Research Article

Development and Optimization of a Condition-Based Maintenance Policy with Sustainability Requirements for Production System

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In the field of condition-based maintenance, maintenance costs and system reliability criteria are the primary considerations for traditional maintenance management. These methods lack consideration of the environmental impact caused by equipment degradation, such as excessive emissions and energy consumption. In addition, because equipment degradation has various impacts on the ecological environment, companies with excessive emissions and energy consumption can receive huge fines, making it of great value to study ecoconscious maintenance strategies. In this paper, we propose a condition-based maintenance strategy considering energy consumption and carbon dioxide emissions. The major objective of the research is to extend a model which integrates ecological aspects with maintenance decision-making and optimization. The simulation and sensitivity analyses conducted verify that the model proposed can minimize total costs, as well as the environmental impact.

1. Introduction

Maintenance refers to maintaining and repairing equipment and systems to ensure they are in good condition. It is an indispensable activity related to machines, ranging from grinding or replacing a small unit to precisely moderating or repairing the whole system. When it comes to the lengthy operation of systems, such as social infrastructure, military equipment, production system, household electrical appliances, and vehicles, maintenance plays a crucial role in ensuring their safety and reliability. If the equipment and system cannot be maintained, its continuous use will lead to deterioration (e.g., aging, wear, corrosion, and creep) resulting in declining performance, reduced reliability, and even a shortened life. Within this context, condition-based maintenance strategies have been studied and widely applied by organizations as they propose an optimal maintenance policy based on the deterioration state of the system.

At present, most organizations focus solely on the relative maintenance cost when establishing a maintenance strategy for normal operation and system reliability. Minimizing cost

is the primary task, while the impact of the system on the environment is seldom considered. Take, for example, a ship company that finds belt relaxation in various motor boilers, blowers, and belt conveyors or inefficiency of door adjustment in the induced draft fan, where shutdown will not happen. The company will not undertake immediate maintenance considering the cost criteria; however, its continued use will seriously affect the critical value of combustion conditions (i.e., the excess coefficient of air). Generally, when the critical value is too large, the exhaust heat loss and electricity consumption of the draft fan increases. Conversely, when the value is too small, combustion is not complete and harmful gas emissions increase, causing serious environmental pollution.

As energy and environmental issues become increasingly severe, awareness of ecological impacts is progressing in the international community. Since the late 1980s human society has been advocating a “green era,” where protecting the environment, embracing nature, and promoting sustainable development are the center of all activities. In this context, green maintenance models have become the new trend

for saving energy and resources, implementing environmental protection, and achieving sustainable development. The questions of how to achieve environmentally-friendly and efficient production and maintenance and of how to improve resource utilization rate and competitive power to create a green image have now become main concerns of many companies. In the field of maintenance management research, traditional, condition-based maintenance is mainly concerned with maintenance costs and system reliability-optimization, while ignoring the subsequent excess wasted gas emissions and excess energy consumption. This focus does not consider the efficient use of resources, impacts on the environment, and sustainable development. Energy, resources and the environment are all in the scope of ecology [1]; hence, studying condition-based maintenance while taking ecological factors into account can be of great value.

2. Literature Review

The traditional research on CBM focused on the optimization of maintenance itself but lacked consideration of ecological aspects. With the increasingly serious energy and environmental problems over the past decade, literature on ecologically aware maintenance optimization has gradually emerged. More scholars are beginning to integrate energy conservation and emission reduction into CBM management.

Van Horenbeek et al. proposed a CBM model in 2014 with the first integration of ecological factors into decision-making and optimization [1]. The model is an economic and ecological analysis tool that incorporates several maintenance strategies (e.g., failure-based, block-based, and use-based maintenance). Using this analysis tool, we can obtain the optimal maintenance interval considering economic, ecological, and integrated aspects. The author verified the model and validated that it can be effectively applied to the cutting tool of a turning machine. The results also reveal that the ecological optimal maintenance interval is much smaller than the economic optimal maintenance interval. Integrated cost is mainly determined by economic costs, but the ecological impact cannot be ignored. In addition, a Pareto analysis showed that, by varying the preventive maintenance interval, a slight increase in the economic cost of maintenance will significantly reduce the ecological impact.

Xu and Cao [2] proposed to use energy efficiency performance as an objective for maintenance by updating periodic maintenance on energy efficiency evolution, but mainly on mastering the deterioration of machine tool (a Markov process application). Shafiee et al. [3] use the deterioration level of multibladed wind turbine system to rise/trigger the unplanned maintenance action for any failure items, while electrical energy as the useful output of this system is not fully taken into account. So energy efficiency is seen only as the auxiliary result of these previous CBM plans. In addition, Mora et al. [4] point out that energy can be saved by optimal periodic preventive maintenance. Yildirim and Nezami [5] also think that energy can impact preventive maintenance by considering energy consumption in the case

of minimal repair. To summarize, energy consumption in link with maintenance is mainly considered as an item to adapt a preventive maintenance plan in terms of scheduled one but not in case of CBM strategy.

Therefore, Hoang et al. [6] presented an optimization model for the CBM strategy in 2016 based on the stochastic process theory. This model assumes that the degradation increment is subject to the Gamma process and that the deterioration state, along with available output, determines the Energy Efficiency Index (EEI). If the EEI exceeds the threshold level in a periodic inspection, preventive maintenance action will be undertaken. The number of different maintenance actions in the objective function of this optimization model is affected by the EEI threshold, which fully integrates the energy consumption problem into the maintenance activity. Moreover, the total cost is divided into maintenance cost and energy consumption cost so that both of them are considered when making maintenance decisions. Compared with the traditional CBM strategy based on the deterioration of the system, the CBM strategy based on EEI proposed by the literature [6] can more efficiently reduce costs and save energy.

In addition to energy consumption, some scholars also integrate environmental degradation into the CBM strategy. Martorell et al. [7] proposed integrated multicriteria decision-making (IMCDM) in order to achieve better scores in reliability, availability, maintainability, safety, and cost (RAMS+C) at nuclear power plants. The proposed methodology integrates, among others, the risk to environment. The IMCDM is formulated as a multiobjective optimization where the decision variables are preventive maintenance interval, surveillance test interval, and maximum allowed outage time. Vassiliadis and Pistikopoulos [8] developed an optimization framework which takes into account the environmental risks and the operability characteristics at the early stage of process design. This framework permits the identification of optimal preventive maintenance schedules.

The excess of environmental damage generated by production systems is, in numerous situations, caused indirectly by the deterioration of those systems (e.g., degradation and/or corrosion yielding leakages and wear causing excess of energy consumption). For instance, in a nuclear power plant, the degradation of the mechanical shaft seal of the refrigeration compressor induces toxic refrigerant leakages [9]. Yan and Hua [10] showed that the degradation of machine tools causes an increase of energy consumption which can be converted to carbon emission by referring to the standard developed by the Climate Registry.

Therefore, when it is technically possible, condition-based monitoring, repair, and maintenance are the most appropriate activities to be adopted. It allows monitoring the degradation level and the resulting environmental damage in order to take the appropriate preventive actions and limit the risk of penalties and sometimes catastrophes. Chouikhi et al. [11] pointed out that, with the global enhancement of environmental awareness, many countries have introduced policies of severely punishing companies with excess emissions, which is overlooked by the traditional CBM strategy.

This means the exhaust emission may have exceeded the standard before maintenance. In order to remedy this problem, Chouikhi et al. proposed a modeling method of CBM in 2012 based on probabilistic statistical theory to integrate environmental issues into maintenance actions. The two parameters (the time interval from equipment in good condition to the time point at which the exhaust gas leaks go beyond the warning line, in addition to the time interval from exceeding the warning line to the downtime) are assumed to be subject to a joint probability density function, so then the probability of preventive maintenance (when the leak exceeds the critical line), corrective maintenance, and inspection can be obtained; thus, we get the expected total cost per time unit during the renewal cycle.

Tlili et al. [12] made some improvement on the method [11] and proposed a threshold lower than the critical level fixed by legislation as a criterion for judging whether to perform preventive maintenance, thereby avoiding the huge amount of economic penalties for excess exhaust gas. Furthermore, Tlili et al. [12] integrated the penalty cost for excess emissions into the cost function, making the total cost in the objective function more comprehensive. In addition, they compared periodic inspection with nonperiodic inspection, with the results showing that the nonperiodic inspection policy can save more costs and induce less environmental pollution.

