Hindawi
Mathematical Problems in Engineering
Volume 2018, Article ID 3905879, 10 pages
https://doi.org/10.1155/2018/3905879

Research Article

An ADRC Method for Noncascaded Integral Systems Based on Algebraic Substitution Method and Its Structure

Zhijian Huang,[1,2] **Yudong Li**,[1] **Yihua Liu**,[1] **Wenbo Sui**,[1] **and Guichen Zhang**[1]

[1] *Lab of Intelligent Control and Computation, Shanghai Maritime University, Shanghai 201306, China*
[2] *Department of Electrical, Computer, and Biomedical Engineering, University of Rhode Island, Kingston, RI 02881, USA*

Correspondence should be addressed to Zhijian Huang; zjhuang@shmtu.edu.cn

Received 11 August 2017; Accepted 28 May 2018; Published 14 June 2018

Academic Editor: R. Aguilar-López

The Active Disturbance Rejection Control (ADRC) prefers the cascaded integral system for a convenient design or better control effect and takes it as a typical form. However, the state variables of practical system do not necessarily have a cascaded integral relationship. Therefore, this paper proposes an algebraic substitution method and its structure, which can convert a noncascaded integral system of PID control into a cascaded integral form. The adjusting parameters of the ADRC controller are also demonstrated. Meanwhile, a numerical example and the oscillation control of a flexible arm are demonstrated to show the conversion, controller design, and control effect. The converted system is proved to be more suitable for a direct ADRC control. In addition, for the numerical example, its control effect for the converted system is compared with a PID controller under different disturbances. The result shows that the converted system can achieve a better control effect under the ADRC than that of a PID. The theory is a guide before practice. This converting method not only solves the ADRC control problem of some noncascaded integral systems in theory and simulation but also expands the application scope of the ADRC method.

1. Introduction

The Active Disturbance Rejection Control (ADRC) has begun to be used in many areas recently [1–23]. This theory was first proposed by *Han* [24, 25]. The central idea is that the internal dynamic and external disturbance of a controlled system can be estimated and compensated in real time with a tracking differentiator (TD), extended state observer (ESO), nonlinear state error feedback (NLSEF), compensator, etc. Thus, the ADRC may promote the control quality and speed where PID is used [25].

For the ADRC, its ESO is a cascaded integral form, its TD tracks the system state and derivative, and its NLSEF is based on the ESO and TD. Thus, these characteristics make the ADRC suitable for a cascaded integral system. This is because the system order, system variables, and known states are explicit to the ESO and TD for the system in a cascaded integral form. Then, the ADRC usually selects the cascaded integral system as a typical form for an easy design or better control effect. For example, in 2015, *Shao* thought that the ADRC was available to a cascaded integral system, such as a motion control system [10]. In 2006, *Gao* also thought that if a plant model was in a cascaded integral form, the ESO could be established, and the ADRC could has a full state feedback from the ESO [26].

However, in practical control systems, there are many cases of noncascaded integral forms. When necessary, a converting method is needed to get the cascaded integral form. At present, the research in this field is as follows: (1) Some scholars adopted a converting method. The Differential Geometry is one of them [27]. This method combines a nonlinear state conversion and linearization using its object model. Also in 2014, *Huang* converted a two-order state space form into the cascaded integral system by a mathematic transform [28]. In 2014, *Huang* used the same method to convert a multiorder state space system [28]. In 2014, *Ramírez-Neria* utilized the decoupling property of the object model and decomposed it into the cascaded connection of two independent blocks [29]. (2) Some ADRC applications were limited to the control system with an implicit cascaded

integral form, such as the differential equation, rational proper fraction, or state space form, as shown in [12, 13, 26, 30]. (3) Some ADRC applications were limited to the control system with an explicit cascaded integral system. For example, the nonlinear ADRC has been applied to the fast tool servo systems [14, 31], which are cascaded integral systems with two stages.

As the real control systems have various forms, there are many styles to be converted, and their converting method may also be different. In practical application, the PID feedback control is the most widely used. If the PID control object of a noncascaded integral form can be converted into the cascaded integral system, it would be highly representative.

Therefore, an algebraic substitution method and its structure are proposed in this paper to convert the noncascaded integral system of a PID control object into the cascaded integral form. The adjusting parameters of the ADRC controller are also demonstrated. A numerical example and the oscillation control of a flexible arm are simulated to show the conversion and ADRC control effect. The converted system is proved to be more suitable for a direct ADRC control. The ADRC can achieve a nonovershoot tracking control while satisfying the rapidity under disturbances. In addition, the control effect of the numerical example is compared under the periodical, white noise and inaccurate model disturbances during a step input response while the controller and its parameters are kept invariable. The results show that the converted system can achieve a better control effect for the ADRC than that of a PID.

Thus, this paper presents an approach, which can transform a noncascaded integral system into the cascaded integral form for an easier and better ADRC control. The converted ADRC control system has a good antidisturbance and adaptive effect. The theory is a guide before practice. This converting method solves the ADRC control problems of some noncascaded integral systems in theory and simulation. It also expands the application scope of the ADRC method.

2. The ADRC Control Method

For a continuous system, the ADRC control method adopts the following four steps:

(1) Arranging a transient process for the control reference with the TD:

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= fhan\left(x_1 - v, x_2, r, h_0\right) \\
fhan &= -r\left(\frac{a}{d}\right) fsg\left(a, d\right) \\
&\quad - r\,\mathrm{sign}\left(a\right)\left(1 - fsg\left(a, d\right)\right) \\
d &= r \cdot h_0^2 \\
a_0 &= h_0 \cdot x_2\left(t\right) \\
y &= x_1\left(t\right) + a_0 \\
a_1 &= \sqrt{d\left(d + 8\left|y\right|\right)} \\
a_2 &= a_0 + \frac{\mathrm{sign}\left(y\right)\left(a_1 - d\right)}{2} \\
a &= \left(a_0 + y\right) fsg\left(y, d\right) + a_2\left(1 - fsg\left(y, d\right)\right) \\
fsg\left(x, d\right) &= \frac{\left(\mathrm{sign}\left(x + d\right) - \mathrm{sign}\left(x - d\right)\right)}{2}
\end{aligned} \tag{2}$$

In it, x_1, x_2 are the system state and its first derivative, respectively; v is the control reference; h_0 is the filtering factor; r is the time ruler; sign is a sign function; and $fhan$ is a constructed nonlinear function shown in (2) [16, 32].

