Research Article

Contractual Efficiency of PPP Infrastructure Projects: An Incomplete Contract Model

Lei Shi 1, Lu Zhang 1, Masamitsu Onishi 2, Kiyoshi Kobayashi 3, and Dashuang Dai 4

1 Department of Construction Management, Dalian University of Technology, Dalian 116024, China
2 Disaster Prevention Research Institute, Kyoto University, Kyoto 611-0011, Japan
3 Graduate School of Management, Kyoto University, Kyoto 606-8501, Japan
4 Faculty of Management and Economics, Dalian University of Technology, Dalian 116024, China

Correspondence should be addressed to Lei Shi; leishi@dlut.edu.cn

Received 19 June 2017; Accepted 18 February 2018; Published 30 April 2018

Academic Editor: Ibrahim Zeid

Copyright © 2018 Lei Shi et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

This study analyses the contractual efficiency of public-private partnership (PPP) infrastructure projects, with a focus on two financial aspects: the nonrecourse principal and incompleteness of debt contracts. The nonrecourse principal releases the sponsoring companies from the debt contract when the special purpose vehicle (SPV) established by the sponsoring companies falls into default. Consequently, all obligations under the debt contract are limited to the liability of the SPV following its default. Because the debt contract is incomplete, a renegotiation of an additional loan between the bank and the SPV might occur to enable project continuation or liquidation, which in turn influences the SPV’s ex ante strategies (moral hazard). Considering these two financial features of PPP infrastructure projects, this study develops an incomplete contract model to investigate how the renegotiation triggers ex ante moral hazard and ex post inefficient liquidation. We derive equilibrium strategies under service fees endogenously determined via bidding and examine the effect of equilibrium strategies on contractual efficiency. Finally, we propose an optimal combination of a performance guarantee, the government’s termination right, and a service fee to improve the contractual efficiency of PPP infrastructure projects.

1. Introduction

Public-private partnerships (PPPs) are innovative arrangements enabling the procurement of infrastructure services by governments with private participation. Pioneered in the United Kingdom through the private finance initiative in 1992 [1], PPP arrangements have since been adopted in countries with various levels of wealth on all continents [2–4]. Despite their popularity, PPP infrastructure projects have produced diverse results. On the one hand, many projects in a broad range of sectors have been successfully developed using PPPs, with significantly improved efficiency [5–7]. On the other hand, various problems have been encountered in PPP infrastructure development projects worldwide [8, 9]. For example, PPPs have resulted in high contracting costs, which reduce project efficiency. There is also evidence suggesting that a bilateral monopoly relationship between the government and private companies creates hold-up opportunities, which can lead the projects to be either abandoned or taken in-house [10]. This point suggests that incentives for private companies to enhance contractual efficiencies are important in PPP infrastructure projects.

A PPP infrastructure project usually involves a large upfront investment. A special purpose vehicle (SPV) set up by the sponsoring companies raises project funds primarily via project financing, which includes features such as the nonrecourse principal and incompleteness of debt contracts. The nonrecourse principal releases the sponsoring companies from the debt contract in the event that the SPV falls into bankruptcy. Consequently, obligations under the debt contract after an SPV’s bankruptcy are limited to the liability of the SPV. In addition, debt contracts in PPP projects are incomplete contracts [11]. As Hart and Moore [12] noted, in relation to incompleteness of debt contracts, considerable
importance has been attached to the question of whether contract efficiency can be achieved when considering default risks. The focus on incomplete debt contracts emphasizes the fact that renegotiation is an important mechanism for the allocation of control rights across states [13, 14], which thus creates incentives for borrowers to strive to avoid adverse states [15].

This study aims to analyse the effects of renegotiation between the SPV and the bank on ex ante moral hazard and ex post inefficient liquidation by considering the nonrecourse principal and incompleteness of debt contracts features. Assume a situation in which the SPV falls into default because of cost overruns during a PPP project. A renegotiation with the bank will be necessary if the SPV is to obtain an additional loan. On the one hand, the option to renegotiate might be harmful to ex post efficiency. In general, if the PPP infrastructure project has large external benefits, a social loss will occur if the bank, which makes decisions based on the future revenues and risks of the project, refuses to provide financial support. On the other hand, the renegotiation might also worsen the ex ante asset substitution problem [16]. In PPP projects, asset substitution, which is a type of moral hazard of the SPV, refers to a situation in which the SPV prefers a risky project strategy (i.e., a strategy that leads to lower operating costs in the good states but higher costs in the bad states) to a safe strategy because of limited liability and contractual incompleteness. Suppose there is an adverse state in which the bank believes that the project is worth more if it continues as a going concern in the hands of the current SPV. The SPV can renegotiate an additional loan, and a debt forgiveness agreement is likely to be reached because of the nonrecourse principal and the limited liability of the SPV. Hence, the SPV can capture some of the going-concern surplus, and its payoff in the insolvency state will be nonnegative. Thus, the incentive for the SPV to adopt a suboptimal project strategy increases.

In particular, we build an incomplete contract model for a PPP project developing infrastructure services that are paid for by the government as a three-stage incomplete contract model. In the basic model, given the exogenous service fee, we investigate how the renegotiation between the SPV and the bank, which occurs after cost overruns have placed the project at risk, affects the SPV’s choice of strategy (moral hazard) before it falls into default and possible project liquidation after it falls into default. Moreover, we consider the case in which the service fee is endogenously determined by competitive bidding. The basic model is then extended by introducing a performance guarantee into the concession contract and termination rights implemented by the government when the SPV falls into bankruptcy. In most concession contracts in relation to PPP projects, the government asks the SPV to deposit an amount of money as a performance guarantee to safeguard against project risk [17]. If the SPV fails to fulfil its contractual responsibilities because of bankruptcy, the government has the right to terminate the concession contract and confiscate the performance guarantee. Thus, the performance guarantee is viewed as a hedge by governments against the possible bankruptcy of the project [18]. However, the effect of a performance guarantee on the ex ante moral hazard of the SPV has not been investigated sufficiently. We examine how the combination of termination right and a performance guarantee improves both the ex ante and ex post efficiency of PPP projects.

In the PPP finance literature, the focus thus far has been on the trade-off between public and private financing [19]. The benefits of private financing are explained as lowering the shadow cost of public financing [20], taking full advantage of lender knowledge and expertise to evaluate project risks [21] and improve incentives [22], and efficient termination of bad projects [23], whereas the costs of private financing include higher interest rates, exacerbated moral hazard by introducing further risk-sharing [22], and loss of the consumer surplus that the higher prices set by private companies generate [20]. Our contribution here is that we focus on the two essential features of private financing, namely, the limited recourse principal and incompleteness of debt contracts. We present an incomplete contract model to examine the causes of ex ante moral hazard of the SPV and ex post project liquidation.

