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Research Article

A Novel Design of Sparse FIR Multiple Notch Filters with Tunable Notch Frequencies

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We focus on the design of finite impulse response (FIR) multiple notch filters. To reduce the computational complexity and hardware implementation complexity, a novel algorithm is developed based on the mixture of the tuning of notch frequencies and the sparsity of filter coefficients. The proposed design procedure can be carried out as follow: first, since sparse FIR filters have lower implementation complexity than full filters, a sparse linear phase FIR single notch filter with the given rejection bandwidth and passband attenuation is designed. Second, a tuning procedure is applied to the computed sparse filter to produce the desired sparse linear phase FIR multiple notch filter. When the notch frequencies are varied, the same tuning procedure can be employed to render the new multiple notch filter instead of designing the filter from scratch. The effectiveness of the proposed algorithm is demonstrated through three design examples.

1. Introduction

The multiple notch filters, which can highly attenuate some frequency components in the input signal while leaving the others relatively unchanged, are widely used in many applications. Important examples include radar systems, control and instrumentation systems, communications systems, medical applications, biomedical engineering, and indoor localization [1, 2].

Various methods [3–8] have been reported to design FIR multiple notch filters. In general, the multiple notch filters derived from these algorithms are not sparse. Compared with full FIR filters, sparse filters can significantly reduce the implementation complexity in the hardware. In [9], we proposed an iterative reweighed OMP algorithm to compute sparse notch filters. However, when the notch frequencies are varied, it requires one to design the whole filter from scratch, hence increasing the computational complexity of this scheme.

Recently, in [10–12], a number of algorithms are proposed to design FIR filters based on LMS minimization or Monte Carlo methods. The disadvantage of these approaches is the suboptimality in terms of the filter length related to its selectivity. Another disadvantage is that the attenuation at the notch frequency changes during the adaptation process; therefore, a strong attenuation of the disturbing signal at the notch frequency is not guaranteed. Moreover, the actual value of the attenuation at notch frequency is caused by the adaptation process.

In this brief, the design problems of sparse FIR multiple notch filters with tunable notch frequencies are studied. To reduce the computational complexity and the hardware complexity, a novel algorithm is developed based on the mixture of the tuning of notch frequencies and the sparsity of filter coefficients. The sparse FIR multiple notch filters can significantly reduce the number of the adders and multipliers used in the hardware implementation. However, the design of FIR sparse filter always involves iterative procedures and numerical optimization, which results in a high computational complexity for the practice system. The tuning of notch frequencies is a useful operation for the design of FIR multiple notch filter. In the case of variable notch frequencies, the same tuning process is implemented to obtain the new multiple notch filter instead of designing the filter from scratch. Therefore, the tuning feature can significantly reduce the computational complexity. We demonstrate the effectiveness of this approach through three design examples.

2. Problem Formulation

Given the design parameters of linear phase FIR multiple notch filter, which include a set of the notch frequencies $\{\overline{\omega}_i\}_{i=1}^{r}$, rejection bandwidth $\Delta\omega$, and passbands attenuation α, the given notch frequencies $\{\overline{\omega}_i\}_{i=1}^{r}$ satisfying $\overline{\omega}_i < \overline{\omega}_{i+1}$ for $1 \le i \le r$ are allowed to be nonuniformly distributed in the set $[0, \pi]$. The ideal multiple notch filter amplitude response $H_d(\omega)$ satisfies

$$H_d(\omega) = \begin{cases} 0 & \omega \in \Omega^0 \\ 1 & \omega \in \Omega^1, \end{cases} \tag{1}$$

where Ω^0 and Ω^1 are, respectively, defined as

$$\begin{aligned} \Omega^0 &= \left\{\omega \mid \left|\omega - \overline{\omega}_i\right| \le \frac{\Delta\omega}{2},\ 1 \le i \le r\right\}, \\ \Omega^1 &= [0, \pi] - \Omega^0. \end{aligned} \tag{2}$$

To simplify the presentation, we focus on the design of Type-I linear phase FIR filter $H(e^{j\omega}) = e^{-jM\omega}H_0(\omega)$; that is, the filter order $N = 2M$ is even and $h(m) = h(N-m)$ for all $0 \le m \le N$. For other types of filter, our design method presented in this letter is feasible. For the case of Type-I filter, the zero-phase amplitude response $H_0(\omega)$ can be expressed as

$$H_0(\omega) = h(M) + 2\sum_{m=1}^{M} h(M-m)\cos(m\omega), \tag{3}$$

with $M = N/2$.

