tau = \frac{P_n}{A} \] (5)

where \( P_n \) is the normal contact force that equals \( p_{n0} \) and \( p_{n1} \) in magnitude and \( A \) is the area of contact surface between ball and raceway.

The sliding distance of ball screw in each stroke can be calculated according to the movement principle

\[ S = \frac{\pi d_n n_r}{\cos y} \] (6)

where \( d_n \) is the nominal diameter of guide screw and \( n_r \) is the turning laps of ball screw in each stroke.

Considering the initial wear volume \( W_0 \) and the initial stroke number \( N_0 \) of ball screw, take the integral to both sides of formula (4), and the calculation formula of fatigue...
worn volume of ball screw can be derived by substituting into formula (2), (5), and (6)

\[ W = K_1 \left( \frac{\pi d N r}{z_A A \sin \alpha \cos^2 \gamma} \right)^{9/2} F_n^{9/2} (N - N_0) + W_0 \]  

(7)

It can be learned from the fatigue wear volume formula of ball screw that once the type of ball screw is determined (the intrinsic parameters of ball screw including \( K_1, d, N, z, \alpha, \gamma, A \) are fixed), the fatigue wear volume is in proportion to stroke number \( N \) and nonlinearly related to axial load \( F_n \), which is in accordance with that proposed in [33]. Stroke number \( N \) is used to express the useful life of ball screw in this paper.

### 4. Degradation Model Derivation

#### 4.1. Derivation of Degradation Model Based on Wear Volume Formula

There are many factors affecting the instantaneous degradation, including initial degradation-level, amount of harmful material, material properties, operating conditions, and environmental conditions (temperature and humidity) [36]. The initial degradation-level, material properties, and environmental conditions are fixed for ball screw with decided type and operating environment. Therefore, the degradation of ball screw is affected by the amount of harmful material and operating condition. Amount of harmful material can be measured by total degradation for ball screw. The wear rate of ball screw is related to axial load \( F_n \) according to formula (7), which is a representation of operating condition. Therefore, both the total degradation and wear rate will affect the instantaneous degradation rate of ball screw [37].

Based on these theories, it is assumed that the degradation rate of ball screw is proportional to degradation and wear rate. The degradation model of ball screw will be derived based on this assumption in this paper. The rationality of this assumption will be validated through the derived degradation model, and the degradation model will be verified by analyzing degradation data collected from run-to-failure test of ball screw. The equation is listed according to the above assumption

\[ \frac{dD}{dN} = c_1 D \frac{dW}{dN} \]  

(8)

where \( dD/dN \) is the degradation rate of ball screw, \( D \) is the degradation, \( dW/dN \) is the instantaneous wear rate for ball screw whose wear volume is \( W \), and \( c_1 \) is a constant.

Consider the initial degradation of ball screw as \( D_0 \). Take the integral to both sides of formula (8) to get the simplified formula.

\[ D = \exp \left[ c_1 (W - W_0) \right] + D_0 \]  

(9)

The degradation model is derived by substituting the wear volume formula (7) into formula (9).

\[ D = \exp \left\{ c_1 \left( K_1 \left( \frac{\pi d N r}{z_A A \sin \alpha \cos^2 \gamma} \right)^{9/2} F_n^{9/2} N_0 \right) \right\} \]  

(10)

In order to make the structure clearer and easier to understand, measures are taken to simplify the derived degradation model.

Define \( k_1 \) and \( k_2 \) as follows.

\[ k_1 = \exp \left( -c_1 K_1 \left( \frac{\pi d N r}{z_A A \sin \alpha \cos^2 \gamma} \right)^{9/2} F_n^{9/2} N_0 \right) \]  

(11)

\[ k_2 = c_1 K_1 \left( \frac{\pi d N r}{z_A A \sin \alpha \cos^2 \gamma} \right)^{9/2} F_n^{9/2} N_0 \]  

Then, substitute \( k_1 \) and \( k_2 \) into formula (10) to simplify the degradation model as follows.

\[ D = D_0 + k_1 \exp (k_2 N) \]  

(12)

\( k_1 \) and \( k_2 \) are both constant once the type of ball screw is determined and the axial load \( F_n \) is invariable. The degradation \( D \) is the monotonic function of stroke number \( N \) in this situation. Stochastic effect during degradation is considered in the degradation model [8, 28]. The degradation model of ball screw can be expressed as

\[ D(N) = D_0 + k_1 \exp (k_2 N) + \sigma B(N) \]  

(13)

where \( B(N) \) is the standard Brownian motion and \( \sigma \) is the diffusion coefficient.

