Numerical Analysis of Stress Fields and Crack Growths in the Floor Strata of Coal Seam for Longwall Mining

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This paper predicts the possibility of water inrush from a confined aquifer under the action of mining activities and water pressure. The study uses numerical analyses to evaluate stress redistribution and crack growth which result from coal extraction operations. Two models are presented in this study. By simplifying the distribution of the disturbed vertical stress on the coal seam and floor around a working face, a model is established to analyze the additional stresses in the floor strata induced by mining activities. And some distribution features of all the additional stress components are described. By using the superposition principle in fracture mechanics, another model is developed to analyze the crack growth in the floor strata under the action of disturbed stresses and water pressure. And the stress intensity factors at the crack tip are presented and the process of crack growth is obtained in the advancement of a working face. Because of discretizing only loading areas and crack surfaces, the present methods can obtain the accurate numerical results. Finally, some suggestions are made for preventing the water inrush from a confined aquifer.

1. Introduction

In the coalfields of North China, coal seams are underlain by Ordovician limestone, which is strongly karstified and highly permeable. In most cases, the limestone aquifer contains an abundant supply of water and is confined. The activities of mining these coal seams are threatened with the confined water from the Ordovician limestone. Water inrush accidents result in heavy losses and casualties [1, 2]. With coal mining being executed in deeper sites, the pressure of confined water becomes larger and this type of water inrushes may occur more frequently.

In order to avoid water inrushes and keep the mines safe, large-scale dewatering or depressurizing of the Ordovician aquifer was chosen. In practice, dewatering causes serious environmental problems such as surface subsidence and underground water pollution and reduces the amount of water available for supply in the area. To solve this dilemma, some researches were devoted to make full use of the uncontaminated karst water obtained from dewatering for water supply [3]. Another measure to prevent water inrushes is to use the floor strata between the coal seam and the aquifer as a protective rock pillar. However, once the strength of the floor strata is difficult to resist the invasion of high-pressure water, the confined water floods into working faces. In some cases, grouting into the rock layers is used to improve the strength of floor strata to resist the invasion of confined water. By taking these measures, the water in the Ordovician limestone can be preserved to some extent and the coal seams can be extracted in a relatively safe environment.

As early as in 1979, the field investigations of the floor strata were executed in Fengfeng coalfield, China, where mining activities were threatened by confined aquifers [4]. After that, more field investigations of floor strata were performed in different types of mines. In Jingxing coalfield, China, it was found that the joints in floor strata were impermeable before mining, and confined water passed through the joints and flooded into the mines after mining [5]. This phenomenon may be explained by the joint growth induced by mining activities and water pressure from a confined aquifer. Additionally, the test of water injection in boreholes was used for understanding the permeability variations in...

2.1. Yue’s Solution Based Numerical Method for Elastic Fields in a Layered Medium. Xiao et al. [14] developed a numerical method for assessing the elastic fields in an arbitrarily depth-homogeneous medium under complex loads using Yue’s solution. Yue’s solution [15] is a fundamental singular solution for the generalized Kelvin problems of a multilayered elastic solid of infinite extent subject to concentrated point body vectors. The stresses and displacements at any points of a layer medium are described as

\[
\sigma_{ij}(Q) = \int_{S} \sigma_{ijk}^{*}(Q, P) t_k(P) dS(P), \quad i, j, k = x, y, z \quad (1a)
\]

\[
\mathbf{u}_i(Q) = \int_{S} \mathbf{u}_{ijk}^{*}(Q, P) t_k(P) dS(P), \quad i, k = x, y, z \quad (1b)
\]

where \(\sigma_{ijk}^{*}(Q, P)\) and \(\mathbf{u}_{ijk}^{*}(Q, P)\) are, respectively, stresses and displacements of Yue’s solution for the field point \(Q\) due to the unit force along the \(k\) direction at the source point \(P\); \(t_k(P)\) is the traction at the source point \(P\); the integral domain \(S\) is the loading area. It should be noted that the subscript \(k\) is a dummy index.

