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Research Article

Improved Stabilization Conditions for Nonlinear Systems with Input and State Delays via T-S Fuzzy Model

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This paper focuses on the problem of nonlinear systems with input and state delays. The considered nonlinear systems are represented by Takagi-Sugeno (T-S) fuzzy model. A new state feedback control approach is introduced for T-S fuzzy systems with input delay and state delays. A new Lyapunov-Krasovskii functional is employed to derive less conservative stability conditions by incorporating a recently developed Wirtinger-based integral inequality. Based on the Lyapunov stability criterion, a series of linear matrix inequalities (LMIs) are obtained by using the slack variables and integral inequality, which guarantees the asymptotic stability of the closed-loop system. Several numerical examples are given to show the advantages of the proposed results.

1. Introduction

In recent years, T-S fuzzy system has become a very active research direction in the fields of fuzzy control [1–8]. T-S fuzzy model is useful for approximating complex nonlinear systems by local linear submodel and fuzzy membership functions. It has been proved that T-S fuzzy model can approximate any smooth nonlinear dynamic systems. Therefore, T-S fuzzy model is a very effective and powerful tool for the study of nonlinear systems. Currently, many valuable results on stability analysis, controller design, filter design, and fault detection of T-S fuzzy systems have been widely reported in [9–19].

It is well known that time-delay is an important source of system instability or oscillation [20–27]. Lyapunov-Krasovskii functional (LKF) technique is an effective method to handle stability analysis and controller design of T-S fuzzy time-delay system. Various methods are introduced to obtain less conservative stability conditions for T-S fuzzy time-delay system. There are two main relaxed techniques for T-S fuzzy time-delay system, for example, constructing new LKF with more system information and using new bounding inequalities. A variety of results have been presented for T-S fuzzy time-delay system by combining LKF and bounding inequality approaches [28–37]. However, the existing results are still very conservative due to the form of LKF, and thus constructing new LKF with more system information is a challenging issue to reduce the conservativeness of stability analysis for T-S fuzzy time-delay system.

The aforementioned results only consider the state delays in T-S fuzzy systems. Therefore, these approaches mentioned above may be invalid when they are used to control the systems with input delays. It is well known that input delays always occur in practical control systems. Therefore, it is very significant to study stability and controller design for T-S fuzzy systems with state and input delays. Therefore, a few studies have been developed for T-S fuzzy systems with state and input delays [38–40]. However, the existing results are very conservative because LFK including double integral terms are only used to derive stability conditions in the form of LMIs.

In this paper, the stabilization problem of nonlinear systems with input and state delays is studied by using T-S fuzzy model. Novel stabilization conditions are presented by Lyapunov stability theory. The main contributions of this paper can be summarized as follows. (1) A new LKF with triple integral terms is for the first time constructed for T-S fuzzy model with input and state delays. (2) Wirtinger-based double integral inequality is employed to derive LMI-based stabilization conditions. (3) Using LKF with more system information, Wirtinger-based double integral inequality, and matrix transformation technique, less conservative stabilization conditions are proposed for nonlinear systems with input and state delays. The paper is organized as follows. The problem descriptions and some useful lemmas are presented in Section 2. The main derivation process is given in Section 3. Several numerical examples are provided to demonstrate the effectiveness of the proposed approach in Section 4. The conclusion is drawn in the last section.

2. Preliminaries

2.1. Problem Description. Consider a nonlinear system described by T-S fuzzy model with input and state delays. The rules are defined as follows.

Plant Rule i. If $\theta_1(t)$ is M_{i1} and $\theta_2(t)$ is M_{i2} and … and $\theta_p(t)$ is M_{ip}, then

$$\begin{gathered}\dot{x}(t) = A_i x(t) + A_{di} x(t-h) + B_i u(t-\tau), \\ x(t) = \phi(t), \quad t \in [-\max(h,\tau), 0], \\ i = 1, 2, \ldots, r,\end{gathered} \tag{1}$$

where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^{\mathrm{T}} \in R^n$ is the state; $\phi(t)$ is the initial condition defined on $t \in [-\max(h,\tau), 0]$; M_{ij} is the fuzzy set; r is the number of plant rules; and $\theta(t) = [\theta_1(t), \theta_2(t), \ldots, \theta_p(t)]$ is the antecedent variable vector. h and τ, respectively, indicate the state delay and input delay. A_i, B_i, and A_{di} are known real constant matrices with appropriate dimensions.

The overall fuzzy model can be given by

$$\dot{x}(t) = \sum_{i=1}^{r} \lambda_i(\theta(t)) \left[A_i x(t) + A_{di} x(t-h) + B_i u(t-\tau)\right], \tag{2}$$

where

$$\begin{gathered}\sum_{i=1}^{r} \lambda_i(\theta(t)) = 1, \\ \lambda_i(\theta(t)) = \frac{\omega_i(\theta(t))}{\sum_{i=1}^{r} \omega_i(\theta(t))}, \\ \omega_i(\theta(t)) = \prod_{j=1}^{r} M_{ij}(\theta_j(t)),\end{gathered} \tag{3}$$

and $M_{ij}(\theta_j(t))$ indicates the membership degree of $\theta_j(t)$ in M_{ij}.

A tight T-S fuzzy model can be written as

$$\begin{gathered}\dot{x}(t) = A(\lambda) x(t) + A_d(\lambda) x(t-h) + B(\lambda) u(t-\tau), \\ x(t) = \phi(t), \quad t \in [-\max(h,\tau), 0],\end{gathered} \tag{4}$$

where $A(\lambda) = \sum_{i=1}^{r} \lambda_i(\theta(t)) A_i$, $A_d(\lambda) = \sum_{i=1}^{r} \lambda_i(\theta(t)) A_{di}$, and $B(\lambda) = \sum_{i=1}^{r} \lambda_i(\theta(t)) B_i$.

Moreover, we define the fuzzy state feedback control rules using the following fuzzy parallel distributed compensation strategy.

Controller Rule i. If $\theta_1(t)$ is M_{i1} and $\theta_2(t)$ is M_{i2} and … and $\theta_p(t)$ is M_{ip}, then

$$u(t) = K_i x(t), \quad i = 1, 2, \ldots, r, \tag{5}$$

where $x(t)$ is the input, $u(t) \in R^m$ is the output, and K_i is the gain matrix of the controller.

So the overall output of the fuzzy state feedback controller can be represented as

$$u(t) = \sum_{i=1}^{r} \lambda_i(\theta(t)) K_i x(t). \tag{6}$$

The compact form can be written as

$$u(t) = K(\lambda) x(t), \tag{7}$$

where $K(\lambda) = \sum_{i=1}^{r} \lambda_i(\theta(t)) K_i$.

Therefore, we can get the closed-loop system formed by T-S fuzzy system with input delay and state delays in (2) and state feedback controller in (6), which is as follows:

$$\begin{aligned}\dot{x}(t) = {} & \sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i(\theta(t)) \lambda_j(\theta(t)) \\ & \cdot \left[A_i x(t) + A_{di} x(t-h) + B_i K_j x(t-\tau)\right].\end{aligned} \tag{8}$$

The compact form of the closed-loop system is represented as

$$\begin{aligned}\dot{x}(t) = {} & A(\lambda) x(t) + A_d(\lambda) x(t-h) \\ & + B(\lambda) K(\lambda) x(t-\tau).\end{aligned} \tag{9}$$

The goal of this paper is to find gains K_i such that the closed-loop system (9) is asymptotically stable.

2.2. Useful Lemmas. The following lemmas are useful to obtain the main results of this paper.

Lemma 1 (see [25]). *For any constant matrix $M > 0$, given scalars a and b satisfying $a < b$, the following inequality holds for all continuously differentiable function φ in $[a,b] \to R^n$:*

$$\begin{aligned}& (b-a) \int_a^b \varphi^{\mathrm{T}}(s) M \varphi(s)\, ds \\ & \geq \left(\int_a^b \varphi(s)\, ds\right)^{\mathrm{T}} M \left(\int_a^b \varphi(s)\, ds\right) + 3\Theta_d^{\mathrm{T}} M \Theta_d,\end{aligned} \tag{10}$$

where $\Theta_d = \int_a^b \varphi(s) ds - (2/(b-a)) \int_a^b \int_a^s \varphi(u) du\, ds = -\int_a^b \varphi(s) ds + (2/(b-a)) \int_a^b \int_s^b \varphi(u) du\, ds$.

Lemma 2 (see [26]). *For a given matrix $M > 0$, given scalars a and b satisfying $a < b$, the following inequality holds for all continuously differentiable function x in $[a, b] \to R^n$:*

$$\begin{aligned}
&\frac{(b-a)^2}{2} \int_a^b \int_s^b x^{\mathrm{T}}(u) M x(u)\, du\, ds \\
&\quad \geq \left(\int_a^b \int_s^b x(u)\, du\, ds \right)^{\mathrm{T}} M \left(\int_a^b \int_s^b x(u)\, du\, ds \right) \\
&\qquad + 2\Theta_d^{\mathrm{T}} M \Theta_d,
\end{aligned} \tag{11}$$

where $\Theta_d = -\int_a^b \int_s^b x(u) du\, ds + (3/(b - a)) \int_a^b \int_s^b \int_u^b x(v) dv\, du\, ds$.

