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Research Article

Robust Preview Control for a Class of Uncertain Discrete-Time Lipschitz Nonlinear Systems

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This paper considers the design of the robust preview controller for a class of uncertain discrete-time Lipschitz nonlinear systems. According to the preview control theory, an augmented error system including the tracking error and the known future information on the reference signal is constructed. To avoid static error, a discrete integrator is introduced. Using the linear matrix inequality (LMI) approach, a state feedback controller is developed to guarantee that the closed-loop system of the augmented error system is asymptotically stable with H_∞ performance. Based on this, the robust preview tracking controller of the original system is obtained. Finally, two numerical examples are included to show the effectiveness of the proposed controller.

1. Introduction

Preview control is an important control technique for improving the tracking performance of the closed-loop system by utilizing the known future information about reference signals or disturbances [1–3]. Its study began in the 1960s. Compared with other control methods, the main advantages of preview control theory consist of enhancing the transient response of the closed-loop system and reducing energy consumption. In the past few years, a substantial amount of research on preview control design has been reported in the literature. In [4], the information fusion estimation technique was adopted to address the discrete linear preview tracking problem. In [5–7], the robust preview control schemes were established to realize the problem of asymptotic output tracking for several types of uncertain linear discrete-time systems. In [8, 9], the design of optimal preview controller for continuous- and discrete-time linear descriptor systems was developed via the classical difference method. In [10], the stochastic linear quadratic optimal tracking problem with preview compensation for linear continuous-time Markovian jump systems was investigated. In addition to the theoretical progresses, practical applications can be found in many realistic physical systems such as vehicle suspension systems [11, 12], robot systems [13], and rigid body motion control systems [14, 15].

The Lipschitz system is a typical nonlinear system in which the nonlinearity satisfies the Lipschitz condition. Many physical models can be described by Lipschitz systems, such as robotic manipulator [16] and Chua's circuit [17]. Due to its clear physical meaning, this class of systems has aroused considerable interest [18–20]. Meanwhile, the uncertainties and external disturbance frequently appear in control systems, and they can degrade performance of the systems or even lead to instability. In this situation, robust control theory plays an important role in the field of practical engineering. Recently, the robust tracking problem of Lipschitz systems has been widely studied and different approaches have been proposed. In [21], a novel nonlinear feedback controller was presented to address the issue of robust output tracking with disturbance rejection for a class of Lipschitz nonlinear systems. By combining adaptive principle with sliding mode control

method, an LMI-based robust tracker design for Lipschitz time-delay systems was developed in [22]. In [23], H_∞ control principle was applied to the robust tracking control problem of Lipschitz switched dynamic systems. However, to the best of our knowledge, few studies have been reported in the literature concerning the robust output tracking problem for uncertain Lipschitz nonlinear systems via preview control method. This motivates the present investigation.

This paper considers the problem of designing a robust tracking controller with preview action for a class of discrete-time uncertain Lipschitz nonlinear systems. According to preview control theory, our primary task is to construct an augmented error system which incorporates the tracking error and the reference preview information. In the literature, a classical approach consists of taking the forward or backward difference operator on the system state and control input. Unfortunately, this approach is not applicable in this paper due to the uncertain and nonlinear characteristics of system. To tackle this issue, we adopt an auxiliary variable method to successfully derive the augmented error system. Then, the robust preview tracking problem of the original system is reduced into a robust control problem of the augmented error system. Next, a state feedback controller is developed, and some criteria are established using the LMI technique to ensure that the closed-loop system is asymptotically stable with H_∞ performance. Based on the criteria, the robust tracking controller design with preview action for the original system is derived. Finally, two numerical examples authenticate the effectiveness of the proposed controller. The main contributions of the present study lie in the following aspects: (i) in real world, the system uncertainties and external disturbances are unavoidable and, thus, they are taken into account; (ii) in contrast to the conventional approach [7, 24], a novel auxiliary variable approach is provided for the first time to construct the augmented error system; (iii) the robust tracking controller design via preview control method is proposed for a class of uncertain Lipschitz systems.

Notations. R^n denotes the n-dimensional Euclidean space; $R^{n\times m}$ denotes the $n\times m$ matrix space; For a matrix P, $P>0$ means that P is positive definite; for matrices P and Q, $P>Q$ stands for $P-Q>0$; I and 0 denote the identity matrix and the zero matrix with appropriate dimension, respectively; $l_2[0,\infty)$ refers to the space of square summable infinite vector sequence and for $\omega(k)\in l_2[0,\infty)$, its norm is given by $\|\omega(k)\|_2=\sqrt{\sum_{k=0}^{\infty}\omega^T(k)\omega(k)}$; $f'(x)$ represents the Jacobi matrix of the vector function $f(x)$.

2. Problem Formulation

Consider the discrete-time nonlinear system

$$\begin{aligned} x(k+1) &= f(x(k)) + (A+\Delta A)x(k) + (B+\Delta B)u(k) \\ &\quad + L\omega(k), \\ y(k) &= Cx(k), \end{aligned} \tag{1}$$

where $x(k)\in R^n$ is the state and $u(k)\in R^m$ is the control input; $y(k)\in R^p$ is the output and $\omega(k)\in R^q$ is the external disturbance belonging to $l_2[0,\infty)$; A,B,L,C are constant matrices with appropriate dimensions and C is of full row rank. $\Delta A=\Delta A(k,x,\delta)$ and $\Delta B=\Delta B(k,x,\delta)$ are uncertain matrices that depend on time k, the state x, or some parameter vector δ. $f(x)\in R^n$ is a nonlinear function vector.

The following assumptions are made for system (1).

Assumption 1. There exist real constant matrices with appropriate dimensions D_i, E_i $(i=1,2)$ and uncertain matrices $\Sigma_i=\Sigma_i(k,x,\delta)$ $(i=1,2)$ such that

$$\begin{aligned} \Delta A &= D_1\Sigma_1E_1, \\ \Delta B &= D_2\Sigma_2E_2, \\ \Sigma_i^T\Sigma_i &\le I. \end{aligned} \tag{2}$$

Assumption 1 describes the matching condition on the parameter uncertainties, and it is a rather general assumption for robust control problems.

Assumption 2 (see [25–27]). $f(x)$ is a nonlinear perturbation satisfying $f(0)=0$, and $f'(x)$ exists and is continuous; furthermore,

$$f'(x) = MF(x)N, \tag{3}$$

where $M\in R^{n\times r}$, $N\in R^{s\times n}$ are well-defined real matrices and $F(x)\in R^{r\times s}$ is a norm-bound matrix satisfying $F^T(x)F(x)\le I$.

Remark 3. Note that the class of systems satisfying Assumption 2 is a subset of the class of Lipschitz nonlinear systems and it widely exists in the literature [25–28]. As commented in [25, 26], the Lipschitz property formulated in (3) does not involve any approximation of nonlinearity by its norm; thus this important formulation shall help to obtain less conservative conditions, especially when the nonlinearity has high Lipschitz constant. Moreover, we do not require M,N to be $n\times n$ dimensional matrices; thus, the condition about the nonlinearity $f(x)$ in this paper is more general than that in [25–27].

The reference signal is $r(k)$ and satisfies the following assumption.

Assumption 4 (see [5, 6]). The reference signal $r(k)$ is previewable, and the preview length is M_r; that is, at each time k, M_r future values $r(k+1), r(k+2),\cdots, r(k+M_r)$, and the current reference signal $r(k)$ are available. The future values of the reference signal beyond $k+M_r$ are assumed to be zeros, namely,

$$r(k+i)=0,\quad i=M_r+1, M_r+2,\cdots. \tag{4}$$

Moreover, there exists a constant vector r such that

$$\lim_{k\longrightarrow\infty} r(k) = r. \tag{5}$$

Remark 5. Assumption 4 describes the preview property of the reference signal and is a basic assumption in preview

control theory. References [1, 5] have shown that only the recent previewable signal significantly affects the system performance, and this time period is regarded as the preview interval of the reference signal. The reference information within the preview interval is known beforehand, while the future reference values exceeding the preview interval are unknown and generally assumed to be constant or zero.