The remainder of this study is organized as follows. Section 2 presents related literature about the financial features of PPP infrastructure projects and sources of moral hazard behaviours of the SPV. Section 3 formalizes the PPP project contract scheme, which includes a debt contract and a concession contract, as an incomplete contract model. Section 4 analyses the mechanism for determining concession prices under competitive bidding systems. Section 5 investigates the effect of a performance guarantee on the efficiencies of PPP projects. Section 6 presents our conclusions and outlines issues to be examined in future studies. The Appendix provides proofs of our conclusions.

2. Literature Review

2.1. Financial Features of PPP Infrastructure Projects. In a typical PPP infrastructure project, all financing is run through the SPV created for the sole purpose of developing the project. This firm is managed by the sponsors, who are equity investors responsible for the construction and operation of the project. Because the typical PPP infrastructure project involves a large initial investment, the equity usually cannot cover the total investment; thus, the sponsors must raise a large amount of funding via debt. This arrangement leaves the sponsors highly leveraged, typically with banks providing 70 percent to 90 percent of their funds. This financial arrangement for a PPP infrastructure project has two important features. First, the sponsors provide no guarantees beyond the right to be paid from the cash flows of the project [24]. When the SPV falls into default, the banks will recover the debt only from the revenue of the project [25, 26]. Moreover, the SPV’s payoff in the default state will be nonnegative from the limited liability. This feature therefore leaves an opportunity for risk-taking behaviour by the SPV. In other words, the SPV might prefer a high-risk, high-return project strategy; that is, moral hazard behaviour.

Second, the contractual term of a PPP infrastructure project is usually approximately 20–30 years [27]. At the contract negotiation stage, the bank and the SPV cannot
2.2. Contractual Efficiency and Moral Hazard of the SPV. In a PPP infrastructure project, the government is concerned with both the social and financial efficiency of the project contract. Social efficiency refers to the project being implemented efficiently; for example, the SPV chooses an optimal project strategy to reduce project cost and enhance quality. We analyse this contractual efficiency from two aspects: ex ante efficiency and ex post efficiency. The former is affected by SPV’s moral hazard behaviour. The latter is related to the continuation or liquidation of the project after the cost overrun occurs. Because the PPP infrastructure project is characterized by huge externality, project liquidation will cause a loss of ex post contractual efficiency. Conversely, financial efficiency refers to the minimum expenditure paid by the government for purchasing the infrastructure services. In this study, we focus on the PPP project developing infrastructure services that are paid for by the government. How to achieve the best project value with minimum money, that is, achieve the value for money (VFM) of the PPP project, is the primary goal of the government.

The causes of a moral hazard of the SPV can be divided into two aspects: internal causes and external causes. The internal causes come from the precondition that the SPV is a rational economic actor and behaves opportunistically [30]. If the objectives of the government and the SPV coincide, the SPV is expected to reach decisions that maximize the government’s interests because such decisions will also maximize its own interests. However, because the SPV’s main objective is to maximize its profit obtained from the project, which diverges from that of the government, the SPV might behave opportunistically regardless of the government’s interest [31].

The external causes are related to asymmetric information, contractual incompleteness, and inadequate monitoring. First, neither the government nor the bank can observe the strategy or the behaviour of the SPV after signing the contract. In other words, information about the strategy and the behaviour is private information of the SPV [32]. It is difficult for the government or the bank to judge or verify the real effort level of the SPV. Thus, the SPV can take advantage of this asymmetrical information to maximize its interests at a cost to the government, for example, can choose the risky strategy or shirk in the construction or operation stage. This moral hazard behaviour of the SPV might further make the cost overrun occur more easily, which triggers the liquidation or termination of the project.

Second, the cost overrun caused by the moral hazard of the SPV cannot be written into the PPP contract [25], which is therefore an incomplete contract. In practice, the design of PPP contracts is often affected by the challenge of including the “appropriate” level of flexibility—with too much flexibility, a moral hazard of the SPV is likely to occur; with too little flexibility, opportunities for welfare-enhancing renegotiations will be lost [33]. When the cost overrun occurs, the SPV must renegotiate with the bank to procure additional funding to enable the continuation of the project. The renegotiation not only has an effect on the ex post efficiency of the project but also influences the ex ante moral hazard of the SPV. If the renegotiation causes a large loss to the SPV, that loss will be effective for deterring the moral hazard of the SPV. However, the nonrecourse principal and limited liability of the SPV lead the renegotiation to impose little negative effect on the SPV’s payoff. Consequently, it will increase the possibility of a moral hazard of the SPV.

Finally, during the contractual term of a PPP, the SPV has a large measure of freedom to manage the infrastructure [6]. The government pays the predetermined service fee to buy the infrastructure services provided by the SPV. The government expects the SPV to use its professional expertise to improve the efficiency of the project, but it is difficult for the government to monitor or supervise the behaviour of the SPV. This insufficient monitoring or supervision is also an external cause that can trigger a moral hazard problem of the SPV. Under insufficient monitoring, the SPV who aims to maximize its own interests will prefer to choose risky project strategy or lower effort level.

In summary, the internal and external causes of the moral hazard problem suggest that the government should provide the SPV with proper incentives to improve contractual efficiency. Many studies lying fully in the realm of incomplete contract literature have focussed on the effects of allocation of property rights to the project efficiency [34, 35]. Their findings showed that the optimal allocation of property rights, depending upon the characteristics of tasks involved in PPP projects, will create proper incentives for the private company to improve project efficiency [36, 37]. Complementing these studies, this study focusses on incompleteness of debt contracts of PPP projects and proposes an optimal combination of performance guarantee, termination right, and service fee determined via competitive bidding to deter the moral hazard of the SPV.

3. Basic Model

3.1. Assumptions. To examine the effects of incompleteness of the debt contract on project efficiencies, we assume that the SPV has no equity in the investment and raises all funds by means of a debt contract. The only role of the government in the basic model is to purchase the infrastructure services provided by the SPV. The PPP project is interpreted as a three-period game, as shown in Figure 1. At date \( t = 0 \), the SPV is selected by a tender. Then, a concession contract is
concluded between the government and the SPV, and a debt contract is concluded between the bank and the SPV. After choosing the project strategy (e.g., construction method), the SPV begins the construction of the facility. We consider the case in which project risks exist (e.g., construction risk). The project costs are determined at date \( t = 1 \). In this stage, whether the project is continued is determined under the initial debt contract. When it is impossible to continue the project, a renegotiation in which the bank decides whether to modify the initial contract and provide an additional loan to the SPV is held between the SPV and the bank. If no additional loan is provided by the bank, the SPV cannot continue the project, and control rights are transferred to the bank. The bank then decides whether to continue or liquidate the project. The operation begins at date \( t = 2 \). If the project continues, the government pays the SPV the service fee. The SPV then repays the debt, and the project terminates at \( t = 2 \).

After signing the concession contract with the government, the SPV enters into a debt contract and starts the project at time \( t = 0 \). The initial investment \( I \) and repayment \( D \) are described in the debt contract. Because the repayment \( D \) includes not only the principal but also the risk premium of the project, \( D > I \) holds. The SPV then chooses the project strategy, which is classified into two types, denoted by \( e_i \) (i = s, d). Project strategy \( e_i \) represents the safe strategy, whereas \( e_d \) represents the risky one.