Traditional CBM does not take into account the environmental attributes of maintenance activities. In view of this problem, although some scholars integrate environmental issues into CBM strategy, the factors involved are not comprehensive. The literature [11, 12] quantified the mutual effects of exhaust emissions and maintenance activities but was limited to the emissions. In reality, maintenance activities also induce energy consumption, so the decision optimization should also consider this ecological factor to reflect how maintenance activities and ecology impact each other. This paper proposes a CBM strategy that combines energy consumption and CO₂ emissions with the deterioration state of the system, establishing a single-unit system maintenance optimization model based on the probabilistic characteristics of the combination of energy consumption and CO₂ emissions. The model makes maintenance decisions based on the threshold of these two ecological factors; in this way, the eco-factors are integrated into maintenance activities, enabling the maintenance activities both to save economic costs and to effectively conserve energy and protect the environment, which can ultimately achieve the goal of sustainable development.

3. Integrative Condition-Based Maintenance Model for Single-Unit Systems Based on Sustainability Requirements

3.1. Problem Description and Model Assumptions

3.1.1. Problem Description. This paper considers the ecological impact of a production system assimilated to a single-unit system as it continuously degrades; that is, it

considers excessive carbon dioxide emissions and excessive energy consumption. Since in many cases environmental degradation generated by the production system has a direct (or indirect) relationship with the system degradation, we can study carbon dioxide emissions and energy consumption in replacement of the deterioration state of the system [11]. It is assumed that the system's carbon dioxide emissions and energy consumption are, respectively, subject to a Wiener process and that the degradation level of the system can only be known through inspection. This paper concerns a system that is periodically inspected. If inspection results show that any one of the system's CO₂ emissions or energy consumption exceeds the critical value, U_i (U_1 refers to the critical level of CO₂ emissions fixed by legislation and U_2 refers to the critical level of energy consumption fixed by legislation), the enterprise has to pay a heavy penalty. Meanwhile, a tremendous impact on the environment will be averted, along with a waste of resources, as when a CM action must be performed. In order to lower the possibility of getting into such an unfavorable situation, a threshold level L_i lower than U_i is set to trigger a PM action (L_1 refers to PM threshold for CO₂ emission and L_2 refers to PM threshold for energy consumption). In case an inspection shows that any of the environmental indicators exceeds the PM threshold, a PM action should be undertaken; otherwise, no maintenance action will be taken before the next inspection.

3.1.2. Model Assumptions. Notations section explains the notations related to the model of the CBM strategy considering carbon dioxide emissions and energy consumption, for ease of understanding.

For the proposed model, the following assumptions are made.

- (1) CO₂ emissions and energy consumption of the system are subject to two independent Wiener processes.
- (2) The time of CO₂ emissions and energy consumption from the moment when the degradation hits the PM threshold level L to the instant at which it reaches the critical level are subject to two independent IG processes.
- (3) The inspection is periodic and the period is $\tau = k\Delta t$, $k = 1, 2, 3, \dots$, where Δt is unit of time.
- (4) The inspection is perfect and time spent is negligible.
- (5) The actions of preventive maintenance and corrective maintenance return the equipment to its original state and the maintenance time is negligible.
- (6) According to the inspection results, only one of the three following events is possible: undertake a PM action, undertake a CM action, or do nothing before next inspection.
- (7) All maintenance costs, inspection costs, and penalty costs are included in the average cost and assumed to be constants.
- (8) A renewal cycle is from current time to the end of maintenance.

- (9) The time from current time to the instant at which the degradation hits the PM threshold and the time from the instant at which it reaches the critical level follow independent normal distributions.

3.2. Model Establishment. Tlili et al. [12] assume the environmental degradation process of the equipment (namely, the carbon dioxide emission of the equipment) obeys a Wiener process:

$$X(t) = \lambda t + \sigma W(t), \quad t > 0, \quad (1)$$

where $X(t)$ denotes standard Brownian motion, λ is the drift coefficient, σ is the diffusion coefficient, and the mean and variance $X(t)$ are expressed by μt and $\sigma^2 t$, respectively. $X(t)$ has the following properties:

(a) $X(0) = 0$.

(b) For any time sequence t_i , $i = 1, 2, 3, \dots, n$ ($0 < t_1 < t_2 < \dots < t_n$), the random increments $X(t_1)$, $X(t_2 - t_1)$, $\dots$, $X(t_n - t_{n-1})$, where $X(t_j - t_{j-1}) = X(t_j) - X(t_{j-1})$, are independent, and any $X(t) - X(r)$, $t > 0$, $r > 0$ follows a normal distribution $N(0, \sigma^2 |t - r|)$.

(c) The paths of $X(t)$ are continuous with probability one.

The time T_U from current time to the moment when the degradation reaches the critical level, U , follows the inverse Gaussian distribution and can be defined as

$$T_U = \inf \{t \geq 0 \mid X(t) \geq U\}. \quad (2)$$

The pdf of T_U is expressed as

$$f_{T_U}(t) = \frac{U}{\sqrt{2\pi t^3 \sigma^2}} \exp\left(-\frac{(U - \lambda t)^2}{2\sigma^2 t}\right) \quad (3)$$

and the cdf of T_U is given by

$$F_{T_U}(t) = \Phi\left(\frac{-U + \lambda t}{\sigma \sqrt{t}}\right) + \exp\left(\frac{2\lambda U}{\sigma^2}\right) \Phi\left(\frac{-U - \lambda t}{\sigma \sqrt{t}}\right). \quad (4)$$

The time T_S from the instant at which the degradation hits the PM threshold level L to the moment when it reaches the critical level, U , can be defined as

$$\Delta T_S = T_U - T_L = \inf \{s \geq 0 \mid X(t + s) \geq U\}. \quad (5)$$

If $L < U$, otherwise $\Delta T_S = 0$.

The degradation increments are independent based on property (b) of the Wiener process; therefore, if $L < U$, T_S can be expressed as follows:

$$T_S = T_U - T_L = \inf \{s \geq 0 \mid X(t + s) \geq U\} \\ = \inf \{s \geq 0 \mid X(t + s) \geq U - L\}. \quad (6)$$

The pdf of T_S is written as follows:

$$f_{T_U - T_L}(t) = \frac{U - L}{\sqrt{2\pi t^3 \sigma^2}} \exp\left(-\frac{(U - L - \lambda t)^2}{2\sigma^2 t}\right). \quad (7)$$

And the cdf of T_S is given by

$$F_{T_U - T_L}(t) \\ = \Phi\left(\frac{-(U - L) + \lambda t}{\sigma \sqrt{t}}\right) \\ + \exp\left(\frac{2\lambda(U - L)}{\sigma^2}\right) \Phi\left(\frac{-(U - L) - \lambda t}{\sigma \sqrt{t}}\right). \quad (8)$$

This paper will take into account energy consumption on the basis of Tlili et al.'s literature. Assuming the CO2 emissions and energy consumption of the system follow two independent Wiener processes, the threshold level of CO2 emissions and energy consumption are L_1 and L_2 , respectively, and the critical level of CO2 emissions and energy consumption fixed by legislation are U_1 and U_2 , respectively.

The Wiener process can be expressed as

$$X_i(t) = \lambda_i t + \sigma_i W(t), \quad t > 0, \quad (9)$$

where $X(t)$ denotes standard Brownian motion, λ_i is the drift coefficient, σ_i is the diffusion coefficient, and the mean and variance of $X(t)$ are expressed as μt and $\sigma^2 t$, respectively.

The time T_{U_i} from current time to the instant at which the degradation hits the critical level, U_i , follows the inverse Gaussian distribution and can be defined as

$$T_{U_i} = \inf \{t \geq 0 \mid X_i(t) \geq U_i\}, \quad i = 1, 2. \quad (10)$$

The pdf of T_{U_i} is expressed as

$$f_{T_{U_i}}(t) = \frac{U_i}{\sqrt{2\pi t^3 \sigma_i^2}} \exp\left(-\frac{(U_i - \lambda_i t)^2}{2\sigma_i^2 t}\right), \quad i = 1, 2. \quad (11)$$

And the cdf of T_{U_i} is given by

$$F_{T_{U_i}}(t) = \Phi\left(\frac{-U_i + \lambda_i t}{\sigma_i \sqrt{t}}\right) \\ + \exp\left(\frac{2\lambda_i U_i}{\sigma_i^2}\right) \Phi\left(\frac{-U_i - \lambda_i t}{\sigma_i \sqrt{t}}\right). \quad (12)$$

The time T_{S_i} from the instant at which the degradation hits the PM threshold level L to the moment when it reaches the critical level U can be defined as

$$T_{S_i} = T_{U_i} - T_{L_i} = \inf \{s_i \geq 0 \mid X(t + s_i) \geq U_i\}. \quad (13)$$

If $L_i < U_i$, otherwise $T_{S_i} = 0$.