The tracking error signal $e(k)$ is defined as

$$e(k) = y(k) - r(k). \tag{6}$$

At the same time, we introduce the following quadratic performance index

$$J = \sum_{k=0}^{\infty} \left[e^T(k) Q_e e(k) + u^T(k) H u(k) \right], \tag{7}$$

where $Q_e > 0, H > 0$ are given weighting matrices.

Our objective is to design a controller with preview action such that the output $y(k)$ tracks the reference signal $r(k)$ without static error even in the presence of uncertainties and external disturbance, that is,

$$\lim_{k\longrightarrow\infty} e(k) = \lim_{k\longrightarrow\infty} \left(y(k) - r(k) \right) = 0. \tag{8}$$

Remark 6. The tracking control problems for continuous-time Lipschitz systems have been widely studied in [21–23]. Nevertheless, there is very limited work on discrete-time counterparts. It is well known that discrete-time systems play an important role in the field of practical engineering. Indeed, real-time monitoring and control are essentially based on discrete-time dynamic systems. This has motivated us to investigate the tracking control problem in discrete-time. Moreover, our results are also applied to continuous-time systems via discretization methods. Among them, Euler approximation is the preferred technique.

Let us now recall three useful lemmas for the development of our work.

Lemma 7 (see [7]). *System $x(k+1) = Ax(k)$ is asymptotically stable if there exist matrices $P > 0$ and G such that*

$$\begin{bmatrix} P - G - G^T & G^T A^T \\ AG & -P \end{bmatrix} < 0. \tag{9}$$

Lemma 8 (see [29]). *Let M, N, and F be real matrices of appropriate dimensions, with F satisfying $F^T F \le I$. Then, the following inequality holds for any constant $\mu > 0$:*

$$MFN + (MFN)^T \le \mu^{-1} M M^T + \mu N^T N. \tag{10}$$

Lemma 9 ([30] (Schur complement lemma)). *Symmetric matrix $\begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{bmatrix} < 0$ if and only if one of the following two conditions is satisfied:*

(i) $S_{11} < 0, S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$;

(ii) $S_{22} < 0, S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0$.

3. Construction of the Augmented Error System

Let $A_1 = \int_0^1 MF((1-\lambda)x(k))N \mathrm{d}\lambda$. From the Mean-Value theorem [31], we have

$$\begin{aligned} f(x(k)) &= \int_0^1 MF((1-\lambda)x(k)) N x(k) \mathrm{d}\lambda \\ &= A_1 x(k). \end{aligned} \tag{11}$$

Substituting (11) into the state equation of system (1) leads to

$$\begin{aligned} x(k+1) &= \left(A + \Delta A + A_1\right) x(k) + (B + \Delta B) u(k) \\ &\quad + L\omega(k), \\ y &= Cx(k). \end{aligned} \tag{12}$$

The current goal is to construct the augmented error system by incorporating the tracking error and the preview information. Note that the traditional difference approach [7, 24] is not applicable here because the system uncertainty is related to time k. To overcome this difficulty, we propose a novel auxiliary variable method. Since C is of full row rank, there exists a matrix T such that

$$CT = I. \tag{13}$$

Hence, the auxiliary variable is selected as

$$x^*(k) = Tr(k). \tag{14}$$

Define a new state vector $x_s(k) = x(k) - x^*(k)$. It yields from (12) and (14) that

$$\begin{aligned} x_s(k+1) &= \left(A + \Delta A + A_1\right) x_s(k) + (B + \Delta B) u(k) \\ &\quad + L\omega(k) + \left(A + \Delta A + A_1\right) Tr(k) \\ &\quad - Tr(k+1), \end{aligned} \tag{15}$$

Using the output equation of systems (1), (6), and (15), one can obtain

$$\begin{aligned} e(k+1) &= C\left(A + \Delta A + A_1\right) x_s(k) \\ &\quad + C(B + \Delta B) u(k) + CL\omega(k) \\ &\quad + C\left(A + \Delta A + A_1\right) Tr(k) - r(k+1). \end{aligned} \tag{16}$$

Taking the preview information about reference signal into account, we define the following vector:

$$x_r(k) = \begin{bmatrix} r(k) \\ r(k+1) \\ \vdots \\ r(k+M_r) \end{bmatrix}. \tag{17}$$

From Assumption 4, it is easily seen that

$$x_r(k+1) = A_r x_r(k), \tag{18}$$

where

$$A_r = \begin{bmatrix} 0 & I & 0 & \cdots & 0 \\ 0 & 0 & I & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & I \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}. \tag{19}$$

Note that the vector $x_r(k) \in R^{p(M_r+1)}$ is given by preview and all future information on the reference signal available at the time k is summarized in (18). In this way, the reference preview information can be incorporated into the controller design conveniently.

Define an augmented state vector $x_m(k) = \left[e^T(k) \;\; x_s{}^T(k) \;\; x_r^T(k)\right]^T$. From (15)-(18), one yields

$$\begin{aligned} x_m(k+1) &= \left(A_m + \Delta A_m\right) x_m(k) \\ &\quad + \left(B_m + \Delta B_m\right) u(k) + L_m \omega(k), \end{aligned} \tag{20}$$

where

$$\begin{aligned} A_m &= \begin{bmatrix} 0 & CA & S_e \\ 0 & A & S_x \\ 0 & 0 & A_r \end{bmatrix}, \\ \Delta A_m &= \begin{bmatrix} 0 & C\left(\Delta A + A_1\right) & \Delta S_e \\ 0 & \Delta A + A_1 & \Delta S_x \\ 0 & 0 & 0 \end{bmatrix}, \\ B_m &= \begin{bmatrix} CB \\ B \\ 0 \end{bmatrix}, \\ \Delta B_m &= \begin{bmatrix} C\Delta B \\ \Delta B \\ 0 \end{bmatrix}, \\ L_m &= \begin{bmatrix} CL \\ L \\ 0 \end{bmatrix} \end{aligned} \tag{21}$$

and

$$\begin{aligned} S_e &= \left[CAT \;\; -I \;\; 0 \;\; \cdots \;\; 0\right], \\ S_x &= \left[AT \;\; -T \;\; 0 \;\; \cdots \;\; 0\right], \\ \Delta S_e &= \left[C\left(\Delta A + A_1\right)T \;\; 0 \;\; 0 \;\; \cdots \;\; 0\right], \\ \Delta S_x &= \left[\left(\Delta A + A_1\right)T \;\; 0 \;\; 0 \;\; \cdots \;\; 0\right]. \end{aligned} \tag{22}$$

Also, $\Delta A_m, \Delta B_m$ satisfy $\Delta A_m = \int_0^1 M_m \Pi_\lambda N_m \, d\lambda$, $\Delta B_m = D_m \Sigma_2 E_2$, where

$$\begin{aligned} M_m &= \begin{bmatrix} CD_1 & CM \\ D_1 & M \\ 0 & 0 \end{bmatrix}, \\ \Pi_\lambda &= \begin{bmatrix} \Sigma_1 & 0 \\ 0 & F\left((1-\lambda)x(k)\right) \end{bmatrix}, \\ N_m &= \begin{bmatrix} 0 & E_1 & E_1T & 0 & 0 & \cdots & 0 \\ 0 & N & NT & 0 & 0 & \cdots & 0 \end{bmatrix}, \\ D_m &= \begin{bmatrix} CD_2 \\ D_2 \\ 0 \end{bmatrix}. \end{aligned} \tag{23}$$

Remark 10. In the derivation of system (20), (11) plays an important role in transforming the original nonlinear uncertain system into a linear uncertain system. This facilitates the preview controller design of Lipschitz nonlinear systems.