3. The Proposed Sparse Linear Phase FIR Multiple Notch Filter Design

In this section, a novel design method is presented to produce the sparse FIR multiple notch filter. The procedure of computing the linear phase FIR multiple notch filter starts with the estimation of the initial order N of the filter $F(e^{j\omega})$ through

$$N = \max_{i \in \{1,\ldots,r\}} N_i. \tag{4}$$

From [13, eq. (20)], N_i is computed as

$$N_i = \max\left\{\widehat{N}\left(\omega_{p_{1i}}, \Delta F, \delta_p, \delta_s\right), \widehat{N}\left(\omega_{p_{2i}}, \Delta F, \delta_p, \delta_s\right)\right\}, \tag{5}$$

where $\Delta F = \Delta\omega/2$ and the function $\widehat{N}(\cdot)$ is determined by [13, eq. (15)]. The arguments of $\widehat{N}(\cdot)$ can be computed as

$$\begin{aligned} \omega_{p_{1i}} &= \frac{\overline{\omega}_i - \Delta F}{2}, \\ \omega_{p_{2i}} &= \frac{1 - \overline{\omega}_i - \Delta F}{2}, \\ \delta_p &= \delta_s = \frac{1-\alpha}{1+\alpha}. \end{aligned} \tag{6}$$

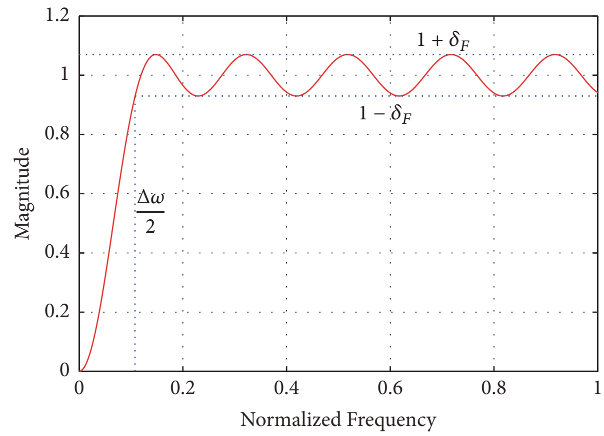

Figure 1: The illustration for the amplitude response of the desired sparse single notch filter.

The following design procedure is mainly comprised of two stages: in the first stage, a sparse linear phase FIR single notch filter $F(e^{j\omega})$ with the given rejection bandwidth and passband attenuation is designed as a fixed sparse filter. In the next stage, a tuning process is carried out to compute the desired multiple notch filter with the given notch frequencies based on the filter $F(e^{j\omega})$.

3.1. Sparse Linear Phase FIR Single Notch Filter Design. In this section, a sparse linear phase FIR single notch filter $F(e^{j\omega})$ of order N with the notch frequency $\overline{\omega}_1 = 0$ is designed. Let $F(e^{j\omega}) = e^{-jM\omega}F_0(\omega)$ represent the single notch filter, as shown in Figure 1; the real-valued amplitude response $F_0(\omega)$ satisfies

$$\begin{aligned} &F_0(\omega) = 0, \quad \omega = 0, \\ &0 < F_0(\omega) < 1 - \delta_F, \quad 0 < |\omega| < \frac{\Delta\omega}{2}, \\ &\left|F_0(\omega) - 1\right| < \delta_F, \quad \frac{\Delta\omega}{2} \le |\omega| \le \pi. \end{aligned} \tag{7}$$

The passband ripple δ_F of the single notch filter $F(e^{j\omega})$ and the attenuation in the passbands α are related through

$$\delta_F = \frac{1-\alpha}{2r(1+\alpha)}. \tag{8}$$

Equation (8) is a conservative choice of δ_F which ensures the multiple notch filter yielded from this choice to satisfy the design specifications. In most cases, δ_F can be chosen between $(1-\alpha)/2r(1+\alpha)$ and $(1-\alpha)/(1+\alpha)$.