The corresponding computer program LayerSmart3D was written in FORTRAN. The techniques adopted in LayerSmart3D primarily involve the discretization of the loading area into a finite number of quadrilateral elements. Values of the loads are inputted at the node points of the discretized area. Numerical verification of LayerSmart3D indicates that numerical solutions of high accuracy can be efficiently calculated for elastic fields induced by the distributed loads in a layered medium.
2.3. The Analytical Methods of Crack Problems in Layered Media. In the following, the two numerical methods mentioned above, i.e., LayerSmart3D and LayerDDM3D, and the superposition principle in fracture mechanics are utilized to analyze crack problems in a layered medium. LayerSmart3D is firstly employed to obtain the stress fields of a layered medium without a crack under the action of distributed loadings within layered media. Using the superposition principle [18], the tractions, whose values are equal to the mining-induced stress and which have opposite directions, are then applied on the crack surfaces in the layered medium without the above distributed loadings. LayerDDM3D is finally employed to obtain the discontinuous displacements of the crack surfaces under the action of the above tractions. The SIF values at the crack tip can be calculated by using the discontinuous displacements of the crack surfaces and the relationship between SIFs and discontinuous displacements.

3. The Additional Stresses of Floor Strata without Cracks Induced by Mining

3.1. Geological and Mining Conditions of the Working Face. Jing et al. [4] investigated the deformation and failure of floor strata in mining the coal seam threatened by confined water in No. 2 Coal Mine of Fengfeng coalfield. No. 2710 working face in this mine was chosen for field measurements. Table 1 describes the roof and floor strata of the extracted coal seam. The coal seam is about 48 m away from the Ordovician limestone in which the water pressure ranges from 1.5 to 2 MPa. The working face is located at a depth of about 145 m and the longwall panel is 90 m long and 350 m wide. The coal seam is 1.5 m thick and the dip angle is about 1–3°. For the sake of simplicity, the rock strata are treated as horizontal media in developing a mechanical model. The working face was advanced along the strike direction of the coal seam. From August 1979 to February 1980, comprehensive measurements of floor deformation and failure were performed in the advancing process of the working face. In the field measurements, large amounts of data were obtained and will be further employed to analyze the water inrush from confined aquifers here.

3.2. A Model of Analyzing Additional Stresses of Floor Strata Induced by Mining

3.2.1. The Mining-Disturbed Area on the Interface of Coal Seam and Floor. Figure 1 illustrates the disturbed area ABCD on the interface between the mined coal seam and the floor. It is assumed that the domain out of the area ABCD is not disturbed by coal mining. According to the in situ investigations, the disturbed area ABCD is 150 m long and
150 m wide. The vertical stress on the interface of the coal seam and the floor strata, which are not disturbed by mining, can be calculated using the following formula:

\[ p_0 = \gamma h = 25 \times 145 = 3625 \text{kN/m}^2 \]  

(3)

where \( \gamma \) is the average unit weight of the overlying strata and \( h \) is the depth of the extracted coal seam below the ground surface.

Mining activities induce the stress concentration on the coal pillars and floor around the working face. In the field investigations at No. 2710 working face, the disturbed area and stress concentration on the coal seams were obtained. Based on these data, we develop a model for the analysis of the influence of coal mining on floor strata.

The vertical stresses along different cross sections on the disturbed area are assumed by using the measured data. The concentration factors of the vertical stress ahead of and behind the working face along the cross section \( A_1-A_2 \) are taken to be \( k_1 = 3 \) and \( k_3 = 1 \), respectively. And the stress concentration factors along the cross sections \( B_1-B_2 \) and \( C_1-C_2 \) are taken to be \( k_3 = 3 \). On the floor strata near the roadways and the working face, the stresses are highly concentrated and the stress concentration factor is taken to be \( k_3 = 3.5 \). The stress concentration factors along other cross sections can be obtained by using the methods of linear or quadratic interpolation.

Figure 2 illustrates the process of obtaining the additional vertical stress on the interface of coal seam and floor along the cross section \( A_1-A_2 \) shown in Figure 1. It can be found that the additional vertical stress in Figure 2(c) is obtained by subtracting out the weight \( p_0 \) of the overburden above the mined coal seam in Figure 2(b) from the vertical pressure in Figure 2(a). Figure 3 presents the additional vertical stresses on the interface between coal seam and floor along three different cross sections. Based on the data from in situ measurements, the parameters of lengths shown in Figure 3(a) are presented as follows: \( a_1 = 40 \text{m}, a_2 = 10 \text{m}, a_3 = 5 \text{m}, a_4 = 5 \text{m}, a_5 = 30 \text{m}, \) and \( a_6 = 60 \text{m} \). The parameters of lengths shown in Figure 3(b) are presented as follows: \( L = 90 \text{m} \) and \( b_1 = 30 \text{m} \). The parameters of lengths shown in Figure 3(c) are presented as follows: \( L = 90 \text{m}, c_1 = 30 \text{m}, c_2 = 30 \text{m}, \) and \( c_3 = 30 \text{m} \).