Notice that the control input of system (20) is $u(k)$ rather than the difference of $u(k)$. If we directly develop a state feedback controller for system (20), this controller will do not include the integral of tracking error. Then, the corresponding closed-loop system does not contain the integral control action that is capable of handling the static error [5]. To achieve the desired robust tracking performance and eliminate the static error, a discrete integrator is introduced as

$$v(k+1) = v(k) + e(k), \tag{24}$$

where the initial value $v(\cdot)$ can be arbitrarily assigned and generally taken as zero.

Combining (20) and (24) leads to

$$\begin{aligned} \overline{x}(k+1) &= \left(\overline{A} + \Delta\overline{A}\right)\overline{x}(k) + \left(\overline{B} + \Delta\overline{B}\right)u(k) \\ &\quad + \overline{L}\omega(k), \end{aligned} \tag{25}$$

where

$$\overline{x}(k) = \begin{bmatrix} x_m(k) \\ v(k) \end{bmatrix},$$
$$\overline{A} = \begin{bmatrix} A_m & 0 \\ S_m & I \end{bmatrix},$$
$$S_m = \begin{bmatrix} I & 0 & 0 \end{bmatrix},$$
$$\Delta\overline{A} = \begin{bmatrix} \Delta A_m & 0 \\ 0 & 0 \end{bmatrix}, \tag{26}$$
$$\overline{B} = \begin{bmatrix} B_m \\ 0 \end{bmatrix},$$
$$\Delta\overline{B} = \begin{bmatrix} \Delta B_m \\ 0 \end{bmatrix},$$
$$\overline{L} = \begin{bmatrix} L_m \\ 0 \end{bmatrix}.$$

Moreover, $\Delta\overline{A}, \Delta\overline{B}$ satisfy

$$\Delta\overline{A} = \int_0^1 \overline{M}\Pi_\lambda \overline{N}\,\mathrm{d}\lambda,$$
$$\Delta\overline{B} = \overline{D}\Sigma_2 E_2, \tag{27}$$

where $\overline{M} = \left[\begin{smallmatrix} M_m \\ 0 \end{smallmatrix}\right]$, $\overline{N} = \begin{bmatrix} N_m & 0 \end{bmatrix}$, $\overline{D} = \left[\begin{smallmatrix} D_m \\ 0 \end{smallmatrix}\right]$.

The corresponding performance index for system (25) is then changed to

$$\overline{J} = J + \sum_{k=0}^{\infty} v^T(k) Q_v v(k) = \sum_{k=0}^{\infty} \left[e^T(k) Q_e e(k) + u^T(k) H u(k) + v^T(k) Q_v v(k) \right] \tag{28}$$

Define the performance signal

$$z(k) = E\overline{x}(k) + Du(k), \tag{29}$$

$$E = \begin{bmatrix} Q_e^{1/2} & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_v^{1/2} \\ 0 & 0 & 0 & 0 \end{bmatrix},$$
$$D = \begin{bmatrix} 0 \\ 0 \\ H^{1/2} \end{bmatrix}. \tag{30}$$

Then, the performance index (28) is further transformed into

$$\overline{J} = \sum_{k=0}^{\infty} z^T(k) z(k) = \|z\|_2^2. \tag{31}$$

In preview control theory, system (25) is usually called an augmented error system. So far, the preview tracking problem of system (1) has been reduced to a robust control problem of system (25) under performance index (31).

4. Design of the Robust Preview Controller

For a prescribed scalar $\gamma > 0$, we aim to construct a state feedback controller

$$u(k) = K\overline{x}(k) \tag{32}$$

for system (25) such that the resulting closed-loop system

$$\overline{x}(k+1) = \left(\overline{A} + \Delta\overline{A} + \overline{B}K + \Delta\overline{B}K\right)\overline{x}(k) + \overline{L}\omega(k) \tag{33}$$

is asymptotically stable. Also, the effect of $\omega(k)$ on the performance signal $z(k)$ is attenuated below a prescribed level in the H_∞ sense, namely,

$$J_\omega = \sum_{k=0}^{\infty} \left(z^T(k) z(k) - \gamma^2 \omega^T(k) \omega(k) \right) < 0 \tag{34}$$

for all nonzero $\omega(k) \in l_2[0,\infty)$ under zero initial conditions.

Theorem 11. *Suppose that Assumptions 1–4 are satisfied. For a prescribed scalar $\gamma > 0$, the closed-loop system (33) is asymptotically stable with disturbance attenuation level γ, if there exist matrices $P > 0$, G and K such that*

$$\begin{bmatrix} P - G - G^T & 0 & G^T\left(\overline{A} + \Delta\overline{A} + \overline{B}K + \Delta\overline{B}K\right)^T & G^T(E + DK)^T \\ 0 & -\gamma^2 I & \overline{L}^T & 0 \\ \left(\overline{A} + \Delta\overline{A} + \overline{B}K + \Delta\overline{B}K\right)G & \overline{L} & -P & 0 \\ (E + DK)G & 0 & 0 & -I \end{bmatrix} < 0. \tag{35}$$

Proof. System (33) with $\omega(k) = 0$ is first considered. We implement a congruence transformation to (35) with an invertible symmetric matrix Γ defined by

$$\Gamma = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & 0 & I \end{bmatrix}. \tag{36}$$

Pre- and postmultiplying (35) by Γ and its transpose, respectively, one can obtain that

$$\begin{bmatrix} P - G - G^T & G^T\left(\overline{A} + \Delta\overline{A} + \overline{B}K + \Delta\overline{B}K\right)^T & 0 & G^T(E + DK)^T \\ \left(\overline{A} + \Delta\overline{A} + \overline{B}K + \Delta\overline{B}K\right)G & -P & \overline{L} & 0 \\ 0 & \overline{L}^T & -\gamma^2 I & 0 \\ (E + DK)G & 0 & 0 & -I \end{bmatrix} < 0. \tag{37}$$

Thus, the following inequality holds

$$\begin{bmatrix} P - G - G^T & G^T\left(\overline{A} + \Delta\overline{A} + \overline{B}K + \Delta\overline{B}K\right)^T \\ \left(\overline{A} + \Delta\overline{A} + \overline{B}K + \Delta\overline{B}K\right)G & -P \end{bmatrix} < 0. \tag{38}$$

In the light of Lemma 7, system (33) is asymptotically stable with $\omega(k) = 0$.

Then consider the Lyapunov function $V(\overline{x}) = \overline{x}^T P^{-1}\overline{x}$. For system (33) under zero initial conditions, J_ω in (34) is equivalent to

$$\begin{aligned} J_\omega &= \sum_{k=0}^{\infty}\left[z^T(k)z(k) - \gamma^2\omega^T(k)\omega(k) + \Delta V(\overline{x}(k))\right] \\ &\quad + V(\overline{x}(0)) - V(\overline{x}(\infty)) \\ &\le \sum_{k=0}^{\infty}\left[z^T(k)z(k) - \gamma^2\omega^T(k)\omega(k) + \Delta V(\overline{x}(k))\right] \\ &= \begin{bmatrix} \overline{x}(k) \\ \omega(k) \end{bmatrix}^T \Omega \begin{bmatrix} \overline{x}(k) \\ \omega(k) \end{bmatrix} \end{aligned} \tag{39}$$

where

$$\Omega = \begin{bmatrix} \overline{A}_c{}^T P^{-1}\overline{A}_c - P^{-1} + (E + DK)^T(E + DK) & \overline{A}_c{}^T P^{-1}\overline{L} \\ \overline{L}^T P^{-1}\overline{A}_c & \overline{L}^T P^{-1}\overline{L} - \gamma^2 I \end{bmatrix} \tag{40}$$

with $\overline{A}_c = \overline{A} + \Delta\overline{A} + \overline{B}K + \Delta\overline{B}K$.

