Research Article

Local Measurement and Diffusion Reconstruction for Signals on a Weighted Graph

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Bandlimited graph signals on an unweighted graph can be reconstructed by its local measurement, which is a generalization of decimation. Since most signals are weighted in real life, we extend and improve the iterative local measurement reconstruction (ILMR) by introducing the diffusion operators to reconstruct bandlimited signals on a weighted graph. We prove that the proposed reconstruction converges to the original signal. Moreover, the simulation results demonstrate that the improved algorithm has better convergence and has robustness against noise.

1. Introduction

In recent years, the demand for processing of large-scale datasets has been increasing, which has promoted the emerging field of signal processing on graphs. Graphs are useful tools for representing large-scale datasets with irregular geometric structures, in which data elements correspond to the vertices, the relationship between data elements is represented by the edge, and the strength of relationship is reflected in the edge weights. The graph signal can be regarded as a function defined on the graph which contains the spatial relationship of the vertices. Signal processing on graphs is an emerging field mainly solving problems in irregular domain [1], such as neuronal networks, social networks, sensor networks [2], image processing [3], and machine learning [4–6].

Sampling and reconstruction play an important role in the processing of graph signals. Some results have been obtained in the sampling and reconstruction of bandlimited graph signals on an unweighted graph. Pesenson [7] put forward the concept of the uniqueness set and proved that bandlimited signals can be exactly reconstructed by samples on a uniqueness set. Based on the frame constructed by a uniqueness set, Narang [8] proposed the iterative least

square reconstruction (ILSR) algorithm. However, since it is not easy to find the uniqueness set, Wang et al. gave the concept of local sets which are easy to be found by greedy algorithm [9]. Moreover, they proposed the iterative weighting reconstruction (IWR) and iterative propagating reconstruction (IPR) algorithms. Note that the unsampled vertices of a graph signal are influenced by all the sampled vertices, which should not be limited in the local sets. Based on a local-mean diffusion operator and a global-bias diffusion operator, Yang et al. [10] proposed the iterative global diffusion reconstruction (IGDR), which improved the IPR. In some potential applications, the obtained data may be a combination of signals associated with several vertices, Wang proposed the iterative local measurement reconstruction (ILMR) to reconstruct the bandlimited graph signals [11]. Since most graphs are weighted in real life and local measurement is more reasonable in practice, such as sensor networks with clustering structure, we mainly generalize the ILMR algorithm to weighted graphs and improve its performance by introducing some diffusion operators.

This paper is organized as follows. In Section 2, some preliminaries on weighted graphs are introduced and the iterative local measurement reconstruction algorithm is reviewed. In Section 3, we propose some diffusion operators associated

with centerless local sets and give an iterative local measurement and diffusion reconstruction algorithm. In Section 4, some simulation experiments are presented for both the synthetic and real-world data.

2. Preliminaries

In this section, we will introduce the concepts of weighted graphs, graph Fourier transform, and the iterative local measurement reconstruction.

2.1. Weighted Graph. A weighted graph $G = \{V, E, W\}$ is composed of a set of vertices V , a set of edges E , and a weight function $W : E \rightarrow R^+$ which assigns a positive weight to each of the edges. In this paper, we just consider the finite graph of N vertices, where $|V| = N < \infty$.

For an undirected weighted graph $G = \{V, E, W\}$, its Laplacian matrix [12] is defined as $L = D - A$, where A is the adjacency matrix of G with elements

$$a_{u,v} = \begin{cases} w(u, v), & u \sim v \in E \\ 0, & \text{otherwise.} \end{cases} \quad (1)$$

$u \sim v$ denotes a side directly connecting nodes u and v . D is the diagonal degree matrix, whose entries are the degree of the corresponding vertices. For a weighted graph, the degree $d(u)$ of each vertex u is defined as

$$d(u) = \sum_{v=1}^N a_{u,v}. \quad (2)$$

2.2. Graph Fourier Transform and Bandlimited Graph Signals. Suppose $\{\lambda_k\}_{1 \leq k \leq N}$ are the eigenvalues and $\{u_k\}_{1 \leq k \leq N}$ are corresponding eigenvectors of the Laplacian matrix L . For graph Fourier transform [13], the eigenvectors $\{u_k\}_{1 \leq k \leq N}$ are considered as the Fourier basis of the frequency-domain, and eigenvalues $\{\lambda_k\}_{1 \leq k \leq N}$ are regarded as frequencies. The graph Fourier transform is defined by

$$\hat{f}(k) = \langle f, u_k \rangle = \sum_{i=1}^N f(i) u_k(i), \quad (3)$$

where $f(i)$ denotes the entry of f associated with vertex i . Similarly, the inverse Fourier transform can be defined as

$$f = \sum_{k=1}^N \hat{f}(k) u_k. \quad (4)$$

A graph signal f is ω -bandlimited if

$$f \in PW_\omega(G) \triangleq \text{span} \{u_i \mid \lambda_i \leq \omega\}. \quad (5)$$