It can be learned from formula (13) that the degradation analysis-based degradation model is a kind of exponential degradation model. It is a typical model in representing the degradation process where cumulative damage has a particular effect on the degradation rate [25, 28]. Exponential model, which was first established by Gebraeel et al. in [24], has been widely used in modeling degradation processes as a kind of experience-based model. Many studies indicate that the exponential model works well in exponential-like degradation processes [25]. In addition, exponential model is also widely used as the degradation model to predict the RUL of bearing [28]. Bearing is similar with ball screw in structure and many mathematical description methods. The success of exponential model in those studies of bearing can preliminarily determine the correctness of the derived degradation model of ball screw.

A diagrammatic curve based on the derived degradation model is drawn to describe the degradation process of ball screw as shown in Figure 3. It is seen that the ball screw degrades rapidly after reaching the critical stroke \( N_c \), which is in accord with the degradation process of machine where cumulative damage exists. Hence the degradation process of ball screw can be divided into two stages based on the critical stroke, i.e., (I) the normal operation stage before \( N \) and (II) the degradation stage after \( N_c \). It is speculated according to Figure 3 that the degradation will propagate quickly once the critical stroke is reached.

#### 4.2. Construction of Degradation Index

After the wear calculation-based degradation model of ball screw is derived, the degradation index needs to be constructed to measure the degradation \( D \). There are two methods for measuring the degradation of ball screw: direct measurement and indirect
The calculation process of Weibull distribution shape parameter of vibration signal envelope can be divided into the following two steps:

1. **Envelope Extraction.** Envelope of raw signal is extracted based on Hilbert transformation, and the formula of calculating the envelope signal is

   \[ b(t) = \sqrt{x(t)^2 + \bar{x}(t)^2} \]  

   where \( b(t) \) is the envelope signal of raw signal, \( x(t) \) is the raw signal, and \( \bar{x}(t) \) is the Hilbert transformation of raw signal. \( \bar{x}(t) \) can be calculated by the following formula.

   \[ \bar{x}(t) = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{x(\tau)}{t-\tau} d\tau \]  

2. **Weibull Distribution Shape Parameter Calculation.** Fit the calculated envelope signal \( b(t) \) into a two-parameter Weibull distribution model to get the shape parameter of envelope signal. The probability density function of the two-parameter Weibull distribution model is

   \[ f(x) = \frac{\beta}{\eta} \left( \frac{x}{\eta} \right)^{\beta-1} \exp \left[ -\left( \frac{x}{\eta} \right)^\beta \right] \]  

   where \( \beta \) is the shape parameter and \( \eta \) is the scale parameter. The shape parameter of Weibull distribution model is calculated by utilizing maximum likelihood estimation and Newton iterative method.

   As the degradation index, the Weibull distribution shape parameter of vibration signal envelope can be used to measure the degradation of ball screw. It can be treated as the degradation \( D \) in the derived degradation model (13) to express the degradation model of ball screw. The Weibull distribution shape parameter of vibration signal envelope is described as \( D_\beta \). Therefore, the derived degradation model (13) of ball screw can be rewritten as the degradation index-based degradation model, which is expressed as follows.

   \[ D_\beta (N) = D_{90} + k_1 \exp (k_2 N) + \sigma B(N) \]  

   Correctness of the derived degradation model (17) and the selected degradation index \( D_\beta \) will be validated in following paragraphs by proposing validation method and collecting degradation data from run-to-failure test.

## 5. Degradation Data-Based Cross-Validation Method

The degradation model of ball screw has been established according to degradation analysis in previous sections. The cross-validation method is proposed to verify the exponential model in this section. This method validates the derived exponential degradation model using cross-validation theory based on experimental data, alternately calculating the goodness of fit between the derived degradation model and real degradation path formed by collected degradation data.
5.1. A Brief Introduction to Cross-Validation Theory and Determination Coefficient. Cross-validation is a kind of statistical analysis method that can be used to verify model by calculating the goodness of fit between model and real process data. It divides the original process dataset into \( n \) groups and then alternately treats \( n-1 \) groups as training set while treating the remaining group as validation set [43]. The cross-validation method used for model verification usually consists of two steps: firstly, estimate unknown parameters of model using \( n-1 \) groups of training set to obtain a known model; secondly, calculate the goodness of fitting between the obtained known model and the remaining validation set, and treat the goodness of fitting as the performance evaluation index. Therefore, the data collected in degradation test of ball screw can be used to verify the derived degradation model according to the cross-validation method.

