Research Article

Event-Based Formation Control of Multiple Quadrotors on $SO(3)$

Chaoyang Dong,¹ Mingyu Ma ,¹ Qing Wang,² and Siqian Ma¹

¹School of Aeronautic Science and Engineering, Beihang University, Beijing 10091, China

²School of Automation Science and Electrical Engineering, Beihang University, Beijing 100191, China

Correspondence should be addressed to Mingyu Ma; mamingyu186@126.com

Received 27 December 2017; Accepted 1 April 2018; Published 13 May 2018

Academic Editor: Calogero Orlando

Copyright © 2018 Chaoyang Dong et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

This paper is concerned with the formation problem of multiple quadrotors, and an event-based control strategy is proposed. The communication topology and relative positions of formation are first considered, and then the model of multiple quadrotors system is developed on the special orthogonal group $SO(3)$. By designing the trigger function, certain events are generated for each quadrotor. Then, the formation controller is driven to update its parameters according to the events. The attitude controller on $SO(3)$ is designed for tracking of the command and stabilization. By the proposed method continuous communication is not required between quadrotors, and it is proved that the quadrotors could achieve the desired formation. Simulation illustrates that the proposed event-based formation control method is effective.

1. Introduction

A team of multiple flight vehicles could have more flexibility, reliability, and efficiency over single vehicle, especially in difficult tasks like search, mapping, and attack missions. Therefore, formation control of multiple vehicles has attracted considerable attention for study [1–3]. As a typical unmanned aerial vehicle (UAV), a quadrotor has simple mechanical and flexible maneuverability. Therefore, quadrotor is usually used as the platform for formation control [4], and various types of implementation and applications such as surveillance [5], fault diagnosis [6], and fault-tolerant control [7] have also been investigated.

As a representative approach, formation control problem in several works [8–10] is studied through kinematics. That is, the quadrotor is considered as a point, and its model is double integrators with position and velocity. The quadrotors formation is studied in [9] through the second-order multiagent system. Similar model is adopted in [10], and the consensus protocol is used for formation. However, the control system of the quadrotor is nonlinear. Its thrust and attitude are coupled. Thus, the dynamics of each quadrotor cannot be easily neglected. Formation control of multiple quadrotors with attitude dynamics can be found in [11–13]. The consensus

protocol is used for position control in [11], and it is then transformed as the attitude commands in the form of Euler angles. In [12], the thrust and attitude controllers were studied through dynamic surface control method, with both leader-follower and leaderless cases. The potential functions were proposed for the quadrotors formation in [13]. It also uses repulsive functions to guarantee obstacle avoidance.

Nonetheless, in the referred works [11–13] the dynamics of quadrotor is described with pitch/yaw/roll angles, and then the attitude commands and controllers are developed separately for each angle. It should be noticed that when the pitch angle is 90 degrees, the attitude system described by Euler angles would become singular [14]. This would cause the failure of the controller, so the methods using pitch/yaw/roll angles may not be applicable for large maneuver of quadrotors. To solve this problem, control methods on special orthogonal group $SO(3)$ for single quadrotor were proposed in [15]. In $SO(3)$ this singularity could be avoided by proper design, and the attitude can be modeled and controlled coherently. For single quadrotor, a globally defined model on $SO(3)$ is introduced in [15, 16], and based on it the geometric tracking controller is provided. In [17] the proportional-derivative control on $SO(3)$ was presented with disturbance compensation. The attitude dynamics was

regarded as second-order systems on $SO(3)$ in [18], and the coordinated attitude control of rigid bodies is studied with undirected communication topology.

It should be also noticed that continuous information exchange between quadrotors is required in the above methods, and the controller is formulated by the real-time states of its neighbors. In practical situations, the communications between quadrotors are difficult to maintain continuous. Event-based control strategy provides a solution that the update of the controller is event-triggered rather than driven by the pass of time [19, 20]. In [21, 22] event-triggered consensus was studied for the multiagent system with single-integrator dynamics. The event-based control for general linear multiagent was discussed in [23]. It provides two sufficient conditions for consensus, with or without continuous communication, and significantly decreases updates of the controller. The event was defined to be triggered at discrete sampling time instants in [24], but a clock for synchronization is necessary in each agent.