The design of the sparse single notch filter $F(e^{j\omega})$ can be formulated as

$$\min_{\mathbf{f}} \quad \|\mathbf{f}\|_0 \tag{9a}$$

$$\text{s.t.} \quad |\mathbf{c}(\omega)\mathbf{f} - 1| \le \delta_F, \quad \omega \in \left[\frac{\Delta\omega}{2}, \pi\right], \tag{9b}$$

$$\mathbf{c}(\omega)\mathbf{f} = 0, \quad \omega = 0, \tag{9c}$$

where we have

$$\begin{aligned}\mathbf{c}(\omega) &= \left[1 \quad \cos(\omega) \quad \cdots \quad \cos(m\omega) \quad \cdots \quad \cos(M\omega)\right], \\ \mathbf{f} &= \left[f(M) \quad 2f(M-1) \quad \cdots \quad 2f(m) \quad \cdots \quad 2f(0)\right]^T,\end{aligned} \tag{10}$$

with $0 \le m \le M$.

To compute a solution of problem (9a), (9b), and (9c), we follow the standard discretization procedure as presented in [14] and replace the continuous parameter ω by L samples (where $L \gg 1$ is a large positive integer) uniformly distributed in the frequency set $[\Delta\omega/2, \pi]$. Thus, the discretization and normalized formulation of problem (9a), (9b), and (9c) is given by

$$\min_{\mathbf{f}} \quad \|\mathbf{f}\|_0 \tag{11a}$$

$$\text{s.t.} \quad \left|\mathbf{A}\mathbf{f} - \mathbf{1}_{L\times 1}\right| \le \delta_{\mathrm{F}} \cdot \mathbf{1}_{L\times 1}, \tag{11b}$$

$$\mathbf{1}_{1\times L}\mathbf{f} = 0, \tag{11c}$$

where we have

$$\mathbf{A} = \begin{bmatrix} \mathbf{c}(\omega_1) \\ \mathbf{c}(\omega_2) \\ \vdots \\ \mathbf{c}(\omega_l) \\ \vdots \\ \mathbf{c}(\omega_L) \end{bmatrix} = \begin{bmatrix} 1 & \cos(\omega_1) & \cdots & \cos(M\omega_1) \\ 1 & \cos(\omega_2) & \cdots & \cos(M\omega_2) \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \cos(\omega_l) & \cdots & \cos(M\omega_l) \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \cos(\omega_L) & \cdots & \cos(M\omega_L) \end{bmatrix}, \tag{12}$$

with $\omega_l \in [\Delta\omega/2, \pi]$ and $1 \le l \le L$.

It is known that this optimization problem is in general NP-hard due to the existence of l_0-norm in its objective function. To tackle this problem, a great deal of effort has been made to develop efficient algorithms. In this paper, we can employ one of these sparse filter algorithms, for example, linear programming [15], iterative second-order cone programming (ISOCP) [16], iterative reweighted l_1 (IRL1) [17], and iterative reweighted OMP (IROMP) schemes [9], to attain the desired sparse FIR single notch filter.

3.2. The Design of the Desired Linear Phase FIR Multiple Notch Filter. In this section, a tuning process is implemented to derive the desired FIR multiple notch filter based on $F(e^{j\omega})$ of the previous stage.

For the given notch frequencies set $\{\overline{\omega}_i\}_{i=1}^r$, the multiple notch filter $H(e^{j\omega})$ can be given as

$$H\left(e^{j\omega}\right) = e^{-jM\omega}\sum_{i=1}^{r}\left[F_0\left(\omega + \overline{\omega}_i\right) + F_0\left(\omega - \overline{\omega}_i\right)\right]. \tag{13}$$

According to the Fourier transform theory, the impulse response $h(n)$ of $H(e^{j\omega})$ can be obtained as

$$h(n) = \sum_{i=1}^{r} f(n)\cos\left(n\overline{\omega}_i\right), \tag{14}$$

where $0 \le n \le N$.