Lemma 3 (see [41]). *Given matrices $v \in R^n$, $\Theta = \Theta^{\mathrm{T}} \in R^{n\times n}$, and $N \in R^{m\times n}$, if* $\operatorname{rank}(N) < n$, *then*

$$v^{\mathrm{T}} \Theta v < 0, \quad \forall N v = 0,\ v \neq 0 \tag{12}$$

if and only if there exists matrix $L \in R^{n\times m}$ such that

$$\Theta + LN + N^{\mathrm{T}} L^{\mathrm{T}} < 0. \tag{13}$$

3. Main Results

In this section, new stability conditions for system (9) will be presented. Now we give the following theorem.

Theorem 4. *Consider the closed-loop system (9) and given scalars h and τ to meet $h > 0$, $\tau > 0$, the system is asymptotically stable, if there exist scalars a_1 and a_2, matrices $M(\lambda)$, X, and positive definite symmetric matrices $\widehat{P}$, $\widehat{Q}$, $\widehat{D}$, $\widehat{T}$, $\widehat{\widehat{Q}}$, $\widehat{\widehat{D}}$, and $\widehat{\widehat{T}}$, such that the following inequality holds:*

$$\widehat{\Xi} + \widehat{L}\widehat{\Gamma}(\lambda) + \widehat{\Gamma}(\lambda)^{\mathrm{T}} \widehat{L}^{\mathrm{T}} < 0, \tag{14}$$

where

$$\begin{aligned}
\widehat{\Xi}_0 &= \operatorname{sym}\left\{ [e_1\ \ e_3\ \ e_5\ \ e_6\ \ e_7\ \ e_9\ \ e_{10}]\, \widehat{P}\, [e_2\ \ e_4\ \ e_1 - e_3\ \ he_1 - e_5\ \ e_8\ \ e_1 - e_7\ \ \tau e_1 - e_9]^{\mathrm{T}} \right\} + [e_1\ \ e_2]\, \widehat{Q}\, [e_1\ \ e_2]^{\mathrm{T}} \\
&\quad - [e_3\ \ e_4]\, \widehat{Q}\, [e_3\ \ e_4]^{\mathrm{T}} + h^2 [e_1\ \ e_2]\, \widehat{D}\, [e_1\ \ e_2]^{\mathrm{T}} + \left(\frac{h^2}{2}\right)^2 e_2 \widehat{T} e_2^{\mathrm{T}} + [e_1\ \ e_2]\, \widehat{\widehat{Q}}\, [e_1\ \ e_2]^{\mathrm{T}} - [e_7\ \ e_8]\, \widehat{\widehat{Q}}\, [e_7\ \ e_8]^{\mathrm{T}} \\
&\quad + \tau^2 [e_1\ \ e_2]\, \widehat{\widehat{D}}\, [e_1\ \ e_2]^{\mathrm{T}} + \left(\frac{\tau^2}{2}\right)^2 e_2 \widehat{\widehat{T}} e_2^{\mathrm{T}}, \\
\widehat{\Xi}_1 &= -[e_5\ \ e_1 - e_3]\, \widehat{D}\, [e_5\ \ e_1 - e_3]^{\mathrm{T}} - 3\left[\left(\frac{2}{h}\right) e_6 - e_5\ \ e_1 + e_3 - \left(\frac{2}{h}\right) e_5 \right] \widehat{D} \left[\left(\frac{2}{h}\right) e_6 - e_5\ \ e_1 + e_3 - \left(\frac{2}{h}\right) e_5 \right]^{\mathrm{T}}, \\
\widehat{\Xi}_{2,1} &= -[he_1 - e_5]\, \widehat{T}\, [he_1 - e_5]^{\mathrm{T}}, \\
\widehat{\Xi}_{2,2} &= -2\left[\left(\frac{h}{2}\right) e_1 + e_5 - \left(\frac{3}{h}\right) e_6 \right] \widehat{T} \left[\left(\frac{h}{2}\right) e_1 + e_5 - \left(\frac{3}{h}\right) e_6 \right]^{\mathrm{T}}, \\
\widehat{\Xi}_3 &= -[e_9\ \ e_1 - e_7]\, \widehat{\widehat{D}}\, [e_9\ \ e_1 - e_7]^{\mathrm{T}} - 3\left[\left(\frac{2}{\tau}\right) e_{10} - e_9\ \ e_1 + e_7 - \left(\frac{2}{\tau}\right) e_9 \right] \widehat{\widehat{D}} \left[\left(\frac{2}{\tau}\right) e_{10} - e_9\ \ e_1 + e_7 - \left(\frac{2}{\tau}\right) e_9 \right]^{\mathrm{T}}, \\
\widehat{\Xi}_{4,1} &= -[\tau e_1 - e_9]\, \widehat{\widehat{T}}\, [\tau e_1 - e_9]^{\mathrm{T}}, \\
\widehat{\Xi}_{4,2} &= -2\left[\left(\frac{\tau}{2}\right) e_1 + e_9 - \left(\frac{3}{\tau}\right) e_{10} \right] \widehat{\widehat{T}} \left[\left(\frac{\tau}{2}\right) e_1 + e_9 - \left(\frac{3}{\tau}\right) e_{10} \right]^{\mathrm{T}}, \\
\widehat{\Xi} &= \widehat{\Xi}_0 + \widehat{\Xi}_1 + \widehat{\Xi}_{2,1} + \widehat{\Xi}_{2,2} + \widehat{\Xi}_3 + \widehat{\Xi}_{4,1} + \widehat{\Xi}_{4,2}, \\
\widehat{L} &= a_1 e_1 + e_2 + a_2 e_3, \\
\widehat{\Gamma}(\lambda) &= [A(\lambda) X\ \ -X\ \ A_d(\lambda) X\ \ B(\lambda) M(\lambda)]\, [e_1, e_2, e_3, e_7]^{\mathrm{T}}.
\end{aligned} \tag{15}$$

In addition, the gain matrix of state feedback controller can be obtained as

$$K(\lambda) = M(\lambda) X^{-1}. \tag{16}$$

Proof. In order to establish a stability condition of system (9), we choose the following Lyapunov-Krasovskii functional:

$$\begin{aligned}
V &= v_1^{\mathrm{T}}(t) P v_1(t) + \int_{t-h}^{t} v_2^{\mathrm{T}}(s) Q v_2(s)\, ds \\
&\quad + h \int_{t-h}^{t} \int_s^t v_2^{\mathrm{T}}(u) D v_2(u)\, du\, ds
\end{aligned}$$

$$\begin{aligned}
&+\frac{h^2}{2}\int_{t-h}^{t}\int_{s}^{t}\int_{u}^{t}\dot{x}^{\mathrm{T}}(v)\,T\dot{x}(v)\,dv\,du\,ds \\
&+\int_{t-\tau}^{t}v_2^{\mathrm{T}}(s)\,\overline{Q}v_2(s)\,ds \\
&+\tau\int_{t-\tau}^{t}\int_{s}^{t}v_2^{\mathrm{T}}(u)\,\overline{D}v_2(u)\,du\,ds \\
&+\frac{\tau^2}{2}\int_{t-\tau}^{t}\int_{s}^{t}\int_{u}^{t}\dot{x}^{\mathrm{T}}(v)\,\overline{T}\dot{x}(v)\,dv\,du\,ds,
\end{aligned} \tag{17}$$

where

$$\begin{gathered}
v_1(t)=\begin{bmatrix} x(t) \\ x(t-h) \\ \int_{t-h}^{t}x(s)\,ds \\ \int_{t-h}^{t}\int_{s}^{t}x(u)\,du\,ds \\ x(t-\tau) \\ \int_{t-\tau}^{t}x(s)\,ds \\ \int_{t-\tau}^{t}\int_{s}^{t}x(u)\,du\,ds \end{bmatrix}, \\
v_2(t)=\begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}, \\
\dot{v}_1(t)=\begin{bmatrix} \dot{x}(t) \\ \dot{x}(t-h) \\ x(t)-x(t-h) \\ hx(t)-\int_{t-h}^{t}x(s)\,ds \\ \dot{x}(t-\tau) \\ x(t)-x(t-\tau) \\ \tau x(t)-\int_{t-\tau}^{t}x(s)\,ds \end{bmatrix}.
\end{gathered} \tag{18}$$

Moreover, $P \in R^{7n\times 7n}$, $Q,\overline{Q} \in R^{2n\times 2n}$, $D,\overline{D} \in R^{2n\times 2n}$, and $T,\overline{T} \in R^{n\times n}$ are positive definite symmetric matrices.