Determination coefficient, which is expressed as \( R^2 \), is usually treated as the measurement of fitting [44]. \( R^2 \) ranges from 0 to 1. Bigger value of \( R^2 \) means better fitting degree. The goodness of fit between degradation model and degradation process data is measured by determination coefficient \( R^2 \). It is calculated as

\[
R^2 = 1 - \frac{\sum_{i=1}^{m} (D_i - \hat{D}_i)^2}{\sum_{i=1}^{m} (D_i - \bar{D})^2} \tag{18}
\]

where \( m \) is the number of degradation data, \( D_i \) is the degradation of the \( i \)th group of degradation data, \( \hat{D}_i \) is the average degradation of all the performance degradation data, and \( \bar{D} \) is the degradation calculated by degradation model in the \( i \)th group of degradation process data.

5.2. Cross-Validation Method Based on Experimental Data. Experimental data collected in the degradation test of ball screw can be utilized to calculate the degradation path to validate the derived degradation model. Validation method is proposed based on cross-validation theory and experimental degradation data. This method divides the degradation data of ball screw into \( n \) groups and alternatively selects \( n-1 \) groups as training set to estimate unknown parameters of the derived degradation model. Then, the goodness of fitting (measured by determination coefficient) between the estimated model and the remaining validation set is calculated to evaluate the derived model.

As shown in Figure 4, the degradation data-based cross-validation method mainly consists of data preprocessing and cross-validation. The overall process of the proposed method is described as follows.

**Step 1.** Divide total raw degradation signal set into \( n \) groups and the grouped vibration signal sets are expressed as \( \{S\}_1, \{S\}_2, \cdots, \{S\}_n \). The raw degradation signals can be grouped according to the repetition times of data collection, which means the number of clusters is the same as the number of repeats.

**Step 2.** Calculate corresponding degradation sets of the \( n \) groups of raw vibration signal sets based on constructed degradation index. The calculated degradation sets, which are respectively expressed as \( \{D\}_1, \{D\}_2, \cdots, \{D\}_n \), can be used to form real degradation path and also can be used to estimate unknown parameters of degradation model.

**Step 3.** Select \( n-1 \) groups of degradation sets from \( \{D\}_1, \{D\}_2, \cdots, \{D\}_n \) for model training, and the remaining degradation set is used for model validation. The selection is alternate. It repeats for \( n \) times to ensure that each degradation set can be used for both model training and validation.

**Step 4.** Estimate unknown parameters of derived degradation model using the selected \( n-1 \) groups of degradation sets. The obtained known degradation model sets can be expressed as \( \{M\}_1, \{M\}_2, \cdots, \{M\}_n \), respectively.

**Step 5.** Calculate determination coefficient sets between the obtained degradation model sets and real degradation path formed by the remaining group of degradation validation set. Cross-validation is required for \( n \) times, and \( n \) groups of determination coefficient sets are obtained. The calculated determination coefficient sets are, respectively, expressed as \( \{R^2\}_1, \{R^2\}_2, \cdots, \{R^2\}_n \).

**Step 6.** Calculate the average of all determination coefficient sets, and the average determination coefficient set is expressed as \( \bar{R}^2 \).

The average determination coefficient set is treated as the index of the derived degradation model in describing the performance degradation process of ball screw. Compared with validating the derived degradation model directly using the degradation data, the cross-validation method makes full use of the collected degradation data through cross-validating among multiple groups of data. This method validates the derived degradation model by calculating the determination coefficient between this model and the real degradation path of ball screw.

6. Experimental Data-Based Degradation Model Validation

In this section, run-to-failure test of ball screw is designed to collect experimental degradation data to generate the real degradation path. The constructed degradation index is validated by using the collected experimental data. Cross-validation method proposed in Section 5 is then utilized to verify the derived exponential degradation model. Besides, RUL of ball screw is predicted based on the proposed exponential model to further validate its correctness and rationality.