That is, the frequency components corresponding to eigenvalues larger than ω are all zero. The subspace $PW_\omega(G)$ of ω -bandlimited signals on graph G is a Hilbert space called Paley-Wiener space [14].

2.3. Iterative Local Measurement Reconstruction. In this subsection, we will introduce the iterative local measurement reconstruction algorithm given in [11].

Step 1 (centerless local sets). For a weighted graph $G = \{V, E, W\}$, V is divided into disjoint local sets $\{N_i\}_{i \in I}$, which satisfy

$$\begin{aligned} \bigcup_{i \in I} N_i &= V, \\ N_i \cap N_j &= \emptyset \quad \forall i, j \in I, i \neq j. \end{aligned} \quad (6)$$

Step 2 (local measurement). A local weight $\varphi_i \in R^N$ associated with a centerless local set N_i satisfies

$$\begin{aligned} \varphi_i(v) &\begin{cases} > 0, & v \in N_i \\ = 0, & v \notin N_i, \end{cases} \\ \sum_{v \in N_i} \varphi_i(v) &= 1. \end{aligned} \quad (7)$$

Then a set of local measurements for a graph signal is $\{\langle f, \varphi_i \rangle\}_{i \in I}$.

Step 3 (ILMR). The iterative process of iterative local measurement reconstruction (ILMR) is

$$\begin{aligned} f^{(0)} &= P_\omega \left(\sum_{i \in I} \langle f, \varphi_i \rangle \delta_{N_i} \right) \\ f^{(k+1)} &= f^{(k)} + P_\omega \left(\sum_{i \in I} \langle f - f^{(k)}, \varphi_i \rangle \delta_{N_i} \right), \end{aligned} \quad (8)$$

where P_ω is an orthogonal projection on $PW_\omega(G)$, and δ_{N_i} is defined as

$$\delta_{N_i}(v) = \begin{cases} 1, & v \in N_i \\ 0, & v \notin N_i. \end{cases} \quad (9)$$

3. Improved Local Measurement Reconstruction

In this section, we propose some diffusion operators associated with centerless local sets and give an iterative local measurement and diffusion reconstruction algorithm.

3.1. The Proposed Diffusion Operators

Definition 1 (diffusion operators). For a given reconstructed residual f_i^s (if f_i^s is the reconstructed residual of k th step, then $f_i^s = \langle f - f^k, \varphi_i \rangle \delta_{N_i}$) of i th centerless local sets, we define the diffusion operator P_{d, N_i} as

$$P_{d, N_i} f_i^s = (u_i') (u_i'')^T f_i^s, \quad (10)$$

where u_i' denotes the modified Laplacian eigenvector matrix in which the rows corresponding to the vertices of i th

FIGURE 1: A weighted graph with three centerless local sets.

centerless local sets are set to the zero sequence and the columns corresponding to out of the bandwidth ω are set to the zero sequence, and u_i'' denotes the modified Laplacian eigenvector matrix in which the rows corresponding to the vertices out of the i th centerless local sets are set to the zero sequence and the columns corresponding to out of the bandwidth ω are set to the zero sequence.

Example 2. As shown in Figure 1, assume that we have three centerless local sets N_i , $i = 1, 2, 3$, and the bandwidth $\lambda_3 \leq \omega < \lambda_4$.

By Definition 1, we obtain

$$u_1' = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ u_{13} & u_{23} & u_{33} & 0 & 0 & 0 \\ u_{14} & u_{24} & u_{34} & 0 & 0 & 0 \\ u_{15} & u_{25} & u_{35} & 0 & 0 & 0 \\ u_{16} & u_{26} & u_{36} & 0 & 0 & 0 \end{bmatrix}, \quad (11)$$

$$u_1'' = \begin{bmatrix} u_{11} & u_{21} & u_{31} & 0 & 0 & 0 \\ u_{12} & u_{22} & u_{32} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Suppose that f_1^s is the reconstructed residual in the k th step, and then