The time-derivative of V can be computed as

$$\begin{aligned}
\dot{V} &= 2v_1^{\mathrm{T}}(t)\,P\dot{v}_1(t)+v_2^{\mathrm{T}}(t)\,Qv_2(t) \\
&\quad -v_2^{\mathrm{T}}(t-h)\,Qv_2(t-h)+h^2v_2^{\mathrm{T}}(t)\,Dv_2(t) \\
&\quad -h\xi_1(t)+\left(\frac{h^2}{2}\right)^2\dot{x}^{\mathrm{T}}(t)\,T\dot{x}(t)-\frac{h^2}{2}\xi_2(t) \\
&\quad +v_2^{\mathrm{T}}(t)\,\overline{Q}v_2(t)-v_2^{\mathrm{T}}(t-\tau)\,\overline{Q}v_2(t-\tau) \\
&\quad +\tau^2v_2^{\mathrm{T}}(t)\,\overline{D}v_2(t)-\tau\xi_3(t) \\
&\quad +\left(\frac{\tau^2}{2}\right)^2\dot{x}^{\mathrm{T}}(t)\,\overline{T}\dot{x}(t)-\frac{\tau^2}{2}\xi_4(t) \\
&= 2v_1^{\mathrm{T}}(t)\,P\dot{v}_1(t)+v_2^{\mathrm{T}}(t)\,Qv_2(t) \\
&\quad -v_2^{\mathrm{T}}(t-h)\,Qv_2(t-h)+h^2v_2^{\mathrm{T}}(t)\,Dv_2(t) \\
&\quad +\left(\frac{h^2}{2}\right)^2\dot{x}^{\mathrm{T}}(t)\,T\dot{x}(t)+v_2^{\mathrm{T}}(t)\,\overline{Q}v_2(t) \\
&\quad -v_2^{\mathrm{T}}(t-\tau)\,\overline{Q}v_2(t-\tau)+\tau^2v_2^{\mathrm{T}}(t)\,\overline{D}v_2(t) \\
&\quad +\left(\frac{\tau^2}{2}\right)^2\dot{x}^{\mathrm{T}}(t)\,\overline{T}\dot{x}(t)-h\xi_1(t)-\frac{h^2}{2}\xi_2(t) \\
&\quad -\tau\xi_3(t)-\frac{\tau^2}{2}\xi_4(t) \\
&= \zeta^{\mathrm{T}}(t)\,\Xi_0\zeta(t)-h\xi_1(t)-\frac{h^2}{2}\xi_2(t)-\tau\xi_3(t) \\
&\quad -\frac{\tau^2}{2}\xi_4(t),
\end{aligned} \tag{19}$$

where

$$\begin{aligned}
\xi_1(t) &= \int_{t-h}^{t}v_2^{\mathrm{T}}(s)\,Dv_2(s)\,ds, \\
\xi_2(t) &= \int_{t-h}^{t}\int_{s}^{t}\dot{x}^{\mathrm{T}}(u)\,T\dot{x}(u)\,du\,ds, \\
\xi_3(t) &= \int_{t-\tau}^{t}v_2^{\mathrm{T}}(s)\,\overline{D}v_2(s)\,ds, \\
\xi_4(t) &= \int_{t-\tau}^{t}\int_{s}^{t}\dot{x}^{\mathrm{T}}(u)\,\overline{T}\dot{x}(u)\,du\,ds, \\
\Xi_0 &= \operatorname{sym}\left\{[e_1\ \ e_3\ \ e_5\ \ e_6\ \ e_7\ \ e_9\ \ e_{10}]\,P\,[e_2\ \ e_4\ \ e_1-e_3\ \ he_1-e_5\ \ e_8\ \ e_1-e_7\ \ \tau e_1-e_9]^{\mathrm{T}}\right\}+[e_1\ \ e_2]\,Q\,[e_1\ \ e_2]^{\mathrm{T}} \\
&\quad -[e_3\ \ e_4]\,Q\,[e_3\ \ e_4]^{\mathrm{T}}+h^2\,[e_1\ \ e_2]\,D\,[e_1\ \ e_2]^{\mathrm{T}}+\left(\frac{h^2}{2}\right)^2 e_2Te_2{}^{\mathrm{T}}+[e_1\ \ e_2]\,\overline{Q}\,[e_1\ \ e_2]^{\mathrm{T}}-[e_7\ \ e_8]\,\overline{Q}\,[e_7\ \ e_8]^{\mathrm{T}}
\end{aligned}$$

$$
\begin{aligned}
&+\tau^2\left[e_1\;\; e_2\right]\overline{D}\left[e_1\;\; e_2\right]^{\mathrm{T}}+\left(\frac{\tau^2}{2}\right)^2 e_2\overline{T}e_2{}^{\mathrm{T}},\\
&\zeta^{\mathrm{T}}(t)=\left[x^{\mathrm{T}}(t),\dot{x}^{\mathrm{T}}(t),x^{\mathrm{T}}(t-h),\dot{x}^{\mathrm{T}}(t-h),\int_{t-h}^{t}x^{\mathrm{T}}(s)\,ds,\int_{t-h}^{t}\int_{s}^{t}x^{\mathrm{T}}(u)\,du\,ds,x^{\mathrm{T}}(t-\tau),\dot{x}^{\mathrm{T}}(t-\tau),\int_{t-\tau}^{t}x^{\mathrm{T}}(s)\,ds,\right.\\
&\left.\int_{t-\tau}^{t}\int_{s}^{t}x^{\mathrm{T}}(u)\,du\,ds\right],
\end{aligned}
\tag{20}
$$

and $e_i \in R^{10n\times n}$ $(i = 1,2,\ldots,10)$ means the block entry matrices; for example, $e_3^{\mathrm{T}}\zeta(t) = x(t-h)$, $e_9^{\mathrm{T}}\zeta(t) = \int_{t-\tau}^{t}x(s)ds$.

Using Lemmas 1 and 2 to four integral terms $\xi_1(t)$, $\xi_2(t)$, $\xi_3(t)$, and $\xi_4(t)$ in (19), we have

$$-h\xi_1(t)\le-\phi_{1,1}^{\mathrm{T}}(t)\,D\phi_{1,1}(t)-3\phi_{1,2}^{\mathrm{T}}(t)\,D\phi_{1,2}(t),\tag{21}$$

$$-\frac{h^2}{2}\xi_2(t)\le-\phi_{2,1}^{\mathrm{T}}(t)\,T\phi_{2,1}(t)-2\phi_{2,2}^{\mathrm{T}}(t)\,T\phi_{2,2}(t),\tag{22}$$

$$-\tau\xi_3(t)\le-\phi_{3,1}^{\mathrm{T}}(t)\,\overline{D}\phi_{3,1}(t)-3\phi_{3,2}^{\mathrm{T}}(t)\,\overline{D}\phi_{3,2}(t),\tag{23}$$

$$-\frac{\tau^2}{2}\xi_4(t)\le-\phi_{4,1}^{\mathrm{T}}(t)\,\overline{T}\phi_{4,1}(t)-2\phi_{4,2}^{\mathrm{T}}(t)\,\overline{T}\phi_{4,2}(t),\tag{24}$$

where

$$
\begin{aligned}
\varphi_{1,1}(t)&=\int_{t-h}^{t}v_2(s)\,ds=\begin{bmatrix}\int_{t-h}^{t}x(s)\,ds\\ x(t)-x(t-h)\end{bmatrix}\\
&=\left[e_5\;\; e_1-e_3\right]^{\mathrm{T}}\zeta(t),\\
\varphi_{1,2}(t)&=\frac{2}{h}\int_{t-h}^{t}\int_{s}^{t}v_2(u)\,du\,ds-\int_{t-h}^{t}v_2(s)\,ds\\
&=\begin{bmatrix}\frac{2}{h}\int_{t-h}^{t}\int_{s}^{t}x(u)\,du\,ds-\int_{t-h}^{t}x(s)\,ds\\ x(t)+x(t-h)-\frac{2}{h}\int_{t-h}^{t}x(s)\,ds\end{bmatrix}\\
&=\left[\frac{2}{h}e_6-e_5\;\; e_1+e_3-\frac{2}{h}e_5\right]^{\mathrm{T}}\zeta(t),\\
\varphi_{2,1}(t)&=\int_{t-h}^{t}\int_{s}^{t}\dot{x}(u)\,du\,ds=hx(t)-\int_{t-h}^{t}x(s)\,ds\\
&=\left(he_1-e_5\right)^{\mathrm{T}}\zeta(t),\\
\varphi_{2,2}(t)&=\frac{3}{h}\int_{t-h}^{t}\int_{s}^{t}\int_{u}^{t}\dot{x}(v)\,dv\,du\,ds\\
&\quad-\int_{t-h}^{t}\int_{s}^{t}\dot{x}(u)\,du\,ds\\
&=\frac{h}{2}x(t)+\int_{t-h}^{t}x(s)\,ds\\
&\quad-\frac{3}{h}\int_{t-h}^{t}\int_{s}^{t}x(u)\,du\,ds\\
&=\left(\frac{h}{2}e_1+e_5-\frac{3}{h}e_6\right)^{\mathrm{T}}\zeta(t),\\
\varphi_{3,1}(t)&=\int_{t-\tau}^{t}v_2(s)\,ds=\begin{bmatrix}\int_{t-\tau}^{t}x(s)\,ds\\ x(t)-x(t-\tau)\end{bmatrix}\\
&=\left[e_9\;\; e_1-e_7\right]^{\mathrm{T}}\zeta(t),\\
\varphi_{3,2}(t)&=\frac{2}{\tau}\int_{t-\tau}^{t}\int_{s}^{t}v_2(u)\,du\,ds-\int_{t-\tau}^{t}v_2(s)\,ds\\
&=\begin{bmatrix}\frac{2}{\tau}\int_{t-\tau}^{t}\int_{s}^{t}x(u)\,du\,ds-\int_{t-\tau}^{t}x(s)\,ds\\ x(t)+x(t-\tau)-\frac{2}{\tau}\int_{t-\tau}^{t}x(s)\,ds\end{bmatrix}\\
&=\left[\frac{2}{\tau}e_{10}-e_9\;\; e_1+e_7-\frac{2}{\tau}e_9\right]^{\mathrm{T}}\zeta(t),\\
\varphi_{4,1}(t)&=\int_{t-\tau}^{t}\int_{s}^{t}\dot{x}(u)\,du\,ds=\tau x(t)-\int_{t-\tau}^{t}x(s)\,ds\\
&=\left(\tau e_1-e_9\right)^{\mathrm{T}}\zeta(t),\\
\varphi_{4,2}(t)&=\frac{3}{\tau}\int_{t-\tau}^{t}\int_{s}^{t}\int_{u}^{t}\dot{x}(v)\,dv\,du\,ds\\
&\quad-\int_{t-\tau}^{t}\int_{s}^{t}\dot{x}(u)\,du\,ds\\
&=\frac{\tau}{2}x(t)+\int_{t-\tau}^{t}x(s)\,ds\\
&\quad-\frac{3}{\tau}\int_{t-\tau}^{t}\int_{s}^{t}x(u)\,du\,ds\\
&=\left(\frac{\tau}{2}e_1+e_9-\frac{3}{\tau}e_{10}\right)^{\mathrm{T}}\zeta(t).
\end{aligned}
\tag{25}
$$