(2) Estimating the system states and total disturbance of the controlled object with the ESO:

$$\begin{aligned}
\dot{z}_1 &= z_2 - \beta_{01} e \\
\dot{z}_2 &= z_3 - \beta_{02} fe + u \\
\dot{z}_3 &= -\beta_{03} fe_1 \\
e &= z_1 - y, \\
fe &= fal\left(e, 0.5, \delta\right), \\
fe_1 &= fal\left(e, 0.25, \delta\right)
\end{aligned} \tag{3}$$

$$fal\left(e, \alpha, \delta\right) = \begin{cases} \dfrac{e}{\delta^{\alpha-1}}, & \left|e\right| \le \delta \\ \left|e\right|^{\alpha}\,\mathrm{sign}\left(e\right), & \left|e\right| > \delta \end{cases} \tag{4}$$

In it, e is the error between the estimated state and system output; z_1, z_2, and z_3 are the estimations of the system states x; β_{01}, β_{02}, and β_{03} are gain coefficients; u is the control signal; y is the system output; δ and α are the parameters of fal function; fe and fe_1 are the outputs of fal function; fal is another constructed nonlinear function shown in (4) [16, 32].

(3) The nonlinear state error feedback with the NLSEF:

$$\begin{aligned}
e_1 &= x_1 - z_1, \\
e_2 &= x_2 - z_2 \\
u_0 &= \beta_1 fal\left(e_1, \alpha_1, \delta_1\right) + \beta_2 fal\left(e_2, \alpha_2, \delta_1\right)
\end{aligned} \tag{5}$$

In it, e_1 and e_2 are the errors between the estimated states and system states; u_0 is an intermediate control output before compensation; β_1 and β_2 are gain coefficients; $\alpha_1, \alpha_2, \delta_1$ are the parameters of fal function.

(4) The disturbance compensation with a compensator:

$$u = u_0 - \frac{z_3}{b_0} \tag{6}$$

In it, b_0 is the amplification factor of the control signal; u is a final control signal.

3. The Algebraic Substitution Method and Its Structure for the ADRC

Remark 1. According to (1) and (3), if x_1, $\dot{x}_1$, x_2, $\dot{x}_2$, and their counterparts z_1, $\dot{z}_1$, $\dot{z}_2$, and $\dot{z}_3$ of the controlled system are

explicit, this system is in a cascaded integral form. Then, the TD, ESO, and NLSEF of the ADRC controller can be easily designed.

The cascaded integral system is a closed state feedback system, and its archetype can be described ideally in

$$\begin{aligned} \dot{x}_1 &= f_1 + x_2 \\ &\vdots \\ \dot{x}_{n-1} &= f_{n-1} + x_n \\ \dot{x}_n &= f_n + bu \\ y &= x_1 \end{aligned} \tag{7}$$

In it, $x_i, i = 1, 2, \cdots, n$ are the state variables of the controlled system; $f_i, i = 1, 2, \cdots, n$ are the unknown system functions; u is the control signal; y is the system output; and the coefficient $b \in \mathbf{R}^n$. This kind of cascaded integral system is a typical form suitable for the ADRC.

Then, a method is needed to construct an object system and convert the noncascaded integral system into the above cascaded integral form. The two-order control system in (8) is taken for example.

$$\begin{aligned} \ddot{x} &= -a_1 x - a_2 \dot{x} + u + \omega \\ y &= x \end{aligned} \tag{8}$$

In it, u is the control signal; y is the system output; x is the system state; and ω is the system disturbance. v is set as the control reference. Thus, the error between the system state and control reference is

$$e = v - x = v - y \tag{9}$$

Then, the proposed algebraic substitution method and its structure adopt the following six steps as shown in Figure 1:

(1) The PID control output for the controlled system is given as

$$u = K\left(\frac{1}{T_i}\int_0^t e(t)\,dt + e + T_d\dot{e}\right) \tag{10}$$

In it, K, T_i, T_d are the feedback gain, integral time, and differential time of the PID controller, respectively.

(2) In order to construct a cascaded integral system, let

$$\int_0^t e(t)\,dt = e_0(t) \tag{11}$$

(3) Then, the derivative of (11) is deduced as

$$\dot{e}_0(t) = e(t) \tag{12}$$

Then,

$$\ddot{e}(t) = \frac{d^2(v-x)}{dt^2} = -\ddot{x} = a_1 x + a_2\dot{x} - u - \omega \tag{13}$$

(4) By substituting (10) into (13), the following can be obtained:

$$\begin{aligned} \ddot{e}(t) &= \frac{d^2(v-x)}{dt^2} = -\ddot{x} \\ &= a_1 x + a_2\dot{x} - K\left(\frac{1}{T_i}\int_0^t e(t)\,dt + e + T_d\dot{e}\right) - \omega \\ &= a_1(x-v) + a_1 v + a_2\dot{x} \\ &\quad - K\left(\frac{1}{T_i}\int_0^t e(t)\,dt + e + T_d\dot{e}\right) - \omega \\ &= a_1(x-v) + a_1 v - a_2(\dot{v} - \dot{e}) \\ &\quad - K\left(\frac{1}{T_i}\int_0^t e(t)\,dt + e + T_d\dot{e}\right) - \omega \\ &= a_1(x-v) + a_1 v + a_2\dot{e} \\ &\quad - K\left(\frac{1}{T_i}\int_0^t e(t)\,dt + e + T_d\dot{e}\right) - \omega \\ &= -(a_1 + K)e - (KT_d - a_2)\dot{e} + a_1 v \\ &\quad - \frac{K}{T_i}\int_0^t e(t)\,dt - \omega \end{aligned} \tag{14}$$

(5) According to (12), the substitution relationships of the variables are set as

$$\dot{e}_0(t) = e(t) = e_1(t), \tag{15}$$

$$\dot{e}(t) = e_2(t) \tag{16}$$

(6) After arrangement, the above equations can be converted into a cascaded integral system as follows:

$$\begin{aligned} &\dot{e}_0 = e_1, \quad e_0(0) = 0 \\ &\dot{e}_1 = e_2, \quad e_1(0) = v(0) \\ &\dot{e}_2 = -\frac{K}{T_i}e_0 - (K + a_1)e_1 - (KT_d - a_2)e_2 + a_1 v - \omega, \\ &\qquad\qquad e_2(0) = 0 \\ &y = v - e_1 \end{aligned} \tag{17}$$

Thus, the structure of a cascaded integral ADRC control system is shown in Figure 2. In the virtual frame, it is a cascaded integral object system of n orders. The other part is an error-feedback ADRC controller consisting of the TD, ESO, NLSEF, and disturbance compensator. The TD, ESO, NLSEF, disturbance compensator, and their variables are seen in ((1)-(6)).