3.2.2. Element Mesh of the Disturbed Area and Comparison of Numerical and Field Results. In order to analyze the elastic fields using the software LayerSmart3D, the disturbed area ABCD is discretized into the mesh with 2610 nodes and 8037 elements, shown in Figure 4. The additional vertical stresses on the disturbed area ABCD (see Figure 3) are applied on the nodes of the element mesh. Figure 5 illustrates the distribution of the additional vertical stress on the interface of coal seam and floor. It can be found that the distribution of the additional vertical stress shown in Figure 5 is in good agreement with the one shown in Figure 3.

After presenting the mechanical parameters of floor strata in Table 1 and the additional vertical stress in Figure 5, the
Figure 4: Element mesh of the disturbed area ABCD, the projection abcd of the calculating area at \( z = 15 \) m, and the plane at \( y = 20 \) m for calculating the stresses.

Figure 5: Additional vertical stress on the disturbed area ABCD.

Software LayerSmart3D is used to obtain the stress and displacement fields of floor strata. Table 2 lists the vertical displacements at some points in the floor strata from numerical analyses and in situ measurements. It can be found that the displacements from the two methods are much close to each other if the differences of the geological and mining conditions for two methods are considered.

3.3 Variations of Additional Stresses in Floor Strata Induced by Mining

3.3.1 Distribution of Additional Stresses at the Depth \( z = 15 \) m. As shown in Figure 4, the model is symmetric with respect to the coordinate plane \( Oxz \). Thus, the rectangular area, i.e., \( 60 \leq x \leq 110 \) m, \( 0 \leq y \leq 45 \) m, and \( z = 15 \) m, is chosen to calculate the additional stresses. This area has a projection \( abcd \) on the coordinate plane \( Oxy \) shown in Figure 4. Figures 6–11 illustrate the distribution of all the stress components on the calculating area.

Figures 6–8 show the variations of three normal stresses in the floor strata. In Figure 6, the rectangular area can be divided into three areas by considering the distribution features of \( \sigma_{xx} \). Below the goaf area, \( \sigma_{xx} \) is tensile and has small values. Below the coal seam in the front of the working face,
Table 2: Comparison of maximum vertical displacements from in situ measurements and numerical analyses along the advancing direction (unit: mm).

<table>
<thead>
<tr>
<th>Positions ((y, z))</th>
<th>ahead of the face wall</th>
<th>behind the face wall</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>In situ measurements</td>
<td>This study</td>
</tr>
<tr>
<td>((0, 4.9))</td>
<td>8</td>
<td>9.9995</td>
</tr>
<tr>
<td>((0, 6.2))</td>
<td>8</td>
<td>9.7000</td>
</tr>
<tr>
<td>((0, 10.2))</td>
<td>7</td>
<td>8.9472</td>
</tr>
<tr>
<td>((40, 2.4))</td>
<td>13</td>
<td>10.3224</td>
</tr>
<tr>
<td>((40, 4.7))</td>
<td>14</td>
<td>9.3394</td>
</tr>
<tr>
<td>((40, 7.2))</td>
<td>14</td>
<td>9.6763</td>
</tr>
</tbody>
</table>

**Figure 7:** Contours of the additional stress \(\sigma_{yy}\) at the depth \(z = 15\) m.

**Figure 8:** Contours of the additional stress \(\sigma_{xz}\) at the depth \(z = 15\) m.

\(\sigma_{xx}\) is compressive and its minimum value is about -1.6 MPa. And below the coal pillar outside the goaf area, \(\sigma_{xx}\) is tensile and its maximum value is about 1.6 MPa. In Figure 7, the rectangular area can also be divided into three areas. Below the goaf area, \(\sigma_{yy}\) approaches zero. Below the coal seam in the front of the working face, \(\sigma_{yy}\) is tensile and its minimum value is about 1.6 MPa. And below the coal pillar outside the goaf area, \(\sigma_{yy}\) is compressive and its minimum value is about 1.6 MPa. In Figure 8, the rectangular area can be divided into two areas. \(\sigma_{xz}\) is compressive below the goaf area and is tensile away from the goaf area.