Combining (21)–(24), (19) can be rewritten as

$$
\begin{aligned}
&\dot{V}\le\zeta^{\mathrm{T}}(t)\left(\Xi_0+\Xi_1+\Xi_{2,1}+\Xi_{2,2}+\Xi_3+\Xi_{4,1}+\Xi_{4,2}\right)\\
&\cdot\zeta(t)=\zeta^{\mathrm{T}}(t)\,\Xi\zeta(t),
\end{aligned}
\tag{26}
$$

where

$$\begin{aligned}
\Xi_1 &= -\left[e_5 \;\; e_1 - e_3\right] D \left[e_5 \;\; e_1 - e_3\right]^{\mathrm{T}} - 3\left[\left(\frac{2}{h}\right) e_6 - e_5 \;\; e_1 + e_3 - \left(\frac{2}{h}\right) e_5\right] D \left[\left(\frac{2}{h}\right) e_6 - e_5 \;\; e_1 + e_3 - \left(\frac{2}{h}\right) e_5\right]^{\mathrm{T}}, \\
\Xi_{2,1} &= -\left[h e_1 - e_5\right] T \left[h e_1 - e_5\right]^{\mathrm{T}}, \\
\Xi_{2,2} &= -2\left[\left(\frac{h}{2}\right) e_1 + e_5 - \left(\frac{3}{h}\right) e_6\right] T \left[\left(\frac{h}{2}\right) e_1 + e_5 - \left(\frac{3}{h}\right) e_6\right]^{\mathrm{T}}, \\
\Xi_3 &= -\left[e_9 \;\; e_1 - e_7\right] \overline{D} \left[e_9 \;\; e_1 - e_7\right]^{\mathrm{T}} - 3\left[\left(\frac{2}{\tau}\right) e_{10} - e_9 \;\; e_1 + e_7 - \left(\frac{2}{\tau}\right) e_9\right] \overline{D} \left[\left(\frac{2}{\tau}\right) e_{10} - e_9 \;\; e_1 + e_7 - \left(\frac{2}{\tau}\right) e_9\right]^{\mathrm{T}}, \\
\Xi_{4,1} &= -\left[\tau e_1 - e_9\right] \overline{T} \left[\tau e_1 - e_9\right]^{\mathrm{T}}, \\
\Xi_{4,2} &= -2\left[\left(\frac{\tau}{2}\right) e_1 + e_9 - \left(\frac{3}{\tau}\right) e_{10}\right] \overline{T} \left[\left(\frac{\tau}{2}\right) e_1 + e_9 - \left(\frac{3}{\tau}\right) e_{10}\right]^{\mathrm{T}}, \\
\Xi &= \Xi_0 + \Xi_1 + \Xi_{2,1} + \Xi_{2,2} + \Xi_3 + \Xi_{4,1} + \Xi_{4,2}.
\end{aligned} \tag{27}$$

Then, a new stability condition for system (9) can be described as

$$\zeta^{\mathrm{T}}(t)\,\Xi\zeta(t) < 0 \quad \text{s.t.}\; \Gamma(\lambda)\,\zeta(t) = 0,\; \zeta(t) \neq 0, \tag{28}$$

where $\Gamma(\lambda) = \left[A(\lambda) \;\; -I \;\; A_d(\lambda) \;\; B(\lambda)K(\lambda)\right]\left[e_1 \;\; e_2 \;\; e_3 \;\; e_7\right]^{\mathrm{T}}$.

Based on Lemma 3, formula (28) can be rewritten as

$$\Xi + L\Gamma(\lambda) + \Gamma(\lambda)^{\mathrm{T}} L^{\mathrm{T}} < 0, \tag{29}$$

where $L = a_1 e_1 L_0 + e_2 L_0 + a_2 e_3 L_0$.

Now, let $L_0 = X^{-\mathrm{T}}$ and $K(\lambda) = M(\lambda)X^{-1}$. Applying matrix inequality (29) to the left multiplication $\mathrm{diag}[X, X, X, X, X, X, X, X, X, X]^{\mathrm{T}}$ and right multiplication $\mathrm{diag}[X, X, X, X, X, X, X, X, X, X]$, we have

$$\widehat{\Xi} + \widehat{L}\widehat{\Gamma}(\lambda) + \widehat{\Gamma}(\lambda)^{\mathrm{T}}\widehat{L}^{\mathrm{T}} < 0, \tag{30}$$

where

$$\begin{aligned}
\widehat{\Xi}_0 &= \mathrm{sym}\left\{\left[e_1 \;\; e_3 \;\; e_5 \;\; e_6 \;\; e_7 \;\; e_9 \;\; e_{10}\right] \widehat{P} \left[e_2 \;\; e_4 \;\; e_1 - e_3 \;\; h e_1 - e_5 \;\; e_8 \;\; e_1 - e_7 \;\; \tau e_1 - e_9\right]^{\mathrm{T}}\right\} + \left[e_1 \;\; e_2\right] \widehat{Q} \left[e_1 \;\; e_2\right]^{\mathrm{T}} \\
&\quad - \left[e_3 \;\; e_4\right] \widehat{Q} \left[e_3 \;\; e_4\right]^{\mathrm{T}} + h^2 \left[e_1 \;\; e_2\right] \widehat{D} \left[e_1 \;\; e_2\right]^{\mathrm{T}} + \left(\frac{h^2}{2}\right)^2 e_2 \widehat{T} e_2^{\mathrm{T}} + \left[e_1 \;\; e_2\right] \widehat{\overline{Q}} \left[e_1 \;\; e_2\right]^{\mathrm{T}} - \left[e_7 \;\; e_8\right] \widehat{\overline{Q}} \left[e_7 \;\; e_8\right]^{\mathrm{T}} \\
&\quad + \tau^2 \left[e_1 \;\; e_2\right] \widehat{\overline{D}} \left[e_1 \;\; e_2\right]^{\mathrm{T}} + \left(\frac{\tau^2}{2}\right)^2 e_2 \widehat{\overline{T}} e_2^{\mathrm{T}}, \\
\widehat{\Xi}_1 &= -\left[e_5 \;\; e_1 - e_3\right] \widehat{D} \left[e_5 \;\; e_1 - e_3\right]^{\mathrm{T}} - 3\left[\left(\frac{2}{h}\right) e_6 - e_5 \;\; e_1 + e_3 - \left(\frac{2}{h}\right) e_5\right] \widehat{D} \left[\left(\frac{2}{h}\right) e_6 - e_5 \;\; e_1 + e_3 - \left(\frac{2}{h}\right) e_5\right]^{\mathrm{T}}, \\
\widehat{\Xi}_{2,1} &= -\left[h e_1 - e_5\right] \widehat{T} \left[h e_1 - e_5\right]^{\mathrm{T}}, \\
\widehat{\Xi}_{2,2} &= -2\left[\left(\frac{h}{2}\right) e_1 + e_5 - \left(\frac{3}{h}\right) e_6\right] \widehat{T} \left[\left(\frac{h}{2}\right) e_1 + e_5 - \left(\frac{3}{h}\right) e_6\right]^{\mathrm{T}}, \\
\widehat{\Xi}_3 &= -\left[e_9 \;\; e_1 - e_7\right] \widehat{\overline{D}} \left[e_9 \;\; e_1 - e_7\right]^{\mathrm{T}} - 3\left[\left(\frac{2}{\tau}\right) e_{10} - e_9 \;\; e_1 + e_7 - \left(\frac{2}{\tau}\right) e_9\right] \widehat{\overline{D}} \left[\left(\frac{2}{\tau}\right) e_{10} - e_9 \;\; e_1 + e_7 - \left(\frac{2}{\tau}\right) e_9\right]^{\mathrm{T}}, \\
\widehat{\Xi}_{4,1} &= -\left[\tau e_1 - e_9\right] \widehat{\overline{T}} \left[\tau e_1 - e_9\right]^{\mathrm{T}}, \\
\widehat{\Xi}_{4,2} &= -2\left[\left(\frac{\tau}{2}\right) e_1 + e_9 - \left(\frac{3}{\tau}\right) e_{10}\right] \widehat{\overline{T}} \left[\left(\frac{\tau}{2}\right) e_1 + e_9 - \left(\frac{3}{\tau}\right) e_{10}\right]^{\mathrm{T}}, \\
\widehat{\Xi} &= \widehat{\Xi}_0 + \widehat{\Xi}_1 + \widehat{\Xi}_{2,1} + \widehat{\Xi}_{2,2} + \widehat{\Xi}_3 + \widehat{\Xi}_{4,1} + \widehat{\Xi}_{4,2},
\end{aligned}$$