4. The Parameters of ADRC Controller

Although the ADRC has many parameters to determine its control effect, it is not difficult to determine them. This is because many parameters have their universal values that are suitable for most conditions.

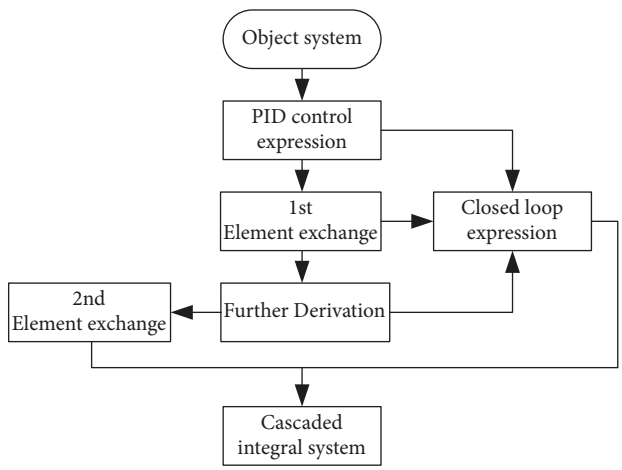

Figure 1: The principle of the proposed algebraic substitution method and its structure.

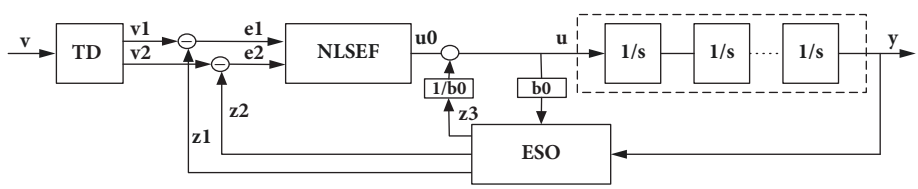

Figure 2: The structure of a cascaded integral ADRC control system.

4.1. The Adjusting Parameters of the TD. For the TD, the discrete time step h usually takes $h = 0.01$, h_0 usually takes 20~30 times of the h, and r usually takes several tens [33, 34].

Figure 3 shows that, for a greater r, the TD output $x_1(t)$ is closer to the input signal, and the TD output $x_2(t)$ is closer to the derivative and vice versa. However, if r is too large, the tracking quality will be deteriorated by the noise in the input signal.

For a smaller h_0, a similar simulation can also show that the TD has a better tracking effect and vice versa. If h_0 is too small, the tracking quality will also be deteriorated by the noise in the input signal.

Figure 4 shows that, as long as the discrete time step $h \leq h_0$, the stable oscillation in the TD outputs can always be eliminated.

4.2. The Adjusting Parameters of the ESO, NLSEF, and Compensator [35, 36]. A successful z_1, z_2, z_3 output of the ESO under $\beta_{01} = 100, \beta_{02} = 300, \beta_{03} = 1000, \alpha_1 = 0.5, \alpha_2 = 0.25$, and $\delta = 0.05$ is shown in Figure 5.

The α in the *fal* function of the ESO and NLSEF is the power of its exponential function. When it does not equal 1, the ADRC is a nonlinear controller. The δ in the *fal* is only effective for a nonlinear system. A lot of simulations show that a small change in these parameters will greatly affect the setting of other parameters. Thus, these parameters should not be modified after the ADRC controller has been started, while the other parameters $\beta_{01}, \beta_{02}, \beta_{03}, b_0$ can be modified online.

In general, the α takes a fixed value of $\alpha_1 = 0.5, \alpha_2 = 0.25$ for the ESO or $\alpha_1 = 0.75, \alpha_2 = 1.25$ for the NLSEF based on a practical experience, which is to reduce its power calculation. If the δ is too large, the ADRC can only work in a linear region. If the δ is too small, the ADRC may have oscillation phenomenon. Thus, the δ generally takes 0.1 at first, and then it is fine-tuned according to the control effect.

The β_{01}, β_{02}, and β_{03} affect the estimation of control system states and disturbance, respectively. If the disturbance is large, the $\beta_{01}, \beta_{02}, \beta_{03}$ should also be large. In addition, the larger the β_{03}, the smaller the delay. However, if β_{03} is too large, it probably causes the estimated value to diverge. An appropriate increase of β_{01}, β_{02} can suppress this divergence. However, if β_{01}, β_{02} are too large, the estimated value will also diverge. Thus, these parameters should be coordinately adjusted.

A small change of b_0 will also lead to a jumping change in the control output. The b_0 usually takes a larger value for a delay system. A larger b_0 can also effectively compensate the disturbance and model uncertainty.

5. The ADRC Control Effect of the Converted System

Example 2. A two-order dynamic system

$$\begin{aligned} \ddot{x} &= -6\dot{x} - 3x + \omega + u \\ y &= x \end{aligned} \tag{18}$$

is taken for example. The control reference is a unit step signal of $v = 1$ from time 1 and initial value 0. This is because the