Figures 9–11 show the variations of three shear stresses in the floor strata. In Figure 9, \(\sigma_{xy}\) has a maximum value of

**Figure 9:** Contours of the additional stress \(\sigma_{xy}\) at the depth \(z = 15\) m.

**Figure 10:** Contours of the additional stress \(\sigma_{xz}\) at the depth \(z = 15\) m.
about -1.0 MPa at the point \((x, y, z) = (94 \text{ m}, 46 \text{ m}, 15 \text{ m})\) and \(\sigma_{xy}\) becomes small away from this point. In Figure 10, \(\sigma_{yz}\) is concentrated in the front of the working face and becomes small away from this zone and its maximum value is about 1.0 MPa. In Figure 11, \(\sigma_{yz}\) is concentrated below the coal pillar outside the goaf area and becomes small away from this zone.

3.3.2. Distribution of the Additional Stresses in the Plane \(y = 20 \text{ m}\)

Figures 12–17 illustrate the distribution of all the stress components in the plane \(y = 20 \text{ m}\) for different \(x\) and \(z\) values. As shown in Figure 4, this plane is along the advancing direction of the working face and has a horizontal distance of 25 m from the coal pillar outside the goaf. For the sake of simplification, the stress variations are referred to as the ones with the depth increasing in the following analyses. From these figures, we can have the following observations:

(i) At \(x = 0 \text{ m}\), all the stresses have very small values because the position is away from the maximum loadings. In the first and second layers, the stresses vary obviously with depth. In the other layers, all the stresses are very small and vary weakly with depth.

(ii) \(\sigma_{xx}\) has small values in the first layer and the absolute values of \(\sigma_{xx}\) increase in the second layer. In the first layer, \(\sigma_{yy}\) increases at \(x = 100 \text{ m}\) whilst \(\sigma_{xy}\) decreases at the other \(x\) values. In the second layer, \(\sigma_{yy}\) is negative and decreases. At \(x = 40, 81 \text{ m}\), \(\sigma_{xz}\) is positive and decreases. At \(x = 90 \text{ m}\), \(\sigma_{xz}\) is positive in the first layer and negative in the other remaining layers and decreases. \(\sigma_{xz}\) is negative and decreases at \(x = 95 \text{ m}\), and \(\sigma_{xz}\) is negative and increases at \(x = 100 \text{ m}\).

(iii) At \(x = 0, 40 \text{ m}, \sigma_{xy}\) decreases in the first, second, and third layers and increases in the remaining two layers. The variation of \(\sigma_{xy}\) at \(x = 81, 90, 95, 100 \text{ m}\) is different from the one at \(x = 0, 40 \text{ m}, \sigma_{xz}\) is negative at \(x = 0, 40 \text{ m}\) whilst \(\sigma_{xz}\) is positive at \(x = 81, 90, 95, 100 \text{ m}\). In the different layers, \(\sigma_{yz}\) first increases and then decreases for a given \(z\) value from \(x = 81\) to 100 m. The absolute values of \(\sigma_{xz}\) first increase and then decrease. \(\sigma_{yz}\) is always positive for all the \(x\) values. For \(x = 0 \text{ m}, \sigma_{yz}\) increases. Except for \(x = 0 \text{ m}, \) in the first and second layers, \(\sigma_{yz}\) first increases and then decreases. And from the third layer to the fifth layer, \(\sigma_{yz}\) first increases and then decreases.

4. A Model of Analyzing Crack Problems in Floor Strata

As shown in Figure 18, a rectangular crack is located in floor strata and is subject to mining-disturbed stresses and water pressure. A global coordinate system \(Oxyz\), which is completely the same as the one shown in Figure 1, is attached to the model to be analyzed and a local coordinate system \(O'x'y'z'\) is attached on the crack surface. The rectangular crack is 2 m long and 1 m wide.
wide (AB and CD sides) and 4 m long (AD and BC sides). Assume that the crack surfaces are parallel to the advancing direction of the working face and the crack center is located at the point \((x, y, z) = (d_1, d_2, 46 m)\).