$$\begin{aligned}
\widehat{P} &= \left(\operatorname{diag}[X, X, X, X, X, X, X]^{\mathrm{T}}\right) P \left(\operatorname{diag}[X, X, X, X, X, X, X]\right),\\
\widehat{Q} &= \left(\operatorname{diag}[X, X]^{\mathrm{T}}\right) Q \left(\operatorname{diag}[X, X]\right),\\
\widehat{\overline{Q}} &= \left(\operatorname{diag}[X, X]^{\mathrm{T}}\right) \overline{Q} \left(\operatorname{diag}[X, X]\right),\\
\widehat{D} &= \left(\operatorname{diag}[X, X]^{\mathrm{T}}\right) D \left(\operatorname{diag}[X, X]\right),\\
\widehat{\overline{D}} &= \left(\operatorname{diag}[X, X]^{\mathrm{T}}\right) \widehat{D} \left(\operatorname{diag}[X, X]\right),\\
\widehat{T} &= X^{\mathrm{T}} T X,\\
\widehat{\overline{T}} &= X^{\mathrm{T}} \overline{T} X.
\end{aligned} \tag{31}$$

Since

$$\begin{aligned}
&\left(\operatorname{diag}[X, X, X, X, X, X, X, X, X, X]^{\mathrm{T}}\right) L\Gamma(\lambda)\\
&\quad\cdot \left(\operatorname{diag}[X, X, X, X, X, X, X, X, X, X]\right) = \left(a_1 e_1 + e_2\right.\\
&\quad\left. + a_2 e_3\right) \left[A(\lambda) X \quad -X \quad A_d(\lambda) X \quad B(\lambda) M(\lambda)\right] \left[e_1,\right.\\
&\quad\left. e_2, e_3, e_7\right]^{\mathrm{T}} = \widehat{L}\widehat{\Gamma}(\lambda),
\end{aligned} \tag{32}$$

we have $\widehat{L} = a_1 e_1 + e_2 + a_2 e_3$ and $\widehat{\Gamma}(\lambda) = [A(\lambda)X \quad -X \quad A_d(\lambda)X \quad B(\lambda)M(\lambda)]\,[e_1, e_2, e_3, e_7]^{\mathrm{T}}$.

Thus, we transform inequality (29) into the form of linear matrix inequality, which is defined in (14). The whole proof is completed. ☐

Theorem 4 is dependent on time-varying information λ, which cannot be computed by LMI. In the following analysis, our goal is to convert Theorem 4 into LMI.

Theorem 5. *Consider the closed-loop system (9) and given scalars h and τ to meet $h > 0$, $\tau > 0$, the system is asymptotically stable, if there exist scalars a_1 and a_2, matrices M_j, X, and positive definite symmetric matrices $\widehat{P}$, $\widehat{Q}$, $\widehat{D}$, $\widehat{T}$, $\widehat{\overline{Q}}$, $\widehat{\overline{D}}$, and $\widehat{\overline{T}}$, such that the following LMI holds:*

$$\begin{aligned}
&\widehat{\Xi} + \widehat{L}\widehat{\Gamma}(i, i) + \widehat{\Gamma}(i, i)^{\mathrm{T}} \widehat{L}^{\mathrm{T}} < 0, \quad i = 1 \cdots r,\\
&\widehat{\Xi} + \widehat{L}\widehat{\Gamma}(i, j) + \widehat{\Gamma}(i, j)^{\mathrm{T}} \widehat{L}^{\mathrm{T}} + \widehat{\Xi} + \widehat{L}\widehat{\Gamma}(j, i)\\
&\quad + \widehat{\Gamma}(j, i)^{\mathrm{T}} \widehat{L}^{\mathrm{T}} < 0 \quad 1 \le i < j \le r,
\end{aligned} \tag{33}$$

where

$$\begin{aligned}
\widehat{\Xi}_0 &= \operatorname{sym}\left\{[e_1 \ \ e_3 \ \ e_5 \ \ e_6 \ \ e_7 \ \ e_9 \ \ e_{10}]\, \widehat{P}\, [e_2 \ \ e_4 \ \ e_1 - e_3 \ \ h e_1 - e_5 \ \ e_8 \ \ e_1 - e_7 \ \ \tau e_1 - e_9]^{\mathrm{T}}\right\} + [e_1 \ \ e_2]\, \widehat{Q}\, [e_1 \ \ e_2]^{\mathrm{T}}\\
&\quad - [e_3 \ \ e_4]\, \widehat{Q}\, [e_3 \ \ e_4]^{\mathrm{T}} + h^2 [e_1 \ \ e_2]\, \widehat{D}\, [e_1 \ \ e_2]^{\mathrm{T}} + \left(\frac{h^2}{2}\right)^2 e_2 \widehat{T} e_2{}^{\mathrm{T}} + [e_1 \ \ e_2]\, \widehat{\overline{Q}}\, [e_1 \ \ e_2]^{\mathrm{T}} - [e_7 \ \ e_8]\, \widehat{\overline{Q}}\, [e_7 \ \ e_8]^{\mathrm{T}}\\
&\quad + \tau^2 [e_1 \ \ e_2]\, \widehat{\overline{D}}\, [e_1 \ \ e_2]^{\mathrm{T}} + \left(\frac{\tau^2}{2}\right)^2 e_2 \widehat{\overline{T}} e_2{}^{\mathrm{T}},\\
\widehat{\Xi}_1 &= -[e_5 \ \ e_1 - e_3]\, \widehat{D}\, [e_5 \ \ e_1 - e_3]^{\mathrm{T}} - 3\left[\left(\frac{2}{h}\right) e_6 - e_5 \ \ e_1 + e_3 - \left(\frac{2}{h}\right) e_5\right] \widehat{D} \left[\left(\frac{2}{h}\right) e_6 - e_5 \ \ e_1 + e_3 - \left(\frac{2}{h}\right) e_5\right]^{\mathrm{T}},\\
\widehat{\Xi}_{2,1} &= -[h e_1 - e_5]\, \widehat{T}\, [h e_1 - e_5]^{\mathrm{T}},\\
\widehat{\Xi}_{2,2} &= -2\left[\left(\frac{h}{2}\right) e_1 + e_5 - \left(\frac{3}{h}\right) e_6\right] \widehat{T} \left[\left(\frac{h}{2}\right) e_1 + e_5 - \left(\frac{3}{h}\right) e_6\right]^{\mathrm{T}},\\
\widehat{\Xi}_3 &= -[e_9 \ \ e_1 - e_7]\, \widehat{\overline{D}}\, [e_9 \ \ e_1 - e_7]^{\mathrm{T}} - 3\left[\left(\frac{2}{\tau}\right) e_{10} - e_9 \ \ e_1 + e_7 - \left(\frac{2}{\tau}\right) e_9\right] \widehat{\overline{D}} \left[\left(\frac{2}{\tau}\right) e_{10} - e_9 \ \ e_1 + e_7 - \left(\frac{2}{\tau}\right) e_9\right]^{\mathrm{T}},\\
\widehat{\Xi}_{4,1} &= -[\tau e_1 - e_9]\, \widehat{\overline{T}}\, [\tau e_1 - e_9]^{\mathrm{T}},\\
\widehat{\Xi}_{4,2} &= -2\left[\left(\frac{\tau}{2}\right) e_1 + e_9 - \left(\frac{3}{\tau}\right) e_{10}\right] \widehat{\overline{T}} \left[\left(\frac{\tau}{2}\right) e_1 + e_9 - \left(\frac{3}{\tau}\right) e_{10}\right]^{\mathrm{T}},
\end{aligned}$$

$$\begin{aligned}
&\widehat{\Xi} = \widehat{\Xi}_0 + \widehat{\Xi}_1 + \widehat{\Xi}_{2,1} + \widehat{\Xi}_{2,2} + \widehat{\Xi}_3 + \widehat{\Xi}_{4,1} + \widehat{\Xi}_{4,2},\\
&\widehat{L} = a_1 e_1 + e_2 + a_2 e_3,\\
&\widehat{\Gamma}(i,j) = [A_i X \quad -X \quad A_{di} X \quad B_i M_j]\,[e_1, e_2, e_3, e_7]^{\mathrm{T}}.
\end{aligned} \tag{34}$$