$$P_{d,N_1} f_1^s = (u_1') (u_1'')^T f_1^s = [(u_{11}u_{13} + u_{21}u_{23} + u_{31}u_{33} + u_{12}u_{13} + u_{22}u_{23} + u_{32}u_{33}) \delta_3 + (u_{11}u_{14}$$

$$+ u_{21}u_{24} + u_{31}u_{34} + u_{12}u_{14} + u_{22}u_{24} + u_{32}u_{34}) \delta_4 + (u_{11}u_{15} + u_{21}u_{25} + u_{31}u_{35} + u_{12}u_{15} + u_{22}u_{25} + u_{32}u_{35}) \delta_5 + (u_{11}u_{16} + u_{21}u_{26} + u_{31}u_{36} + u_{12}u_{16} + u_{22}u_{26} + u_{32}u_{36}) \delta_6] \langle f - f^{(k)}, \varphi_1 \rangle \quad (12)$$

As the above example shows, the diffusion is to transmit the local measurement error of some centerless local set to all other vertices out of this centerless local set.

3.2. Iterative Local Measurement and Diffusion Reconstruction. In this subsection, we give a new reconstruction method based on local measurement and diffusion operators, which is iterative local measurement and diffusion reconstruction (ILMDR). The proposed algorithm can be written as follows:

$$f^{(0)} = P_\omega \left(\sum_{i \in I} \langle f, \varphi_i \rangle \delta_{N_i} \right) + P_\omega \left(\sum_{i \in I} P_{d,N_i} (\langle f, \varphi_i \rangle \delta_{N_i}) \right) \quad (13)$$

$$f^{(k+1)} = f^{(k)} + P_\omega \left(\sum_{i \in I} \langle f - f^{(k)}, \varphi_i \rangle \delta_{N_i} \right) + P_\omega \left(\sum_{i \in I} P_{d,N_i} (\langle f - f^{(k)}, \varphi_i \rangle \delta_{N_i}) \right).$$

Theorem 3. For given centerless local sets and the associated local weights $\{(N_i, \varphi_i)\}_{i \in I}$, suppose that $\omega_{\min}$ is the smallest weight on the graph $G = \{V, E, W\}$, P_{N_i} is the sampling operator on local set N_i , and $D_i^0 := \max_{u,v \in N_i} \text{dist}^0(u, v)$ is the maximal number of edges in N_i , where $\text{dist}^0(u, v)$ denotes the number of edges in the shortest path connecting u and v . If

$$r := C_{\max} \sqrt{\frac{\omega}{\omega_{\min}}} + \sum_{i \in I} \|\varphi_i\| \|P_\omega (I - P_{N_i}) P_\omega \delta_{N_i}\| < 1, \quad (14)$$

where $C_{\max} = \max_{i \in I} \sqrt{|N_i| D_i^0}$, then any signals $f \in PW_\omega(G)$ can be reconstructed by algorithm (13), with the error at the k th iteration satisfying

$$\|f - f^{(k)}\| \leq r^k \|f - f^{(0)}\|. \quad (15)$$

Proof. The proof is postponed to the Appendix. $\square$

4. The Simulation Results

In this paper, we choose the simulation data and real data to analyze the performance of the proposed algorithm, which is evaluated from convergence rate and robustness against additive noise. In the simulation data, we choose the Minnesota path graph [15] with 700 vertices. In real data, we

FIGURE 2: The convergence behavior of ILMR and ILMDR for various division of centerless local sets with uniform weight (left) and random weight (right).

choose the data structure diagram of air quality index (AQI) [16] of the 31 capital cities in China and the temperature (2017.10.1) data [17, 18] of 80 cities of Guangdong province and Guangxi province of China. Since all the selected graphs are unweighted, we take random numbers in $[0.5, 1]$ as the weights on the edges of the corresponding graph. Moreover, we use greedy method to partition centerless local sets with maximal cardinality $N_{max} = \max_{i \in I} |N_i|$. Three kinds of weights for local measurement are considered

- (1) uniform weight, where $\varphi_i(v) = 1/|N_i|$, $\forall v \in N_i$;
- (2) random weight, where $\varphi_i(v) = \varphi'_i(v) / \sum_{u \in N_i} \varphi'_i(u)$, $\forall v \in N_i$, $\varphi'_i(u) \sim U(0, 1)$;
- (3) supposing that the observed signal associated with each vertex is corrupted by additive noise. If noises associated with vertices are independent and follow zero-mean Gaussian distributions $n(v) \sim N(0, \sigma^2(v))$, the optimal weight [11] is

$$\varphi_i(v) = \frac{(\sigma^2(v))^{-1}}{\sum_{v \in N_i} (\sigma^2(v))^{-1}}, \quad \forall v \in N_i. \quad (16)$$