Using the superposition principle in fracture mechanics [18], the disturbed stresses induced by mining activities can be transformed to the tractions on the crack surfaces. The disturbed stresses should be the summation of the horizontal normal stress \(\sigma_h\) induced by rock weights and the additional stresses \((\sigma_{yy}, \sigma_{xy}, \sigma_{xz})\) induced by mining activities. Additionally, the crack surfaces are subject to the water pressure \(p_w\) from confined aquifer. Thus, the crack surfaces are subjected to the following tractions:

\[
\begin{align*}
   f_x &= -\sigma_{xy} \\
   f_y &= -(\sigma_{yy} + \sigma_h) + p_w \quad (4a) \\
   f_z &= -\sigma_{yz} \\
\end{align*}
\]

where \(p_w\) is taken to be 2 MPa.

In (4b), \(\sigma_h\) is calculated by the following formula:

\[
\sigma_h = \frac{\gamma}{1-\gamma} \rho h
\]

where \(\gamma\) and \(\gamma\) are, respectively, the average values of Poisson’s ratio and unit weight of rock strata and \(h\) is the depth of the
crack center below the ground surface. Take $y = 25\text{kN/m}^2$, $v = 0.28$, and $h = 192\text{m}$. Thus, $\sigma_h = 1.8667\text{MPa}$.

The crack surface is discretized into 435 nodes and 98 nine-node elements with 60 nine-node isoparametric elements, 34 discontinuous elements of type I, and four discontinuous elements of type II. The discontinuous displacements of the crack surfaces can be obtained using the element mesh, the tractions of the crack surfaces can be calculated using the discontinuous elements of type II. The discontinuous displacements, 34 discontinuous elements of type I, and four discontinuous elements with 60 nine-node isoparametric elements can be solved using the computer code LayerDDM3D.

5. SIFs of Cracks under the Action of Water Pressure and Disturbed Stresses

5.1. General. The stress intensity factors (SIF) values at the crack tip can be calculated by using the discontinuous displacements of crack surfaces. When the crack is not located at the interfaces of multilayered solids, the crack tip field singularities and angular distributions are the same as those in a homogeneous medium and the material parameters exert an influence on the magnitudes of the SIFs. Based on the relationship of displacements and the SIFs, the formulae of the SIFs along the coordinate plane $Oxz$ are as follows:

\[
K_1 = \frac{E}{4(1-v^2)} \left( \frac{\pi}{2r} \right) \Delta \mu_{y', z'} (r, \theta = \pm \pi, \varphi = 0),
\]
\[
K_{II} = \frac{E}{4(1-v^2)} \left( \frac{\pi}{2r} \right) \Delta \mu_{x', y'} (r, \theta = \pm \pi, \varphi = 0),
\]
\[
K_{III} = \frac{E}{4(1+v)} \left( \frac{\pi}{2r} \right) \Delta \mu_{x', z'} (r, \theta = \pm \pi, \varphi = 0),
\]

where $(r, \theta, \varphi)$ is the spherical coordinates located at the crack front and $\theta$ is an angle in the coordinate plane normal to the crack line of crack front.

The following two cases are analyzed: Case 1: a rectangular crack is located in the coordinate plane $Oxz$, i.e., $d_2 = 0$; $d_1$, $d_{20} = 50, 60, 70, 80\text{m}$; $\theta = 0.28\text{radian}$; $\varphi = 0$.

5.2. Case 1: The Crack Is Located in the Coordinate Plane $Oxz$. In this case, the shear stresses $\sigma_{xy}$ and $\sigma_{yz}$ in the coordinate plane $Oxz$ are equal to zero because of the symmetry of the model. As a result, $K_{II}$ and $K_{III}$ are equal to zero. Figure 19 illustrates the variations of $K_I$ for the crack at different positions along the advancing direction of the working face. For the crack subject to water pressure, the maximum value of $K_I$ is 0.1134 MPa·m$^{1/2}$. However, for the crack subject to water pressure and disturbed stresses, the maximum value of $K_I$ is 0.1351 MPa·m$^{1/2}$ at $d_1 = 0$. With $d_1$ increasing, $K_I$ increases, increases at maximum values at $d_1 = 40, 50$, and then decreases. For $d_1 = 80$ and $\delta_y \geq 0.5\text{m}$, $K_I$ is equal to zero and $K_{II}$ along the whole BC side is equal to zero at $d_1 \geq 90$. These findings mean that the crack below the coal seam is closed and the crack below the goaf is opened under the action of disturbed stresses and water pressure. When the crack is located at some horizontal distance of 40-50m after the coal wall of the working face, there are the maximum opening displacements. In this case, the crack becomes easy to open and induces the uplifting of the confined water.