Based on property (b) of the Wiener process, the degradation increments are independent; hence, if $L < U$, T_{S_i} can be written as follows:

$$\begin{aligned} T_{S_i} &= T_{U_i} - T_{L_i} = \inf \{S_i \geq 0 \mid X(t + S_i) \geq U_i\} \\ &= \inf \{S_i \geq 0 \mid X(t + S_i) \geq U_i - L_i\}, \quad i = 1, 2. \end{aligned} \quad (14)$$

The pdf of T_{S_i} is expressed as

$$f_{T_{U_i}-T_{L_i}}(t) = \frac{U_i - L_i}{\sqrt{2\pi t^3 \sigma_i^2}} \exp\left(-\frac{(U_i - L_i - \lambda_i t)^2}{2\sigma_i^2 t}\right). \quad (15)$$

And the cdf of T_{S_i} is given by

$$\begin{aligned} F_{T_{U_i}-T_{L_i}}(t) &= \Phi\left(\frac{-(U_i - L_i) + \lambda_i t}{\sigma_i \sqrt{t}}\right) \\ &+ \exp\left(\frac{2\lambda_i (U_i - L_i)}{\sigma_i^2}\right) \Phi\left(\frac{-(U_i - L_i) - \lambda_i t}{\sigma_i \sqrt{t}}\right). \end{aligned} \quad (16)$$

We suppose that periodic inspections are conducted at times $i\tau$, $i = 1, 2, 3, \dots$. The objective is to determine the optimal PM threshold level of CO2 emission L_1^* , optimal PM threshold level of energy consumption L_2^* , and the interinspection period τ^* .

According to classical renewal theory, the average long-run cost rate per time unit can be expressed over a renewal cycle S , defined as the interval between successive maintenance actions.

Hence, the expression of the average long-run cost rate is given by

$$EC(L_1, L_2, \tau) = \frac{E[C_T(L_1, L_2, \tau)]}{E[C_{\text{cycle}}(L_1, L_2, \tau)]}, \quad (17)$$

where $E[C_T]$ denotes the expected cost during the cycle and $E[C_{\text{cycle}}]$ denotes the expected renewal cycle length.

3.2.1. The Expected Renewal Cycle Length. Suppose the renewal period of the system denotes $T = i\tau$, $i = 1, 2, 3, \dots$. Since the system is considered to be as good as new after both maintenance actions (PM or CM), there are three possible events determined by the system's CO2 emissions and energy consumption, which are now described.

(a) The time T_{L_1} from current time to the instant at which the CO2 emission $X_1(t)$ reaches the PM threshold level is found $(i-1)\tau < T_{L_1} \leq i\tau$, while the time T_{L_2} from current time to the instant at which the energy consumption $X_2(t)$ reaches the PM threshold level is found $T_{L_2} \geq i\tau$. At $i\tau$, PM or CM action is performed since the measured CO2 emission $X_1(t)$ has exceeded the PM threshold level L_1 . Express this event as $A_1(i, \tau) = \{(i-1)\tau < T_{L_1} \leq i\tau, T_{L_2} \geq i\tau\}$, where $i = 1, 2, 3, \dots$, and its probability of occurrence is given by

$$\begin{aligned} P(A_1(i, \tau)) &= \int_{(i-1)\tau}^{i\tau} \int_{i\tau}^{\infty} F(u, v) du dv = \int_{(i-1)\tau}^{i\tau} \int_{i\tau}^{\infty} f_{T_{L_1}}(u) f_{T_{L_2}}(v) du dv \\ &= \int_{(i-1)\tau}^{i\tau} \frac{L_1}{\sqrt{2\pi u^3 \sigma_1^2}} \exp\left(-\frac{(L_1 - \lambda_1 u)^2}{2\sigma_1^2 u}\right) du \int_{i\tau}^{\infty} \frac{L_2}{\sqrt{2\pi v^3 \sigma_2^2}} \exp\left(-\frac{(L_2 - \lambda_2 v)^2}{2\sigma_2^2 v}\right) dv. \end{aligned} \quad (18)$$

(b) The time T_{L_2} from current time to the instant at which the energy consumption $X_2(t)$ reaches the PM threshold is found, $(i-1)\tau < T_{L_2} \leq i\tau$, while the time T_{L_1} from current time to the instant at which the CO2 emission $X_1(t)$ reaches the PM threshold is found, $T_{L_1} \geq i\tau$. At $i\tau$, PM or CM

action is performed since the measured energy consumption $X_2(t)$ has exceeded the PM threshold level L_2 . Express this event as $A_2(i, \tau) = \{(i-1)\tau < T_{L_2} \leq i\tau, T_{L_1} \geq i\tau\}$, where $i = 1, 2, 3, \dots$, and its probability of occurrence is given by

$$\begin{aligned} P(A_2(i, \tau)) &= \int_{i\tau}^{\infty} \int_{(i-1)\tau}^{i\tau} F(u, v) du dv = \int_{i\tau}^{\infty} \int_{(i-1)\tau}^{i\tau} f_{T_{L_1}}(u) f_{T_{L_2}}(v) du dv \\ &= \int_{i\tau}^{\infty} \frac{L_1}{\sqrt{2\pi u^3 \sigma_1^2}} \exp\left(-\frac{(L_1 - \lambda_1 u)^2}{2\sigma_1^2 u}\right) du \int_{(i-1)\tau}^{i\tau} \frac{L_2}{\sqrt{2\pi v^3 \sigma_2^2}} \exp\left(-\frac{(L_2 - \lambda_2 v)^2}{2\sigma_2^2 v}\right) dv. \end{aligned} \quad (19)$$

(c) The time from current time to the instant at which $X_1(t)$ and $X_2(t)$ reaches the threshold level are found $(i-1)\tau < T_{L_1} \leq i\tau$ and $(i-1)\tau < T_{L_2} \leq i\tau$, respectively. At $i\tau$, PM or CM action is performed since both $X_1(t)$ and $X_2(t)$ have exceeded the PM threshold level L_1 .

Express this event as $A_3(i, \tau) = \{(i-1)\tau < T_{L_1} \leq i\tau, (i-1)\tau < T_{L_2} \leq i\tau\}$, where $i = 1, 2, 3, \dots$, and its probability of occurrence is given by

$$\begin{aligned} P(A_3(i, \tau)) &= \int_{(i-1)\tau}^{i\tau} \int_{(i-1)\tau}^{i\tau} F(u, v) du dv = \int_{(i-1)\tau}^{i\tau} \int_{(i-1)\tau}^{i\tau} f_{T_{L_1}}(u) f_{T_{L_2}}(v) du dv \\ &= \int_{(i-1)\tau}^{i\tau} \frac{L_1}{\sqrt{2\pi u^3 \sigma_1^2}} \exp\left(-\frac{(L_1 - \lambda_1 u)^2}{2\sigma_1^2 u}\right) du \int_{(i-1)\tau}^{i\tau} \frac{L_2}{\sqrt{2\pi v^3 \sigma_2^2}} \exp\left(-\frac{(L_2 - \lambda_2 v)^2}{2\sigma_2^2 v}\right) dv. \end{aligned} \quad (20)$$

Finally, the expected renewal cycle length is expressed as

$$\begin{aligned} E[C_{\text{cycle}}(L_1, L_2, \tau)] &= \sum_{i=1}^{\infty} (P(A_1(i, \tau)) + P(A_2(i, \tau)) + P(A_3(i, \tau))) \\ &= \int_{(i-1)\tau}^{i\tau} \frac{L_1}{\sqrt{2\pi u^3 \sigma_1^2}} \exp\left(-\frac{(L_1 - \lambda_1 u)^2}{2\sigma_1^2 u}\right) du \int_{i\tau}^{\infty} \frac{L_2}{\sqrt{2\pi v^3 \sigma_2^2}} \exp\left(-\frac{(L_2 - \lambda_2 v)^2}{2\sigma_2^2 v}\right) dv \\ &\quad + \int_{i\tau}^{\infty} \frac{L_1}{\sqrt{2\pi u^3 \sigma_1^2}} \exp\left(-\frac{(L_1 - \lambda_1 u)^2}{2\sigma_1^2 u}\right) du \int_{(i-1)\tau}^{i\tau} \frac{L_2}{\sqrt{2\pi v^3 \sigma_2^2}} \exp\left(-\frac{(L_2 - \lambda_2 v)^2}{2\sigma_2^2 v}\right) dv \\ &\quad + \int_{(i-1)\tau}^{i\tau} \frac{L_1}{\sqrt{2\pi u^3 \sigma_1^2}} \exp\left(-\frac{(L_1 - \lambda_1 u)^2}{2\sigma_1^2 u}\right) du \int_{(i-1)\tau}^{i\tau} \frac{L_2}{\sqrt{2\pi v^3 \sigma_2^2}} \exp\left(-\frac{(L_2 - \lambda_2 v)^2}{2\sigma_2^2 v}\right) dv. \end{aligned} \quad (21)$$

3.2.2. Expected Total Cost within a Cycle. The expected total cost within a cycle can be expressed as

$$\begin{aligned} E[(L_1, L_2, \tau)] &= C_C P_P + C_P P_P + C_I E[N] \\ &\quad + C_{\text{CO}_2} E[T_{\xi}] + C_{\text{energy}} E[T_{\delta}]. \end{aligned} \quad (22)$$

The three first elements in the formula are PM, CM, and the inspection average costs within a cycle, respectively. $C_{\text{CO}_2} E[T_{\xi}]$ denotes the penalty cost for excess CO2 emissions within a cycle and $C_{\text{energy}} E[T_{\delta}]$ denotes the penalty cost for excess energy consumption within a cycle.