6.1. Experiment Study Description and Degradation Data Acquisition. FFZD4010R-3 type of ball screw is chosen to do run-to-failure test by mounting on the acceleration performance degradation test bench, which is designed to simulate the whole performance degradation process of ball screw from new to failure during processing. As shown in Figure 5,
Figure 4: Framework for degradation data-based cross-validation method.
this test bench consists of driving part and loading part and is composed of driving motor, slide guide, ball screw, rack and pinion unit, and magnetic power brake. In the driving part, drive torque and rotational speed of ball screw are provided by driving motor. In the loading part, the axial load of ball screw is provided by magnetic powder brake via rack and pinion drive and could be varied by changing the input current of magnetic powder brake. Industrial PC and PC-based control system are used in the test bench to carry out motion control, load control, and data collection as shown in Figure 5.

Three accelerometers are used to collect three groups of vibration signals at the same time. Two accelerometers are mounted on ball nut to collect signals in radial and axial direction, respectively, and the other accelerometer is mounted on bearing seat to collect signals in radial direction of screw. Accelerometers 1, 2, and 3, respectively, represent the sensors mounted on radial direction of bearing seat, radial direction of ball nut, and axial direction of ball nut. Vibration signals of each degradation degree are collected in 9 working conditions, which are the all cross combination of three rotational speeds and three axial loads as shown in Table 1. Each group of data collection repeats for 3 times in order to reduce the influence of random factors and satisfy the repeated design principle of experiment. Therefore, 81 groups of vibration signals are collected by 3 accelerometers for 9 working conditions in every degradation degree. The sampling interval between two adjacent degradation degrees is 1000 strokes, and 217 sets of degradation data are collected when the ball screw degraded. Constant axial working load is 2kN and rotational speed is 300 r/min in wear condition. The sampling frequency is 5000Hz. In conclusion, the degradation vibration signals of ball screw consist of 17577 groups of samples. The measured vibration signals consist of main signals that reflect the degradation of ball screw and noise from environment and testing equipment. All the collected samples are utilized in validating the derived degradation model of ball screw.

6.2. Degradation Index Verification and Data Preprocessing.
The degradation index constructed in Section 4 needs to be verified before data preprocessing. After that, the degradation sets can be calculated based on the validated degradation index and raw degradation signals in the data preprocessing.

6.2.1. Degradation Index Verification. The degradation index of ball screw that has been constructed in Section 4 is the Weibull distribution shape parameter of vibration signal envelope. In order to verify whether the degradation index could well reflect the degradation of ball screw, data collected in the run-to-failure test is utilized. For reducing computation, working conditions 1, 5, and 9 are selected because these three working conditions contain all these 3 axial loads and 3 rotational speeds. Therefore, vibration signals collected in working condition 9 by accelerometer 1, working condition 5 by accelerometer 2, and working condition 1 by accelerometer 3 are selected to validate the degradation index in this section.

Table 1: Working conditions for degradation signals collection.

<table>
<thead>
<tr>
<th>Working condition</th>
<th>Axial load (kN)</th>
<th>Rotational speed (r/min)</th>
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<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>100</td>
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<td>7</td>
<td>0</td>
<td>800</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>800</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td>800</td>
</tr>
</tbody>
</table>
Figure 6 shows the temporal vibration signals of the tested ball screw at different degradation stages, including the normal operation stage, the degradation stage, and the failure stage. It is seen that the amplitude of collected temporal vibration signals increases over degradation. This indicates that the vibration signals can reflect the performance degradation process of ball screw and play a significant role in the performance degradation assessment.

The degradation sets of corresponding vibration signals are also calculated according to the construction procedure of the Weibull distribution shape parameter of signal envelope proposed in Section 4 to verify the degradation index. Figure 7 shows the performance degradation path of ball screw formed by the degradation sets. Degradation calculated by degradation data collected in one working condition by one sensor can be drawn to a line to form the performance degradation path. It is seen from Figure 7 that the degradation increases relatively slow in early stage and rise rapidly in later stage, which shows a monotonous increasing trend. Monotonicity is usually utilized to evaluate an excellent degradation index. Therefore, the monotonicity of the calculated degradation index indicates that the proposed Weibull distribution shape parameter of signal envelope can well reflect the degradation process of ball screw.

Figure 7 is also compared with Figure 6 to further validate the effectiveness of the proposed degradation index. It is seen that the degradation index-based degradation path has higher monotonicity and tendency and is more direct and clear in describing the degradation process of ball screw than the temporal vibration signals. The constructed Weibull distribution shape parameter of signal envelope performs well in describing the degradation process of ball screw and is suitable to be treated as the degradation index.