Based on the above analysis, $J_\omega \le 0$ holds if $\Omega < 0$ is satisfied. Next, we will prove that condition (35) guarantees $\Omega < 0$.

From (35) and the inequality $-G^T P^{-1} G \le P - G - G^T$, one can obtain that

$$\begin{bmatrix} -G^T P^{-1} G & 0 & G^T\left(\overline{A} + \Delta\overline{A} + \overline{B}K + \Delta\overline{B}K\right)^T & G^T(E + DK)^T \\ 0 & -\gamma^2 I & \overline{L}^T & 0 \\ \left(\overline{A} + \Delta\overline{A} + \overline{B}K + \Delta\overline{B}K\right)G & \overline{L} & -P & 0 \\ (E + DK)G & 0 & 0 & -I \end{bmatrix} < 0. \tag{41}$$

Performing a congruence transformation to the above inequality with the invertible matrix $\mathrm{diag}(G^{-T}, I, I, I)$ yields

$$\begin{bmatrix} -P^{-1} & 0 & \left(\overline{A} + \Delta\overline{A} + \overline{B}K + \Delta\overline{B}K\right)^T & (E + DK)^T \\ 0 & -\gamma^2 I & \overline{L}^T & 0 \\ \overline{A} + \Delta\overline{A} + \overline{B}K + \Delta\overline{B}K & \overline{L} & -P & 0 \\ E + DK & 0 & 0 & -I \end{bmatrix} < 0. \tag{42}$$

Notice that $\mathrm{diag}(-P, -I) < 0$; then applying Lemma 9 to (42) leads to $\Omega < 0$. The proof is completed. □

Indeed, the inequality (35) provided in Theorem 11 cannot be applied directly due to the presence of some uncertain terms. Thanks to Lemmas 8 and 9, the condition in Theorem 11 can be converted into the computationally tractable LMI condition.

Theorem 12. *Suppose that Assumptions 1–4 are satisfied. For a prescribed scalar $\gamma > 0$, system (33) is asymptotically stable with disturbance attenuation level γ, if there exist matrices $P > 0$, G, and W and a scalar $\varepsilon > 0$ such that*

$$\begin{bmatrix} P-G-G^T & 0 & G^T\overline{A}^T + W^T\overline{B}^T & G^TE^T + W^TD^T & G^T\overline{N}^T & W^TE_2^T \\ 0 & -\gamma^2 I & \overline{L}^T & 0 & 0 & 0 \\ \overline{A}G + \overline{B}W & \overline{L} & -P + \varepsilon\left(\overline{M}\,\overline{M}^T + \overline{D}\,\overline{D}^T\right) & 0 & 0 & 0 \\ EG + DW & 0 & 0 & -I & 0 & 0 \\ \overline{N}G & 0 & 0 & 0 & -\varepsilon I & 0 \\ E_2W & 0 & 0 & 0 & 0 & -\varepsilon I \end{bmatrix} < 0. \tag{43}$$

Furthermore, the feedback gain matrix K is given by $K = WG^{-1}$.

Proof. Let the left-hand side of inequality (35) be Φ. From (27), Φ can be expressed as

$$\Phi = \Psi + \int_0^1 \left(M_l\Sigma_\lambda N_l + N_l{}^T\Sigma_\lambda^T M_l{}^T\right) d\lambda, \tag{44}$$

where

$$\begin{aligned} \Psi &= \begin{bmatrix} P-G-G^T & 0 & G^T\left(\overline{A}+\overline{B}K\right)^T & G^T(E+DK)^T \\ 0 & -\gamma^2 I & \overline{L}^T & 0 \\ \left(\overline{A}+\overline{B}K\right)G & \overline{L} & -P & 0 \\ (E+DK)G & 0 & 0 & -I \end{bmatrix}, \\ M_l &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \overline{M} & \overline{D} \\ 0 & 0 \end{bmatrix}, \\ \Sigma_\lambda &= \begin{bmatrix} \Pi_\lambda & 0 \\ 0 & \Sigma_2 \end{bmatrix}, \\ N_l &= \begin{bmatrix} \overline{N}G & 0 & 0 & 0 \\ E_2KG & 0 & 0 & 0 \end{bmatrix}. \end{aligned} \tag{45}$$

It can be verified that $\Sigma_\lambda{}^T\Sigma_\lambda \le I$; then from Lemma 8, it follows that

$$M_l\Sigma_\lambda N_l + N_l{}^T\Sigma_\lambda^T M_l{}^T \le \varepsilon M_l M_l{}^T + \varepsilon^{-1}N_l{}^T N_l. \tag{46}$$

Notice that the right-hand side of inequality (46) is irrelevant to λ; thus its integral over $[0,1]$ is itself. By exploiting the property of integral, one has

$$\begin{aligned} &\int_0^1 \left(M_l\Sigma_\lambda N_l + N_l{}^T\Sigma_\lambda^T M_l{}^T\right) d\lambda \\ &\quad \le \varepsilon M_l M_l{}^T + \varepsilon^{-1}N_l{}^T N_l. \end{aligned} \tag{47}$$

From (44) and (47), the following inequality is obtained:

$$\Phi \le \Psi + \varepsilon M_l M_l{}^T + \varepsilon^{-1}N_l{}^T N_l. \tag{48}$$

Thus, $\Phi < 0$ if $\Psi + \varepsilon M_l M_l{}^T + \varepsilon^{-1}N_l{}^T N_l < 0$. Applying Lemma 9 again, $\Psi + \varepsilon M_l M_l{}^T + \varepsilon^{-1}N_l{}^T N_l < 0$ is equivalent to

$$\begin{bmatrix} \Psi + \varepsilon M_l M_l{}^T & N_l{}^T \\ N_l & -\varepsilon I \end{bmatrix} < 0. \tag{49}$$

Letting $W = KG$, then (49) can be formulated in the form of (43). Thus, if LMI (43) holds, then inequality (35), i.e., $\Phi < 0$, holds. Theorem 12 is derived immediately by Theorem 11. This completes the proof. □

In order to make the controller structure clear and highlight the preview action of reference information, we partition the control gain matrix K so that

$$\begin{aligned} K &= \left[K_e \;\vdots\; K_x \;\vdots\; k_r(0)\;\; k_r(1)\;\; \cdots\;\; k_r(M_r) \;\vdots\; K_v\right]. \end{aligned} \tag{50}$$

Then (32) can be rewritten as

$$\begin{aligned} u(k) &= K_e e(k) + K_x x_s(k) + \sum_{i=0}^{M_r} k_r(i)\, r(k+i) \\ &\quad + K_v v(k). \end{aligned} \tag{51}$$

Based on the above analysis, the main result of this paper is summarized below.

Theorem 13. *Suppose that Assumptions 1–4 are satisfied. If LMI (43) in Theorem 12 is feasible, then the robust preview controller of system (1) is*

$$u(k) = K_e e(k) + K_x x(k) + \sum_{i=0}^{M_r} k_r(i) r(k+i) + K_v \left(\sum_{i=0}^{k-1} e(i) + v(0) \right) - K_x T r(k). \tag{52}$$

where control gains $K_e, K_x, k_r(0), k_r(1), \cdots, k_r(M_r), K_v$ are determined by (50). Under this controller, the output $y(k)$ can track the reference signal $r(k)$ without static error.