(1) *The Probability P_P That the Cycle Ends with a Preventive Maintenance Action.* Suppose a PM action is undertaken in the system at $i\tau$. There are three possible events, as follows.

(a) At $i\tau$, the CO2 emission $X_1(t)$ was higher than the PM threshold level L_1 , but lower than the critical level U_1 , while the energy consumption $X_2(t)$ has not reached the PM threshold level L_2 . Express this event as $B_1(i, \tau) = \{(i-1)\tau < T_{L_1} \leq i\tau < T_{U_1}, T_{L_2} \geq i\tau\}$, then its probability of occurrence is given by

$$\begin{aligned} P(B_1(i, \tau)) &= \int_{(i-1)\tau}^{i\tau} \int_{i\tau-u}^{\infty} \int_{i\tau}^{\infty} F(u, s, v) du ds dv \\ &= \int_{(i-1)\tau}^{i\tau} \int_{i\tau-u}^{\infty} \int_{i\tau}^{\infty} f_{T_{L_1}}(u) f_{T_{U_1}-T_{L_1}}(s) f_{T_{L_2}}(v) du ds dv \\ &= \int_{(i-1)\tau}^{i\tau} \int_{i\tau-u}^{\infty} f_{T_{L_1}}(u) \cdot f_{T_{U_1}-T_{L_1}}(s) du ds \int_{i\tau}^{\infty} f_{T_{L_2}}(v) dv \\ &= \int_{(i-1)\tau}^{i\tau} f_{T_{L_1}}(u) \left[1 - F_{T_{U_1}-T_{L_1}}(i\tau - u)\right] du \int_{i\tau}^{\infty} f_{T_{L_2}}(v) dv \\ &= \int_{(i-1)\tau}^{i\tau} \frac{L_1}{\sqrt{2\pi u^3 \sigma_1^2}} \exp\left(-\frac{(L_1 - \lambda_1 u)^2}{2\sigma_1^2 u}\right) \left[1 - \Phi\left(\frac{-(U_1 - L_1) + \lambda_1 u}{\sigma_1 \sqrt{u}}\right) - \exp\left(\frac{2\lambda_1 (U_1 - L_1)}{\sigma_1^2}\right) \cdot \Phi\left(\frac{-(U_1 - L_1) - \lambda_1 u}{\sigma_1 \sqrt{u}}\right)\right] du \int_{i\tau}^{\infty} \frac{L_2}{\sqrt{2\pi v^3 \sigma_2^2}} \exp\left(-\frac{(L_2 - \lambda_2 v)^2}{2\sigma_2^2 v}\right) dv. \end{aligned} \quad (23)$$

(b) At $i\tau$, the energy consumption $X_2(t)$ was higher than the PM threshold level L_2 , but lower than the critical level U_2 , while the CO2 emissions $X_1(t)$ has not reached the PM

threshold level L_1 . Express this event as $B_2(i, \tau) = \{(i-1)\tau < T_{L_2} \leq i\tau < T_{U_2}, T_{L_1} \geq i\tau\}$, then its probability of occurrence is given by

$$\begin{aligned}
 P(B_2(i, \tau)) &= \int_{i\tau}^{\infty} \int_{(i-1)\tau}^{i\tau} \int_{i\tau-v}^{\infty} F(u, t, v) du dv dt = \int_{i\tau}^{\infty} \int_{(i-1)\tau}^{i\tau} \int_{i\tau-v}^{\infty} f_{T_{L_1}}(u) f_{T_{L_2}}(v) f_{T_{U_2}-T_{L_2}}(t) du dv dt \\
 &= \int_{i\tau}^{\infty} f_{T_{L_1}}(u) du \int_{(i-1)\tau}^{i\tau} \int_{i\tau-v}^{\infty} f_{T_{L_2}}(v) f_{T_{U_2}-T_{L_2}}(t) dv dt \\
 &= \int_{i\tau}^{\infty} f_{T_{L_1}}(u) du \int_{(i-1)\tau}^{i\tau} f_{T_{L_2}}(v) [1 - F_{T_{U_2}-T_{L_2}}(i\tau - v)] dv \\
 &= \int_{i\tau}^{\infty} \frac{L_1}{\sqrt{2\pi u^3 \sigma_1^2}} \exp\left(-\frac{(L_1 - \lambda_1 u)^2}{2\sigma_1^2 u}\right) du \cdot \int_{(i-1)\tau}^{i\tau} \frac{L_2}{\sqrt{2\pi v^3 \sigma_2^2}} \exp\left(-\frac{(L_2 - \lambda_2 v)^2}{2\sigma_2^2 v}\right) dv \cdot \left[1 - \Phi\left(\frac{-(U_2 - L_2) + \lambda_2 v}{\sigma_1 \sqrt{v}}\right) - \exp\left(\frac{2\lambda_2 (U_2 - L_2)}{\sigma_2^2}\right) \Phi\left(\frac{-(U_2 - L_2) - \lambda_2 v}{\sigma_2 \sqrt{v}}\right)\right] dv.
 \end{aligned} \tag{24}$$

(c) At $i\tau$, both $X_1(t)$ and $X_2(t)$ were higher than the PM threshold level L_i , but lower than the critical value U_i . Express

this event as $B_3(i, \tau) = \{(i-1)\tau < T_{L_1} \leq i\tau < T_{U_1}, (i-1)\tau < T_{L_2} \leq i\tau < T_{U_2}\}$, then its probability of occurrence is given by

$$\begin{aligned}
 P(B_3(i, \tau)) &= \int_{(i-1)\tau}^{i\tau} \int_{i\tau-u}^{\infty} \int_{(i-1)\tau}^{i\tau} \int_{i\tau-v}^{\infty} F(u, s, v, t) du ds dv dt \\
 &= \int_{(i-1)\tau}^{i\tau} \int_{i\tau-u}^{\infty} \int_{(i-1)\tau}^{i\tau} \int_{i\tau-v}^{\infty} f_{T_{L_1}}(u) f_{T_{U_1}-T_{L_1}}(s) f_{T_{L_2}}(v) f_{T_{U_2}-T_{L_2}}(t) du ds dv dt = \int_{(i-1)\tau}^{i\tau} \int_{i\tau-u}^{\infty} f_{T_{L_1}}(u) \\
 &\cdot f_{T_{U_1}-T_{L_1}}(s) du ds \int_{(i-1)\tau}^{i\tau} \int_{i\tau-v}^{\infty} f_{T_{L_2}}(v) f_{T_{U_2}-T_{L_2}}(t) dv dt = \int_{(i-1)\tau}^{i\tau} f_{T_{L_1}}(u) \\
 &\cdot [1 - F_{T_{U_1}-T_{L_1}}(i\tau - u)] du \cdot \int_{(i-1)\tau}^{i\tau} f_{T_{L_2}}(v) [1 - F_{T_{U_2}-T_{L_2}}(i\tau - v)] dv = \int_{(i-1)\tau}^{i\tau} \frac{L_1}{\sqrt{2\pi u^3 \sigma_1^2}} \exp\left(-\frac{(L_1 - \lambda_1 u)^2}{2\sigma_1^2 u}\right) du \\
 &\cdot \left[1 - \Phi\left(\frac{-(U_1 - L_1) + \lambda_1 u}{\sigma_1 \sqrt{u}}\right) - \exp\left(\frac{2\lambda_1 (U_1 - L_1)}{\sigma_1^2}\right) \Phi\left(\frac{-(U_1 - L_1) - \lambda_1 u}{\sigma_1 \sqrt{u}}\right)\right] du \\
 &\cdot \int_{(i-1)\tau}^{i\tau} \frac{L_2}{\sqrt{2\pi v^3 \sigma_2^2}} \exp\left(-\frac{(L_2 - \lambda_2 v)^2}{2\sigma_2^2 v}\right) dv \cdot \left[1 - \Phi\left(\frac{-(U_2 - L_2) + \lambda_2 v}{\sigma_1 \sqrt{v}}\right) - \exp\left(\frac{2\lambda_2 (U_2 - L_2)}{\sigma_2^2}\right) \Phi\left(\frac{-(U_2 - L_2) - \lambda_2 v}{\sigma_2 \sqrt{v}}\right)\right] dv.
 \end{aligned} \tag{25}$$