Thus, it is suitable to introduce the event-based method to the formation control of quadrotors. Though there are established works on the event-based cooperative control strategy, these methods mainly focus the geometric motion of general agents. It is noted that the event-based formation control with dynamics of quadrotors on $SO(3)$ has not been fully studied. Extending basic multiagent systems and single quadrotor to the formation control of multiple quadrotors is not a trivial task. Due to the different structure of attitude dynamics on $SO(3)$, the event-based design and formation stability analysis would become more complex.

Motivated by these ideas, this paper focuses on the multiple quadrotors system, and the dynamics of each quadrotor on $SO(3)$ is considered. The event-based control strategy in multiagent system is introduced for quadrotors formation. Based on local information, a triggering function with time-varying threshold is proposed to generate the events. Since the states of other quadrotors cannot be obtained during non-triggering time, this paper employs the states at last triggering time instants to build dynamical models and construct the event-based formation controller. To avoid singularity, the attitude controller is formulated with the desired commands on $SO(3)$. Then, the stability of the formation and dynamics of quadrotors are analyzed together as an entire closed-loop system. In contrast to existing methods that depend on continuous communications, the proposed event-based method only requires information exchanges when certain events are triggered. It reduces the communication loads and would be more advantageous for application. A simulation with five quadrotors formations is given to illustrate the effectiveness of the proposed method.

2. Preliminaries

2.1. Modeling of the Quadrotors Formation on $SO(3)$. The model of a quadrotor is illustrated in Figure 1. The coordinates of the inertial frame are denoted by (x, y, z) , and the directions of the three axes of the body frame are represented by $b_1^{(i)}$, $b_2^{(i)}$, and $b_3^{(i)}$.

FIGURE 1: The model of the quadrotor.

The attitude describes the orientation from the body frame to inertial frame, and this rotation in three dimensions can be expressed by a rotation matrix $R \in \mathbb{R}^{3 \times 3}$. Consider the special orthogonal group:

$$SO(3) = \{R \in \mathbb{R}^{3 \times 3} \mid RR^T = I_3, \det R = 1\}. \quad (1)$$

Each attitude can be mapped to an element R in $SO(3)$, and this mapping is bijective. Therefore, in this paper we consider R to describe the attitude of the quadrotor. For a vector $\Omega = [\omega_1 \ \omega_2 \ \omega_3]^T$, define the hat map as

$$\widehat{\Omega} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}. \quad (2)$$

Its inverse operation is called the vee map, and it is defined as

$$\begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}^\vee = [\omega_1 \ \omega_2 \ \omega_3]^T. \quad (3)$$

Let $e_1 = [1 \ 0 \ 0]^T$, $e_2 = [0 \ 1 \ 0]^T$, and $e_3 = [0 \ 0 \ 1]^T$. According to the definition of R , the directions of b_k^i can be described by $R_i e_k$; that is, $b_k^i = R_i e_k$, $k = 1, 2, 3$. Then, the thrust in inertial frame is $-f_i R_i e_3$. Based on these discussions, we then present the $SO(3)$ model of the i th quadrotor:

$$\begin{aligned} \dot{p}_i &= v_i, \\ m_i \dot{v}_i &= m_i g e_3 - f_i R_i e_3, \\ \dot{R}_i &= R_i \widehat{\Omega}_i, \end{aligned} \quad (4)$$

$$J_i \dot{\Omega}_i + \Omega_i \times J_i \Omega_i = M_i,$$

where p_i and $v_i \in \mathbb{R}^3$ are the position and velocity, respectively, m_i is the mass, g is the gravity acceleration, $f_i = \sum_{k=1}^4 f_k^{(i)} \in \mathbb{R}$ is the total thrust, $J_i \in \mathbb{R}^3$ is the inertia matrix, $\Omega_i \in \mathbb{R}^3$ is the angular velocity, and $M_i \in \mathbb{R}^3$ is the control moment, $i = 1, 2, \dots, N$.