In addition, the gain matrix of state feedback controller can be described as

$$K_j = M_j X^{-1}. \tag{35}$$

Proof. Consider the following forms and substitute them into inequality (14):

$$\begin{aligned}
A(\lambda) &= \sum_{i=1}^{r} \lambda_i(\theta(t))\, A_i,\\
A_d(\lambda) &= \sum_{i=1}^{r} \lambda_i(\theta(t))\, A_{di},\\
B(\lambda) &= \sum_{i=1}^{r} \lambda_i(\theta(t))\, B_i,\\
M(\lambda) &= \sum_{i=1}^{r} \lambda_i(\theta(t))\, M_i.
\end{aligned} \tag{36}$$

Thus, inequality (14) can be rewritten as

$$\begin{aligned}
&\sum_{i=1}^{r}\sum_{j=1}^{r} \lambda_i(\theta(t))\,\lambda_j(\theta(t))\left(\Xi + \widehat{L}\widehat{\Gamma}(i,j) + \widehat{\Gamma}(i,j)^{\mathrm{T}}\widehat{L}^{\mathrm{T}}\right)\\
&\quad < 0,
\end{aligned} \tag{37}$$

where $\widehat{\Gamma}(i,j) = [A_i X \quad -X \quad A_{di} X \quad B_i M_j]\,[e_1, e_2, e_3, e_7]^{\mathrm{T}}$.

Now we can rewrite (37) as

$$\begin{aligned}
&\sum_{i=1}^{r} \lambda_i^2(\theta(t))\left(\widehat{\Xi} + \widehat{L}\widehat{\Gamma}(i,i) + \widehat{\Gamma}(i,i)^{\mathrm{T}}\widehat{L}^{\mathrm{T}}\right)\\
&\quad + \sum_{i=1}^{r}\sum_{i<j}^{r} \lambda_i(\theta(t))\,\lambda_j(\theta(t)) \times \left(\widehat{\Xi} + \widehat{L}\widehat{\Gamma}(i,j)\right.\\
&\quad \left. + \widehat{\Gamma}(i,j)^{\mathrm{T}}\widehat{L}^{\mathrm{T}} + \widehat{\Xi} + \widehat{L}\widehat{\Gamma}(j,i) + \widehat{\Gamma}(j,i)^{\mathrm{T}}\widehat{L}^{\mathrm{T}}\right) < 0.
\end{aligned} \tag{38}$$

Thus, we can get the following linear matrix inequality:

$$\begin{aligned}
&\widehat{\Xi} + \widehat{L}\widehat{\Gamma}(i,i) + \widehat{\Gamma}(i,i)^{\mathrm{T}}\widehat{L}^{\mathrm{T}} < 0, \quad i = 1\cdots r,\\
&\widehat{\Xi} + \widehat{L}\widehat{\Gamma}(i,j) + \widehat{\Gamma}(i,j)^{\mathrm{T}}\widehat{L}^{\mathrm{T}} + \widehat{\Xi} + \widehat{L}\widehat{\Gamma}(j,i)\\
&\quad + \widehat{\Gamma}(j,i)^{\mathrm{T}}\widehat{L}^{\mathrm{T}} < 0 \quad 1 \le i < j \le r.
\end{aligned} \tag{39}$$

The whole proof is completed. □

Remark 6. T-S fuzzy time-delay systems have been widely studied in previous papers. However, the existing literature only considers one delay factor, for example, state delay or input delay. Currently, a few results are presented for T-S fuzzy system with state and input delays. Unlike the existing results, a new Lyapunov-Krasovskii functional including triple integral terms is employed to derive less conservative stability conditions. In the next section, some simulation examples will be provided to illustrate the effectiveness of the proposed method.

In order to show the effectiveness of the Lyapunov-Krasovskii functional with triple integral terms and Wirtinger-based double integral inequality, we use the Lyapunov-Krasovskii functional of (17) without triple integral terms to derive new stabilization conditions. Now we give the following corollary.

Corollary 7. *Consider the closed-loop system (9) and given scalars h and τ to meet $h > 0$, $\tau > 0$, the system is asymptotically stable, if there exist scalars a_1 and a_2, matrices M_j, X, and positive definite symmetric matrices $\widehat{P}$, $\widehat{Q}$, $\widehat{D}$, $\widehat{\overline{Q}}$, $\widehat{\overline{D}}$, such that the following LMI holds:*

$$\begin{aligned}
&\widehat{\Xi} + \widehat{L}\widehat{\Gamma}(i,i) + \widehat{\Gamma}(i,i)^{\mathrm{T}}\widehat{L}^{\mathrm{T}} < 0, \quad i = 1\cdots r,\\
&\widehat{\Xi} + \widehat{L}\widehat{\Gamma}(i,j) + \widehat{\Gamma}(i,j)^{\mathrm{T}}\widehat{L}^{\mathrm{T}} + \widehat{\Xi} + \widehat{L}\widehat{\Gamma}(j,i)\\
&\quad + \widehat{\Gamma}(j,i)^{\mathrm{T}}\widehat{L}^{\mathrm{T}} < 0 \quad 1 \le i < j \le r,
\end{aligned} \tag{40}$$

where

$$\begin{aligned}
\widehat{\Xi}_0 ={}& \mathrm{sym}\left\{[e_1 \quad e_3 \quad e_5 \quad e_6 \quad e_7 \quad e_9 \quad e_{10}]\,\widehat{P}\,[e_2 \quad e_4 \quad e_1 - e_3 \quad he_1 - e_5 \quad e_8 \quad e_1 - e_7 \quad \tau e_1 - e_9]^{\mathrm{T}}\right\} + [e_1 \quad e_2]\,\widehat{Q}\,[e_1 \quad e_2]^{\mathrm{T}}\\
&- [e_3 \quad e_4]\,\widehat{Q}\,[e_3 \quad e_4]^{\mathrm{T}} + h^2 [e_1 \quad e_2]\,\widehat{D}\,[e_1 \quad e_2]^{\mathrm{T}} + [e_1 \quad e_2]\,\widehat{\overline{Q}}\,[e_1 \quad e_2]^{\mathrm{T}} - [e_7 \quad e_8]\,\widehat{\overline{Q}}\,[e_7 \quad e_8]^{\mathrm{T}} + \tau^2 [e_1 \quad e_2]\,\widehat{\overline{D}}\,[e_1 \quad e_2]^{\mathrm{T}},\\
\widehat{\Xi}_1 ={}& -[e_5 \quad e_1 - e_3]\,\widehat{D}\,[e_5 \quad e_1 - e_3]^{\mathrm{T}} - 3\left[\left(\frac{2}{h}\right)e_6 - e_5 \quad e_1 + e_3 - \left(\frac{2}{h}\right)e_5\right]\widehat{D}\left[\left(\frac{2}{h}\right)e_6 - e_5 \quad e_1 + e_3 - \left(\frac{2}{h}\right)e_5\right]^{\mathrm{T}},\\
\widehat{\Xi}_3 ={}& -[e_9 \quad e_1 - e_7]\,\widehat{\overline{D}}\,[e_9 \quad e_1 - e_7]^{\mathrm{T}} - 3\left[\left(\frac{2}{\tau}\right)e_{10} - e_9 \quad e_1 + e_7 - \left(\frac{2}{\tau}\right)e_9\right]\widehat{\overline{D}}\left[\left(\frac{2}{\tau}\right)e_{10} - e_9 \quad e_1 + e_7 - \left(\frac{2}{\tau}\right)e_9\right]^{\mathrm{T}},\\
\widehat{\Xi} ={}& \widehat{\Xi}_0 + \widehat{\Xi}_1 + \widehat{\Xi}_3,
\end{aligned}$$

$$\widehat{L} = a_1 e_1 + e_2 + a_2 e_3,$$
$$\widehat{\Gamma}(i,j) = \begin{bmatrix} A_i X & -X & A_{di} X & B_i M_j \end{bmatrix} \left[e_1, e_2, e_3, e_7 \right]^{\mathrm{T}}. \quad (41)$$

In addition, the gain matrix of state feedback controller can be described as

$$K_j = M_j X^{-1}. \quad (42)$$

4. Numerical Examples

In this section, we provide three numerical examples to illustrate the effectiveness of the stabilization criteria developed by this paper. The first examples show the improvement of our results. The second and third examples are used to demonstrate the effectiveness of the controller design.

Example 1. Consider a two-rule T-S fuzzy system (2). The system matrices are given as follows:

$$A_1 = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 1 & 0 \\ 1 & -2 \end{bmatrix},$$
$$B_1 = B_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad (43)$$
$$A_{d1} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},$$
$$A_{d2} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

In this example, only input delay is considered. When the parameters are defined as $a_1 = a_2 = 0.1$, the maximum input delay and the corresponding gain matrix of the system are obtained by Theorem 5. Meanwhile, the proposed method is compared with that in [42], as shown in Table 1. Clearly, the method of Theorem 5 is more superior than the existing results. Moreover, we cannot find feasible solution by Corollary 7. Therefore, the Lyapunov-Krasovskii functional with triple integral terms and Wirtinger-based double integral inequality are important to reduce the conservativeness of stability analysis for T-S fuzzy systems with state and input delays.