6.2.2. Degradation Index-Based Data Preprocessing. There are mainly two steps for calculating the degradation based on the proposed degradation index: firstly, use Hilbert transformation to extract envelope of vibration signal in each group; then, calculate Weibull distribution shape parameter of the extracted envelope, and treat it as the degradation.

Raw vibration signals are collected in 9 working conditions by 3 accelerometers in the whole degradation process. Data collection repeats for 3 times, and therefore the raw experimental signals are divided into 3 groups according
to the repeated time. Wavelet denoising method is applied to denoise the raw experimental signals before calculating degradation index. Degradation sets of these 3 groups of raw experimental signals are calculated as \( \{D_{\beta 1}\}, \{D_{\beta 2}\}, \) and \( \{D_{\beta 3}\}, \) respectively. These 3 groups of degradation sets have the same structure because they are calculated by the repeated collection of signals. Each degradation set consists of the Weibull distribution shape parameter of vibration signals envelope collected in 9 working conditions by 3 accelerometers. Two of these 3 groups of degradation sets are alternately selected as the training sets while the remaining group is treated as the validation set. There are 3 times of cross-validations; hence the selection is repeated for 3 times, and every selection is different from the others. The average of these two selected training sets is calculated to estimate unknown parameters in degradation model, obtaining known degradation models which are, respectively, expressed as \( \{M\}_1, \{M\}_2, \) and \( \{M\}_3. \) The remaining shape parameter set is used to generate the degradation path. The fitting degree between the known degradation model and the degradation path is calculated to measure the derived degradation model of ball screw.

Two hundred and seventeen groups of degradation vibration signals are collected in one working condition by one accelerometer for one repetition; hence 217 degradation values are calculated to form a degradation group. Each degradation set of \( \{D_{\beta 1}\}, \{D_{\beta 2}\}, \) and \( \{D_{\beta 3}\}, \) contains 27 such degradation groups because signals in each set are collected in 9 working conditions by 3 accelerometers. In conclusion, preprocessed data totally contain 81 degradation groups, and each group contains 217 Weibull distribution shape parameters of the degradation signal envelope. Each degradation group can generate one corresponding degradation path, which could be used to validate the derived degradation model by calculating one determination coefficient. Therefore, 3 groups of determination coefficients will be calculated because of the 3 degradation sets, and each group contains 27 determination coefficients.

6.3 Cross-Validation for the Derived Degradation Model

6.3.1 Fitting between the Degradation Model and Degradation Path. In the proposed cross-validation method, the fitting degree between the degradation model and the real degradation path is used to evaluate the effectiveness of the derived degradation model. Vibration signals collected in degradation test are utilized to calculate the degradation path and update unknown parameters of degradation model.

In order to show the fitting process visually, degradation data collected in different working conditions by different accelerometers are selected in this section to calculate degradation path and corresponding degradation model. Fitting
curves of the calculated degradation model and degradation path are drawn. Both the degradation model and the degradation path in these figures are normalized. Working conditions 1, 5, and 9 are selected to form fitting curves because these three working conditions contain all these 3 axial loads and 3 rotational speeds. Fitting curves of these 3 working conditions are, respectively, shown in Figures 8, 9, and 10. Figures 8(a), 8(b), and 8(c), respectively, represent fitting curves formed by signals collected in different repetitions by different accelerometers for working condition 1. To simplify graphics, Figure 8(a) represents fitting curves of cross-validation 1 and accelerometer 3, Figure 8(b) represents fitting curves of cross-validation 2 and accelerometer 2 and and Figure 8(c) represent fitting curves of cross-validation 3 and accelerometer 1. Figures 9 and 10 have the same structure as Figure 8 and, respectively, represent fitting curves of working condition 5 and 9.

In addition, determination coefficients $R^2$ of working conditions 1, 5, and 9 are also calculated as listed in Table 2, to measure the fitting degree between degradation model and fitting curves. Determination coefficients that correspond to fitting curves in Figures 8, 9, and 10 are marked in bold black font as shown in Table 2. Each group of vibration signals collected in one working condition by one accelerometer could get three determination coefficients because three repetitive cross-validations are applied. The average of these three determination coefficients is calculated to measure the goodness of fit between the derived degradation model and the real degradation path in this working condition. Accelerometer is abbreviated as Acc in Table 2.