5.3. Case 2: The Crack Is Located in the Plane $y = 40\text{m}$, i.e., $d_2 = 40\text{m}$. In this case, the crack is close to the coal pillars outside the working face and has a horizontal distance of 5m from the coal pillar. As discussed in Section 3, the shear stresses induced by mining exist at this position and as a result the crack has three types of fracture modes. Figures 20–22 show the variations of $K_I$, $K_{II}$, and $K_{III}$ along the crack front BC, respectively.

In Figure 20, the $K_I$ values for $d_1 = 0, 40\text{m}$ are very close to the ones of the crack only subject to water pressure whilst...
The disturbed stresses do not exist.
The disturbed stresses do exist.

\[ K_I (\text{MPa} \cdot \text{m}^{1/2}) \]

\[ -1.5 \quad -1.0 \quad -0.5 \quad 0.0 \quad 0.5 \quad 1.0 \quad 1.5 \quad 2.0 \]

\[ x (\text{m}) \]

\[ d_2 = 40 \text{ m} \]
\[ d_1 = 0 \text{ m} \]
\[ d_1 = 30 \text{ m} \]
\[ d_1 = 50 \text{ m} \]
\[ d_1 = 70 \text{ m} \]
\[ d_1 = 90 \text{ m} \]
\[ d_1 = 110 \text{ m} \]
\[ d_1 = 130 \text{ m} \]
\[ d_1 = 150 \text{ m} \]
\[ d_1 = 170 \text{ m} \]
\[ d_1 = 190 \text{ m} \]
\[ d_1 = 210 \text{ m} \]

Figure 19: Variation of \( K_I \) values at the crack side (BC) for Case 2.

The disturbed stresses do not exist.
The disturbed stresses do exist.

\[ K_I (\text{MPa} \cdot \text{m}^{1/2}) \]

\[ -1.5 \quad -1.0 \quad -0.5 \quad 0.0 \quad 0.5 \quad 1.0 \quad 1.5 \quad 2.0 \]

\[ x (\text{m}) \]

\[ d_2 = 40 \text{ m} \]
\[ d_1 = 0 \text{ m} \]
\[ d_1 = 30 \text{ m} \]
\[ d_1 = 50 \text{ m} \]
\[ d_1 = 70 \text{ m} \]
\[ d_1 = 90 \text{ m} \]
\[ d_1 = 110 \text{ m} \]
\[ d_1 = 130 \text{ m} \]
\[ d_1 = 150 \text{ m} \]

Figure 20: Variation of \( K_I \) values at the crack side (BC) for Case 2.

\[ K_{II} (\text{MPa} \cdot \text{m}^{1/2}) \]

\[ -1.0 \quad -0.5 \quad 0.0 \quad 0.5 \quad 1.0 \quad 1.5 \quad 2.0 \]

\[ x (\text{m}) \]

\[ d_2 = 40 \text{ m} \]
\[ d_1 = 0 \text{ m} \]
\[ d_1 = 30 \text{ m} \]
\[ d_1 = 50 \text{ m} \]
\[ d_1 = 70 \text{ m} \]
\[ d_1 = 90 \text{ m} \]
\[ d_1 = 110 \text{ m} \]
\[ d_1 = 130 \text{ m} \]
\[ d_1 = 150 \text{ m} \]

Figure 21: Variation of \( K_{II} \) values at the crack side (BC) for Case 2.

\[ K_{III} (\text{MPa} \cdot \text{m}^{1/2}) \]

\[ -1.5 \quad -1.0 \quad -0.5 \quad 0.0 \quad 0.5 \quad 1.0 \quad 1.5 \quad 2.0 \]

\[ x (\text{m}) \]

\[ d_2 = 40 \text{ m} \]
\[ d_1 = 0 \text{ m} \]
\[ d_1 = 30 \text{ m} \]
\[ d_1 = 50 \text{ m} \]
\[ d_1 = 70 \text{ m} \]
\[ d_1 = 90 \text{ m} \]
\[ d_1 = 110 \text{ m} \]
\[ d_1 = 130 \text{ m} \]
\[ d_1 = 150 \text{ m} \]

Figure 22: Variation of \( K_{III} \) values at the crack side (BC) for Case 2.