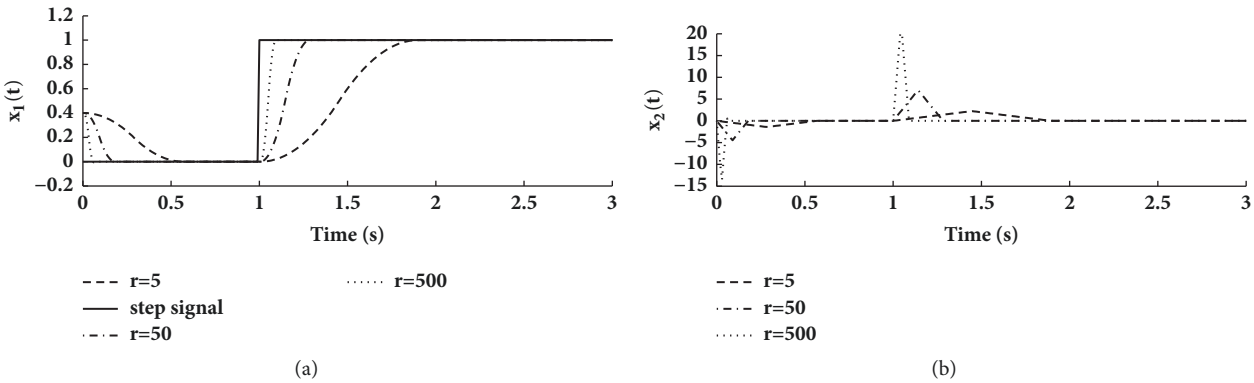

Figure 3: The tracking and derivative outputs of a TD with $r = 5, 50, 500$, respectively, and $h = 0.01$ and $h_0 = 0.01$ under a step response.

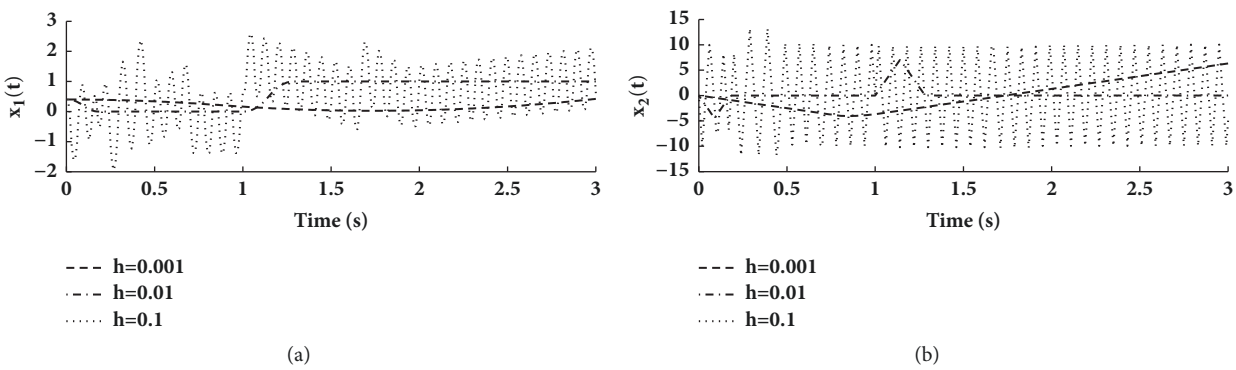

Figure 4: The tracking and derivative outputs of a TD with $h = 0.001, 0.01, 0.1$, respectively, and $r = 50$, $h_0 = 0.01$ under a step response.

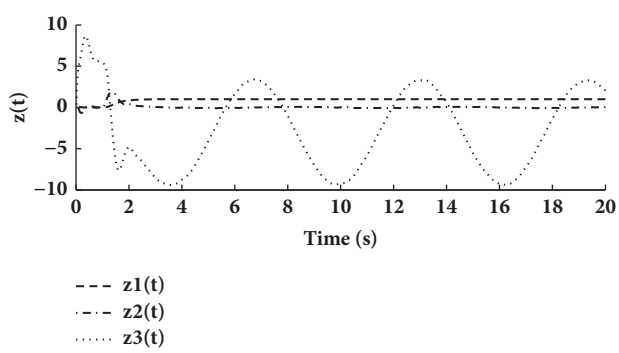

Figure 5: A successful output of the ESO with $\beta_{01} = 100, \beta_{02} = 300, \beta_{03} = 1000, \alpha_1 = 0.5, \alpha_2 = 0.25$, and $\delta = 0.05$.

step response is the most destructive to a dynamic system. In (18), u is the control signal, and

$$\omega = \sin(t) + \gamma n(t) \tag{19}$$

The ω is an added disturbance; $n(t)$ is a white noise; its mean value is 0; and variance is 1; $\gamma = 0.1$.

Then, the control effect will be tested under no disturbance, a periodical disturbance, white noise, or inaccurate model disturbance from time 0. If the controller can resist all these kinds of disturbances while keeping the controller design and its parameters invariable, it has a good adaptive ability and can work on most occasions.

5.1. The Conversion and the ADRC Controller Design. Now, the control system of (18) will be converted into a cascaded integral form according to ((9)-(17)), that is, the algebraic substitution method and its structure presented in Section 3. First, v is set as the control reference. The error between the system state and control reference is

$$e = v - x = v - y \tag{20}$$

Then, the conversion adopts the following six steps:

(1) The PID control output for the controlled system is given as

$$\begin{aligned} u &= K\left(\frac{1}{T_i}\int_0^t e(t)\,dt + e + T_d\dot{e}\right) \\ &= 34\left(\frac{1}{17}\int_0^t e(t)\,dt + e + 0.8\dot{e}\right) \end{aligned} \tag{21}$$

In it, the controller parameters take $K = 34$, $T_i = 17$, and $T_d = 0.8$, which are the same as that of the PID controller in Section 5.4 for a convenient comparing.

(2) In order to construct a cascaded integral system, let

$$\int_0^t e(t)\,dt = e_0(t) \tag{22}$$

(3) Then, the derivative of (22) is deduced as

$$\dot{e}_0(t) = e(t) \tag{23}$$

Then

$$\ddot{e}(t) = \frac{d^2(v-x)}{dt^2} = -\ddot{x} = 3x + 6\dot{x} - u - \omega, \tag{24}$$

(4) By substituting (21) into (24) and according to (14), the following can be obtained:

$$\begin{aligned} \ddot{e}(t) &= \frac{d^2(v-x)}{dt^2} = -\ddot{x} \\ &= 3x + 6\dot{x} - K\left(\frac{1}{T_i}\int_0^t e(t)\,dt + e + T_d\dot{e}\right) - \omega \\ &= -37e - 21.2\dot{e} + 3v - 2\int_0^t e(t)\,dt - \omega \end{aligned} \tag{25}$$