Note that the total thrust and moment satisfy

$$\begin{bmatrix} f_i \\ M_1^{(i)} \\ M_2^{(i)} \\ M_3^{(i)} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -d^{(i)} & 0 & d^{(i)} \\ d^{(i)} & 0 & -d^{(i)} & 0 \\ -c_{\tau f}^{(i)} & c_{\tau f}^{(i)} & -c_{\tau f}^{(i)} & c_{\tau f}^{(i)} \end{bmatrix} \begin{bmatrix} f_1^{(i)} \\ f_2^{(i)} \\ f_3^{(i)} \\ f_4^{(i)} \end{bmatrix}, \quad (5)$$

where $d^{(i)}$ is the distance between the center of mass and center of rotor on the $b_1^{(i)}-b_2^{(i)}$ plane. $c_{\tau f}^{(i)}$ is a positive constant and $(-1)^k c_{\tau f}^{(i)} f_k^{(i)}$ represents the moment produced by the k th rotor. It is learned that the matrix in (5) is invertible when $d^{(i)}, c_{\tau f}^{(i)} \neq 0$. Note that by (5) each $f_k^{(i)}$ can be calculated for given f_i and M_i . Therefore, the total thrust f_i and moment M_i are considered as the control inputs in this paper.

2.2. Communication Topology and Formation of Multiple Quadrotors. The communication topology indicates the information exchange between quadrotors, and it is described by graph theory. In this paper the information link is considered to be directed. Let $\mathcal{G} = \mathcal{G}(\mathcal{V}, \mathcal{E})$ be a directed graph with N nodes; each node represents one quadrotor. $\mathcal{V} = \{1, 2, \dots, N\}$ is the vertex set and $\mathcal{E} = \{(i, j) : i, j \in \mathcal{V}\}$ is the edge set. A directed edge from i to j is denoted by the pair (i, j) , and it means that the i th quadrotor can receive messages from the j th quadrotor. In this case the j th quadrotor is said to be the neighbor of i th quadrotor. The neighbor set is denoted by $\mathcal{N}_i$, where $\mathcal{N}_i = \{j \in \mathcal{V} \mid (j, i) \in \mathcal{E}\}$. The adjacency matrix $\mathcal{A}$ of $\mathcal{G}$ is a $N \times N$ matrix, whose ij th entry a_{ij} is greater than zero if (j, i) is an edge of G and zero if it is not. The Laplacian $\mathcal{L} = [l_{ij}]$ is defined as follows:

$$\mathcal{L} = \begin{cases} l_{ij} = -a_{ij}, & i \neq j \\ l_{ii} = \sum_{j=1, j \neq i}^n a_{ij}. \end{cases} \quad (6)$$

A directed tree is a directed graph, where every node has exactly one parent except for the root, and the root is the only node with no parent but has a direct path to every other node. A spanning tree of a digraph is a directed tree formed by graph edges that connects all the nodes of the graph.

In the three-dimensional space, the formation of multiple quadrotors can be described by the relative positions between each other. As shown in Figure 2, let O_r be the center of the formation; $r_i(r_j)$ is the vector $r_i(r_j)$ that denotes the distance from the $i(j)$ th quadrotor to O_r . Therefore, the relative position of these two vehicles is $r_{ij} = r_i - r_j$. By selecting certain r_{ij} , it would formulate the desired formation.

3. Event-Based Formation Control Design

In this section, the event-based control strategy for quadrotors formation is presented. Each quadrotor communicates with its neighbors only when it satisfies its own trigger conditions. In nontrigger time instants, the quadrotors need no communication, and it maintains to be functional with

FIGURE 2: The relative positions between quadrotors.

its own controllers. Moreover, the trigger conditions of each quadrotor are allowed to be different in the proposed method, and thus the multiple quadrotors system is triggered asynchronously.