Remark 14. Note that the controller design strategy provided in this paper consists of five parts. As shown in (52), the first part represents the compensation action on the tracking error, the second part represents the state feedback control action, the third part is the preview compensator with respect to the reference signal, the fourth part is the integral action on the tracking error, and the last one depends on the current reference information. It should be pointed out that the key point for improving tracking performance consists of the efficient use of the reference preview information, i.e., the preview compensator, which is not taken into account in [21–23].

Remark 15. Notice that, without the nonlinearity and the external disturbance, system (1) is reduced to a discrete-time linear system with norm-bounded parameter uncertainties. The stabilization problem for such a system has been deeply investigated in [32–34]. It is noteworthy that, in this paper, the zero solution of the closed-loop system of system (1) is asymptotically stable in the case, where $r(k)$ is identically equal to zero. Note that in this situation $x(k)$ becomes a part of the augmented state vector due to $x_s(k) = x(k) - x^*(k) = x(k) - Tr(k) = x(k)$. Thus, if the reference signal is $r(k) \equiv 0$, the stabilization of system (1) can be achieved. That is, the stabilization issue is a special case of this paper. It should be emphasized that our proposed controller design is developed via the error system method; therefore, this method which is used to study stability problem is quite different from that in [32–34]. In addition, it should be mentioned that the controller design schemes discussed in [18, 35] are only effective for achieving stabilization of Lipschitz systems and not suitable to deal with the tracking problem considered here.

Remark 16. The choice of the design parameters will directly influence the tracking performance. To achieve satisfactory control effect, the related design parameters can be selected using the classic trial-and-error technique [36–38] but subject to satisfaction of all the requirements made in the paper.

5. Numerical Examples

Example 1. Consider the single-link flexible joint robot system [39–42]

$$\begin{aligned} \dot{x}(t) &= g(x(t)) + Tx(t) + Du(t), \\ y(t) &= Cx(t), \end{aligned} \tag{53}$$

where

$$\begin{aligned} g(x(t)) &= \begin{bmatrix} 0 \\ 0 \\ 0 \\ -3.33\sin(x_3(t)) \end{bmatrix}, \\ T &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ -48.6 & -1.25 & 48.6 & 0 \\ 0 & 0 & 0 & 1 \\ 19.5 & 0 & -19.5 & 0 \end{bmatrix}, \\ D &= \begin{bmatrix} 0 \\ 21.6 \\ 0 \\ 0 \end{bmatrix}, \\ C &= \begin{bmatrix} 0.7 \\ 0 \\ 2.9 \\ 0 \end{bmatrix}^T. \end{aligned} \tag{54}$$

We suppose that system (53) is subject to parameter uncertainties and external disturbance. Under this assumption, system (60) can be rewritten under the form

$$\begin{aligned} \dot{x}(t) &= g(x(t)) + (T + \Delta T) x(t) + (D + \Delta D) u(t) \\ &\quad + B_\omega \omega(t), \\ y(t) &= Cx(t), \end{aligned} \tag{55}$$

where $B_\omega = \begin{bmatrix} -0.1 & 0 & 0.2 & 0 \end{bmatrix}^T$, $\Delta T = D_1 \Sigma_1 E_1$, $\Delta D = D_2 \Sigma_2 E_2$ with

$$D_1 = \begin{bmatrix} 0.01 & 0 & 0 & 0 \\ 0 & 0.01 & 0 & 0 \\ 0 & 0 & 0.01 & 0 \\ 0 & 0 & 0 & 0.01 \end{bmatrix},$$

$$\Sigma_1 = \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & 0.1\sin(10\pi t) + a & 0 & 0 \\ 0 & 0 & 0.2\sin(10\pi t) + a & 0 \\ 0 & 0 & 0 & 0.01 \end{bmatrix},$$

$$E_1 = \begin{bmatrix} 0.1 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0.1 \end{bmatrix},$$

$$D_2 = \begin{bmatrix} 0.1 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Sigma_2 = 0_{4\times 4},$$

$$E_2 = \begin{bmatrix} 0.01 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \tag{56}$$

where a is an uncertain parameter, which varies between -0.5 and 0.5, i.e., $-0.5 \le a \le 0.5$. The external disturbance is taken as $\omega(t) = \sin(t)\exp(-2t)$. The other parameters $g(x(t)), T, D, C$ are defined as described previously.

By applying the Euler discretization to system (55) with the sample time δ, a discrete-time system is obtained in the form

$$\begin{aligned} x(\delta(k+1)) &= f(x(\delta k)) + (A + \Delta A)\, x(\delta k) \\ &\quad + (B + \Delta B)\, u(\delta k) + L\omega(\delta k), \\ y(\delta k) &= Cx(\delta k), \end{aligned} \tag{57}$$

where $f(x) = \delta g(x)$, $A = I + \delta T$, $\Delta A = \delta \Delta T$, $B = \delta D$, $\Delta B = \delta \Delta D$, $L = \delta B_\omega$.

It is clear that C is of full row rank, $\omega(\delta k) = \sin(\delta k)\exp(-2\delta k) \in l_2[0,\infty)$, and Σ_i $(i = 1, 2)$ satisfies $\Sigma_i^T \Sigma_i \le I$. Moreover, $f(0) = 0$ and some matrices related to (3) are obtained as follows:

$$M = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \sqrt{3.33} \end{bmatrix}, \tag{58}$$

$$N = \delta \begin{bmatrix} 0 & 0 & -\sqrt{3.33} & 0 \end{bmatrix},$$

$$F(x) = \cos(x_3).$$

Set $\delta = 0.05s$, $H = 0.01$, $Q_e = 0.01$, $Q_v = 0.1$ and $\gamma = 1.5$. $T = \begin{bmatrix} 1 & 0 & 3/29 & 0 \end{bmatrix}^T$ is a solution of (13). To compare the effect of the preview length on the tracking performance, three cases are considered, including $l_r = 0s$ (i.e., $M_r = 0$), $l_r = 0.25s$ $(M_r = 5)$, and $l_r = 0.4s$ $(M_r = 8)$. By solving LMI (43) in Theorem 12, the desired controller can be obtained. Then, the closed-loop output is derived.

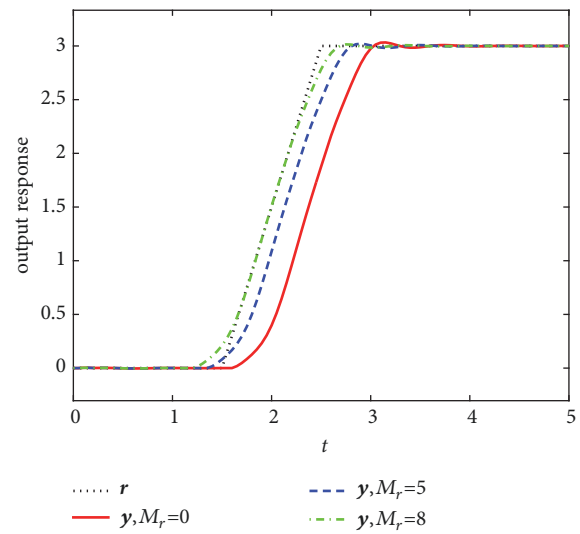

Figure 1: The output response to reference signal (59).