The project costs determined at \( t = 1 \) are denoted by the function \( C(e_i, \omega_j) \) (i = s, d; j = 1, 2, 3), which depends upon both the project strategy \( e_i \) (i = s, d) and the state variable \( \omega_j \) (j = 1, 2, 3) realized with positive probabilities \( p_j = \text{Prob}[\omega = \omega_j] > 0 \). The project cost is uncertain and is thus referred to as the cost risk. We assume that the cost risk is known to both the SPV and the bank. However, the realization of \( \omega_j \) (j = 1, 2, 3) is observable but not verifiable to outsiders. Thus, the situations contingent on the cost risk cannot be described in the debt contract. Therefore, the debt contract is an incomplete contract. Suppose that the financial market is perfectly competitive, and both the SPV and the bank are risk neutral. Assume that

\[
C(e_d, \omega_1) = 0
\]

\[
C(e_s, \omega_1) = C(e_s, \omega_2) = C_1 
\]

\[
C(e_d, \omega_2) = C(e_d, \omega_3) = C(s, \omega_3) = C_2
\]

\( C_2 > C_1 > 0 \). (2)

That is, the project cost associated with the risky project strategy is lower than that associated with the safe one when the state variable \( \omega_1 \) is realized. The bank cannot observe the project strategy chosen by the SPV, but it can observe the costs. Given (1), if the state variable \( \omega_1 \) is realized, the project cost is determined as \( C_2 \), which is independent of the project strategy. Thus, the bank fails to identify the type of project strategy chosen by the SPV; that is, the information concerning the project strategy is private information held by the SPV.

When the project proceeds, the SPV earns the service fee paid by the government, denoted by \( R \), at \( t = 2 \). We assume that the service fee is determined exogenously in the basic game. Furthermore, we assume that

\[
C_2 > R \geq 0. \tag{3}
\]

If \( R \geq C_2 \) holds, the SPV earns nonnegative payoffs independent of the project strategy employed. Assumption (3) implies that the government intends to maximize the VFM of the project. Suppose that the repayment occurs after the SPV is paid. The discount rate is assumed zero.

After the project cost occurs at \( t = 1 \), a renegotiation can occur between the SPV and the bank. At the beginning of \( t = 1 \), all of the borrowing \( I = C_1 \) is invested into the project. When the cost becomes \( C_2 \), the SPV cannot continue the project unless the bank provides an additional loan. Thus, the SPV will propose a renegotiation with the bank. If the bank decides to provide the additional loan, a new debt contract will be written, and the project is continued by the SPV. However, the project will be liquidated if the bank refuses to provide further financial support. At this point, the bank will suffer a loss equal to the initial investment \( C_1 \).

The last assumption states that the safe project strategy is socially optimal; namely, \( (p_s + p_d)(R - C_1) + p_2(R - C_2) \geq p_s R + (p_2 + p_3)(R - C_2) \), which is simplified as

\[
\Lambda = p_2C_2 - (p_1 + p_2)C_1 \geq 0, \tag{4}
\]

where \( \Lambda \) represents the social cost of a moral hazard.

3.2. Equilibrium Solutions of the Basic Model. The basic model is a game with complete information, whereas the information on project strategy is asymmetric between the SPV and the bank. We solve the subgame-perfect equilibria by backward induction. First, let us focus on the subgame wherein the bank and the SPV renegotiate whether to continue the project and derive two lemmas.

**Lemma 1.**

If

\[
R + I - C(e_i, \omega_j) \geq D \quad (5a)
\]

\[
C(e_i, \omega_j) \leq I \quad (5b)
\]

holds, the renegotiation never occurs.

Lemma 1 shows that if the SPV's profit is nonnegative and the project cost does not cover the initial investment amount, the SPV should complete the project under the initial debt contract and make repayments. If (5a) holds, any renegotiation request from the SPV should be rejected because the bank knows that the SPV can fully repay the contract amount. However, (5a) or (5b) might not hold contingent on the realization of the state variable \( \omega_1 \). In this case, the project cannot proceed unless the bank extinguishes part of the debt and provides an additional loan.

**Lemma 2.** In the renegotiation, the bank provides an additional loan if and only if

\[
R \geq C_2 - C_1 \tag{6}
\]

holds.
When the project cost is determined as \( C_2 \), the SPV needs an additional loan to continue the project. Suppose that the bank has all of the bargaining power in the renegotiation. Then, the new repayment \( D' \) is written as
\[
D' = R. \tag{7}
\]
The left-hand side of (7) represents the payoff earned by the bank, and the right-hand side represents the additional loan required. Lemma 2 shows that the bank can recover part of the debt by continuing the project. If (6) is satisfied, the bank will provide an additional loan. Note that the initial loan \( I \) is a sunk cost; thus, it is not considered in the renegotiation. Conversely, if (6) is not satisfied, the bank refuses to provide an additional loan, which results in the liquidation of the project.

Different subgame-perfect equilibria are available depending upon the amount of the service fee. Thus, two scenarios arise:

Scenario 1: \( R \geq C_2 - C_1 \).

Scenario 2: \( R < C_2 - C_1 \).

In Scenario 1, the SPV will continue the project to date 2 independent of cost risk. In Scenario 2, the project will be liquidated when the project cost is determined as \( C_2 \).

Scenario 1. First, consider the case in which the SPV chooses the safe project strategy \( e_s \) at \( t = 0 \). When the state variables \( \omega_1, \omega_2 \) are realized, the project costs become \( C(e_s, \omega_1) = C(e_s, \omega_2) = C_1 \). The SPV can complete the project without any additional loan from the bank. Given the repayment \( D \), the payoffs earned by the SPV are \( \pi^{spv}(\omega_1) = \pi^{spv}(\omega_2) = R + I - C_1 - D \). However, when state variable \( \omega_3 \) is realized, an additional loan equal to \( C_2 - C_1 \) is necessary for the project to continue. From Lemma 2, the bank chooses to provide an additional loan and captures the entire service fee \( R \), which is paid at date 2. Thus, after \( \omega_3 \) is realized, the SPV obtains the payoff written as \( \pi^{spv}_e(\omega_3) = 0 \). Correspondingly, the bank's payoffs are represented by \( \pi^{bank}_e(\omega_1) = \pi^{bank}_e(\omega_2) = D - I > 0 \) and \( \pi^{bank}_e(\omega_3) = R - C_2 < 0 \).

In the case in which the SPV chooses the risky project strategy \( e_r \), a lack of funds is realized for the state variable \( \omega_2 \) or \( \omega_5 \). The payoffs of the SPV and the bank are represented by \( \pi^{spv}_e(\omega_1) = R + I - D, \pi^{spv}_e(\omega_2) = \pi^{spv}_e(\omega_3) = 0, \pi^{bank}_e(\omega_1) = D - I, \pi^{bank}_e(\omega_2) = \pi^{bank}_e(\omega_3) = R - C_2 \), respectively.