Example 2. Consider the following complex nonlinear continuous-time system studied in [43]:

$$\dot{x}_1(t) = x_1(t) + x_2(t) + \sin x_3(t) - 0.1 x_4(t) + \left(x_1^2(t) + 1 \right) u(t),$$
$$\dot{x}_2(t) = x_1(t) - 2 x_2(t),$$
$$\dot{x}_3(t) = x_1(t) + x_1^2(t) x_2(t) - 0.3 x_3(t),$$
$$\dot{x}_4(t) = \sin x_3(t) - x_4(t),$$
$$y_1(t) = x_2(t) + \left(x_1^2(t) + 1 \right) x_4(t),$$
$$y_2(t) = x_1(t), \quad (44)$$

where a and b are positive numbers, and assume $x_1(t) \in [-a, a]$, $x_3(t) \in [-b, b]$.

The nonlinear system is exactly represented by the following T-S fuzzy model.

Plant Rule 1. If $x_1(t)$ is M_1^1 and $x_3(t)$ is M_3^1, then

$$\dot{x}(t) = A_1 x(t) + B_1 u(t)$$
$$y(t) = C_1 x(t). \quad (45)$$

Plant Rule 2. If $x_1(t)$ is M_1^1 and $x_3(t)$ is M_3^2, then

$$\dot{x}(t) = A_2 x(t) + B_2 u(t)$$
$$y(t) = C_2 x(t). \quad (46)$$

Plant Rule 3. If $x_1(t)$ is M_1^2 and $x_3(t)$ is M_3^1, then

$$\dot{x}(t) = A_3 x(t) + B_3 u(t)$$
$$y(t) = C_3 x(t). \quad (47)$$

Plant Rule 4. If $x_1(t)$ is M_1^2 and $x_3(t)$ is M_3^2, then

$$\dot{x}(t) = A_4 x(t) + B_4 u(t)$$
$$y(t) = C_4 x(t), \quad (48)$$

where

$$A_1 = \begin{bmatrix} 1 & 1 & 1 & -0.1 \\ 1 & -2 & 0 & 0 \\ 1 & a^2 & -0.3 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix},$$

Table 1: Maximum allowable time delay and feedback gains.

Literature	τ	Feedback gains
[42]	0.5953	$K_1 = \begin{pmatrix}-0.9379 & -0.6779\end{pmatrix}$ $K_2 = \begin{pmatrix}-0.8903 & -0.2171\end{pmatrix}$
Theorem 5	0.9799	$K_1 = \begin{pmatrix}-0.0030 & -1.0151\end{pmatrix}$ $K_2 = \begin{pmatrix}-1.0094 & 0.0171\end{pmatrix}$

$$A_2 = \begin{bmatrix} 1 & 1 & \dfrac{(\sin b)}{b} & -0.1 \\ 1 & -2 & 0 & 0 \\ 1 & a^2 & -0.3 & 0 \\ 0 & 0 & \dfrac{(\sin b)}{b} & -1 \end{bmatrix},$$

$$A_3 = \begin{bmatrix} 1 & 1 & 1 & -0.1 \\ 1 & -2 & 0 & 0 \\ 1 & 0 & -0.3 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix},$$

$$A_4 = \begin{bmatrix} 1 & 1 & \dfrac{(\sin b)}{b} & -0.1 \\ 1 & -2 & 0 & 0 \\ 1 & 0 & -0.3 & 0 \\ 0 & 0 & \dfrac{(\sin b)}{b} & -1 \end{bmatrix},$$

$$B_1 = B_2 = \begin{bmatrix} 1 + a^2 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

$$B_3 = B_4 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}. \tag{49}$$

To verify the effectiveness of the proposed method, the other system matrices are given as

$$A_{d1} = A_{d2} = \begin{bmatrix} 0.1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$
$$A_{d3} = A_{d4} = \begin{bmatrix} 0.1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{50}$$

In particular, assuming $a = 1.4$ and $b = 0.7$ and setting parameters $a_1 = 1$ and $a_2 = 0.01$, we can get the maximum allowable delays of input and state as $h_{\max} = 68$ and $\tau_{\max} = 0.277$, respectively. However, we cannot find feasible solution by Corollary 7. According to Theorem 5, taking $h = 10$ and $\tau = 0.27$, we have the following feedback gains:

$$\begin{aligned} K_1 &= \begin{bmatrix}-0.7593 & -0.3889 & -0.3436 & 0.0275\end{bmatrix}, \\ K_2 &= \begin{bmatrix}-0.7391 & -0.3767 & -0.3174 & 0.0265\end{bmatrix}, \\ K_3 &= \begin{bmatrix}-1.9484 & -0.5514 & -0.8764 & 0.0701\end{bmatrix}, \\ K_4 &= \begin{bmatrix}-1.9354 & -0.5713 & -0.8440 & 0.0692\end{bmatrix}. \end{aligned} \tag{51}$$

For simulation, we choose

$$\begin{aligned} \lambda_1(\theta(t)) &= M_1^1 M_3^1, \\ \lambda_2(\theta(t)) &= M_1^1 M_3^2, \\ \lambda_3(\theta(t)) &= M_1^2 M_3^1, \\ \lambda_4(\theta(t)) &= M_1^2 M_3^2, \end{aligned} \tag{52}$$

where the premise membership functions are as follows:

$$\begin{aligned} M_1^1 &= \frac{x_1^2}{a^2}, \\ M_1^2 &= 1 - M_1^1, \\ M_3^1 &= \begin{cases} \dfrac{b \sin x_3 - x_3 \sin b}{x_3 (b - \sin b)}, & x_3 \neq 0 \\ 1, & x_3 = 0, \end{cases} \\ M_3^2 &= 1 - M_3^1. \end{aligned} \tag{53}$$

Suppose the initial condition $\varphi^{\mathrm{T}}(t) = [-1.2 \;\; 0.5 \;\; 0.7 \;\; -0.6]^{\mathrm{T}}$, $h = 10$, and $\tau = 0.27$; the state responses of the closed-loop control system are shown in Figure 1. From Figure 1, the proposed state feedback controller can stabilize the original nonlinear system with input and state delays.

Example 3. Consider the following T-S fuzzy system with two rules [39]. The system matrix parameters are given as follows:

$$A_1 = \begin{bmatrix} -a\dfrac{v\bar{t}}{Lt_0} & 0 & 0 \\ a\dfrac{v\bar{t}}{Lt_0} & 0 & 0 \\ -a\dfrac{v^2\bar{t}^2}{2Lt_0} & \dfrac{v\bar{t}}{t_0} & 0 \end{bmatrix},$$