Compute the attenuation $\widehat{\alpha}$ in the passbands of the linear phase FIR multiple notch filter $H(e^{j\omega})$ as

$$\widehat{\alpha} = \frac{\min\left(H_0(\omega)\right)}{\max\left(H_0(\omega)\right)}, \quad \omega \in \Omega^1. \tag{15}$$

If $\widehat{\alpha} \le \alpha$, then the computed filter $\{h(n)\}_{n=0}^N$ is a sparse solution for the given specifications. Otherwise, the following linear program optimization is run to minimize the attenuation in the passbands of the obtained filter:

$$\min_{\mathbf{h},\mu} \quad \mu \tag{16a}$$

$$\text{s.t.} \quad \left|\mathbf{B}\mathbf{h} - \mathbf{1}_{L\times 1}\right| \le (\delta + \mu) \cdot \mathbf{1}_{L\times 1}, \tag{16b}$$

$$\mathbf{c}\left(\overline{\omega}_i\right)\mathbf{h} = 0, \quad i = 1, 2, \ldots, r, \tag{16c}$$

$$h(n) = 0, \quad n \in \mathscr{Z}, \tag{16d}$$

where $\mathscr{Z}$ represents the set of indices at which $h(n) = 0$ based on (14) and matrix $\mathbf{B}$ can be written as

$$\mathbf{B} = \begin{bmatrix} \mathbf{c}\left(\omega_1'\right) \\ \mathbf{c}\left(\omega_2'\right) \\ \vdots \\ \mathbf{c}\left(\omega_l'\right) \\ \vdots \\ \mathbf{c}\left(\omega_L'\right) \end{bmatrix}, \quad \omega_l' \in \Omega^1. \tag{17}$$

If the optimal objective value μ of (16a), (16b), (16c), and (16d) is negative, that is, $\mu \le 0$, the obtained filter $\mathbf{h}$ is a sparse solution for the given specifications. Otherwise, the sparsity pattern $\mathscr{Z}$ is infeasible to the given specifications of the multiple notch filter; then the largest element is eliminated from $\mathscr{Z}$ and the linear program (16a), (16b), (16c), and (16d) is solved with the new set $\mathscr{Z}$ until $\mu \le 0$.

When the notch frequencies are changed, the same tuning process is implemented to yield the new multiple notch filter instead of designing the filter from scratch. Figure 2 outlines the main steps of the proposed algorithm.

4. Simulation

In this section, we confirm the effectiveness of our multiple notch filter design scheme through three examples.

Example 1. Let us design a multiple notch filter specified by a set of notch frequencies $\{0.25\pi, 0.49\pi, 0.78\pi\}$, $\alpha = -0.80$ (passbands attenuation), and $\Delta\omega = 0.05\pi$ (the rejection bandwidths).

By substituting the design specifications into [9, eq. (11)], we obtain the initial order $N = 174$. In this simulation, we employ the IROMP scheme [9] to design the sparse single notch filter. As shown in Figure 3, the amplitude response of the multiple notch filter is derived by following