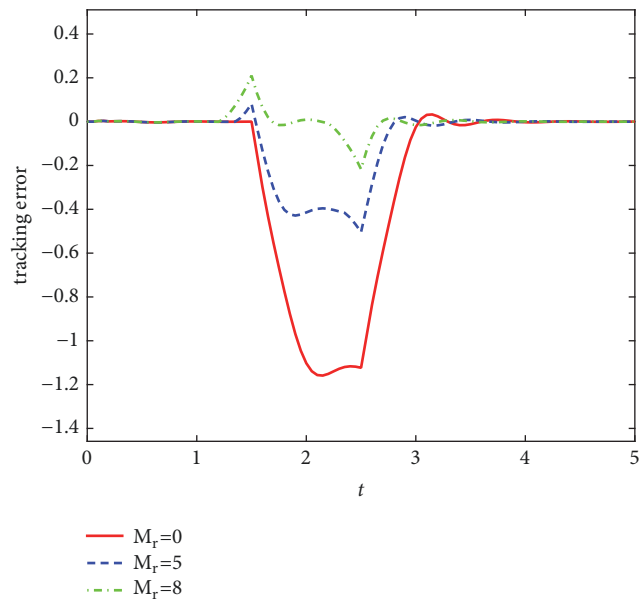

Figure 2: The tracking error to reference signal (59).

For the purpose of simulation, the reference signal $r(t)$ is taken as

$$r(t) = \begin{cases} 0, & t < 1.5 \\ 3(t - 1.5), & 1.5 \le t \le 2.5 \\ 3, & t > 2.5. \end{cases} \tag{59}$$

We assume that the preview length of $r(t)$ is l_r and denote $M_r = l_r/\delta$.

The closed-loop output and the tracking error are presented in Figures 1 and 2, respectively. Figure 3 shows the control input. It is observed that all output trajectories can track the reference signal without static error, irrespective of

Table 1: Performance index results.

Performance Index	Standard H_∞ controller	Preview controller ($M_r = 5$)	Preview controller ($M_r = 8$)
IAE	22.9754	8.7716	1.9887
ISE	21.5247	3.2716	0.2359
ITAE	1025.2860	378.9774	81.4084
ITSE	954.8567	141.3613	9.4950

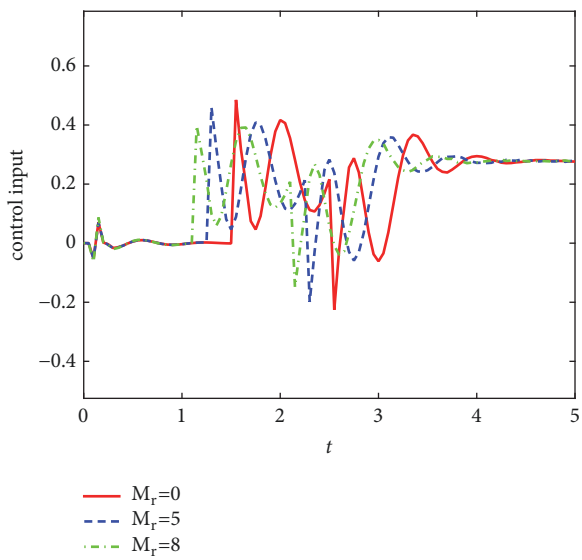

Figure 3: The control input to reference signal (59).

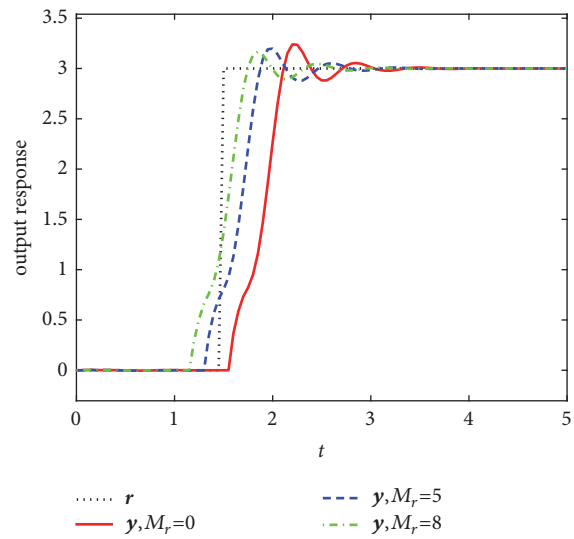

Figure 4: The output response to reference signal (60).

the uncertainties and disturbances. The simulations authenticate the fine and robust performance of the controllers. Moreover, compared with the controller without preview, the controller with preview action provides better tracking performance even in the presence of uncertainties and external disturbance. Furthermore, as shown in Table 1, the effectiveness of the controller is evaluated and compared with standard H_∞ controller using error performance indices. We can clearly see that all these performance indices values are decreased when using the preview controller. This is because our proposed controller not only has disturbance rejection ability but also includes preview compensator for improving tracking quality.

When the reference signal is taken as

$$r(t) = \begin{cases} 0, & t < 1.5 \\ 3, & t \geq 1.5 \end{cases} \tag{60}$$

the simulation results are shown in Figures 4, 5, and 6.

Figure 4 shows the output response of the closed-loop system. Figure 5 illustrates the tracking error between the actual and desired outputs, and Figure 6 plots the control input. It is concluded from these figures that the control techniques in three cases are all capable of overcoming uncertainties and external disturbances and provide asymptotic tracking of the reference signal. The difference is that our proposed preview controller produces faster response, shortens the settling time, and reduces the overshoot simultaneously compared to the controller without preview. This is because in the design of the controller the compensation action on the reference signal is taken into account in addition to robustness. Table 2

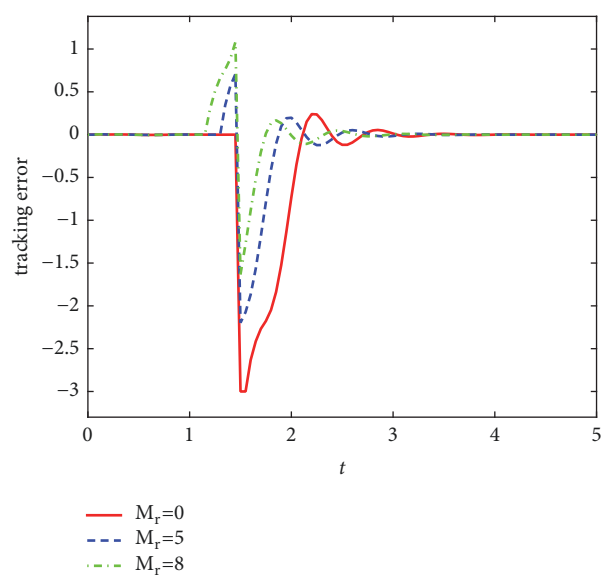

Figure 5: The tracking error to reference signal (60).

Table 2: Performance index results.

Performance Index	Standard H_∞ controller	Preview controller ($M_r = 5$)	Preview controller ($M_r = 8$)
IAE	24.9509	13.4162	10.0432
ISE	52.7359	18.3726	8.8157
ITAE	877.6702	446.0528	311.4555
ITSE	1756.4146	580.1884	261.7284

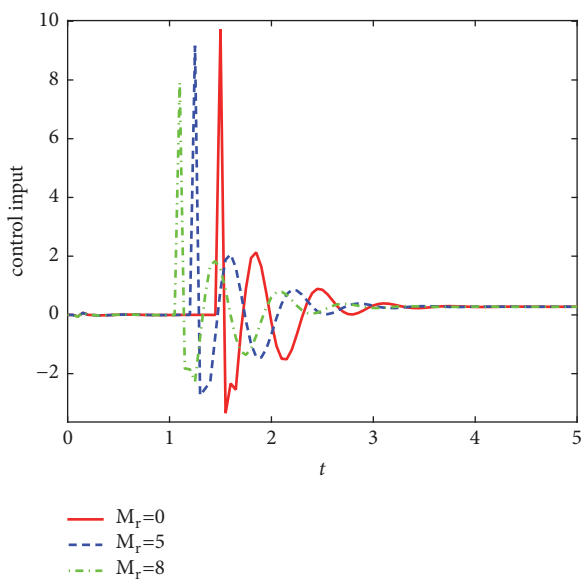

Figure 6: The control input to reference signal (60).

shows that using the preview controller helps to reduce the values of error performance indexes. Also, the closed-loop system can achieve better performance by adjusting the preview length.