(5) According to (23), the substitution relationships of the variables are set as

$$\dot{e}_0(t) = e(t) = e_1(t), \tag{26}$$

$$\dot{e}(t) = e_2(t) \tag{27}$$

(6) After arrangement, the control system of (16) can be converted into a new cascaded integral form as follows:

$$\begin{aligned} &\dot{e}_0 = e_1, \quad e_0(0) = 0 \\ &\dot{e}_1 = e_2, \quad e_1(0) = v(0) \\ &\dot{e}_2 = -2e_0 - 37e_1 - 21.2e_2 + 3v - \omega, \quad e_2(0) = 0 \\ &y = v - e_1 \end{aligned} \tag{28}$$

In (28), the new system states are e_0, e_1, and e_2. Then, the new system of the cascaded integral form in (28) can be directly controlled with the ADRC controller in Section 2. The control reference is still the unit step signal of $v = 1$ from time 1 and initial value 0. The adopted ADRC controller is in ((1)-(6)). The ADRC parameters for the best control effect can be found as $r = 30$ and $h_0 = 0.3$; $\beta_{01} = 100$, $\beta_{02} = 300$, $\beta_{03} = 1000$, and $\delta = 0.05$; $\alpha_1 = 0.75$, $\alpha_2 = 1.25$, $\beta_1 = 100$, $\beta_2 = 10$, and $\delta_1 = 0$; $b_0 = 1$ with a trial and error method. The discrete time step is $h = 0.01$.

5.2. The ADRC Control Effect of the Converted System. When the disturbance is $\omega = 0$ and the ADRC controller parameters are set according to Section 5.1, the control effect for the unit step input $v = 1$ at time 1 is shown in Figure 6. Then, for the same control condition, Figure 6 also shows the control effect: (1) under a periodical disturbance of $\omega = \sin(t)$; (2) under a periodical and white noise disturbances of $\omega = \sin(t) + \gamma n(t)$. The simulation shows that the ADRC can achieve a nonovershoot unit step tracking control while satisfying the rapidity under all the disturbances. This control effect is very close to that of no disturbance. Thus, the ADRC control for the converted cascaded integral system is feasible.

5.3. The ADRC Control Effect under Model Parameter Error of 100%. Assume that the model parameters a_1 and a_2 and other parameters K, T_i, and T_d in (17) cannot be accurately estimated. For example, these parameters are reduced to 100% error from their true values. Then, under the same ADRC controller and its parameters as well as the same unit step input $v = 1$ at time 1, Figure 7 shows the control effect: (1) under a periodical disturbance of $\omega = \sin(t)$; (2) under a periodical and white noise disturbances of $\omega = \sin(t) + \gamma n(t)$. The simulation shows that the ADRC controller can still achieve a nonovershoot unit step tracking control while satisfying the rapidity under all the disturbances. This is because the parameter errors of the object model can also be seen as a disturbance and then be estimated and compensated by the ESO and compensator. Thus, the noncascaded integral system with an inaccurate object model and parameter can still achieve the conversion and ADRC control, as long as the system order can be determined. This is also an advantage of the ADRC controller.

If the model parameters a_1 and a_2 and other parameters K, T_i, and T_d are increased to 100% error from their true values, the simulation shows that the ADRC controller can still achieve the above control effect under the same ADRC controller and disturbances. This will not be repeated here. The reason is also the same. The results also show that the cascaded integral ADRC control system has a strong adaptive ability. Thus, the antidisturbance ability is good enough to let the controller parameters invariable. This ability can resist the periodical, white noise, and inaccurate model disturbance during the step response while keeping controller and its parameters unchanged.

5.4. The Comparing PID Control for the Original System. A PID controller can also be designed to control the system of (18) as

$$\begin{aligned} &e = v - x, \\ &u = K\left(\frac{1}{T_i}\int_0^t e(t)\,dt + e + T_d\dot{e}\right) \end{aligned} \tag{29}$$

The parameters of the best control effect can be found as $K = 34$, $T_i = 17$, and $T_d = 0.8$ with a trial and error method.

When the disturbance is $\omega = 0$, the control effect for the unit step input $v = 1$ at time 1 is shown in Figure 8. Then, for the same control condition, Figure 8 also shows

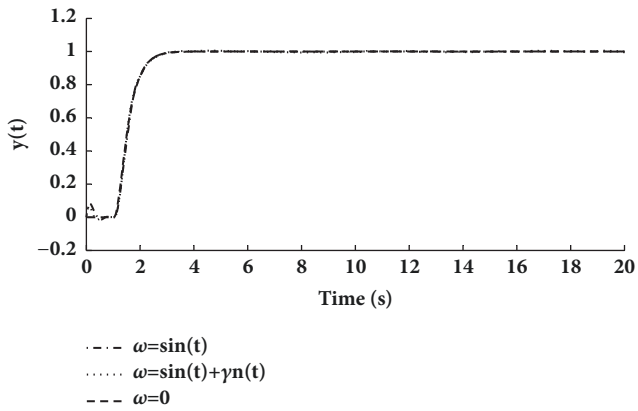

Figure 6: The ADRC control effect with parameters $r = 30$ and $h_0 = 0.3$; $\beta_{01} = 100$, $\beta_{02} = 300$, $\beta_{03} = 1000$, and $\delta = 0.05$; $\alpha_1 = 0.75$, $\alpha_2 = 1.25$, $\beta_1 = 100$, $\beta_2 = 10$, and $\delta_1 = 0$; $b_0 = 1$ for (a) under no disturbance; (b) under a periodical disturbance of $\omega = \sin(t)$; (c) under a periodical and white noise disturbances of $\omega = \sin(t) + \gamma n(t)$.

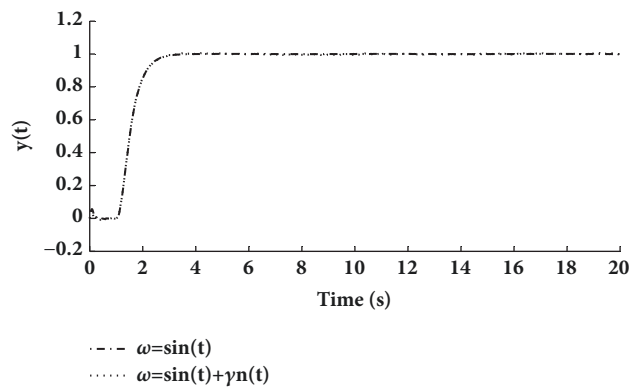

Figure 7: The control effect under the same ADRC controller and its parameters with 100% model error when there is (a) a periodical disturbance of $\omega = \sin(t)$; (b) a periodical and white noise disturbances of $\omega = \sin(t) + \gamma n(t)$.

the control effect: (1) under a periodical disturbance of $\omega = \sin(t)$; (2) under a periodical and white noise disturbance of $\omega = \sin(t) + \gamma n(t)$. The results show that the PID controller can only achieve an overshoot unit step tracking control under no disturbance. The antidisturbance ability of the PID controller is very limited for the periodical and white noise disturbances if its parameters cannot be adjusted adaptively. Thus, compared with Figures 6-7, the ADRC for the converted system has a much better adaptive ability and control effect than that of the PID controller.