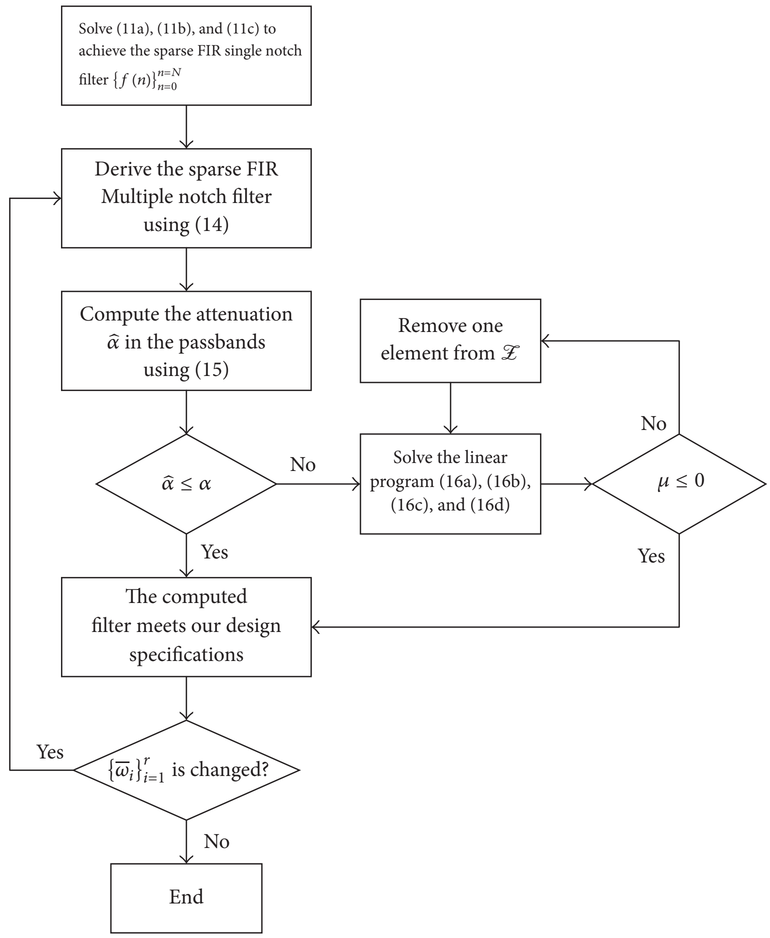

Figure 2: Flowchart of the proposed design algorithm.

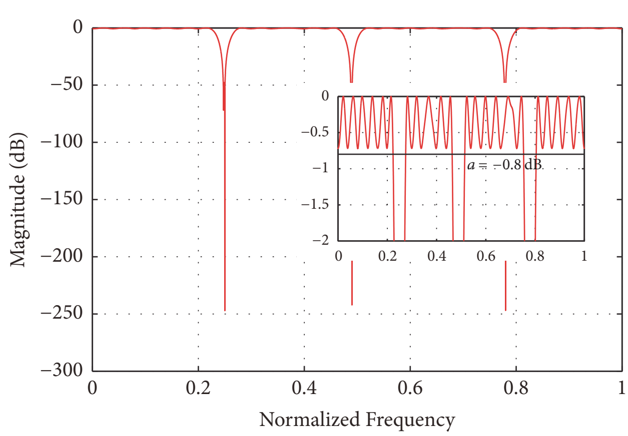

Figure 3: The amplitude response of the filter yielded from our design method for Example 1.

steps in Figure 2. It is obvious that the specification is well satisfied. The nonzero tap weights of the multiple notch filter yielded from our design method are listed in Table 1. The filter order, number of nonzero taps, rejection bandwidth, passband attenuation, and attenuation at the notch frequency are listed in Table 2.

Example 2. We only change the notch frequencies from $\{0.25\pi, 0.49\pi, 0.78\pi\}$ of Example 1 to $\{0.34\pi, 0.43\pi, 0.72\pi\}$ but use the same rejection bandwidth and attenuation in the passbands.

Since the same rejection bandwidth and attenuation in the passbands as Example 1 are used, the sparse single notch filter $F(e^{j\omega})$ of (11a), (11b), and (11c) with $N = 174$ can be identical to the one computed in Example 1. Following the tuning procedure from (14) to (16a), (16b), (16c), and (16d), we compute the sparse multiple notch filter with this new set of the notch frequencies. Figure 4 shows the performance of the sparse multiple notch filter yielded from our scheme. The nonzero tap weights of the multiple notch filter yielded from our design method are listed in Table 3. The filter order, number of nonzero taps, rejection bandwidth, passband attenuation, and attenuation at the notch frequency are listed in Table 2.

Example 3. Change the set of notch frequencies in Example 1 to $\{0.25\pi, 0.49\pi, 0.61\pi, 0.78\pi\}$, while α and $\Delta\omega$ remain the same.

Since α and $\Delta\omega$ are kept constant, we start with the sparse single notch filter $F(e^{j\omega})$ which is the same as that derived in

Table 1: Nonzero coefficients of the designed filter in Example 1.