Then, consider how the SPV chooses the project strategy at \( t = 0 \). Given the amount of repayment \( D \), the SPV's expected payoffs are written as \( \Pi^{spv}_{e_s}(D) = (p_1 + p_2)(R + I - C_1 - D) \) and \( \Pi^{spv}_{e_r}(D) = p_1(R + I - D) \). The conditions that ensure that the SPV chooses the safe project strategy are written as
\[
\Pi^{spv}_{e_s}(D) - \Pi^{spv}_{e_r}(D) = p_2(R + I - C_1 - D) - p_1C_1 \geq 0 \tag{8a}
\]
\[
R \geq C_1 + D - I \tag{8b}
\]
\[
R \geq C_2 - C_1. \tag{8c}
\]
Condition (8a) is the incentive-compatible constraint, and condition (8b) is the participation constraint corresponding to the safe project strategy. In contrast, the conditions for choosing the risky project strategy are
\[
\Pi^{spv}_{e_r}(D) - \Pi^{spv}_{e_s}(D) = p_2(R + I - C_1 - D) - p_1C_1 < 0 \tag{9a}
\]
\[
R \geq D - I \tag{9b}
\]
\[
R \geq C_2 - C_1. \tag{9c}
\]

Before the SPV chooses the project strategy, the bank determines the amount of repayment \( D \) at date 0. With respect to the project strategies chosen by the SPV, the bank's expected payoffs are \( \Pi^{bank}_{e_s}(D) = (p_1 + p_2)(R + I - D) + p_3(R - C_2) \) and \( \Pi^{bank}_{e_r}(D) = p_1(R + I - D) + p_3(R - C_2) \). Suppose that the financial market is perfectly competitive. Then, the repayments necessary for the bank to break even are \( D_s = I + p_3(C_2 - R)/(1 - p_2) \) and \( D_d = I + (1 - p_1)(C_2 - R)/p_1 \). The second terms denoted in \( D_s \) and \( D_d \) represent the premiums for undertaking the risk of debt forgiveness.

Introducing \( D_s \) and \( D_d \) into (8a)–(8c) and (9a)–(9c), respectively, we obtain the following equilibrium solutions (the proof is included in Appendix B).

Equilibrium A: \( D^* = D_s, \ e^* = e_s \),
\[
\text{if } (C_2 > R) \max \{v_1, v_2\} \tag{10}
\]

Equilibrium B: \( D^* = D_d, \ e^* = e_d \),
\[
\text{if } v_1 > R \max \{v_2, v_3\}. \tag{10}
\]

Define \( v_1 = ((1 - p_1)^2)/p_2C_1 + p_3C_2, \ v_2 = C_2 - C_1, \) and \( v_3 = (1 - p_1)C_2 \).

Scenario 2. From Lemma 2, the bank's decision-making in the renegotiation differs between Scenarios 2 and 1 when the project cost is determined as \( C_2 \).

When the project cost is determined as \( C_2 \), the bank will refuse to provide an additional loan, and the project will be liquidated. Because of the limited liability of the SPV, its payoff is 0, even when the project is liquidated. Thus, the SPV obtains the payoffs shown in Scenario 1. Given the repayment \( D' \), the expected payoffs obtained by the SPV are \( \Pi^{spv}_{e_s}(D') = (p_1 + p_2)(R + I - C_1 - D') \) and \( \Pi^{spv}_{e_r}(D') = p_1(R + I - D') \).

However, the bank's payoffs differ from those shown in Scenario 1. When the project cost is determined as \( C_2 \), the bank chooses to liquidate the project and obtains the payoff represented by \( -I \). Thus, given the project strategy chosen by the SPV, the break-even repayments are written as \( D'_s = I/(1 - p_3) \) and \( D'_d = I/p_1 \).
Finally, we derive the following equilibrium solutions, which are summarized in Table 1 (the proof is provided in Appendix B).

**equilibrium C:** \( D^* = D_{tr}, \quad e^* = e_i, \) if \( v_2 > R \geq v_4 \)

**equilibrium D:**  
\[ D^* = D_{tr}, \quad e^* = e_{dt}, \]
\[
\text{if } \min \{v_2, v_4\} > R \geq v_3.
\]

Define \( v_4 = \frac{1}{(1 - p_1)} + p_1/p_2)C_1 \) and \( v_5 = (1 - p_1)/p_2)C_1 \).

<table>
<thead>
<tr>
<th>Equilibria</th>
<th>Establishing conditions</th>
<th>( e^* )</th>
<th>( \Pi_{spv}^\omega )</th>
<th>( \omega )</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>( R \geq \max{v_1, v_2} )</td>
<td>( e_i )</td>
<td>( R - (1 - p_3)C_1 - p_3C_2 )</td>
<td>( \omega_1 )</td>
<td>( \bigcirc )</td>
</tr>
<tr>
<td>B</td>
<td>( v_1 &gt; R \geq \max{v_2, v_3} )</td>
<td>( e_d )</td>
<td>( R - (1 - p_3)C_2 )</td>
<td>( \omega_2 )</td>
<td>( \Delta )</td>
</tr>
<tr>
<td>C</td>
<td>( v_2 &gt; R \geq v_4 )</td>
<td>( e_i )</td>
<td>( (1 - p_3)R - C_1 )</td>
<td>( \omega_3 )</td>
<td>( \bigcirc )</td>
</tr>
<tr>
<td>D</td>
<td>( \min{v_2, v_4} &gt; R \geq v_3 )</td>
<td>( e_d )</td>
<td>( p_1R - (1 - p_3)C_1 )</td>
<td>( \omega_4 )</td>
<td>( \Delta )</td>
</tr>
</tbody>
</table>

**Note.** The establishing condition column shows the establishing conditions of the relevant equilibrium solutions. \( \Pi_{spv}^\omega \) shows the expected payoffs earned by the SPV under equilibrium strategy \( e^* \). The \( \bigcirc \) sign in the default column implies that the debt contract is not modified; the \( \Delta \) sign means that the bank extinguishes part of the debt and the project continues. The \( \times \) sign means that the SPV is liquidated. Define \( v_1 = (1 - p_3)/(p_2)C_1 + p_2C_2, v_2 = C_2 - C_1, v_3 = (1 - p_1)/p_1)C_1, v_4 = (1/(1 - p_3) + p_1/p_2)C_1, \) and \( v_5 = ((1 - p_1)/p_1)C_1 \).