6. Application in the Oscillation Control of a Flexible Arm

Example 3. For the oscillation control of a flexible arm, its mathematical model can be formulated as the following parallel system of (30). The oscillation is decomposed according to its frequency spectrum. Assume that the first three terms of the oscillation are as follows:

$$\begin{aligned}
\ddot{x}_1 &= -\omega_1^2 x_1 - 2\xi_1\omega_1\dot{x}_1 + b_1 u = \frac{1}{176}u \\
\ddot{x}_2 &= -\omega_2^2 x_2 - 2\xi_2\omega_2\dot{x}_2 + b_2 u \\
&= -4.2849x_2 - 0.01242\dot{x}_2 + \frac{1.28}{176}u \\
\ddot{x}_3 &= -\omega_3^2 x_3 - 2\xi_3\omega_3\dot{x}_3 + b_3 u \\
&= -561.69x_3 - 0.1422\dot{x}_3 + \frac{0.03}{176}u \\
y &= x_1 + x_2 + x_3
\end{aligned} \tag{30}$$

In it, x_1, x_2, and x_3 are the oscillation of the fundamental, second harmonic, and third harmonic frequency $(\omega_1, \omega_2, \omega_3)$ for the flexible arm, respectively, ξ_1, ξ_2, ξ_3 is the elastic coefficient of each frequency, u is the added control signal to suppress the oscillation of the flexible arm, and $b_i, i = 1, 2, 3$, is the coefficient of each control signal.

Here, the control reference is to make the system output $y = x_1 + x_2 + x_3$ close to 0 as soon as possible, that is, $y^* = 0$, and its initial value is assumed as $y(0) = 1.0$. As (30) is an underdriven control system, adopting a single-input single-output ADRC controller directly cannot achieve its control effect. Because of the coupling relationship among

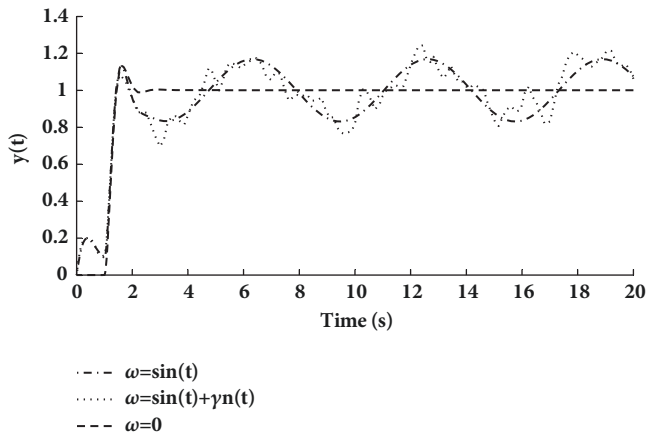

Figure 8: The PID control effect with the parameters $K = 34$, $T_i = 17$, and $T_d = 0.8$ for (a) under no disturbance; (b) under a periodical disturbance of $\omega = \sin(t)$; (c) under a periodical and white noise disturbances of $\omega = \sin(t) + \gamma n(t)$.

the three oscillation variables, the control effect either cannot be achieved by adopting three ADRC controllers in parallel.

According to (30), it has $\omega_1 = 0$, $\omega_2 = 2.07$, $\omega_3 = 23.7$, and $\xi_1 = \xi_2 = \xi_3 = 0.003$. Taking the constructing intermediate variables $\omega_0 = 0.18079747$, and $\xi_0 = 0.003$, (30) can be converted into a system of following form:

$$\begin{aligned}
\ddot{x}_1 &= -\omega_0^2 x_1 - 2\xi_0\omega_0\dot{x}_1 + b_1 u - \left(\omega_1^2 - \omega_0^2\right) x_1 \\
&\quad - 2\left(\xi_1\omega_1 - \xi_0\omega_0\right)\dot{x}_1 = \frac{1}{176}u \\
\ddot{x}_2 &= -\omega_0^2 x_2 - 2\xi_0\omega_0\dot{x}_2 + b_2 u - \left(\omega_2^2 - \omega_0^2\right) x_2 \\
&\quad - 2\left(\xi_2\omega_2 - \xi_0\omega_0\right)\dot{x}_2 \\
&= -\omega_0^2 x_2 - 2\xi_0\omega_0\dot{x}_2 + \frac{1.28}{176}u - \left(2.07^2 - \omega_0^2\right) x_2 \\
&\quad - 2\left(0.003 \cdot 2.07 - \xi_0\omega_0\right)\dot{x}_2 \\
\ddot{x}_3 &= -\omega_0^2 x_3 - 2\xi_0\omega_0\dot{x}_3 + b_3 u - \left(\omega_3^2 - \omega_0^2\right) x_3 \\
&\quad - 2\left(\xi_3\omega_3 - \xi_0\omega_0\right)\dot{x}_3 \\
&= -\omega_0^2 x_3 - 2\xi_0\omega_0\dot{x}_3 + \frac{0.03}{176}u - \left(23.7^2 - \omega_0^2\right) x_3 \\
&\quad - 2\left(0.003 \cdot 23.7 - \xi_0\omega_0\right)\dot{x}_3 \\
y &= x_1 + x_2 + x_3
\end{aligned} \tag{31}$$

By combining the similar terms in each line of (31), the following can be obtained:

$$\begin{aligned}
\ddot{y} &= -\omega_0^2 y - 2\xi_0\omega_0\dot{y} + bu + \omega(x, \dot{x}) \\
\omega(x, \dot{x}) &= -\sum_1^3 \left[\left(\omega_i^2 - \omega_0^2\right) x_i - 2\left(\xi_i\omega_i - \xi_0\omega_0\right)\dot{x}_i\right] \\
b &= \sum_1^3 b_i = 0.012972
\end{aligned} \tag{32}$$

After this transformation, the proposed algebraic substitution method and its structure can convert (32) into to a cascaded integral system according to ((8)-(17)), which will not be repeated here. Then, the ADRC controller can be directly and easily used to control the converted cascaded integral system. As long as the coefficients $\omega_0^2, 2\xi_0\omega_0$ in (32) change in a certain range, the ADRC controller can always suppress the oscillation of the flexible arm well.