3.1. Design of the Event-Based Formation Control. Define $s_i = [p_i^T \ v_i^T]^T$, and then the dynamics of position in (4) can be rewritten in a state-space form as follows:

$$\dot{s}_i = A s_i + B u_i, \quad (7)$$

where

$$\begin{aligned} u_i &= -\frac{f_i R_i e_3}{m_i} + g e_3, \\ A &= \begin{bmatrix} 0_3 & I_3 \\ 0_3 & 0_3 \end{bmatrix}, \\ B &= \begin{bmatrix} 0_3 \\ I_3 \end{bmatrix}. \end{aligned} \quad (8)$$

The event-based formation control for the thrust f_i is designed as

$$f_i = \left(-mK \sum_{j \in \mathcal{N}_i} a_{ij} (y_{ij}(t) - y_i(t)) + m g e_3 \right) \cdot (R_i e_3), \quad (9)$$

where

$$\begin{aligned} \dot{y}_i(t) &= A y_i(t), \\ y_i(t_k^i) &= s_i(t_k^i) - r_i, \\ \dot{y}_{ij}(t) &= A y_{ij}(t), \\ y_{ij}(t_k^i) &= s_j(t_k^i) - r_j. \end{aligned} \quad (10)$$

“ $\cdot$ ” is the inner product, $K \in \mathbb{R}^{3 \times 6}$ is the control gain, t_k^i is the k th triggering time instant of the i th vehicle, and $y_{ij}(t)$ is the state of j th vehicle in the i th controller.

The proposed controller uses both the states $s_i(t)$ and its neighbors $s_j(t)$. Note that the i th quadrotor can only use the discrete value $s_j(t_k^i)$. At nontrigger times ($t \neq t_k^i$), the real-time values of its neighbors are unavailable and the i th controller works independently. Thus, we consider constructing virtual models $y_i(t)$ and $y_{ij}(t)$ to replace $s_i(t)$ and $s_j(t)$.

To analyze the formation control system (7) with controller (9), denote

$$F_i = -m_i K \sum_{j \in \mathcal{N}_i} a_{ij} (y_{ij}(t) - y_i(t)) + m_i g e_3, \quad (11)$$

$$\sigma_i = \frac{f_i}{e_3^T R_{di}^T R_i e_3} ((e_3^T R_{di}^T R_i e_3) R_i e_3 - R_{di} e_3). \quad (12)$$

Note that $f_i = \|F_i\| R_{di} e_3$. Then, we have

$$\begin{aligned} f_i R_i e_3 &= \frac{f_i}{e_3^T R_{di}^T R_i e_3} R_{di} e_3 + \sigma_i \\ &= \frac{\|F_i\| R_{di} e_3 \cdot R_i e_3}{e_3^T R_{di}^T R_i e_3} R_{di} e_3 + \sigma_i = F_i + \sigma_i. \end{aligned} \quad (13)$$

Thus, system (7) can be expressed as

$$\dot{s}_i = A s_i + B K \sum_{j \in \mathcal{N}_i} a_{ij} (y_{ij}(t) - y_i(t)) + \frac{B \sigma_i}{m_i}. \quad (14)$$

Let $\varepsilon(t) = [\varepsilon_2^T(t) \ \varepsilon_3^T(t) \cdots \varepsilon_N^T(t)]^T$, $\varepsilon_i(t) = \tilde{s}_i(t) - \tilde{s}_1(t)$, $\tilde{s}_i(t) = s_i(t) - \tilde{r}_i$, $\varepsilon_p(t) = [\varepsilon_{p2}^T(t) \ \varepsilon_{p3}^T(t) \cdots \varepsilon_{pN}^T(t)]^T$, and $\varepsilon_{pi}(t) = B \sigma_i / m_i - B \sigma_1 / m_1$. And denote the errors as $e_i = y_i(t) - \tilde{s}_i(t)$ and $e_{ij}(t) = y_{ij}(t) - \tilde{s}_j(t)$. Then, we have