It is seen from these figures that the calculated degradation path and the derived degradation model increase relatively slow in early stage but rise rapidly in later stage, both showing an exponential increase trend. In Table 2, all
the average determination coefficients are over 0.6546 and the maximum of these determination coefficients is 0.8538. All of these figures and the table indicate that the derived degradation model fits well with the real degradation path of ball screw.

For comparison, the linear model is also selected to fit the degradation path of ball screw, which is described as

\[ D = a + \mu N + \sigma_l B_l(N) \]  

(19)

where \( D \) is the degradation of ball screw, \( N \) is the number of stroke, \( a \) and \( \mu \) are model parameters, \( B_l(N) \) is the standard Brownian motion, and \( \sigma_l \) is the diffusion coefficient.

Taking working conditions 1, 5, and 9 as examples, the determination coefficients between linear model and degradation path of ball screw are calculated and listed in Table 3.

It can be seen from Table 3 that most of the determination coefficients obtained by linear model in working conditions 1 and 5 are around 0.5, and the maximum value is 0.5781, which is less than 0.6. In working condition 9, the determination coefficients are relatively large and most of them are greater than 0.7. However, it is also less than the value of the proposed exponential model. The results show that the fitting between linear model and degradation path is worse than that of the proposed model, which means the proposed exponential model performs much better than the linear model in describing the real degradation process of ball screw. The comparison further validates the correctness and rationality of the proposed degradation model.

### 6.3.2. The Results of Cross-Validation

The derived degradation model of ball screw is validated by calculating determination coefficients based on the proposed cross-validation method and collected data in this section. Cross-validation was carried out for 81 times because 81 groups of degradation data are collected in 9 working conditions by 3 accelerometers.
for 3 repetitions. One cross-validation will calculate one determination coefficient; hence 81 determination coefficients are generated to measure the goodness of fit between degradation model and degradation path. The 81 determination coefficients can be divided into 3 sets according to the 3 repetitive data collections, and each set contains 27 determination coefficients. The average $\{R^2\}$ of these three determination coefficient sets is calculated to evaluate the derived degradation model in describing the performance degradation of ball screw. The average determination coefficient set $\{R^2\}$ also contains 27 average determination coefficients because every group of degradation data collected in one working condition by one accelerometer could generate one corresponding average determination coefficient. The values of average determination coefficient set $\{R^2\}$ are shown in Figure 11. The averages of average determination coefficients collected in one working condition are also calculated and shown in Figure 11.

It can be seen from the results that average determination coefficients change in $[0.6633, 0.8594]$, and the average of all average determination coefficients is calculated as 0.7848. The results mean that the derived degradation model fits well with the real degradation path of ball screw. The high fitting degree indicates the correctness of degradation analysis about ball screw in this study. The results also illustrate that the derived exponential degradation model performs well in describing the degradation process of ball screw.

Furthermore, it is learned that degradation models calculated by degradation signals collected in different axial loads in different rotational speeds by different sensors all fit well with corresponding degradation paths, which means the fitting of the degradation model is not affected by the axial load, rotational speed, and position of data collection. The degradation model can describe the performance degradation of ball screw in different axial loads and rotational speeds, which indicates that the derived exponential degradation model can essentially represent the performance degradation process of the ball screw.

![Figure 11: Value of average determination coefficients $\{R^2\}$](image)

**Figure 11:** Value of average determination coefficients $\{R^2\}$.

6.4. Ball Screw RUL Prediction Using the Proposed Degradation Model

6.4.1. RUL Prediction Method of Ball Screw. To further demonstrate the correctness and capability of the proposed exponential degradation model for RUL prediction, the vibration signals collected in working condition 1 by accelerometer 3 are selected to estimate the RUL of ball screw based on the particle filtering (PF) algorithm proposed in [45]. The PF algorithm is utilized to update the model parameters and predict the RUL value by incorporating the measured data, the developed degradation model, and failure threshold. In this paper, RUL is considered as the interval between current time and the first time to reach the predefined failure threshold of monitored degradation data. The flowchart of the RUL prediction method based on PF algorithm is shown in Figure 12.

![Figure 12: Flowchart of PF algorithm-based RUL prediction method.](image)
The initial prediction point of ball screw is defined as the 120th point (120000 strokes), and the failure threshold of ball screw is set as when the amplitude of the acceleration signal overpassed 20g [15]. The PF algorithm is utilized to integrate the degradation model with measured dataset for parameters updating and RUL prediction after the degradation starts. At each inspection interval after initial prediction point, a corresponding RUL distribution can be obtained at this time based on failure threshold and newly collected degradation data. The 50% percentile of the RUL distribution is decided as the predicted remaining useful life of current time.