### 4. Bidding Systems and Equilibrium Solutions

#### 4.1. Bidding Systems

In the basic model, the equilibrium solutions are derived by assuming that the service fee \( R \) is an exogenous variable. However, in practice, the service fee is determined by bidding before the concession contract is signed. This study assumes that a bidding process concerning the concession contract is implemented and that the bidder offering the lowest price is selected as the SPV at the beginning of a PPP project. For example, assume \( n \) firms attend the bid, and the corresponding prices submitted by each firm are denoted by \( p_i (i = 1, 2, \ldots, n) \); then the price submitted by winning bidder firm \( j \) must satisfy \( p_j = \min\{p_1, p_2, \ldots, p_n\} \). In many real-world PPP projects, the winning bid is selected following a comprehensive evaluation of the proposals presented by all bidders. However, the cost of the comprehensive evaluation, which is represented by a trade-off between the evaluation items and the service fee, is generally measured in monetary terms. Suppose that perfect competition is observed during the bidding process. In the real world, there are significant transaction costs, for instance, the costs of participating in the bidding process. Thus, in many cases, the number of bidders participating in the bidding process is restricted, and perfect competition might not be realized. However, because the purpose of this study is to analyse the structure of PPP contracts, we assume that perfect competition is observed in the bidding process. We first analyse a case in which the bid price is determined by perfect competition and no constraints are imposed on the bid price. Assuming perfect competition means that the lowest price bidder is awarded as a winner. The equilibrium solutions in this case are referred to as competitive bidding equilibrium solutions (CBEs). In CBEs, the bidders that choose the safe project strategy might not be selected because of excessive competition.

#### 4.2. Competitive Bidding Equilibrium Solutions (CBEs)

Suppose that competitive bidding is implemented at \( t = 0 \) to determine the service fee \( R \). The government signs the concession contract with the winning bidder, who offers the lowest feasible price. We find the lowest service fee \( (1 - p_1)/(p_2)C_1 + p_2C_2 \) and \( v_3 = (1 - p_1)/p_1)C_1, v_4 = (1/(1 - p_3) + p_1/p_2)C_1, \) and \( v_5 = ((1 - p_1)/p_1)C_1 \).

**CBE 1:** \( e^* = e_{dt}, v^* = v_5 \)

**CBE 2:** \( e^* = e_i, v^* = v_4 \)

**CBE 3:** \( e^* = e_{dt}, v^* = v_3 \)

**CBE 4:** \( e^* = e_{dt}, v^* = v_5 \)
Equilibrium $D$ in the basic model is particularly reflected in CBE 1, in which neither a moral hazard nor inefficient liquidation can be avoided. CBE 2 reflects equilibrium $C$, in which the moral hazard problem is avoided, but the project is liquidated when the project cost is determined as $C_2$. CBE 3 and CBE 4 both reflect equilibrium $B$, in which a moral hazard occurs, and the inefficient liquidation of the project is avoided.

**Proposition 3.** When the service fee is determined via competitive bidding for a PPP concession contract, a moral hazard and inefficient liquidation cannot be avoided simultaneously during the contract period.

### 5. Performance Guarantee Model

#### 5.1. Contract Termination and Performance Guarantee

We have shown that the financial efficiency and social efficiency of the PPP project cannot be achieved simultaneously via the competitive bidding system. Here, we address this issue by adding a performance guarantee and right of termination into the concession contract, as shown in the performance guarantee model. The differences between the basic model and the performance guarantee model are as follows. At the beginning of the project, the SPV is obligated to deposit guarantee money with the government. Assume that the SPV is prohibited from procuring this guarantee money from debt funding. Suppose that the project cost realized at $t = 1$ is $C_2$; thus, the project cannot continue unless an additional loan is provided by the bank. Then, the government exercises its termination right and cancels the concession contract with the SPV. The bank must then decide whether to accept the termination of the concession contract with the SPV. The project is liquidated. The government would then pay the SPV compensation amount $A$, which is equal to repayment $\hat{D}$ for acquiring the assets of the project. After making this payment, the government incurs additional expenditure $C_2 - C_1$ to continue the project. The government must decide whether to find an alternative SPV or to continue the project itself. When the concession contract is terminated, the guarantee money that is confiscated by the government is used wholly or partly for repayments to the bank. Conversely, if the bank rejects the termination of the concession contract, it can modify the debt contract and provide an additional loan to the SPV. Thus, the SPV can continue the project to its conclusion. Finally, if the termination right is not exercised by the government and the project continues until $t = 2$, the guarantee money will be returned to the SPV.

#### 5.2. Equilibrium Solutions of the Performance Guarantee Model

When the project cost is determined as $C_1$, the SPV can complete the project without any additional loan. The amount of performance guarantee money, denoted by $K$, is paid at $t = 0$ and is returned at $t = 2$. The SPV’s payoffs are the same as those in the basic model. However, when the project cost is $C_2$, an additional loan equal to $C_2 - C_1$ is required to enable the continuation of the project. The government exercises its termination right and confiscates the guarantee money. The government then acquires the assets of the project by paying the bank compensation amount $\hat{D}$. The government can then sign a new concession contract with an alternative SPV (or the initial SPV). An alternative SPV will bear the additional cost $C_2 - C_1$ and continue the project. Therefore, when the project cost is determined as $C_2$, the SPV’s payoff is $-K$.

Conversely, the bank’s payoffs are not contingent on the project state because the bank can always receive compensation equal to the repayment $\hat{D}$. Therefore, the bank obtains the expected payoffs, which are independent of the strategy $\hat{e}_i$ ($i = s, d$) chosen by the SPV, represented by $\hat{\Pi}^{\text{bank}}_e = \hat{D} - 1$. Given the competitive financial market, the repayment $\hat{D}$ is determined as $\hat{D} = 1$.

Given guarantee money $K$ and $\hat{D} = 1$, the expected payoffs of the SPV, which chooses either the safe or the risky project strategy, are written as $\hat{\Pi}^{\text{spv}}_e (K) = (p_1 + p_2)(R - C_1) - p_1 K$ and $\hat{\Pi}^{\text{spv}}_s (K) = p_1 R - (p_2 + p_1) K$. The conditions ensuring that the safe project strategy is chosen are written as

\[
\begin{align*}
    p_1 (R + K) &\geq (p_1 + p_2) C_1 \quad (13a) \\
    (p_1 + p_2) (R - C_1) &\geq p_3 K \geq 0. \quad (13b)
\end{align*}
\]

Equations (13a) and (13b) represent the incentive constraint and the participation constraint, respectively, for the choice of the safe project strategy by the SPV. Consider the case in which the participation constraint (13b) holds but the incentive constraint (13a) does not. For the risky project strategy to be feasible, the following two conditions must be satisfied:

\[
\begin{align*}
    p_2 (R + K) &< (p_1 + p_2) C_1 \quad (14a) \\
    p_1 R &\geq (p_2 + p_1) K, \quad (14b)
\end{align*}
\]

where (14a) and (14b) represent the incentive constraint and the participation constraint, respectively, corresponding to the risky project strategy.
ensured when the project cost is determined as $C_1$, because the project will be continued by an alternative SPV after the government terminates the existing concession contract. In equilibrium $E$, the SPV chooses the risky project strategy, whereas in equilibrium $F$, the SPV chooses the safe project strategy, and a moral hazard cannot be prevented.