$$\begin{aligned} \dot{\varepsilon}_i(t) &= \dot{\tilde{s}}_i(t) - \dot{\tilde{s}}_1(t) = \dot{s}_i(t) - \dot{s}_1(t) \\ &= A \varepsilon_i(t) \\ &\quad + B K \sum_{j \in \mathcal{N}_i} a_{ij} (\tilde{s}_j(t) - \tilde{s}_i(t) + e_{ij}(t) - e_i(t)) \\ &\quad - B K \sum_{j \in \mathcal{N}_1} a_{1j} (\tilde{s}_j(t) - \tilde{s}_1(t) + e_{1j}(t) - e_1(t)) \\ &\quad + \frac{B \sigma_i}{m_i} - \frac{B \sigma_1}{m_1} \\ &= (A - d_i B K) \varepsilon_i(t) + B K \sum_{j=1}^N (a_{ij} - a_{1j}) \varepsilon_j(t) \\ &\quad - d_i B K e_i(t) + d_1 B K e_1(t) + \sum_{j \in \mathcal{N}_i} a_{ij} e_{ij}(t) \\ &\quad - \sum_{j \in \mathcal{N}_1} a_{1j} e_{1j}(t) + \varepsilon_{pi}. \end{aligned} \quad (15)$$

Thus, let $\bar{e}_1(t) = [e_1^T(t) \cdots e_1^T(t)]^T$, $\bar{e}_2(t) = [e_2^T(t) \cdots e_N^T(t)]^T$, $\bar{e}_{11}(t) = [e_{11}^T(t) \cdots e_{11}^T(t)]^T$, and $\bar{e}_{22}(t) = [e_{22}^T(t) \cdots e_{NN}^T(t)]^T$, where $e_{ii}(t) = [e_{ij}(t)]$ is $|\mathcal{N}_i|$ -dimensional vectors, $j \in \mathcal{N}_i$. Then, (15) can be rewritten as

$$\begin{aligned} \dot{\varepsilon}(t) &= (I_{N-1} \otimes A - L_1 \otimes B K) \varepsilon(t) - (D_2 \otimes B K) \bar{e}_2(t) \\ &\quad + (D_1 \otimes B K) \bar{e}_1(t) + (A_2 \otimes B K) \bar{e}_{22}(t) \\ &\quad - (A_1 \otimes B K) \bar{e}_{11}(t) + \varepsilon_p(t), \end{aligned} \quad (16)$$

where

$$\begin{aligned} L_1 &= \begin{bmatrix} a_{12} + d_2 & a_{13} - a_{23} & \cdots & a_{1N} - a_{2N} \\ a_{12} - a_{32} & a_{13} - d_3 & \cdots & a_{1N} - a_{3N} \\ \vdots & \vdots & \ddots & \vdots \\ a_{12} - a_{N2} & a_{13} - a_{N3} & \cdots & a_{1N} + d_N \end{bmatrix} \\ D_1 &= \text{diag}\{d_1, \dots, d_1\}, \\ D_2 &= \text{diag}\{d_2, d_3, \dots, d_N\}, \\ A_1 &= \text{diag}\{a_1, \dots, a_1\}, \\ A_2 &= \text{diag}\{a_2, \dots, a_N\}, \\ a_i &= (a_{ij})_{1 \times |\mathcal{N}_i|}, \quad j \in \mathcal{N}_i. \end{aligned} \quad (17)$$