6.4.2. RUL Prediction Results. The RUL prediction results of ball screw based on the derived exponential degradation model and PF algorithm are shown in Figure 13. It can be observed that the predicted RUL converges to the real RUL as time goes, and the RUL can be estimated in a high and stable accuracy after 168000 strokes. At the beginning of predicting process, the predicted RULs deviate a lot from the real values because of the existence of multiple uncertainties due to lack of measurements. The uncertainties reduce as more measured degradation data become available, which enables estimating the RUL in a higher accuracy.

The prediction error of RUL is calculated to evaluate the performance of the prediction method and corresponding degradation model, which is defined as form

$$E_{R_i} = ActRUL_i - PredRUL_i$$  \hspace{1cm} (20)

where $E_{R_i}$ is the RUL prediction error of the $i$th point and $ActRUL_i$ and $PredRUL_i$ are the actual RUL and the predicted RUL of the $i$th point, respectively.

The RUL prediction errors based on the proposed degradation model are shown in Figure 14. It can be learned from Figure 14 that the prediction error becomes smaller as the amount of measured degradation data increase with time. The prediction error reduced to a very low value after 168000 strokes.

For comparison, the linear model is also applied as degradation model to predict the RUL of ball screw. The RUL prediction results and the prediction errors of ball screw based on the linear model-based prediction method are shown in Figures 13 and 14, respectively. As observed from the figures, there exist large gaps between the estimated RULs and the actual RULs of ball screw for a long time at the beginning by using the linear model. The predicted RUL by linear degradation model deviates to some extent until 195000 strokes, which is much larger than 169000 strokes of the proposed model. The linear model-based RUL estimation converges much slower than the proposed exponential degradation model.

In general, the proposed exponential model-based prediction method converges quickly and has small value of prediction error after reaching 169000 strokes, which indicates that the prediction method based on the proposed degradation model can accurately predict the RUL of the ball screw, thus proving the effectiveness and correctness of the proposed exponential degradation model.

7. Conclusions

In this paper, the degradation model of ball screw was derived by degradation analysis and further validated by the collected
experimental degradation data. The degradation modeling process of ball screw can be structured in the following stages. Firstly, wear volume of ball screw was calculated. Fatigue wear was analyzed to be the predominant failure mode for properly loaded, lubricated, installed ball screw. Based on the wear type analysis and contact force analysis, the wear volume for qualified ball screw whose parameters are fixed was calculated as the function of working load and stroke number in reasonable using condition. Then, the degradation model of ball screw is derived. The influence of total degradation and wear rate on the degradation rate of ball screw was expressed as one differential equation. The degradation model was derived based on this equation. It was obtained as an exponential model by inputting the wear volume formula. Stochastic effect during degradation was considered in the derived exponential degradation model. The Weibull distribution shape parameter of vibration signal envelope was chosen as the degradation index of the degradation model, which has been validated to perform well in describing the degradation process of ball screw. Thirdly, cross-validation method based on degradation data was proposed to validate the exponential degradation model. Determination coefficient was calculated as the index of the derived degradation model in describing the performance degradation process of ball screw. Run-to-failure test of ball screw was carried out to collect degradation data in 9 working conditions. The average determination coefficients calculated from all the degradation data vary between 0.6633 and 0.8594, and the average of all determination coefficients is 0.7848, which indicates the proposed model can well fit the actual degradation path of ball screw. In addition, the proposed model was applied to predict RUL of the tested ball screw with the help of particle filtering (PF) algorithm by using collected data. The RUL of the ball screw was predicted in a high and stable accuracy after 168000 strokes based on the proposed exponential degradation model. For further validation, comparative study with linear model was also performed. All results indicated that the exponential degradation model is reasonable and correct in reflecting the degradation process of ball screw, which also illustrated the rationality of the degradation analysis in this paper. In general, the correctness of the proposed exponential degradation model has been fully demonstrated by theoretical derivation, experimental verification, RUL prediction validation, and comparison.

In the future, intensive study on the RUL prediction of ball screw based on the proposed degradation model will be the focus of our work. Multiple signals like current signal, acoustic emission signal, and temperature will be collected to combine with vibration signal to further research the degradation model of ball screw.

Data Availability

The data used to support the findings of this study have not been made available because the authors can use but do not have the right to share the data.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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