4.1. The Convergence Performance for the Selection of Maximal Cardinality N_{max} . The convergence of the proposed algorithm is verified for various of partition of centerless local sets. The Minnesota path graph is divided into 244, 192, and 150 centerless local sets for N_{max} being 3, 4, and 5, respectively. The local weights are tested including cases (1) and (2). As shown in Figure 2, the convergence is accelerated when the graph is divided into more local sets and has a smaller N_{max} . Moreover, the proposed ILMDR algorithm always convergences faster than the ILMR algorithm for both local weights.

4.2. Robustness against Gaussian Noise. We assume that the observation of the sampled signals is corrupted by noise, and the independent zero-mean Gaussian noise is added

to each vertex with different variance. All of the vertices are randomly divided into three groups with the standard deviations of the noise chosen as 1×10^{-4} , 2×10^{-4} , and 5×10^{-4} , respectively. The performance of the proposed algorithm against independent zero-mean Gaussian noise is tested for three kinds of local weights including cases (1), (2), and (3). The graph is partitioned into 150 centerless local sets with N_{max} equaling 5. The robustness of the proposed algorithm is given in the left side of Figure 3, which shows that the steady-state relative error for the uniform weight is smaller than those of random weight and optimal weight.

For the special case of independent and identically distributed Gaussian noise, the robustness of the proposed algorithm against noise with different SNR is considered. It demonstrates in the right side of Figure 3 that ILMDR has almost the same performance for three kinds of local weights.

4.3. The Analysis of Real Data. In this experiment, we use real data to test the performance of the proposed ILMDR algorithm. As an example of real data, we adopt the day's AQI of 31 capital cities in October 1, 2017. We represent these capital cities by an undirected two-nearest neighbor graph as in Figure 4(a), in which every capital city corresponds to a vertex and two cities are connected by an edge if their AQI are close. The cut-off frequency considered here is 0.5. Another example of real data is the daily temperature data of 80 cities of Guangdong province and Guangxi province in October 1, 2017. The graph in Figure 4(b) is randomly generated from the 80 cities under the requirement that the degree of each node is not more than 3. The graph cut-off frequency is set to be 0.65. We can see from Figure 5 that the ILMDR algorithm has much faster convergence than the ILMR algorithm for both local weights.

5. Conclusion

Most graphs are weighted graphs in real life, so this paper mainly studies the sampling and reconstruction of bandlimited signals on a weighted graph. We propose a new definition

FIGURE 3: The robustness of ILMDR against noise.

FIGURE 4: Graphs of real data.

of diffusion operators, where the diffusion is to transmit the local measurement error of some centerless local set to all other vertices out of this centerless local set. Based on the local measurement and the proposed diffusion operators, an iterative local measurement and diffusion reconstruction method is given to reconstruct the missing data from the observed local samples. We also present the analysis of the convergence for the proposed algorithm. Moreover, the numerical results demonstrate that the original iterative local measurement reconstruction algorithm can still be used for the reconstruction of weighted graphs. However, our improved algorithm has better convergence and also has noise resistance.

Appendix

Lemma A.1 (see [19]). *For any signals f on a weighted graph $G = \{V, E, W\}$,*

$$\begin{aligned}
 & \sum_{p,q \in E} w(p,q) |f(p) - f(q)|^2 \\
 &= \sum_{(p,q) \in E} d(p) |f(p)|^2 \\
 &\quad - 2 \sum_{(p,q) \in E} f(p) f(q) w(p,q).
 \end{aligned} \tag{A.1}$$

FIGURE 5: The convergence behavior of ILMR and ILMDR for air quality index (AQI) (up) and temperature data (down) with uniform weight (left) and random weight (right).