The practical example above confirms the effectiveness of preview controller in improving the tracking performance of the system. A continuous-time system is considered in Example 1. Next, a discrete-time uncertain Lipschitz system will be presented.

Example 2. Considering system (1), the relevant parameters are as follows:

$$A = \begin{bmatrix} 1.3 & 0 \\ 0.2 & 0.85 \end{bmatrix},$$

$$B = \begin{bmatrix} 0.2 \\ 0 \end{bmatrix},$$

$$L = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 0.05 & 0.5 \end{bmatrix},$$

$$D_1 = \begin{bmatrix} -0.1 & 0 \\ 0 & 0.2 \end{bmatrix},$$

$$E_1 = \begin{bmatrix} 0.01 & 0 \\ 0 & -0.01 \end{bmatrix},$$

$$\Sigma_1 = \begin{bmatrix} 0.1a & 0.3\sin(0.5\pi k) + a \\ 0.2\cos(0.3\pi k) + a & -0.01 \end{bmatrix},$$

$$D_2 = \Sigma_2 = 0_{2\times 2},$$

$$E_2 = 0_{2\times 1}. \tag{61}$$

The uncertain parameter a varies between -0.5 and 0.5, i.e., $-0.5 \le a \le 0.5$. The nonlinear function and the external disturbance are taken as $f(x) = \begin{bmatrix} 0.002\arctan(x_2) \\ 0.001\sin(x_1)+0.004(x_2) \end{bmatrix}$ and $\omega(k) = k\exp(-0.2k) \in l_2[0,\infty)$, respectively.

Clearly, C has full row rank and Σ_i $(i = 1, 2)$ satisfies (2). Moreover, $f(0) = 0$ and the matrices related to (3) are obtained as follows:

$$M = \begin{bmatrix} 0 & 0.1 \\ 0.2 & 0 \end{bmatrix},$$

$$F(x) = \begin{bmatrix} 0.5\cos(x_1) & 0.2 \\ 0 & \dfrac{0.2}{1 + x_2^2} \end{bmatrix}, \tag{62}$$

$$N = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.1 \end{bmatrix}.$$

Set $\gamma = 0.32$, $H = 0.05$, $Q_e = 0.1$, $Q_v = 1$, and $T = \begin{bmatrix} 0 & 2 \end{bmatrix}^T$. For the ease of comparison, three cases are discussed, including $M_r = 0$, $M_r = 2$, and $M_r = 5$. In the same way, the controller gain matrix is determined by solving LMI (43) in Theorem 12 and then the output of the closed-loop system is derived.

To carry out the simulation, the reference signal is taken as

$$r(k) = \begin{cases} 0, & k < 20 \\ 0.15(k - 20), & 20 \le k \le 40 \\ 3, & t > 40 \end{cases} \tag{63}$$

The numerical simulation results are shown in Figures 7, 8, and 9.

Figures 7 and 8 illustrate the closed-loop output and the tracking error, respectively. Figure 9 shows the control input.

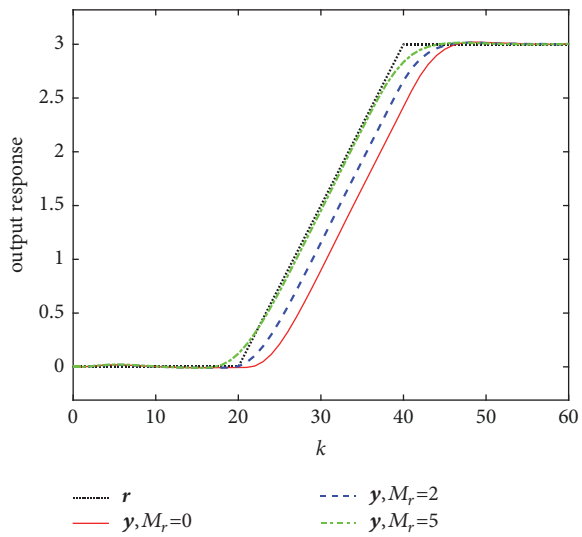

Figure 7: The output of the system to reference signal (63).

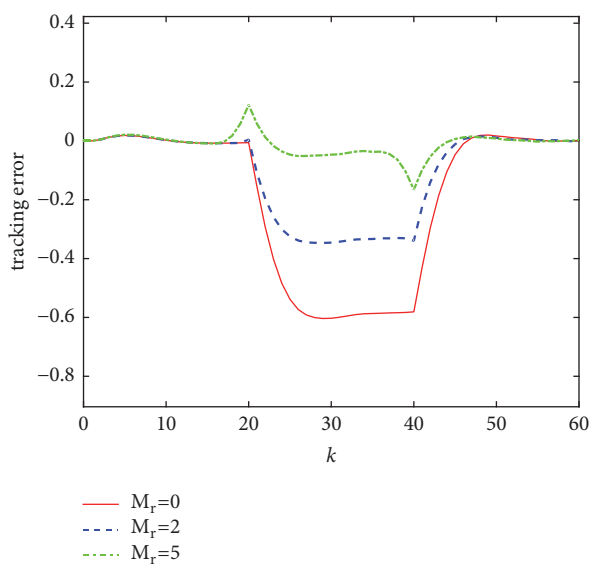

Figure 8: The tracking error to reference signal (63).

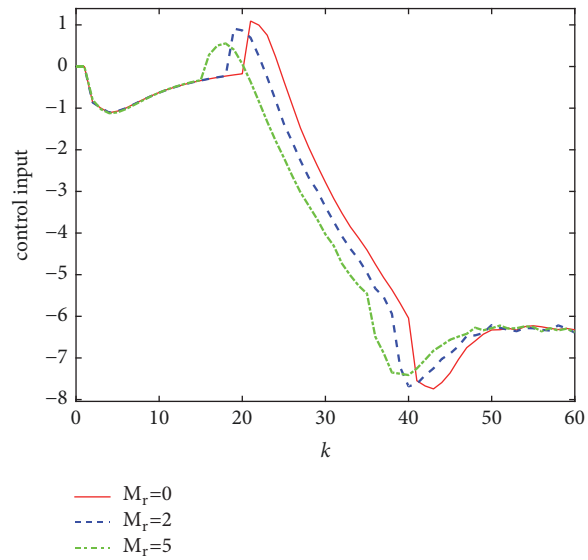

Figure 9: The control input to reference signal (63).

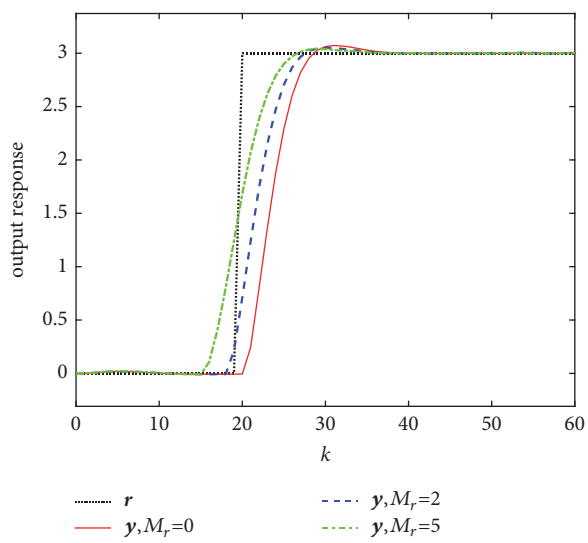

Figure 10: The output of the system to reference signal (64).