5.3. Competitive Bidding Equilibrium Solutions of the Performance Guarantee Model. To find the optimal value of the performance guarantee, we consider competitive bidding to determine the service fee endogenously. We refer to the competitive bidding equilibrium solutions of the performance guarantee model as P-CBEs. Because (13a) and (13b) can be rewritten as

\[ R \geq -K + \frac{p_1 + p_2}{p_2} C_1 \]  
\[ R \geq \left( \frac{1}{p_1 + p_2} - 1 \right) K + C_1, \]

we conclude that the area $\Omega$ shown in Figure 2 satisfies (16a) and (16b) simultaneously. Conversely, from (14a) and (14b), the conditions corresponding to the risky project strategy are rewritten as

\[ R < -K + \frac{p_1 + p_2}{p_2} C_1 \]  
\[ R \geq \frac{p_2 + p_3}{p_1} K. \]

From (17a) and (17b), the area labelled $Y$ in Figure 2 represents the risky project strategy. When the service fee, denoted by $R^*(K)$, is determined by competitive bidding, the following two cases occur.

First, if $K < (p_1(p_1+p_2)/p_2)C_1$, then $R^*(K)$ is equal to the lowest value existing in equilibrium $F$. From (17b), we obtain

\[ R^*(K) = \frac{p_2 + p_3}{p_1} K. \]  

Second, if the performance guarantee is determined as $K \geq (p_1(p_1+p_2)/p_2)C_1$, then $R^*(K)$ is feasible in equilibrium $E$. From (16b), the service fee $R^*(K)$ is written as

\[ R^*(K) = \left( \frac{1}{p_1 + p_2} - 1 \right) K + C_1. \]

As shown in Table 3, the following P-CBEs are derived:

<table>
<thead>
<tr>
<th>P-CBEs</th>
<th>Formation condition $I$</th>
<th>Equilibrium of the performance guarantee model $E$</th>
<th>Bidding price</th>
<th>Liquidation</th>
<th>Moral hazard</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$K &lt; \frac{p_1(p_1 + p_2)}{p_2} C_1$</td>
<td>$\tilde{D}^* = I, \tilde{e}^* = e_s$, if (13a) and (13b) hold</td>
<td>$F$</td>
<td>$\frac{p_2 + p_3 K}{p_1}$</td>
<td>$\circ \times$</td>
</tr>
<tr>
<td>2</td>
<td>$K \geq \frac{p_1(p_1 + p_2)}{p_2} C_1$</td>
<td>$\tilde{D}^* = I, \tilde{e}^* = e_d$, if (14a) and (14b) hold.</td>
<td>$E$</td>
<td>$\frac{\left( \frac{1}{p_1 + p_2} - 1 \right) K + C_1}{p_1}$</td>
<td>$\circ \circ$</td>
</tr>
</tbody>
</table>

In P-CBE 1, a moral hazard and liquidation are avoided, whereas a moral hazard occurs in P-CBE 2.

**Proposition 4.** In competitive bidding for a concession contract, if the amount of performance guarantee $K$ satisfies

\[ K \geq \frac{p_1(p_1 + p_2)}{p_2} C_1, \]

then the socially optimal PPP project is possible. At the same time, maximum VFM is also realized.

Proposition 4 suggests that the performance guarantee is not only an instrument to hedge risks but also a means of...
increasing social and financial efficiencies by disciplining the SPV. The optimal performance guarantee will increase if \( p_1 \) on the right-hand side of (21) is larger or \( p_2 \) is smaller. Meanwhile, the social cost \( \Lambda \) arising from a moral hazard will decrease with a larger \( p_1 \) or smaller \( p_2 \). Note that the choice of the safe project strategy will be socially efficient only if the equity cost of raising the guarantee money is small. However, if the equity cost is greater than the social cost arising from a moral hazard, it might be preferable to choose the risky project strategy. Conversely, because we assume that the government does not behave strategically and pays the bank a compensation equal to the initial loan, the bank has no incentive to exercise its step-in right, which is stipulated in some concession contracts for PPP projects [38]. Thus, in future studies, it might be necessary to formulate a performance guarantee model incorporating a step-in right in cases in which the government behaves strategically.

Moreover, the expected payments from the government are \( (p_1 + p_2)C_1 + p_2C_2 \), when the amount of performance guarantee satisfies (21). No additional costs (quasi-rent) are incurred because the SPV’s expected payoff is zero, and no risk premium exists because the bank bears no risk. Thus, the government’s expected payment is lowest; that is, value for money is realized.

5.4. Implications of the Performance Guarantee Policy. Proposition 4 argues that the social optimum and maximization of VFM are realized in PPP projects if the guarantee money that is determined satisfies (21) and competitive bidding is implemented before the concession contract is signed. By comparing the CBEs in the basic model and P-CBEs in the performance guarantee model, we conclude that the performance guarantee system is characterized by the following four desirable features.

First, inefficient liquidation does not occur because the government guarantees the continuation of the project even when it becomes difficult for the initial SPV to continue the project. Second, the government commits to paying compensation to the bank to ensure the continuation of the project when it becomes difficult for the initial SPV to continue the project. Thus, there is no risk premium in the debt contract because the bank bears no risk. Third, the performance guarantee provides an incentive to the SPV to choose the safe project strategy. Moral hazard is avoided when the performance guarantee satisfies (21). Fourth, the additional payment will be avoided if the appropriate guarantee money is collected in advance. According to agency theory, the moral hazard can be avoided at a cost (quasi-rent), which is an additional payment from the principal to the agent. In a PPP project, any additional payment is actually income transferred from the government to the SPV. Thus, the performance guarantee system not only ensures the social welfare but also minimizes the expected payments from the government; that is, VFM is maximized by the introduction of the performance guarantee system.

Furthermore, the performance guarantee has certain characteristics in its practical application. Note that the optimal amount of the guarantee money is not a specific value but rather is represented by a range that satisfies (21). It is not easy to find the exact value of the guarantee money that satisfies (21). Therefore, the government must gather sufficient information about the project risks. In addition, because the guarantee money is obtained from equity funding, the opportunity cost of equity increases as the size of the guarantee money increases. Therefore, it is desirable to set the guarantee money as low as possible within the range satisfying (21). Issues relating to the calculation of the performance guarantee and the opportunity costs of equity have not been addressed in this study. These issues will be considered in future studies.

6. Conclusions

In this study, we formulate an incomplete contract model of PPP projects, which are assumed to provide infrastructure services paid by the government, to analyse the mechanism of ex ante moral hazard and ex post liquidation. When the SPV is selected through competitive bidding, we find that this method cannot prevent a moral hazard and inefficient liquidation from occurring simultaneously. Then, we introduce a performance guarantee and termination right for the government into the project contracts. The findings indicate that the optimal performance guarantee satisfying (21) not only provides the SPV with an incentive to choose the safe project strategy but also minimizes the expected payment from the government. In addition, inefficient liquidation is avoided because the government exercises its right to terminate the concession contract and finds an alternative SPV to continue the project after the existing SPV falls into default. That is, social efficiency and VFM can be achieved by setting the optimal performance guarantee level satisfying (21), which is stipulated in the concession contract.