Therefore, the probability P_P that the cycle ends with a preventive maintenance action is expressed as

$$\begin{aligned}
 P_P(L_1, L_2, \tau) = & \sum_{i=1}^{\infty} [P(B_1(i, \tau)) + P(B_2(i, \tau)) + P(B_3(i, \tau))] = \sum_{i=1}^{\infty} \left\{ \int_{(i-1)\tau}^{i\tau} \frac{L_1}{\sqrt{2\pi u^3 \sigma_1^2}} \exp\left(-\frac{(L_1 - \lambda_1 u)^2}{2\sigma_1^2 u}\right) \right. \\
 & \cdot \left[1 - \Phi\left(\frac{-(U_1 - L_1) + \lambda_1 u}{\sigma_1 \sqrt{u}}\right) - \exp\left(\frac{2\lambda_1(U_1 - L_1)}{\sigma_1^2}\right) \Phi\left(\frac{-(U_1 - L_1) - \lambda_1 u}{\sigma_1 \sqrt{u}}\right) \right] du \int_{i\tau}^{\infty} \frac{L_2}{\sqrt{2\pi v^3 \sigma_2^2}} \\
 & \cdot \exp\left(-\frac{(L_2 - \lambda_2 v)^2}{2\sigma_2^2 v}\right) dv + \int_{i\tau}^{\infty} \frac{L_1}{\sqrt{2\pi u^3 \sigma_1^2}} \exp\left(-\frac{(L_1 - \lambda_1 u)^2}{2\sigma_1^2 u}\right) du \int_{(i-1)\tau}^{i\tau} \frac{L_2}{\sqrt{2\pi v^3 \sigma_2^2}} \exp\left(-\frac{(L_2 - \lambda_2 v)^2}{2\sigma_2^2 v}\right) \\
 & \cdot \left[1 - \Phi\left(\frac{-(U_2 - L_2) + \lambda_2 v}{\sigma_2 \sqrt{v}}\right) - \exp\left(\frac{2\lambda_2(U_2 - L_2)}{\sigma_2^2}\right) \Phi\left(\frac{-(U_2 - L_2) - \lambda_2 v}{\sigma_2 \sqrt{v}}\right) \right] dv \\
 & + \int_{(i-1)\tau}^{i\tau} \frac{L_1}{\sqrt{2\pi u^3 \sigma_1^2}} \exp\left(-\frac{(L_1 - \lambda_1 u)^2}{2\sigma_1^2 u}\right) \\
 & \cdot \left[1 - \Phi\left(\frac{-(U_1 - L_1) + \lambda_1 u}{\sigma_1 \sqrt{u}}\right) - \exp\left(\frac{2\lambda_1(U_1 - L_1)}{\sigma_1^2}\right) \Phi\left(\frac{-(U_1 - L_1) - \lambda_1 u}{\sigma_1 \sqrt{u}}\right) \right] du \\
 & \cdot \int_{(i-1)\tau}^{i\tau} \frac{L_2}{\sqrt{2\pi v^3 \sigma_2^2}} \exp\left(-\frac{(L_2 - \lambda_2 v)^2}{2\sigma_2^2 v}\right) \\
 & \cdot \left[1 - \Phi\left(\frac{-(U_2 - L_2) + \lambda_2 v}{\sigma_2 \sqrt{v}}\right) - \exp\left(\frac{2\lambda_2(U_2 - L_2)}{\sigma_2^2}\right) \Phi\left(\frac{-(U_2 - L_2) - \lambda_2 v}{\sigma_2 \sqrt{v}}\right) \right] dv \Bigg\}. \quad (26)
 \end{aligned}$$

The sum of P_P and P_C equals 1 as follows:

$$P_P + P_C = 1 \quad (27)$$

Probability P_C that the cycle ends with a corrective maintenance action is

$$P_C(L_1, L_2, \tau) = 1 - P_P(L_1, L_2, \tau). \quad (28)$$

(2) *The Expected Number of Inspections during a Cycle.* The expected renewal cycle length can be obtained from (1), which is here considered to be the interval between consecutive maintenance actions. It is assumed that the maintenance action to be performed is determined after the inspection using CO2 emissions or energy consumption and that the maintenance time is negligible. Since the inspection is periodic and the time interval is τ , while the renewal period is an integral multiple of τ , the expected number of inspections during a cycle can be expressed by

$$E[N] = \frac{E_{\text{cycle}}[(L_1, L_2, \tau)]}{\tau}. \quad (29)$$

(3) *The Average Time for Paying a Penalty during a Cycle.* At $i\tau$, in case an inspection result shows that CO2 emissions or energy consumption has exceeded the critical level, the company has to pay a heavy penalty for the excess amount. The discussion here is divided into two circumstances.

(a) *The Average Time for Generation of an Excess Amount of CO2 Emissions.* At $i\tau$, CO2 emissions $X_1(t)$ have exceeded U_1 ; that is, $(i-1)\tau < T_{L_1} \leq T_{U_1} < i\tau$. $T_{\xi}(i) = i\tau - T_{U_1}$ represents the time from the moment when the amount of CO2 emitted exceeds the critical level to the instant of the inspection. Express this event as

$$\begin{aligned}
 C_1(i, \tau) \\
 = \{(i-1)\tau < T_{L_1} \leq T_{U_1} < i\tau \mid T_{\xi}(i) = i\tau - T_{U_1}\}. \quad (30)
 \end{aligned}$$

Then the expected time for paying a penalty is given by

$$\begin{aligned}
 E[T_{\xi}] = & \sum_{i=1}^{\infty} P\{(i-1)\tau < T_{L_1} \leq T_{U_1} < i\tau \mid T_{\xi}(i) = i\tau - T_{U_1}\} = \sum_{i=1}^{\infty} \int_{(i-1)\tau}^{i\tau} \int_0^{z-u} \int_{(i-1)\tau}^z F(u, s) \cdot (i\tau - z) du ds dz \\
 = & \sum_{i=1}^{\infty} \int_{(i-1)\tau}^{i\tau} \int_0^{z-u} \int_{(i-1)\tau}^z f_{T_{L_1}}(u) f_{T_{U_1}-T_{L_1}}(s) \cdot (i\tau - z) du ds dz = \sum_{i=1}^{\infty} \int_{(i-1)\tau}^{i\tau} (i\tau - z) \int_{(i-1)\tau}^z f_{T_{L_1}}(u) F_{T_{U_1}-T_{L_1}}(z-u) du dz
 \end{aligned}$$

$$\begin{aligned}
&= \sum_{i=1}^{\infty} \int_{(i-1)\tau}^{i\tau} (i\tau - z) \int_{(i-1)\tau}^z \frac{L_1}{\sqrt{2\pi u^3 \sigma_1^2}} \exp\left(-\frac{(L_1 - \lambda_1 u)^2}{2\sigma_1^2 u}\right) \\
&\cdot \left[\Phi\left(\frac{-(U_1 - L_1) + \lambda_1 u}{\sigma_1 \sqrt{u}}\right) + \exp\left(\frac{2\lambda_1 (U_1 - L_1)}{\sigma_1^2}\right) \Phi\left(\frac{-(U_1 - L_1) - \lambda_1 u}{\sigma_1 \sqrt{u}}\right) \right] du dz.
\end{aligned} \tag{31}$$

(b) *The Average Time for Excess Amount of Energy Consumption.* At $i\tau$, the energy consumption $X_2(t)$ has exceeded U_2 ; that is, $(i-1)\tau < T_{L_2} \leq T_{U_2} < i\tau$. $T_\delta(i) = i\tau - T_{U_2}$ represents the average time between the instant when the amount of energy consumption exceeds the critical level and the moment of the inspection. Express this event as

$$\begin{aligned}
&C_2(i, \tau) \\
&= \{(i-1)\tau < T_{L_2} \leq T_{U_2} < i\tau \mid T_\delta(i) = i\tau - T_{U_2}\}.
\end{aligned} \tag{32}$$