Next, we are to discuss the selection of the control gain K . If the graph $\mathcal{G}$ contains a spanning tree, then $\text{Re}\{\lambda_i(L_1)\} > 0$, $i = 1, 2, \dots, N-1$, where $\lambda_i(L_1)$ are the eigenvalues of L_1 . Then, let $\lambda_{\min} = \min_{1 \leq i \leq N-1} \{\text{Re}(\lambda_i)\} > 0$, and consider the following algebraic Riccati equation:

$$P A + A^T P - 2 \lambda_{\min} P B B^T P + Q = 0, \quad (18)$$

where $Q > 0$ is a given matrix and $P > 0$ is the solution of the equation. Choose $K = k B^T P$, $k \geq 1$. Then, from (18) it can be verified that $A - \lambda_i(L_1) B K$ is Hurwitz. Further, it is known that $A_L = (I_{N-1} \otimes A - L_1 \otimes B K)$ is also Hurwitz. Denoting the largest eigenvalue of $(1/2)(A_L^T + A_L)$ as $-\lambda_0$, it is learned that $\lambda_0 > 0$.

Based on the previous analysis, consider the bounds of (16). Then, we have

$$\begin{aligned} \|\dot{\varepsilon}(t)\| &\leq \|A_L \varepsilon(t)\| + \|D_2 \otimes B K\| \|\bar{e}_2(t)\| \\ &\quad + \|D_1 \otimes B K\| \|\bar{e}_1(t)\| + \|A_2 \otimes B K\| \|\bar{e}_{22}(t)\| \\ &\quad + \|A_1 \otimes B K\| \|\bar{e}_{11}(t)\| + \|\varepsilon_p(t)\|. \end{aligned} \quad (19)$$

Note that, for the Kronecker product of two matrices A and B , the norm of it satisfies $\|A \otimes B\| = \|A\| \|B\|$. Thus,

$$\begin{aligned} &\|D_2 \otimes B K\| \|\bar{e}_2(t)\| + \|D_1 \otimes B K\| \|\bar{e}_1(t)\| \leq \|B K\| \\ &\quad \cdot (d_{\max} \|\bar{e}_2(t)\| + d_1 \|\bar{e}_1(t)\|) = \|B K\| \\ &\quad \cdot \left(d_{\max} \sqrt{\sum_{i=2}^N e_i^T(t) e_i(t)} \right. \\ &\quad \left. + d_1 \sqrt{(N-1) e_1^T(t) e_1(t)} \right) \leq d_0 \|B K\| \|e(t)\|, \end{aligned} \quad (20)$$

where $d_0 = \sqrt{d_{\max}^2 + d_1^2(N-1)}$, $e(t) = [e_1^T(t) \cdots e_N^T(t)]^T$, and $d_{\max} = \max_{2 \leq i \leq N} \{d_i\}$.

Following a similar method, we have

$$\begin{aligned} &\|A_2 \otimes B K\| \|\bar{e}_{22}(t)\| + \|A_1 \otimes B K\| \|\bar{e}_{11}(t)\| \\ &\leq a_0 \|B K\| \|\bar{e}(t)\|, \end{aligned} \quad (21)$$

where $a_0 = \sqrt{a_{\max}^2 + \|a_1\|^2(N-1)}$, $a_{\max} = \max_{2 \leq i \leq N, j \in \mathcal{N}_i} \{a_{ij}\}$, and $\bar{e}(t) = [e_{11}^T(t) \cdots e_{NN}^T(t)]^T$.

Combining (20) and (21), (19) can be estimated to be

$$\|\dot{e}(t)\| \leq \|A_L \varepsilon(t)\| + \|BK\| (d_0 \|e(t)\| + a_0 \|\bar{e}(t)\|) + \|\varepsilon_p(t)\|. \quad (22)$$

3.2. Design of the Trigger Function. The trigger function is used to generate the events. When the states in the i th quadrotor satisfy some conditions at t_k^i , an event is triggered and the controllers are updated. Thus, the trigger function of i th quadrotor is designed as

$$\begin{aligned} g_i(t, e_i(t), y_i(t)) &= \eta_i \|e_i(t)\| + \mu_i (e^{\|A\|(t-t_k^i)} - e^{\rho(t-t_k^i)}) \\ &\quad - \frac{1}{\sqrt{2}} \left(\sqrt{c_0 \sum_{j \in \mathcal{N}_i} \|y_{ij}(t) - y_j(t)\|^2} + \delta_i(t) \right), \end{aligned} \quad (23)$$