Taps		Nonzero tap weights
0	174	0.004722848554565
24	150	0.002007423266603
33	141	−0.004871513257007
38	136	−0.030966811403170
39	135	−0.022053492615673
40	134	0.014951595120278
41	133	−0.021767842979883
42	132	0.011692029570564
43	131	0.006950807646344
44	130	0.044114166070248
45	129	0.006311408033158
46	128	−0.062066578840798
47	127	−0.005818521570352
48	126	−0.002318412822056
49	125	0.005091217785500
50	124	0.020608511742342
51	123	−0.014404626498885
52	122	0.061394774774335
53	121	0.008629427173138
54	120	−0.072558769634137
55	119	−0.010787517932644
56	118	−0.022623175477306
57	117	0.030869196951431
58	116	0.009932536096148
59	115	−0.014074814764124
60	114	0.094137780017783
61	113	−0.004199271003407
62	112	−0.061953987746997
63	111	−0.047952288148680
64	110	−0.038531961471284
65	109	0.076026372592104
66	108	−0.015111106082247
67	107	0.000183867057454
68	106	0.099331267386019
69	105	−0.012255378374269
70	104	−0.028055658481912
71	103	−0.100962624763570
72	102	−0.038372777955426
73	101	0.098683027527897
74	100	−0.036089958346951
75	99	0.029655283702946
76	98	0.071889586307707
77	97	0.004739347900282
78	96	−0.002403828886942
79	95	−0.153598792283481
80	94	−0.019548891523249
81	93	0.089075079456139
82	92	−0.023403440116641
83	91	0.056274737680820
84	90	0.018813444978966
85	89	0.047899881207932
86	88	0.000409323536052
87		0.911070614139502

Example 1 ($N = 174$). The sparse multiple notch filter with this new notch frequencies is obtained through the tuning process from (14) to (16a), (16b), (16c), and (16d). Figure 5 illustrates the amplitude response of this filter. It is evident that the specification is satisfied. The nonzero tap weights of the multiple notch filter yielded from our design method are listed in Table 4. The filter order, number of nonzero taps,

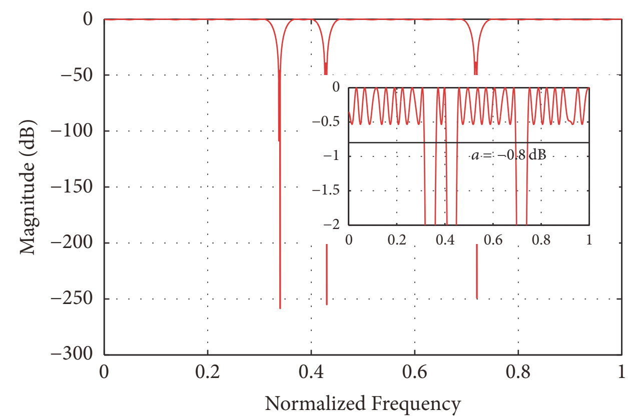

Figure 4: The amplitude response of the filter yielded from our design method for Example 2.

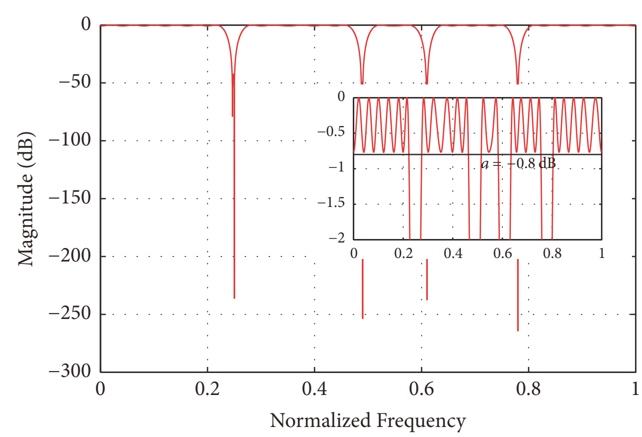

Figure 5: The amplitude response of the filter yielded from our design method for Example 3.

rejection bandwidth, passband attenuation, and attenuation at the notch frequency are listed in Table 2.

5. Conclusion

In this paper, a novel approach has been presented for the design of sparse FIR multiple notch filters with tunable notch frequencies. To further improve the efficiency, the proposed algorithm is based on the mixture of the tuning of notch frequencies and the sparsity of filter coefficients. In the case of variable notch frequencies, the same tuning procedure can be used to render the new multiple notch filter instead of designing the filter from scratch. Therefore, the proposed algorithm can significantly reduce the computational complexity. Three examples are given to show the effectiveness of this approach.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

Table 2: A list of filter order, rejection bandwidth, and attenuation of Examples 1–3.