So the expected time of paying a penalty is given by

$$\begin{aligned}
E[T_\delta] &= \sum_{i=1}^{\infty} P\{(i-1)\tau < T_{L_2} \leq T_{U_2} < i\tau \mid T_\delta(i) = i\tau - T_{U_2}\} = \sum_{i=1}^{\infty} \int_{(i-1)\tau}^{i\tau} \int_0^{w-v} \int_{(i-1)\tau}^w F(v, t) \cdot (i\tau - w) \\
&= \sum_{i=1}^{\infty} \int_{(i-1)\tau}^{i\tau} \int_0^{w-v} \int_{(i-1)\tau}^w f_{T_{L_2}}(v) f_{T_{U_2}-T_{L_2}}(t) \cdot (i\tau - w) dv dt dw = \sum_{i=1}^{\infty} \int_{(i-1)\tau}^{i\tau} (i\tau - w) \\
&\cdot \int_{(i-1)\tau}^w f_{T_{L_2}}(v) F_{T_{U_2}-T_{L_2}}(w-v) dv dw = \sum_{i=1}^{\infty} \int_{(i-1)\tau}^{i\tau} (i\tau - w) \int_{(i-1)\tau}^w \frac{L_2}{\sqrt{2\pi u^3 \sigma_2^2}} \exp\left(-\frac{(L_2 - \lambda_2 v)^2}{2\sigma_2^2 v}\right) \\
&\cdot \left[\Phi\left(\frac{-(U_2 - L_2) + \lambda_2 v}{\sigma_2 \sqrt{v}}\right) + \exp\left(\frac{2\lambda_2 (U_2 - L_2)}{\sigma_2^2}\right) \Phi\left(\frac{-(U_2 - L_2) - \lambda_2 v}{\sigma_2 \sqrt{v}}\right) \right] dv dw.
\end{aligned} \tag{33}$$

3.2.3. *Expected Average Cost per Time Unit during the Renewal Cycle.* Based on the formula above, the expected average cost per time unit during the renewal cycle is given by

$$\begin{aligned}
EC(L_1, L_2, \tau) &= \frac{E[C_T(L_1, L_2, \tau)]}{E[C_{\text{cycle}}(L_1, L_2, \tau)]} \\
&= \frac{C_C P_P + C_P P_P + C_I E[N] + C_{\text{CO}_2} E[T_\xi] + C_{\text{energy}} E[T_\delta]}{E[C_{\text{cycle}}(L_1, L_2, \tau)]},
\end{aligned} \tag{34}$$

where the numerator is expressed as

$$\begin{aligned}
&C_P \sum_{i=1}^{\infty} [P(B_1(i, \tau)) + P(B_2(i, \tau)) + P(B_3(i, \tau))] + C_C \left(1 \right. \\
&\left. - \sum_{i=1}^{\infty} [P(B_1(i, \tau)) + P(B_2(i, \tau)) + P(B_3(i, \tau))] \right) + C_I \\
&\cdot \frac{\sum_{i=1}^{\infty} [(P(A_1(i, \tau)) + P(A_2(i, \tau)) + P(A_3(i, \tau))) i\tau]}{\tau}
\end{aligned}$$

$$\begin{aligned}
&+ C_{\text{CO}_2} \sum_{i=1}^{\infty} \int_{(i-1)\tau}^{i\tau} (i\tau - z) \\
&\cdot \int_{(i-1)\tau}^z \frac{L_1}{\sqrt{2\pi u^3 \sigma_1^2}} \exp\left(-\frac{(L_1 - \lambda_1 u)^2}{2\sigma_1^2 u}\right) \\
&\cdot \left[\Phi\left(\frac{-(U_1 - L_1) + \lambda_1 u}{\sigma_1 \sqrt{u}}\right) + \exp\left(\frac{2\lambda_1 (U_1 - L_1)}{\sigma_1^2}\right) \Phi\left(\frac{-(U_1 - L_1) - \lambda_1 u}{\sigma_1 \sqrt{u}}\right) \right] \\
&\cdot \Phi\left(\frac{-(U_1 - L_1) - \lambda_1 u}{\sigma_1 \sqrt{u}}\right) du dz \\
&+ C_{\text{energy}} \sum_{i=1}^{\infty} \int_{(i-1)\tau}^{i\tau} (i\tau - w) \\
&\cdot \int_{(i-1)\tau}^w \frac{L_2}{\sqrt{2\pi u^3 \sigma_2^2}} \exp\left(-\frac{(L_2 - \lambda_2 v)^2}{2\sigma_2^2 v}\right) \\
&\cdot \left[\Phi\left(\frac{-(U_2 - L_2) + \lambda_2 v}{\sigma_2 \sqrt{v}}\right) + \exp\left(\frac{2\lambda_2 (U_2 - L_2)}{\sigma_2^2}\right) \Phi\left(\frac{-(U_2 - L_2) - \lambda_2 v}{\sigma_2 \sqrt{v}}\right) \right] \\
&\cdot \Phi\left(\frac{-(U_2 - L_2) - \lambda_2 v}{\sigma_2 \sqrt{v}}\right) dv dw.
\end{aligned} \tag{35}$$

The denominator is given by

$$\sum_{i=1}^{\infty} [(P(A_1(i, \tau)) + P(A_2(i, \tau)) + P(A_3(i, \tau))) i \tau]. \quad (36)$$

The denominator can be further developed as follows:

$$\begin{aligned} \sum_{i=1}^{\infty} \left[\left(\int_{(i-1)\tau}^{i\tau} \frac{L_1}{\sqrt{2\pi u^3 \sigma_1^2}} \exp\left(-\frac{(L_1 - \lambda_1 u)^2}{2\sigma_1^2 u}\right) du \int_{i\tau}^{\infty} \frac{L_2}{\sqrt{2\pi v^3 \sigma_2^2}} \exp\left(-\frac{(L_2 - \lambda_2 v)^2}{2\sigma_2^2 v}\right) dv + \int_{i\tau}^{\infty} \frac{L_1}{\sqrt{2\pi u^3 \sigma_1^2}} \exp\left(-\frac{(L_1 - \lambda_1 u)^2}{2\sigma_1^2 u}\right) du \int_{(i-1)\tau}^{i\tau} \frac{L_2}{\sqrt{2\pi v^3 \sigma_2^2}} \exp\left(-\frac{(L_2 - \lambda_2 v)^2}{2\sigma_2^2 v}\right) dv \right. \\ \left. + \int_{(i-1)\tau}^{i\tau} \frac{L_1}{\sqrt{2\pi u^3 \sigma_1^2}} \exp\left(-\frac{(L_1 - \lambda_1 u)^2}{2\sigma_1^2 u}\right) du \int_{(i-1)\tau}^{i\tau} \frac{L_2}{\sqrt{2\pi v^3 \sigma_2^2}} \exp\left(-\frac{(L_2 - \lambda_2 v)^2}{2\sigma_2^2 v}\right) dv \right) i \tau \right]. \quad (37) \end{aligned}$$

3.3. Numerical Procedure. To integrate the calculations illustrated above, after inputting the parameters ΔL_1 , ΔL_2 , $\Delta \tau$, T_u , C_C , C_P , C_i , C_{CO_2} , and C_{energy} , the expected average cost in the renewal cycle can be computed through the numerical procedure in Figure 1.

The detailed steps of the algorithm are described as follows.

Step 1. Initialize $L_1 = 0$, $L_2 = 0$, $t = 0$, $i = 0$ and let $L_1 = L_1 + \Delta L_1$, $L_2 = L_2 + \Delta L_2$, and $t = t + \Delta \tau$.

Step 2. Inspect amount of CO2 emissions $X_1(it)$ and energy consumption $X_2(it)$ at it .

Step 3. If $X_1(it) < L_1$ and $X_2(it) < L_2$, let $i = i + 1$ and go to Step 2; otherwise, go to Step 4.

Step 4. If $t < T_u$, let $t = t + \Delta \tau$ and go to Step 2; otherwise, go to Step 5.

Step 5. Compute $EC(L_1, L_2, it)$ for each it and store the minimum $EC_1(L_1, L_2, it)$.

Step 6. If $L_1 < U_1$, let $L_1 = L_1 + \Delta L_1$ and go to Step 2; otherwise, go to Step 7.

Step 7. Compute $EC_1(L_1, L_2, it)$ for each L_1 and store the minimum $EC_2(L_1, L_2, it)$.

Step 8. If $L_2 < U_2$, let $L_2 = L_2 + \Delta L_2$ and go to Step 2; otherwise, go to Step 9.

Step 9. Compute $EC_2(L_1, L_2, it)$ for each L_2 and store the minimum $EC_{\min}(L_1, L_2, it)$.

Step 10. Output $EC_{\min}(L_1, L_2, it)$ and the corresponding L_1, L_2, it .

Step 11. End.

The above is the fully unfolded formula for the average expected cost per time unit within the renewal cycle. Next, we use the simulation method to obtain the optimal interval of periodic inspection, the PM threshold level of CO2 emissions, and energy consumption and then give the most appropriate maintenance strategy. In addition, sensitivity analyses will be conducted on the penalty cost for excess CO2 emissions, the penalty cost for excess energy consumption, the inspection cost, the PM cost, and the CM cost.

4. Simulation Analysis and Sensitivity Analysis

4.1. Basic Assumption of the Model. The model assumes that the critical level of CO2 emissions and energy consumption fixed by legislation are known constants. The critical level of CO2 emissions will be set at 10 tons with reference to Tlili et al.'s study [12], which are presented in Table 1.