The adjusting parameters of the ADRC controller are taken as $h_0 = 0.05$; $\beta_{01} = 100$, $\beta_{02} = 200$, $\beta_{03} = 800$, and $\delta = 0.05$; $\alpha_1 = 0.75$, $\alpha_2 = 1.25$, $\beta_1 = 100$, $\beta_2 = 10$, and $\delta_1 = 0$; $b_0 = 0.003$ with a trial and error method. As for the r, it can take a proper big value $r = 2.5$ for a good control effect. Although the flexible arm model has $b = 0.012972$, the b is not necessarily known in a real system. Thus, it is better to regard the b as an adjustable parameter. The dividing frequency of the flexible arm model is taken as $\omega_1^2 = 0$, $\omega_2^2 = 4.2849$, and $\omega_3^2 = 561.69$ according to (30), respectively. The simulation results are shown in Figure 9. The oscillation can be suppressed quickly without overshoot, and the control signal is small and reasonable.

If the frequencies of the flexible arm model are changed into $\omega_1^2 = 0$, $\omega_2^2 = 1$, $\omega_3^2 = 16$ or $\omega_1^2 = 0$, $\omega_2^2 = 9$, and $\omega_3^2 = 1600$, the simulation results are very similar to that of Figure 9. The controller parameters for the three cases are fixed, but the oscillation mode change is so large, which shows that the ADRC controller plays a very good antioscillation and adaptive effect with the proposed noncascaded integral ADRC control system.

7. Conclusions

This paper begins with an introduction to the ADRC method and its typical form for the cascaded integral system. Since the real systems do not necessarily have the cascaded integral form, this paper proposes an algebraic substitution method and its structure, which can convert a noncascaded integral system into the cascaded integral form. By this way, the converted system can be controlled directly and easily using

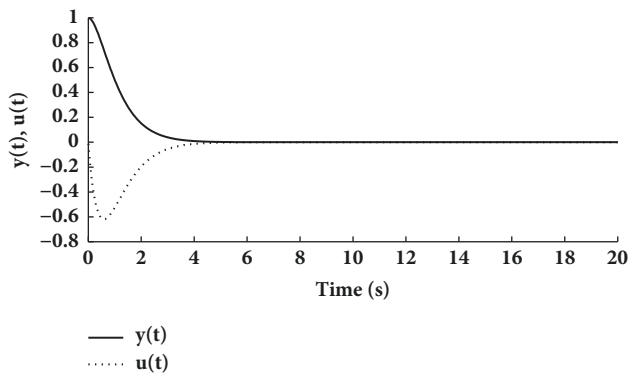

Figure 9: The oscillation control of a flexible arm under $\omega_1^2 = 0$, $\omega_2^2 = 4.2849$, and $\omega_3^2 = 561.69$ with the noncascaded integral ADRC control system.

the ADRC method. Meanwhile, a numerical example and the oscillation control of a flexible arm are simulated to demonstrate the conversion and its control effect. In addition, the control effect of the numerical example is compared between the converted ADRC control system and the PID control system under a variety of disturbances while keeping the two controllers and their parameters invariable.

The research results show the following: (1) This converting method is feasible not only in theory but also in simulation. (2) For the converted cascaded integral system, the ADRC can achieve a nonovershoot tracking control while satisfying the rapidity under many forms of disturbances, and the ADRC controller also has a better control effect than that of the PID controller. (3) For the noncascaded integral system with inaccurate model and parameter, the ADRC can still achieve the conversion and its control effect as long as the system order can be determined. (4) The model and parameter error can also be seen as a disturbance and then be estimated and compensated by the ESO and compensator. This is also an advantage of the proposed method. (5) This paper presents an approach to transform the noncascaded integral system into the cascaded integral form when necessary. (6) This converting method solves the ADRC control problem of some noncascaded integral systems in theory and simulation. It also expands the application scope of the ADRC method.

The theory is a guide before practice. In future, the converting method for non-PID control system should also be researched, and the application to a more complex system can be implemented.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

This work is supported by the NSFC projects of China under Grants no. 61403250, no. 51779136, and no. 51509151, the bureau project of China under Grant no. 2015HT056, and the Science Commission of Shanghai under Grant no. 13510501600.

References

[1] J. J. Rubio, "Structure control for the disturbance rejection in two electromechanical processes," *Journal of The Franklin Institute*, vol. 353, no. 14, pp. 3610–3631, 2016.

[2] H. Dong, Q. Hu, and G. Ma, "Dual-quaternion based fault-tolerant control for spacecraft formation flying with finite-time convergence," *IEEE Transactions on Control Systems Technology*, vol. 61, pp. 87–94, 2016.

[3] E. Sorrentino Ramirez and P. Villafuerte, "Effect of the Control of Generators and Turbines on the Transient Stability of a Power System," *IEEE Latin America Transactions*, vol. 14, no. 3, pp. 1227–1234, 2016.

[4] J. J. Rubio, "Hybrid controller with observer for the estimation and rejection of disturbances," *ISA Transactions®*, vol. 65, pp. 445–455, 2016.

[5] J. T. A. Vianna, L. R. Araujo, and D. R. R. Penido, "High Impedance Fault Area Location in Distribution Systems Based on Current Zero Sequence Component," *IEEE Latin America Transactions*, vol. 14, no. 2, pp. 759–766, 2016.

[6] J. J. Rubio, G. Ochoa, R. Balcazar, and J. Pacheco, "Uniform stable observer for the disturbance estimation in two renewable energy systems," *ISA Transactions®*, vol. 58, pp. 155–164, 2015.