Proof of Theorem 3 . Let $Gf := P_\omega(\sum_{i \in I} \langle f, \varphi_i \rangle \delta_{N_i}) + P_\omega(\sum_{i \in I} P_{d, N_i}(\langle f, \varphi_i \rangle \delta_{N_i}))$. Then, for a signal $f \in PW_\omega(G)$, one has

$$\begin{aligned} \|f - Gf\| &\leq \left\| P_\omega \left(\sum_{i \in I} (f_{N_i} - \langle f, \varphi_i \rangle \delta_{N_i}) \right) \right\| \\ &\quad + \left\| P_\omega \left(\sum_{i \in I} P_{d, N_i}(\langle f, \varphi_i \rangle \delta_{N_i}) \right) \right\| \quad (\text{A.2}) \\ &= I + II, \end{aligned}$$

where

$$f_{N_i}(v) = \begin{cases} f(v), & v \in N_i \\ 0, & v \notin N_i. \end{cases} \quad (\text{A.3})$$

Now, we estimate I and II , respectively. For $i \in I$, one has

$$\begin{aligned} &\|f_{N_i} - \langle f, \varphi_i \rangle \delta_{N_i}\|^2 \\ &= \sum_{v \in N_i} \left| \sum_{p \in N_i} \varphi_i(p) (f(v) - f(p)) \right|^2 \quad (\text{A.4}) \\ &\leq \sum_{v \in N_i} \left(\max_{p \in N_i} |f(v) - f(p)| \right)^2. \end{aligned}$$

Let

$$p_i(v) := \arg \left(\max_{p \in N_i} |f(v) - f(p)| \right)^2. \quad (\text{A.5})$$

Since N_i is connected, there is always a shortest path within N_i from v to $p_i(v)$, which is denoted as $(v, v_1, \dots, v_{k_v}, p_i(v))$, and then

$$\begin{aligned} \left(\max_{p \in N_i} |f(v) - f(p)| \right)^2 &= |f(v) - f(p_i(v))|^2 \\ &\leq D_i^0 \left(|f(v) - f(v_1)|^2 + \dots \right. \\ &\quad \left. + |f(v_{k_v}) - f(p_i(v))|^2 \right). \end{aligned} \quad (\text{A.6})$$

Therefore, we obtain

$$\begin{aligned} \sum_{v \in N_i} \left(\max_{p \in N_i} |f(v) - f(p)| \right)^2 \\ \leq |N_i| D_i^0 \sum_{p \sim q, p, q \in N_i} |f(p) - f(q)|^2. \end{aligned} \quad (\text{A.7})$$

It follows from Lemma A.1 that

$$\begin{aligned} \sum_{(p,q) \in E} |f(p) - f(q)|^2 &= \frac{1}{w_{\min}} \\ &\cdot \sum_{p,q \in E} w_{\min} |f(p) - f(q)|^2 \leq \frac{1}{w_{\min}} \\ &\cdot \sum_{p,q \in E} w(p,q) |f(p) - f(q)|^2 \\ &= \frac{1}{w_{\min}} \left[\sum_{(p,q) \in E} d(p) |f(p)|^2 \right. \\ &\quad \left. - 2 \sum_{(p,q) \in E} f(p) f(q) w(p,q) \right] = \frac{1}{w_{\min}} f^T L f \\ &= \frac{1}{w_{\min}} f^T U \wedge U^T f = \frac{1}{w_{\min}} \sum_{\lambda_i \leq \omega} \lambda_i |\hat{f}(i)|^2 \\ &\leq \frac{\omega}{w_{\min}} \|\hat{f}\|^2 = \frac{\omega}{w_{\min}} \|f\|^2. \end{aligned} \quad (\text{A.8})$$

This together with (A.4) and (A.7) obtains

$$\begin{aligned} I^2 &\leq \sum_{i \in I} \|f_{N_i} - \langle f, \varphi_i \rangle \delta_{N_i}\|^2 \\ &\leq \sum_{i \in I} |N_i| D_i^0 \sum_{p \sim q, p, q \in N_i} |f(p) - f(q)|^2 \\ &\leq C_{\max}^2 \sum_{(p,q) \in E} |f(p) - f(q)|^2 \leq C_{\max}^2 \frac{\omega}{w_{\min}} \|f\|^2. \end{aligned} \quad (\text{A.9})$$

For estimating II , it is easy to verify that

$$P_{d, N_i} \delta_{N_i} = (I - P_{N_i}) P_{\omega} \delta_{N_i}. \quad (\text{A.10})$$

Note that both $I - P_{N_i}$ and P_{ω} are contracting operators. Furthermore, one has

$$\begin{aligned} II &\leq \sum_{i \in I} |\langle f, \varphi_i \rangle| \|P_{\omega} P_{d, N_i} \delta_{N_i}\| \\ &\leq \left(\sum_{i \in I} \|\varphi_i\| \|P_{\omega} (I - P_{N_i}) P_{\omega} \delta_{N_i}\| \right) \|f\|. \end{aligned} \quad (\text{A.11})$$

Consequently, $\|f - Gf\| \leq r \|f\|$, which means the result of Theorem 3. $\square$

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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