In terms of the threshold of energy consumption, it is important to note that since much equipment now uses mixed fuels to reduce environmental pollution (e.g., the mixed boiler which, apart from coal, also consumes diesel and natural gas), the consumption of a single energy source is not proportional to the overall CO2 emissions of the system. In other words, energy consumption and CO2 emissions are independent and thus call for an integrative consideration in this study. Notice each threshold cannot be obtained from the other directly. Here, one energy source will be separated from mixed energy and studied, which is assumed to be coal. Given that China sets different coal limits on various companies, we refer to the coal limit of one specific company as the benchmark and assume the critical level of energy consumption is 4 tons. Again, in reference to Tlili et al.'s study, the critical level of CO2 emissions will be set to 10 tons for the simulation. Other values (PM cost C_P , CM cost C_C , inspection cost C_i , and penalty cost for excess CO2 emission C_{CO_2} , λ_1, σ_1) also refer to Tlili et al.'s study.

The pdf of the time from current time to the moment when CO2 emission reaches the critical value is given by

$$f_{T_{U_1}}(t) = \frac{10}{\sqrt{2\pi t^3 0.35^2}} \exp\left(-\frac{(10 - 1.3t)^2}{2 \cdot 0.35^2 t}\right). \quad (38)$$

Its cdf is expressed as

$$\begin{aligned} F_{T_{U_1}}(t) = \Phi\left(\frac{-10 + 1.3t}{0.35\sqrt{t}}\right) \\ + \exp\left(\frac{2 \cdot 1.3 \cdot 10}{0.35^2}\right) \Phi\left(\frac{-10 - 1.3t}{0.35\sqrt{t}}\right). \end{aligned} \quad (39)$$

The pdf of the time from the instant at which CO2 emissions hit the threshold to the moment when it reaches the critical value is given by

$$\begin{aligned} f_{T_{U_1} - T_{L_1}}(t) \\ = \frac{10 - L_1}{\sqrt{2\pi t^3 0.35^2}} \exp\left(-\frac{(10 - L_1 - 1.3t)^2}{2 \cdot 0.35^2 t}\right). \end{aligned} \quad (40)$$

TABLE 1: Known data.

C_C (USD)	C_P (USD)	C_I (USD)	C_{CO_2} (USD/week)	C_{energy} (USD/week)	U_1	U_2	λ_1	σ_1	λ_2	σ_2
900	500	100	10000	3500	10	4	1.3	0.35	0.416	0.112

FIGURE 1: The numerical procedure for the expected average cost.

Its cdf is expressed as

$$\begin{aligned}
 F_{T_{U_1}-T_{L_1}}(t) &= \Phi\left(\frac{L_1 - 10 + 1.3t}{0.35\sqrt{t}}\right) \\
 &+ \exp\left(\frac{2 \cdot 1.3(10 - L_1)}{0.35^2}\right) \Phi\left(\frac{L_1 - 10 - 1.3t}{0.35\sqrt{t}}\right). \quad (41)
 \end{aligned}$$

Therefore, the pdf of the time from current time to the moment when energy consumption reaches the critical value is given by

$$f_{T_{U_2}}(t) = \frac{4}{\sqrt{2\pi}t^3 \cdot 0.112^2} \exp\left(-\frac{(4 - 0.416t)^2}{2 \cdot 0.112^2 t}\right). \quad (42)$$

Its cdf is expressed as

$$F_{T_{U_2}}(t) = \Phi\left(\frac{-4 + 0.416t}{0.112\sqrt{t}}\right) + \exp\left(\frac{2 \cdot 0.416 \cdot 4}{0.112^2}\right) \Phi\left(\frac{-4 - 0.416t}{0.112\sqrt{t}}\right). \quad (43)$$

The pdf of the time from the instant at which energy consumption hits the threshold to the moment when it reaches the critical value is given by

$$f_{T_{U_2}-T_{L_2}}(t) = \frac{4 - L_2}{\sqrt{2\pi t^3} 0.112^2} \exp\left(-\frac{(4 - L_2 - 0.416t)^2}{2 \cdot 0.112^2 t}\right). \quad (44)$$

Its cdf is expressed as

$$F_{T_{U_2}-T_{L_2}}(t) = \Phi\left(\frac{L_2 - 4 + 0.416t}{0.112\sqrt{t}}\right) + \exp\left(\frac{0.832(4 - L_2)}{0.112^2}\right) \Phi\left(\frac{L_2 - 4 - 0.416t}{0.112\sqrt{t}}\right). \quad (45)$$

4.2. Comparative Analysis. The CO₂ emissions and energy consumption of the system are simulated using the Monte Carlo method and the Particle Swarm Optimization (PSO) algorithm is used to accelerate the operation. The optimal thresholds of CO₂ emissions and energy consumption and the time interval of periodic inspection are obtained by minimizing the expected cost per time unit during the renewal cycle.

Results obtained, which are shown in Table 2, give the optimal threshold of CO₂ emissions as 6 tons and the one of energy consumption as 3 tons and the interval for periodic inspection as 7 week, when the renewal period is 7.002 week. With the limitation of energy consumption, the time of generation of excess CO₂ emissions is relatively low (0.0181 week), compared with Tlili et al. [12], whose time of generation of it is 0.02. The average expected cost per week is \$109.406, which is much lower than the one from Tlili et al. [12], \$123.94. Therefore, the method of this paper performs better in terms of average expected cost.

4.3. Comparative Sensitivity Analysis. This section will vary the penalty cost for excess CO₂ emissions C_{CO_2} , penalty cost for excess energy consumption C_{energy} , inspection cost C_I , and CM cost C_C , respectively, compared with the results of Tlili et al.'s [12] research, to study their influence on the optimal solution and figure out which method performs better.

4.3.1. Penalty Cost for Excess CO₂ Emissions C_{CO_2} . The optimum values are obtained under different circumstances by varying the penalty cost of excess CO₂ emissions. Moreover, the comparison between the results of the paper and Tlili et al. [12] is shown in Table 3.

First, with the increase of C_{CO_2} , its threshold L_1 decreases and the probability of executing a PM action and the frequency of inspection increases, while the average renewal cycle length shrinks, possibly because the increase in penalty for CO₂ emissions forces companies to tightly monitor the emission of CO₂. Companies try to minimize the possibility that CO₂ emissions exceed the critical level by reducing its threshold along with the interval of periodic inspections, so as to avoid the heavy penalty induced. Moreover, it is worth noticing that, in every pair, the time of generation excess CO₂ emission is less than the Tlili et al.'s [12], which may results from the limitation of energy consumption. Furthermore, compared to the method from Tlili et al. [12], in every pair, the expected average cost is much lower.

4.3.2. Penalty Cost for Excess Energy Consumption C_{energy} . The optimum values are obtained under different circumstances by varying the value of the penalty for excessive energy consumption, and the comparison between the results of the paper and Tlili et al. [12] is shown in Table 4.

It can be inferred that, as the penalty cost for excess energy consumption increases, its threshold L_2 decreases and the probability of executing PM action decreases. It is worth noting that the results keep relatively still in case the penalty of excess energy consumption is 0 and \$3500. This probably is because when energy consumption reaches its critical level, the CO₂ emissions have already exceeded its critical level, whose induced penalty could counteract the advantage from the reduction of penalty for excess energy consumption. Meanwhile, the expected time of generation of excess CO₂ emissions increases, probably because the penalty of \$35000 for excess energy consumption weighs way more than the one for excess CO₂ emissions, which makes it worth of taking the cost from the penalty for excess CO₂ emissions. Besides, the table also shows that when the penalty of excess energy consumption is 0 and \$3500, the average expected cost is relatively lower than the result of Tlili et al. [12]. However, when the penalty increases to \$35000, the cost increases so rapidly that exceeds the Tlili et al.'s. We think this is because the penalty of energy consumption contributes too much compared the one of CO₂ emissions.

4.3.3. Inspection Cost C_I . The optimum values are obtained under different circumstances by varying the value of inspection costs. The comparison between the results of the paper and Tlili et al. [12] is shown in Table 5.

This result shows that, as the inspection cost increases, the threshold of CO₂ emissions L_2 decreases and the interval of periodic inspection increases, whereas the average renewal cycle length increases. Since the company extends the interval of periodic inspection to reduce the total cost, less frequent inspections increase the average time of generation of excess CO₂ emissions and energy consumption; therefore, the penalty cost for excess energy consumption contributes substantially to the average cost of the firm. Moreover, compared to Tlili et al. [12], without the limitation of threshold of energy consumption, the penalty of this paper caused by excess CO₂ emissions is much lower than that of Tlili et al.'s [12]. Therefore, the average expected cost of this paper is lower

TABLE 2: Comparison of the obtained optimal solution for CBM policy considering CO2 emission and energy consumption.