[7] X. Yang, J. Cui, D. Lao, D. Li, and J. Chen, "Input Shaping enhanced Active Disturbance Rejection Control for a twin rotor multi-input multi-output system (TRMS)," *ISA Transactions*, vol. 62, pp. 287–298, 2016.

[8] R. Shi, T. He, J. Peng, Y. Zhang, and W. Zhuge, "System design and control for waste heat recovery of automotive engines based on Organic Rankine Cycle," *Energy*, vol. 102, pp. 276–286, 2016.

[9] M. Ran, Q. Wang, and C. Dong, "Stabilization of a class of nonlinear systems with actuator saturation via active disturbance rejection control," *Automatica*, vol. 63, pp. 302–310, 2016.

[10] S. Xingling and W. Honglun, "Back-stepping active disturbance rejection control design for integrated missile guidance and control system via reduced-order ESO," *ISA Transactions®*, vol. 57, pp. 10–22, 2015.

[11] T. Jiang, C. Huang, and L. Guo, "Control of uncertain nonlinear systems based on observers and estimators," *Automatica*, vol. 59, pp. 35–47, 2015.

[12] W. Tan, Y. Hao, and D. Li, "Load frequency control in deregulated environments via active disturbance rejection," *International Journal of Electrical Power & Energy Systems*, vol. 66, pp. 166–177, 2015.

[13] Z. Lei and C. Guo, "Disturbance rejection control solution for ship steering system with uncertain time delay," *Ocean Engineering*, vol. 95, pp. 78–83, 2015.

[14] J. Li, X. Qi, Y. Xia, F. Pu, and K. Chang, "Frequency domain stability analysis of nonlinear active disturbance rejection control system," *ISA Transactions®*, vol. 56, pp. 188–195, 2015.

[15] J. Song, L. Wang, G. Cai, and X. Qi, "Nonlinear fractional order proportion-integral-derivative active disturbance rejection control method design for hypersonic vehicle attitude control," *Acta Astronautica*, vol. 111, article no. 5364, pp. 160–169, 2015.

[16] K. Chang, Y. Xia, K. Huang, and D. Ma, "Obstacle avoidance and active disturbance rejection control for a quadrotor," *Neurocomputing*, vol. 190, pp. 60–69, 2016.

[17] D. Li, C. Li, Z. Gao, and Q. Jin, "On active disturbance rejection in temperature regulation of the proton exchange membrane fuel cells," *Journal of Power Sources*, vol. 283, pp. 452–463, 2015.

[18] Z.-L. Zhao and B.-Z. Guo, "On active disturbance rejection control for nonlinear systems using time-varying gain," *European Journal of Control*, vol. 23, pp. 62–70, 2015.

[19] S. Balajiwale, H. Arya, and A. Joshi, "Study of performance of ADRC for longitudinal control of MAV," *IFAC-PapersOnLine*, vol. 49, no. 1, pp. 585–590, 2016.

[20] G. Shen, Y. Xia, and H. Sun, "A 6DOF mathematical model of parachute in Mars EDL," *Advances in Space Research*, vol. 55, no. 7, pp. 1823–1831, 2015.

[21] Q. Chen, L. Li, M. Wang, and L. Pei, "The precise modeling and active disturbance rejection control of voice coil motor in high precision motion control system," *Applied Mathematical Modelling*, vol. 39, no. 19, pp. 5936–5948, 2015.

[22] Y. Shen, K. Shao, W. Ren, and Y. Liu, "Diving control of autonomous underwater vehicle based on improved active disturbance rejection control approach," *Neurocomputing*, vol. 173, pp. 1377–1385, 2016.

[23] Y. Wu and Q. Zheng, "ADRC or adaptive controller—a simulation study on artificial blood pump," *Computers in Biology and Medicine*, vol. 66, pp. 135–143, 2015.

[24] J. Han, *Active Disturbance Rejection Control Technique–the Technique for Estimating and Compensating the Uncertainties*, The National Defence Industry Press, Beijing, China, 2009.

[25] J. Q. Han, "From PID to active disturbance rejection control," *IEEE Transactions on Industrial Electronics*, vol. 56, no. 3, pp. 900–906, 2009.

[26] Z. Gao, "Active disturbance rejection control: a paradigm shift in feedback control system design," in *Proceedings of the American Control Conference*, pp. 2399–2405, Minneapolis, Minn, USA, June 2006.

[27] A. Isidori, *Nonlinear Control Systems*, Springer, New York, NY, USA, 3rd edition, 1995.

[28] Y. Huang and W. Xue, "Active disturbance rejection control: Methodology and theoretical analysis," *ISA Transactions®*, vol. 53, no. 4, pp. 963–976, 2014.

[29] M. Ramírez-Neria, H. Sira-Ramírez, R. Garrido-Moctezuma, and A. Luviano-Juárez, "Linear active disturbance rejection control of underactuated systems: the case of the Furuta pendulum," *ISA Transactions®*, vol. 53, no. 4, pp. 920–928, 2014.

[30] Z. Gao, "Scaling and bandwidth-parameterization based controller tuning," in *Proceedings of the American Control Conference*, pp. 4989–4996, Denver, Colo, USA, June 2003.

[31] D. Wu and K. Chen, "Frequency-domain analysis of nonlinear active disturbance rejection control via the describing function method," *IEEE Transactions on Industrial Electronics*, vol. 60, no. 9, pp. 3906–3914, 2013.

[32] D. Li, P. Ding, and Z. Gao, "Fractional active disturbance rejection control," *ISA Transactions*, vol. 62, pp. 109–119, 2016.

[33] X. Shao, J. Liu, W. Yang, J. Tang, and J. Li, "Augmented nonlinear differentiator design," *Mechanical Systems Signal Processing*, vol. 90, pp. 268–284, 2017.

[34] H. Feng and B.-Z. Guo, "Active disturbance rejection control: Old and new results," *Annual Reviews in Control*, vol. 44, pp. 238–248, 2017.

[35] Y. Yu, H. Wang, N. Li, Z. Su, and J. Wu, "Automatic carrier landing system based on active disturbance rejection control with a novel parameters optimizer," *Aerospace Science and Technology*, vol. 69, pp. 149–160, 2017.

[36] W. Xue and Y. Huang, "Performance analysis of active disturbance rejection tracking control for a class of uncertain LTI systems," *ISA Transactions®*, vol. 58, pp. 133–154, 2015.