Simulation results in three cases are all robust against uncertainties and time-varying disturbances. Moreover, compared to the controller with no preview, the preview controller makes the closed-loop system have faster dynamic response speed and higher tracking precision. The output tracking with preview action is excellent. Furthermore, a performance comparison between preview controller and standard H_∞ controller is presented in Table 3. The superiority of preview control method is quite clear from this table. Hence, our proposed controller performs significantly better.

When the reference signal is taken as

$$r(k) = \begin{cases} 0, & k < 20 \\ 3, & k \geq 20 \end{cases} \tag{64}$$

the simulation results are shown in Figures 10, 11, and 12.

Figures 10 and 11 show the closed-loop output and the tracking error, respectively. Figure 12 shows the control input. Simulations in three cases confirm the robustness of the controllers. Comparing Figures 10, 11, and 12, we can see that the output tracking with preview action is much better than the one without preview. Due to the consideration of reference preview information, the preview controller achieves better performance indexes than the single H_∞ controller, as shown in Table 4.

In addition, in this example, set $f(x) = 0$ and $\omega(k) = 0$, and the other parameters remain unchanged. The considered system is then converted into an uncertain discrete-time linear system. The stabilization problem for such a system has been addressed in [33]. Here, we take the reference signal as $r(k) \equiv 0$. According to Remark 15, our proposed method also ensures that the closed-loop system is asymptotically stable.

Table 3: Performance index results.

Performance Index	Standard H_∞ controller	Preview controller ($M_r = 2$)	Preview controller ($M_r = 5$)
IAE	12.0682	7.0050	1.6428
ISE	6.3391	2.1067	0.1075
ITAE	388.1932	221.6150	49.7731
ITSE	204.3796	67.0636	3.5105

Table 4: Performance index results.

Performance Index	Standard H_∞ controller	Preview controller ($M_r = 2$)	Preview controller ($M_r = 5$)
IAE	12.6485	7.8533	6.5527
ISE	26.2037	11.1251	5.5557
ITAE	280.9896	170.9029	132.8920
ITSE	558.5424	232.7349	110.0990

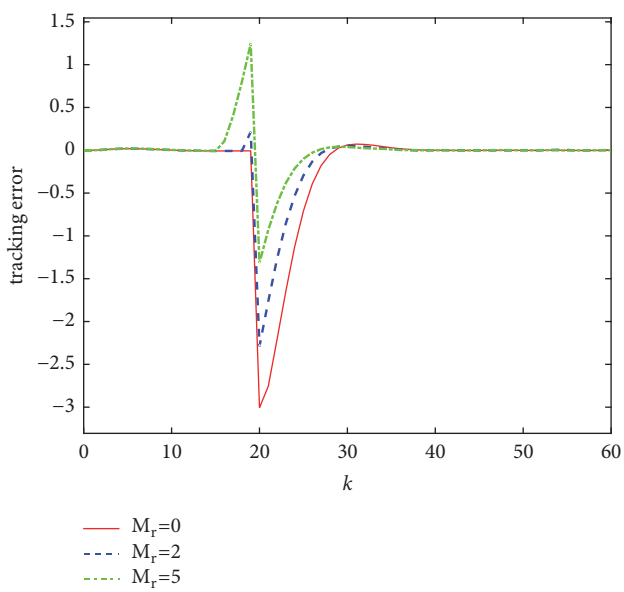

Figure 11: The tracking error to reference signal (64).

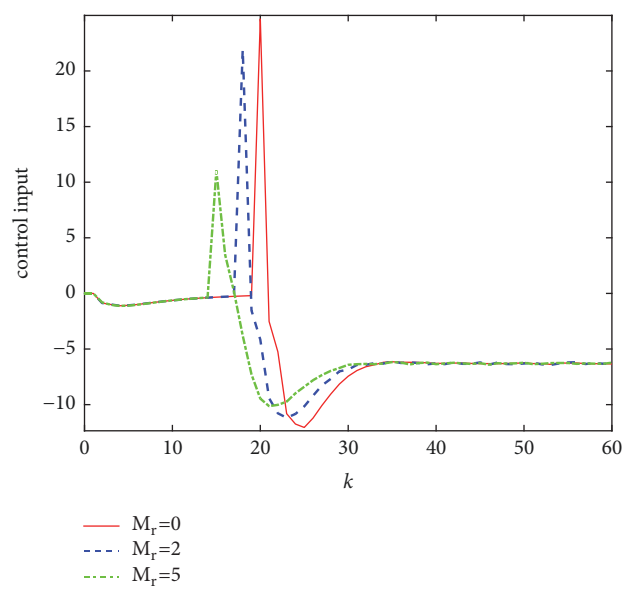

Figure 12: The control input to reference signal (64).

A comparison with relevant result in [33] is presented in Figure 13. The simulation result clearly shows the superiority of the error system method adopted in this paper.

Remark 3. The class of nonlinear systems considered in this paper is a part of the class of Lipschitz systems. Recently, the standard Lipschitz systems [17, 20, 43] and the one-sided Lipschitz systems [42] have been receiving considerable attention owing to their extensive practical applications. How to deal with the preview control problem of these nonlinear systems is a challenging task. Moreover, other complexities like actuator faults [19] and measurement delays [20] can be taken into account in the further studies besides the system uncertainties and the external disturbances. In addition, once the states of system (1) are unavailable, the present control scheme is invalid. Thus, with the help of some novel techniques in [33, 34, 43, 44], the observer-based preview control for Lipschitz systems will be explored in the future work.

6. Conclusion

In this paper, the robust preview tracking controller design for a class of uncertain discrete-time Lipschitz nonlinear systems is investigated. First, we construct an augmented error system including the tracking error and preview information. The Mean-Value theorem plays an important role in dealing with the nonlinearity. To add the integral control action, a discrete integrator is introduced. Next, we develop a state feedback controller for the augmented error system. Using the LMI technique, some criteria on the stability and H_∞ performance are proposed for the closed-loop system. Based on this, the robust preview tracking controller of the original

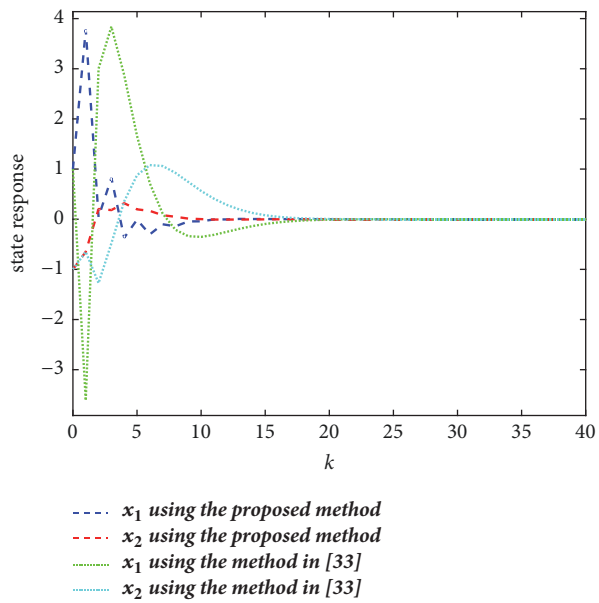

Figure 13: The closed-loop state response.

system is obtained. Finally, the effectiveness of the controller is shown by numerical examples.

Data Availability

Data sharing is not applicable to this article as no datasets were generated or analysed during the current study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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