L_1		L_2	τ	P_P (%)	P_C (%)	$E[N]$	Inspection cost	$E[T_i]$ week	Penalty for excess CO2 emissions	$E[T_\delta]$ week	Penalty for excess energy consumption	$E[C_{cycle}]$ week	EC (USD)
This paper	6	3	7	82.21	1779	1.0003	100.03	0.0181	181	0	0	7.002	109.406
	2	/	7	82.24	1776	1.08	108	0.02	200	/	/	7.1	123.94

TABLE 3: Effect of C_{CO_2} .

C_{CO_2}		L_1	L_2	τ	P_p (%)	P_C (%)	$E[N]$	Inspection cost	$E[T_\xi]$ week	Penalty for excess CO2 emission	$E[T_\delta]$ week	Penalty for excess energy consumption	$E[C_{cycle}]$ week	EC (USD)
0	This paper	7	3	8	31.91	68.09	1.00	100	0.2472	0	0.001	3.5	8	66.9282
	Tlili et al. [12]	9	/	7	38.57	61.43	1.44	144	71	0	/	/	10.06	88.47
10000	This paper	6	3	7	82.21	17.79	1.0003	100.03	0.0181	181	0	0	7002	109.406
	Tlili et al. [12]	2	/	7	82.24	17.76	1.08	108	0.02	200	/	/	71	123.94
100000	This paper	5	3	3	95.75	4.25	1.92	192	0	0	0	0	5.758	119.9967
	Tlili et al. [12]	2	/	5	99.57	0.43	1.00	100	0.007	700	/	/	5	260.37

TABLE 4: Effect of C_{energy} .

C_{energy}	L_1	L_2	τ	P_p (%)	P_c (%)	$E[N]$	Inspection cost	$E[T_\xi]$ week	Penalty for excess CO2 emission	$E[T_\delta]$ week	Penalty for excess energy consumption	$E[C_{\text{cycle}}]$ week	EC (USD)
0	3	4	7	82.23	17.77	1	100	0.0181	181	0	0	7	109.024
3500	6	3	7	82.21	17.79	1.0003	100.03	0.0189	189	0	0	7002	109.406
35000	10	2	4	29.88	70.12	1.85	185	0.2472	2472	0	0	7.4024	417.85
\	2	\	7	82.24	17.76	1.08	108	0.02	200	\	\	7.1	123.94

This
paper
Tlili et
al. [12]

TABLE 5: Effect of C_I .

C_I		L_1	L_2	τ	P_P (%)	P_C (%)	$E[N]$	Inspection cost	$E[T_\xi]$ week	Penalty for excess CO2 emission	$E[T_\delta]$ week	Penalty for excess energy consumption	$E[C_{\text{cycle}}]$ week	EC (USD)
0	This paper	10	1	7	97.75	2.25	2	0	0.001	10	0	0	14	72.637
	Tlili et al. [12]	8	/	1	99.79	0.21	6.65	0	8×10^{-6}	0.08	/	/	6.65	75.35
100	This paper	6	3	7	82.21	17.79	1.0003	100.03	0.0181	181	0	0	7002	109.406
	Tlili et al. [12]	2	/	7	82.24	17.76	1.08	108	0.02	200	/	/	71	123.94
1000	This paper	3	2	8	31.92	68.08	1	1000	0.2472	2472	0.001	3.5	8	488.4407
	Tlili et al. [12]	2	/	9	4.75	95.25	1	1000	1.13	11300	/	/	9	1464.56

than the one of Tlili et al.'s [12], especially when the inspection cost increases to \$1000.

4.3.4. Corrective Maintenance Cost C_C . The optimum values are obtained under different circumstances by varying the value of the CM cost. The comparison between the results of the paper and Tlili et al. [12] is shown in Table 6.

The results indicate that as the CM cost increases, the company will lower the two thresholds so as to reduce the probability of executing CM action. Meanwhile, the interval of periodic inspections and the expected average time of excess CO2 emissions are relatively stable and the inspections keep almost similar frequency, which we think this may be because the balance point for minimizing the expected average cost and its contribution is relative smaller than that of penalty. Since the CM cost increases, as mentioned above, the probability of PM will increase due to its lower cost, which effectively control the time of excess CO2 emissions and energy consumption. Therefore, the penalty will also decrease, which is lower than the Tlili et al.'s [12] results. Thus, this paper's method performs better.

5. Conclusion and Future Directions

The original contribution of this paper is to propose a CBM policy for single-unit systems that integrates ecological factors (CO2 emissions and energy consumption) into a previous model to obtain the optimal thresholds of CO2 emission and energy consumption, as well as the interval of periodic inspection, with the objective function of minimizing average expected cost during the cycle. This paper assumes that the system's CO2 emissions and energy consumption follow two independent normal distributions and that the time from the instant at which one of them hits its corresponding threshold to the moment when it reaches the critical level is independent of the two ecological factors. Based on these two assumptions, the joint pdf of three parameters was established to compute the objective function. We conducted simulations using the Monte Carlo method and algorithm to obtain the optimal threshold of CO2 emissions, optimal threshold of energy consumption, and the interval of periodic inspections. After a sensitivity study for each variable, there were three discoveries. First, the penalty for excess CO2 emissions and excess energy consumption will affect the values of the company's thresholds. Secondly, the PM costs and CM costs will also influence the values of the two thresholds. Lastly, the inspection cost will affect the time interval of periodic inspections. Meanwhile, the comparison between results whether to take energy consumption or not indicates that only considering CO2 emissions still leads to some extra penalty, while adding energy consumption effectively reduces the penalty and probability of CM action, which could result in lower maintenance cost. Therefore, it can be concluded that this paper's method performs better.

This paper leaves much room for further study in two potential directions: (1) to extend CBM policy under periodic inspection to one under nonperiodic inspection; (2) to

extend the research object of this article to a multiunit system; and (3) to establish a model of the relationship between CO2 consumption and mixed energy consumption for a generalized model of optimization of maintenance cost.

Notations

τ :	The interval of periodic inspection
$X_1(t)$:	The Wiener process carbon dioxide emission is subject to
$X_2(t)$:	The Wiener process energy consumption is subject to
L_1 :	PM threshold level of CO2 emission
L_2 :	PM threshold level of energy consumption
U_1 :	Critical level of CO2 emission fixed by legislation
U_2 :	Critical level of energy consumption fixed by legislation
T_U :	The time T_U from present time to the instant one at which the degradation first hits the critical level U
T_S :	The time T_S from the moment when the degradation hits the PM threshold L to the instant at which it reaches the critical level U
$f_{T_U}(t)$:	Probability density function associated with T_U
$F_{T_U}(t)$:	Cumulative distribution function associated with T_U
$f_{T_U-T_L}(t)$:	Probability density function associated T_S
$F_{T_U-T_L}(t)$:	Cumulative distribution associated T_S
S :	The renewal cycle length (A cycle is from current time to the end of maintenance)
$EC(L_1, L_2, \tau)$:	The average cycle cost per time unit
$E[C_T]$:	The expected total cost within a cycle
$E[C_{\text{cycle}}]$:	The expected renewal cycle length
C_C :	Corrective maintenance (CM) action cost
C_P :	Preventive maintenance (PM) action cost
C_I :	Inspection cost
C_{CO_2} :	Penalty cost per time unit for excess CO2 emission incurred once the critical level is exceeded
C_{energy} :	Penalty cost per time unit for excess energy consumption incurred once the critical level is exceeded
P_C :	Probability that the cycle ends with a CM action
P_P :	Probability that the cycle ends with a PM action
$E[N]$:	The expected number of inspections during a cycle
$E[T_\xi]$:	The expected average time of generation of excess amount of CO2 emissions during a cycle
$E[T_\delta]$:	The expected average time of generation of excess amount of energy consumption during a cycle.

TABLE 6: Effect of C_C .

C_C		L_1	L_2	τ	P_P (%)	P_C (%)	$E[N]$	Inspection cost	$E[T_\ell]$ week	Penalty for excess CO2 emission	$E[T_\delta]$ week	Penalty for excess energy consumption	$E[C_{\text{cycle}}]$ week	EC (USD)
600	This paper	8	4	7	71.55	28.45	0.89	89	0.0181	181	0	0	6.252	105
	Tili et al. [12]	4	/	8	32.53	67.47	1	100	0.0262	262	/	/	8	116.25
900	This paper	6	3	7	82.21	17.79	1.0003	100.03	0.0181	181	0	0	7.002	109.406
	Tili et al. [12]	2	/	7	82.24	17.76	1.08	108	0.02	200	/	/	7.1	123.94
1100	This paper	4	1	7	82.23	17.77	1	100	0.0181	181	0	0	7	114.1002
	Tili et al. [12]	2	/	5	99.57	0.43	1	100	0.007	70	/	/	5	134.37

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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