In Figure 21, the \( K_{II} \) values for \( d_1 = 30 \text{ m} \) are larger than the ones of the crack only subject to water pressure. As the \( d_1 \) values increase (\( d_1 \geq 30 \text{ m} \)), the \( K_I \) values decrease. When \( d_1 \geq 90 \text{ m} \), the \( K_I \) values are equal to zero. This means that the crack directly below the goaf is opened and the crack directly below the coal seam is closed under the action of water pressure and disturbed stresses. By comparing the values shown in Figures 19 and 20, it can be found that the \( K_I \) values for \( d_2 = 40 \text{ m} \) are smaller than the ones for \( d_2 = 0 \text{ m} \) at a given \( d_1 \) value.

In Figure 21, the \( K_{II} \) values are, respectively, negative and positive for \( d_1 = 0 \text{ m} \) and \( d_1 = 110 \text{ m} \). For \( d_1 = 0 \text{ m} \), as the \( d_1 \) values increase, the \( K_{II} \) values first decrease, arrive at the minimum values at \( d_1 = 50 \text{ m} \), and then increase. For \( d_1 = 110 \text{ m} \), as the \( d_1 \) values increase, the \( K_{II} \) values first increase, arrive at the maximum values at \( d_1 = 130 \text{ m} \), and then decrease. It is believed that the \( K_{II} \) values tend to zero when the absolute values of \( d_1 \) tend to an infinite value.

In Figure 22, the \( K_{III} \) values for \( d_1 = 0 \text{ m} \) are positive. As the \( d_1 \) values increase, the \( K_{III} \) values decrease along the most part of the crack side BC, become negative values, and arrive at the minimum values at \( d_1 = 90 \text{ m} \). Then, the \( K_{III} \) values increase along the most part of the crack side BC. It is believed that the \( K_{III} \) values tend to zero when the absolute values of \( d_1 \) tend to an infinite value.

### 6. Analysis of Crack Growths Induced by Mining

#### 6.1. Case 1: The Crack Is Located in the Coordinate Plane Oxz

As discussed in Section 5.2, the crack has only mode I deformation type in this case. The crack growth can be analyzed by using the fracture criterion \( K_I = K_{IC} \). Chong et al. [19] investigated the fracture toughness \( K_{IC} \) of layered rocks by using three-point-bend specimens and presented the \( K_{IC} \) values of oil shale. Using an analogical method, the \( K_{IC} \) value of the shale in Table 1 is estimated to be 0.74 MPa \( \cdot \text{m}^{1/2} \). As shown in Figure 19, the maximum \( K_{\text{max}} \) values are about 0.2335 MPa \( \cdot \text{m}^{1/2} \) and \( K_{\text{max}} < K_{IC} \). This means that this type of crack does not grow and the confined water does not uplift under the action of water pressure and disturbed stresses.

#### 6.2. Case 2: The Crack Is Located in the Plane \( y = 40 \text{ m} \), i.e., \( d_2 = 40 \text{ m} \)

As discussed in Section 5.3, the mode I, II, and III deformations of the crack are coupled together. Growth of a crack in elastic solids subject to complex stress states can be assessed using the so-called S-criterion proposed by
Sih [20]. The S-criterion was developed on the basis of the strain energy density concept. It states that local instability is assumed to occur when the local minimum energy factor \(S_{\text{min}}\) reaches a critical value \(S_{\text{cr}}\). Using the fracture criterion and the SIF values in Section 5.3, the crack growth for Case 2 will be discussed in the following.