Example	Filter order	The number of nonzero tap weights	Rejection bandwidth	Passband attenuation	Attenuation at the notch frequency
1	174	105	$0.050\,\pi$	−0.7197 dB	−247 dB
2	174	105	$0.050\,\pi$	−0.5349 dB	−259 dB
3	174	105	$0.050\,\pi$	−0.7687 dB	−264 dB

Table 3: Nonzero coefficients of the designed filter in Example 2.

Taps		Nonzero tap weights
0	174	−0.001484391069741
24	150	0.015053216345025
33	141	0.011203465180753
38	136	0.031930539223127
39	135	−0.011989478832635
40	134	−0.047916512343373
41	133	0.001724422497417
42	132	0.012946318823095
43	131	0.029367872190180
44	130	0.012950777492872
45	129	−0.058861258880195
46	128	−0.027161496236364
47	127	0.038124403599415
48	126	0.005580972074984
49	125	0.024077586822642
50	124	−0.013761958746995
51	123	−0.036380554634187
52	122	0.026803049613006
53	121	−0.001700050810428
54	120	−0.023056804511978
55	119	0.041926654928440
56	118	0.000428542187068
57	117	−0.005253841608839
58	116	−0.008896975496026
59	115	−0.071272465825430
60	114	0.037660569869951
61	113	0.085639588936268
62	112	−0.020291841316044
63	111	−0.035719226300918
64	110	−0.069237101344906
65	109	−0.019718488202998
66	108	0.128901758725893
67	107	0.025726032859600
68	106	−0.073722260607230
69	105	−0.025836288834871
70	104	−0.045891876538100
71	103	0.057310758857846
72	102	0.075489239705611
73	101	−0.063149359304339
74	100	−0.003347070890743
75	99	0.014446702737718
76	98	−0.058103382956618
77	97	0.032069195768895
78	96	0.000558233096476
79	95	0.006719715851877
80	94	0.084935075668951
81	93	−0.076509802170425
82	92	−0.097984593409281
83	91	0.043634372404206
84	90	0.040410785159360
85	89	0.089360899339001
86	88	−0.003266951955140
87		0.915171992267534

Table 4: Nonzero coefficients of the designed filter in Example 3.

Taps		Nonzero tap weights
0	174	0.001163545765695
24	150	0.003165433288996
33	141	0.002091982284630
38	136	−0.047102411486321
39	135	−0.005481276840672
40	134	0.019772136963283
41	133	−0.043110955540852
42	132	0.020979982565140
43	131	0.015127277492607
44	130	0.022611789510971
45	129	0.010481611606251
46	128	−0.039298064713159
47	127	−0.024983870407359
48	126	−0.013239453713585
49	125	0.022935249461144
50	124	0.027344765658964
51	123	−0.037957131548364
52	122	0.075665876343805
53	121	0.024121474369378
54	120	−0.104718797612301
55	119	−0.014707635070068
56	118	0.016302396989146
57	117	0.011567221940061
58	116	−0.010934081396023
59	115	0.019926233327605
60	114	0.089656608769556
61	113	−0.041103476228622
62	112	−0.033265661189825
63	111	−0.026560471743885
64	110	−0.082576601043643
65	109	0.085379844129436
66	108	0.024951523169648
67	107	−0.038897009099235
68	106	0.088237797320545
69	105	0.036518046705842
70	104	−0.053707639239860
71	103	−0.134082248075127
72	102	0.007838779814640
73	101	0.104434475095505
74	100	−0.081493323669535
75	99	0.057503261219777
76	98	0.103032966355301
77	97	−0.046232816359952
78	96	−0.001798639004124
79	95	−0.100284039691277
80	94	−0.057999870693463
81	93	0.060896839992225
82	92	0.036785941586311
83	91	0.041803167312136
84	90	−0.030010449210885
85	89	0.094188919109719
86	88	0.018630470290357
87		0.882383969239141

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