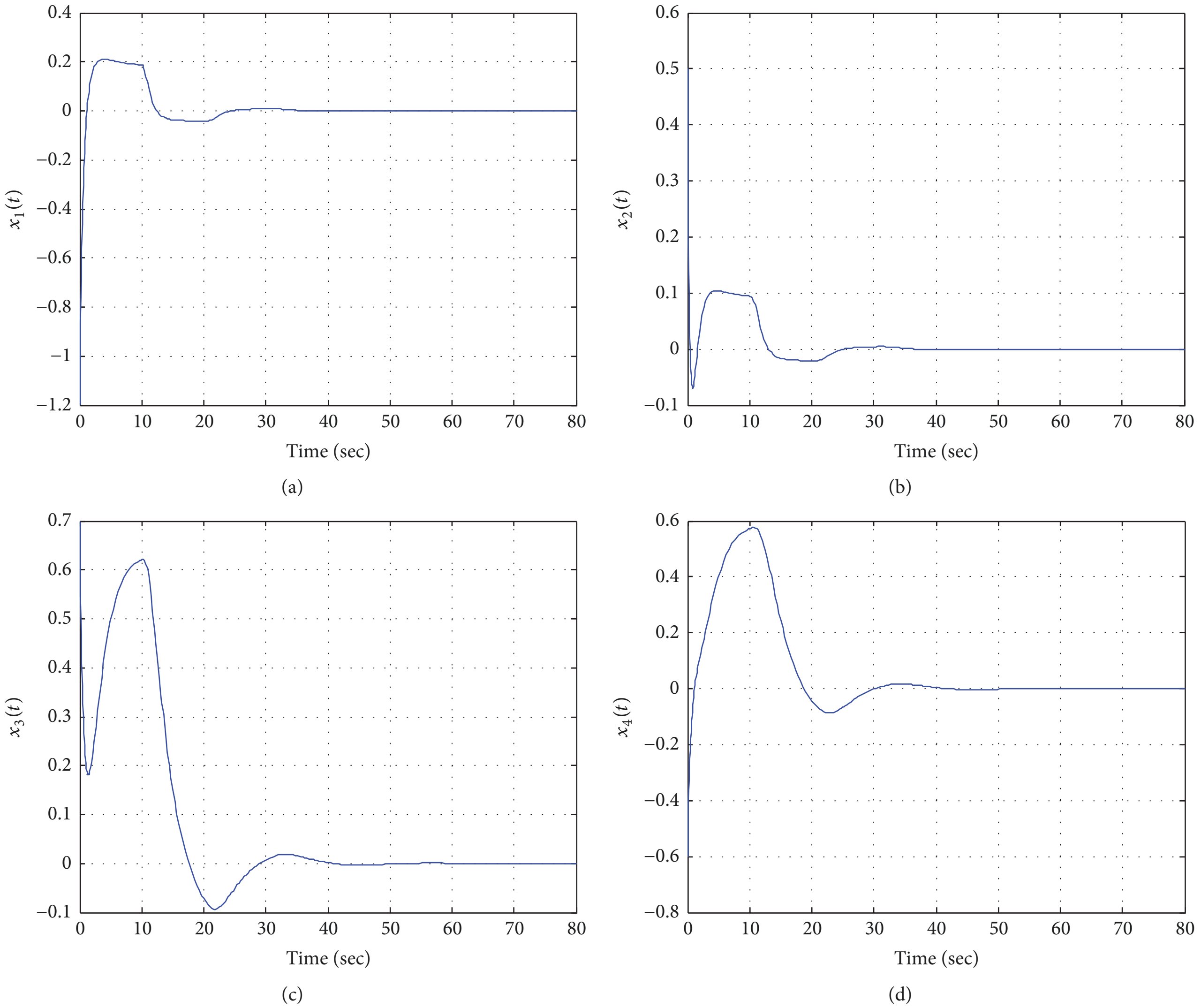


FIGURE 1: State responses of the closed-loop system for Example 2.

$$A_2 = \begin{bmatrix} -a\dfrac{v\bar{t}}{Lt_0} & 0 & 0 \\ a\dfrac{v\bar{t}}{Lt_0} & 0 & 0 \\ -ad\dfrac{v^2\bar{t}^2}{2Lt_0} & \dfrac{dv\bar{t}}{t_0} & 0 \end{bmatrix},$$

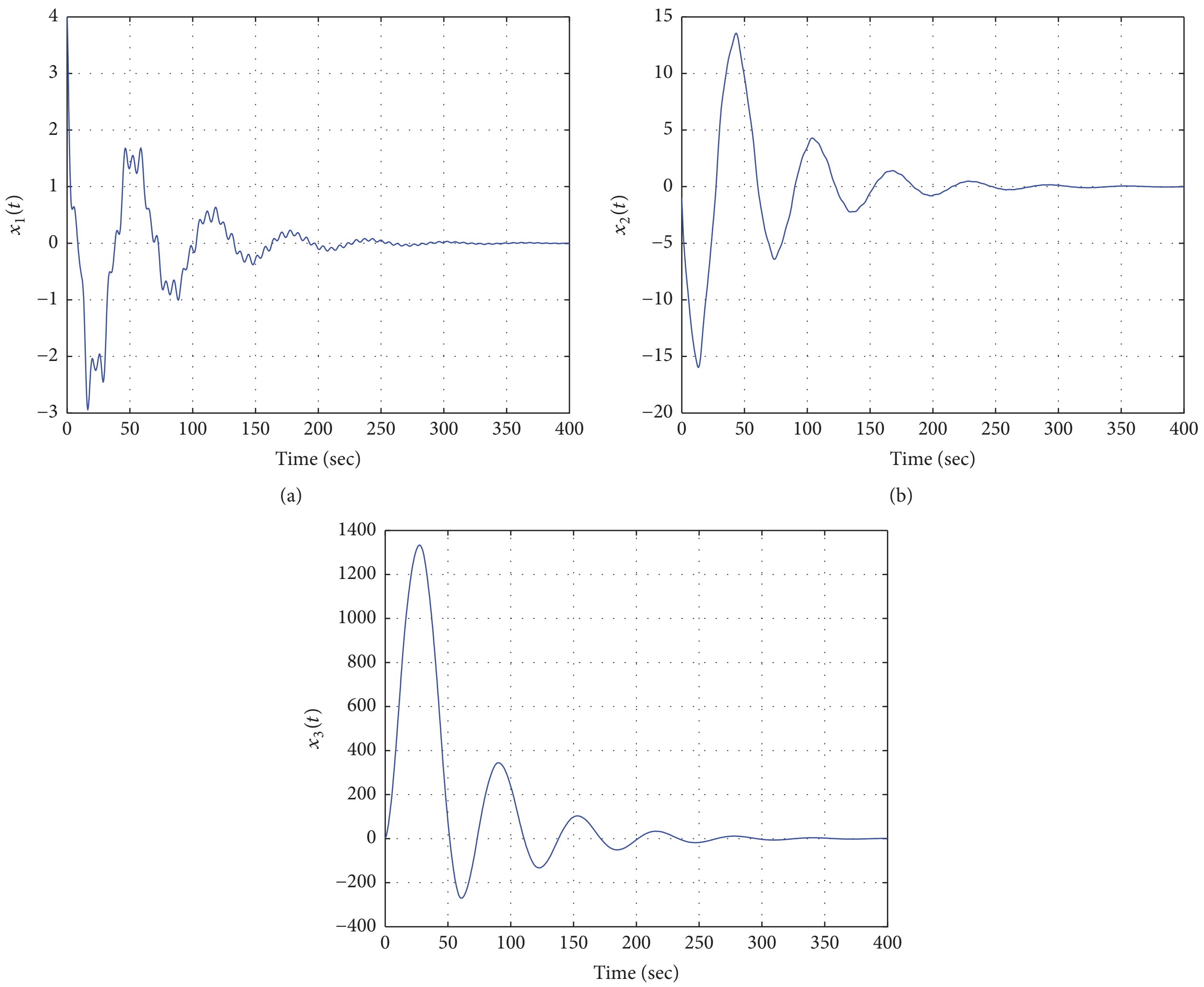
$$B_1 = \begin{bmatrix} \dfrac{v\bar{t}}{lt_0} \\ 0 \\ 0 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} \dfrac{v\bar{t}}{lt_0} \\ 0 \\ 0 \end{bmatrix},$$

$$A_{d1} = \begin{bmatrix} -(1-a)\dfrac{v\bar{t}}{Lt_0} & 0 & 0 \\ (1-a)\dfrac{v\bar{t}}{Lt_0} & 0 & 0 \\ (1-a)\dfrac{v^2\bar{t}^2}{2Lt_0} & 0 & 0 \end{bmatrix},$$

$$A_{d2} = \begin{bmatrix} -(1-a)\dfrac{v\bar{t}}{Lt_0} & 0 & 0 \\ (1-a)\dfrac{v\bar{t}}{Lt_0} & 0 & 0 \\ (1-a)\dfrac{dv^2\bar{t}^2}{2Lt_0} & 0 & 0 \end{bmatrix}, \quad (54)$$

where $l = 2.8$, $L = 5.5$, $v = -1.0$, $\bar{t} = 2.0$, $t_0 = 0.5$, $d = 10t_0/\pi$, $a = 0.7$.



(a)

(b)

(c)

Figure 2: State responses of the closed-loop system for Example 3.

This example is taken from [39], which describes a T-S fuzzy system with state and input delay. In the following analysis, we give the comparison between our method and that proposed by [39]. By the method given in [39], the maximum allowable upper-bound of the time-delays is computed as $h = \tau = 0.167$.

Then, setting parameters $a_1 = 1$ and $a_2 = 0.01$, according to Theorem 5 in our paper, the maximum allowable upper-bounds of input and state delays are, respectively, computed as $h_{\max} = 12.7$ and $\tau_{\max} = 0.925$. Thus, the proposed method obtains less conservative results than that in [39].

Now, for simulation, we set the premise membership function as follows:

$$\begin{aligned} \lambda_1(\theta(t)) &= \frac{1}{1+\exp(x_1+0.5)}, \\ \lambda_2(\theta(t)) &= 1-\lambda_1(\theta(t)). \end{aligned} \tag{55}$$

Taking $h = 12$ and $\tau = 0.85$, and applying Theorem 5, we can obtain the feedback gain matrices as follows:

$$\begin{aligned} K_1 &= \begin{bmatrix} 0.6571 & -0.0459 & 0.0002 \end{bmatrix}, \\ K_2 &= \begin{bmatrix} 0.6576 & -0.0469 & 0.0002 \end{bmatrix}. \end{aligned} \tag{56}$$

Figure 2 shows the state responses of the closed-loop system under the initial condition $\varphi^{\mathrm{T}}(t) = \begin{bmatrix} 4 & -1 & 2 \end{bmatrix}^{\mathrm{T}}$. From Figure 2, it can be seen that the proposed state feedback controller can stabilize the original T-S fuzzy system with input and state delays.

5. Conclusion

The stabilization problem of nonlinear systems with state and input delays is investigated via T-S fuzzy model in this paper. By choosing an appropriate Lyapunov functional, new delay-dependent stabilization criteria are established. Using Wirtinger-based double integral inequality, the proposed stabilization conditions are presented in the form of LMI. The largest allowable input and state delays calculated by the proposed conditions are obviously better than the existing results. Several examples are provided to demonstrate the effectiveness and superiority of the proposed method.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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