From a real-world perspective, a performance guarantee can help to ensure the continuation of PPP infrastructure projects facing project risks because the guarantee money plays an important role in promoting banks to provide additional loans to the SPV. In addition, governments can use performance guarantees to provide SPVs with proper incentives to choose socially efficient project strategies to enhance contractual efficiencies. By setting the optimal performance guarantee level, it is possible to design an optimal contract that will maximize the VFM.

A different approach is necessary under different assumptions. First, after observing the state realized at \( t = 1 \), it might be necessary to develop a real option approach to analyse future behaviour. Second, this study does not consider the possibility that the SPV strategically liquidates the project. For instance, the SPV might strategically default in an attempt to trigger the liquidation of the project. Third, this study assumes that the bank has all of the bargaining power in renegotiation, which occurs when an additional loan is necessary at \( t = 1 \). However, the initial SPV might have considerable bargaining power because of the monopolistic conditions that exist in many international PPP projects. In this case, the performance guarantee would serve to restrain the SPV’s bargaining power. Finally, the government might also behave strategically. We will address the issue of strategic
Appendix

A. Notations

To make the paper more readable so that readers need not recheck the meanings of notations back and forth, this appendix provides a list of some frequently used notations.

$I$: Initial investment

$D$: Debt repayment

$e_i$ ($i = s, d$): Project strategy

$e_s$: Safe project strategy

$e_d$: Risky project strategy

$\omega_j$: State variable

$p_j$: Probability that the state variable $\omega_j$ occurs

$C(e_i, \omega_j)$: Project costs depending upon $e_i$ ($i = s, d$) and $\omega_j$

$R$: Service fee paid by the government to the SPV

$\Lambda$: Social cost of a moral hazard

$\pi_{s}^{pv}(\omega_j)$: Payoff earned by the SPV that chooses $e_s$ under state $\omega_j$

$\pi_{d}^{pv}(\omega_j)$: Bank's payoff when the SPV chooses $e_d$ under state $\omega_j$

$\Pi_{e_i}^{pv}(D)$: Given debt repayment $D$, the expected payoff of the SPV that chooses $e_i$ in Scenario 1

$\Pi_{e_i}^{bank}(D)$: Given debt repayment $D$, the bank's expected payoff when the SPV chooses $e_i$ in Scenario 1

$D_s$: Break-even repayment when the SPV chooses $e_s$ in Scenario 1

$D'_s$: Repayment in Scenario 2

$\Pi_{e_i}^{pv}(D'_s)$: Given debt repayment $D'_s$, the expected payoff of the SPV that chooses $e_i$ in Scenario 2

$D'_{s}$: Break-even repayment when the SPV chooses $e_s$ in Scenario 2

$\Pi_{e_i}^{pv}$: Expected payoffs earned by the SPV under equilibrium strategies in the basic model

$e^*_s$: Equilibrium strategies in the basic model

$D^*_s$: Equilibrium repayments in the basic model

P-CBEs: Competitive bidding equilibrium solutions

$R^A_{min}$: Marginal service fee in equilibrium solution $A$

$R^B_{min}$: Marginal service fee in equilibrium solution $B$

$R^C_{min}$: Marginal service fee in equilibrium solution $C$

$R^D_{min}$: Marginal service fee in equilibrium solution $D$

$A$: Amount of compensation paid by the government to the SPV

$\hat{D}$: Repayment in performance guarantee model

$K$: Amount of performance guarantee money

$\hat{\Pi}_{e_i}^{bank}$: Bank's expected payoff when the SPV chooses $e_i$ in the performance guarantee model

$\hat{\Pi}_{e_i}^{pv}(K)$: Given performance guarantee $K$, the expected payoff of the SPV that chooses $e_i$

$\hat{D}^*$: Equilibrium repayments in the performance guarantee model

$e^*_s$: Equilibrium strategies in the performance guarantee model

$P-CBEs$: Competitive bidding equilibrium solutions of the performance guarantee model

$R^*(K)$: Equilibrium service fees in P-CBEs

B. Proof of Equilibria

B.1. Proof of Equilibria of Basic Model

(a) Proof of Equilibria A and B in Scenario 1 in Which $R \geq C_2 - C_1$ Holds.

Given $D_s = I + p_3(C_2 - R)/(1 - p_3)$, we obtain $R \geq ((1 - p_3)^2/p_2)C_1 + p_3C_2$ and $R \geq (1 - p_3)C_1 + p_3C_2$, respectively, from (8a) and (8b), which represent the incentive-compatible condition and participation condition corresponding to the safe project strategy. Because $((1 - p_3)^2/p_2)C_1 + p_3C_2 \geq (1 - p_3)C_1 + p_3C_2$ holds, the condition for equilibrium $A$ when the safe project strategy $e_s$ is chosen is written as $C_2 > R \geq \max\{v_1, v_2\}$. Define $v_1 = ((1 - p_3)^2/p_2)C_1 + p_3C_2$ and $v_2 = C_2 - C_1$. Under assumption conditions (2), (3), and (4), equilibrium $A$ always exists. Conversely, given $D_s = I + (1 - p_1)(C_2 - R)/p_1$, we obtain $R < v_1$ and $R \geq v_3$ from (9a) and (9b), respectively. Define $v_3 = (1 - p_1)C_2$. Therefore, the condition for equilibrium $B$ in which the risky project strategy $e_d$ is chosen is written as $v_1 > R \geq v_2$. Because $v_2 - v_1 = ((p_1 + p_2)/p_2)\Lambda - C_1$ and $v_1 - v_3 = (p_2/p_1 + p_2)\Lambda - C_1$, the necessary and sufficient condition for equilibrium $B$ to exist is written as $0 < \Lambda \leq \min\{p_2C_1/(p_1 + p_2), p_1(p_1 + p_2)/p_1\}$.

(b) Proof of Equilibria C and D in Scenario 2 in Which $R < C_2 - C_1$ Holds.