The strain energy density factor \(S\) of a three-dimensional crack can be defined as [21]

\[
S(\theta) = a_{11}(\theta) K_1^2 + 2a_{12} K_1 K_2 + a_{22}(\theta) K_2^2 + a_{33} (\theta) K_{\text{III}}^2
\]

(7)

where

\[
a_{11}(\theta) = \frac{1}{16\pi\mu} (3 - 4\mu - \cos \theta) (1 + \cos \theta)
\]

\[
a_{12}(\theta) = \frac{1}{8\pi\mu} \sin \theta (\cos \theta - 1 + 2\nu)
\]

\[
a_{22}(\theta) = \frac{1}{16\pi\mu} [4(1 - \nu)(1 - \cos \theta) + (3 \cos \theta - 1)(1 + \cos \theta)]
\]

\[
a_{33} = \frac{1}{4\pi\mu}
\]

where \(\mu\) is the shear modulus of elasticity, \(\nu\) is Poisson’s ratio, and \(\theta\) is an angle in the coordinate plane normal to the crack line of crack front.

Using the SIF values presented in Section 5.3 and (7), the local minimum energy factor \(S_{\text{min}}\) can be calculated. Figure 23 illustrates the variations of \(S_{\text{min}}\) values with the distance \(d_1\). At \(d_1 = 0, 90, 110, 130, 150\) m, the \(S_{\text{min}}\) values are smaller than the ones at \(d_1 = 30, 50, 60, 70\) m. From \(d_1 = 0\) to \(90\) m, \(S_{\text{min}}\) first increases, arrives at the maximum value at \(d_1 = 50\) m, and then decreases. From \(d_1 = 90\) to \(150\) m, \(S_{\text{min}}\) first decreases, arrives at the minimum value at \(d_1 = 110\) m, and then increases. It can be estimated that \(S_{\text{min}}\) tends to zero as the floor strata are far from the coal wall of the working face.

The following conclusions can be drawn:

(i) For \(d_1 \leq 90\) m, \(S_{\text{min}}\) is the largest at \(d_1 = 50\) m whilst for \(d_1 > 90\) m, \(S_{\text{min}}\) is the largest at \(d_1 = 110\) m. This means that \(S_{\text{min}}\) has two largest values before and after the coal pillar of the working face. Whether the crack grows or not depends on the \(S_{\text{cr}}\) magnitude of the shale.

(ii) The maximum values of \(S_{\text{min}}\) at \(d_1 = 50\) m are larger than the ones at \(d_1 = 110\) m. The crack growth becomes easy when the crack is located below the goaf. It has been found that the \(K_I\) values for Case 1 are the largest at \(d_1 = 40, 50\) m. This means that the growth of any cracks parallel to the coordinate plane \(Oxz\) is easy at some horizontal distance of 40-50 m behind the coal wall of the working face.

7. Conclusions

Numerical methods and mechanical models have been proposed for analyzing the stress fields and crack growths in floor strata for longwall mining. The additional stresses in the floor strata induced by mining are calculated and the crack problems subject to mining-induced stresses and water pressure are investigated. The research results presented in this paper can be summarized as follows:

(1) The additional stresses of the floor strata for longwall mining are described in detail and the distribution features of all the stress components with the depth below the floor and the distance from the working face are obtained.

(2) By analyzing the crack problems under the action of water pressure and disturbed stresses, the SIF values of the crack at different positions of floor strata are obtained. When the crack is located in the coordinate plane \(Oxz\), only the mode I deformation of the crack exists. However, when the crack is not located in the coordinate plane \(Oxz\), the mode I, II, and III deformations of the crack are coupled together.

(3) At some horizontal distance of 40-50 m behind the working face, the vertical crack in the coordinate plane \(Oxz\) has the largest values of \(K_I\) and the vertical crack, which is not located in the coordinate plane \(Oxz\), has the largest values of \(S_{\text{min}}\). At these positions, the crack growths become easier. Therefore, the field measures should be taken in the advancement of the working face to prevent the water inrushes into the goaf behind working faces.

It is the authors’ belief that the analytical methods presented in this paper can be a powerful numerical tool, which can apply to other types of cracks in the floor strata of the coal seam threatened by confined aquifers. Some of the related problems are currently under investigation by the authors for further understanding the complex mechanisms of water inrushes.

Data Availability

The [DATA TYPE] data used to support the findings of this study are included within the article.
Conflicts of Interest

The authors declare that they have no conflicts of interest.

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