Given $\Pi_{e_s}^{pv}(D'_s) = (p_1 + p_2)(R + I - C_1 - D')$ and $\Pi_{e_d}^{pv}(D'_s) = p_1(R + I - D')$, the conditions that guarantee that the SPV chooses the safe project strategy are written as

$$\Pi_{e_s}^{pv}(D'_s) - \Pi_{e_d}^{pv}(D'_s) = p_2(R - D'_s) - p_1 I \geq 0 \quad (B.1a)$$

$$R \geq C_1 + D'_s - I \quad (B.1b)$$

$$R < C_2 - I. \quad (B.1c)$$

Using $D'_s = I/(1 - p_3)$, (B.1a) and (B.1b) are rewritten as $R \geq 1/(1 - p_3) + p_1/p_2C_1$ and $R \geq C_1/(1 - p_3)$, respectively. Because $1/(1 - p_3) + p_1/p_2C_1 \geq C_1/(1 - p_3)$, the safe project strategy is chosen if and only if $v_3 > R \geq v_4$. Define $v_4 = 1/(1 - p_3) + p_1/p_2C_1$. Because $v_2 - v_4 = \Lambda/p_2 - C_1/(p_1 + p_2)$,
the necessary and sufficient condition for equilibrium \( C \) to exist is written as \( \Lambda \geq \frac{p_2}{p_1 + p_2}C_1 \). Conversely, the conditions corresponding to the risky project strategy are written as

\[
\Pi_{\pi_i}^{\text{pr}}(D'_i) - \Pi_{\pi_i}^{\text{pr}}(D'_j) = p_2(R - D'_i) - p_1I < 0 \quad \text{(B.2a)}
\]

\[
R \geq D'_d - I \quad \text{(B.2b)}
\]

\[
R < C_2 - C_1. \quad \text{(B.2c)}
\]

Given \( D'_d = 1/p_1 \), we derive \( \min[v_j, v_j] > R \geq v_j \) from (B.2a) and (B.2b). Define \( v_j = ((1 - p_1)/p_1)C_1 \). Because \( v_j \geq v_j \Leftrightarrow \Lambda \geq \frac{p_2}{p_1 + p_2}C_1 \), and \( v_j \geq v_j \Leftrightarrow (p_2/(p_1 + p_2)C_1 \), and \( v_j \geq v_j \Leftrightarrow (p_2/(p_1 + p_2)C_1 \), the necessary and sufficient condition for equilibrium \( D \) to exist is written as \( \Lambda \geq \max\{p_2/(p_1 + p_2)C_1, 0\} \) and \( p_1(p_1 + p_2)^2 - p_2^2 \geq 0 \).

**B.2. Proof of Competitive Bidding Equilibrium Solutions (CBEs).** Because \( v_j - v_j = ((1 - p_1)/p_1)\Lambda - C_1 \), \( v_j - v_j = \Lambda/p_1 - C_1 \), and \( v_j - v_j = (p_2)/(p_1 + p_2)\Lambda - (p_2)/(p_1 + p_2)C_1 \), the conditions \( v_j \geq v_j \), \( v_j \geq v_j \), and \( v_j \geq v_j \) are equivalent to \( \Lambda \geq \max\{p_2/(p_1 + p_2)C_1, 0\} \) and \( p_1(p_1 + p_2)^2 - p_2^2 \geq 0 \).

In addition, inequality (B.3a)–(B.3d) can be rewritten as

\[
\Lambda \geq \frac{p_1(p + p_2)C_1}{p_2} - \frac{p_1(p + p_2)^2 - p_2^2}{p_2}C_1 \quad \text{(B.5a)}
\]

\[
\Lambda \geq \frac{p_1(p + p_2)C_1}{p_2} - \frac{p_1(p + p_2)^2 - p_2^2}{p_2}C_1 \quad \text{(B.5b)}
\]

\[
\Lambda \geq \frac{p_1(p + p_2)C_1}{p_2} - \frac{p_1(p + p_2)^2 - p_2^2}{p_2}C_1 \quad \text{(B.5c)}
\]

\[
\Lambda \geq \frac{p_1(p + p_2)C_1}{p_2} - \frac{p_1(p + p_2)^2 - p_2^2}{p_2}C_1 \quad \text{(B.5d)}
\]

From (B.4) and (B.5a)–(B.5d), we obtain

\[
\Lambda \geq \frac{p_1(p + p_2)C_1}{p_2} \quad \text{(B.6)}
\]

Therefore, (B.7) can be derived from (B.6) as follows:

\[
v_j \geq v_j \quad \text{implies} \quad v_j \geq v_j \quad v_j \geq v_j \quad v_j \geq v_j. \quad \text{(B.7)}
\]

Equation (B.7) can be rewritten as

\[
v_j \geq v_j \quad \text{implies} \quad v_j \geq v_j \quad v_j \geq v_j. \quad \text{(B.8)}
\]

Next, consider the case in which \( v_j \geq v_j \) and \( v_j \leq v_j \) hold. That is,

\[
\Lambda \geq \frac{p_1(p + p_2)C_1}{p_2} \quad \text{(B.9)}
\]

\[
p_1(p + p_2)^2 - p_2^2 \geq 0.
\]
Comparing the last terms on the right-hand side of (B.5a)–(B.5d), the following inequalities clearly hold:

\[-\left(1 - p_1\right) \left(p_1\left(p_1 + p_2\right)^2 - p_2^2\right)C_1/p_1p_2(1 - p_1 - p_2) \geq \begin{cases} 
\frac{p_2}{p_1\left(p_1 + p_2\right)^2 - p_2^2}C_1 - \frac{p_2}{p_1\left(p_1 + p_2\right)^2 - p_2^2}C_1 \\
\left(1 - p_1 - p_2\right) \left(p_1\left(p_1 + p_2\right)^2 - p_2^2\right)p_2\left(p_1 + p_2\right)(1 - p_1)C_1.
\end{cases} \]

From (B.10), we obtain

\[v_1 \geq v_5 \implies \begin{cases} 
v_2 \geq v_1 \geq v_4 \\
v_2 \geq v_3 \geq v_5 \\
v_3 \geq v_1 \\
v_3 \geq v_4.
\end{cases} \]

Equation (B.11) can be rewritten as

\[v_2 \geq v_3 \geq v_1 \geq v_5 \geq v_4. \]

Equations (B.8) and (B.12) show that the following two patterns concerning the order of the marginal service fees exist:

pattern 1:

\[v_2 > v_3 \geq v_1 \geq v_4 > v_5 \]

pattern 2:

\[v_2 > v_3 \geq v_1 \geq v_5 \geq v_4. \]

The necessary and sufficient conditions for pattern 1 and pattern 2 are represented by (B.4) and (B.9), respectively. Clearly, reversion of the order of service fees in pattern 1 and pattern 2 is possible; that is, \(v_5 \geq v_4 \geq v_1 \geq v_3 \geq v_2\) (pattern 3) and \(v_4 > v_5 \geq v_1 \geq v_3 \geq v_2\) (pattern 4) exist. The necessary and sufficient conditions for pattern 3 and pattern 4 to exist are therefore represented by \(\Lambda < \left(p_1\left(p_1 + p_2\right)/p_2\right)C_1; p_1\left(p_1 + p_2\right)^2 - p_2^2 < 0\) and \(\Lambda < \left(p_1\left(p_1 + p_2\right)/p_2\right)C_1; p_1\left(p_1 + p_2\right)^2 - p_2^2 > 0\), respectively.

**Conflicts of Interest**

The authors declare that there are no conflicts of interest concerning the publication of this paper.

**Acknowledgments**

The authors acknowledge the financial support of the National Natural Science Foundation of China (nos. 71672017 and 71372084).