where $\eta_i = \sqrt{d_0^2 + c_0 |\mathcal{N}_i|}$, c_0 is a constant and $2\|BK\| \sqrt{2c_0 n_{\max}} < \lambda_0$, $n_{\max} = \max\{|\mathcal{N}_i|\}$ is the maximum number of the neighbors among N quadrotors, $\mu_i = \sqrt{(a_0^2 + b_0) \sum_{j \in \mathcal{N}_i} (\mu_k^{ij})^2}$, $\mu_k^{ij} = A_0 \|Bu_j(t_k^i)\| / (\|A\| - \rho)$, $b_0 \geq c_0 > 0$, $\delta_i(t) = \delta_0^i e^{-\beta_i t} + \delta_1^i$, and A_0 , ρ , δ_0^i , δ_1^i , and β_i are given constants.

Therefore, the trigger time instants are defined as follows:

$$t_{k+1}^i = \inf \{t > t_k^i \mid g_i(t, e_i(t), y_i(t)) > 0\}. \quad (24)$$

when $t = t_{k+1}^i$, the i th quadrotor updates its controller. From (23) it can be learned that after the update the trigger function became less than zero again. The trigger function keeps being negative between two successive trigger times.

By models (4) and (22), it is known that if the attitude subsystem is stable, then $\lim_{t \rightarrow \infty} \|\varepsilon_p(t)\| \rightarrow 0$. Thus, the trajectory/formation control of the quadrotor depends on the stability of its attitude. Therefore, before conducting stability analysis of the event-based control method, in the next section we would first discuss the attitude subsystem of the quadrotor.

Remark 1. $\delta_i(t)$ in the trigger function is designed as a time-varying threshold. It could regulate the frequency of triggering the event. In our method $\delta_i(t)$ is set to be in the form of $\delta_0^i e^{-\beta_i t} + \delta_1^i$. Thus, the threshold is relatively large to prevent excessive event-triggering when the system starts; and it became smaller for better accuracy as the system converges.

4. Attitude Control Design on SO(3)

The thrust with both its magnitude and direction determines the flight trajectory. Note that the magnitude of the thrust is given by f_i , and the direction is described by $b_3^{(i)} = R_i e_3$.

Therefore, the thrust in the inertial frame can be expressed as $-f_i R_i e_3$. In order to achieve the desired formation, the attitude needs to be adjusted to provide the correct direction. In this section the attitude commands are first discussed, and then the attitude controllers on SO(3) are proposed to track the commands and stabilize the attitude subsystem.

4.1. Design of the Attitude Command. The attitude command of i th vehicle is denoted by R_{di} , where $R_{di} = [b_{1d}^{(i)} \ b_{2d}^{(i)} \ b_{3d}^{(i)}] \in \text{SO}(3)$. Thus, based on the thrust command f_i in (9), we would design $b_{3d}^{(i)}$ as follows:

$$b_{3d}^{(i)} = \frac{F_i}{\|F_i\|}, \quad (25)$$

where F_i is defined in (11). It should be noticed that $R_{di} \in \text{SO}(3)$, which means that $b_{1d}^{(i)}$, $b_{2d}^{(i)}$, and $b_{3d}^{(i)}$ are orthogonal to each other. Once two of the column vectors in R_{di} are chosen, the third vector is then fixed. After designing the third body axis $b_{3d}^{(i)}$, there is one remaining degree of freedom to adjust the attitude. Thus, we have $b_{2d}^{(i)} = (b_{3d}^{(i)} \times b_{1d}^{(i)}) / \|b_{3d}^{(i)} \times b_{1d}^{(i)}\|$, $b_{1d}^{(i)} = b_{2d}^{(i)} \times b_{3d}^{(i)}$.

The attitude error between R_i and R_{di} on SO(3) is then defined as

$$e_{Ri} = \frac{1}{2} (R_{di}^T R_i - R_i^T R_{di})^\vee \quad (26)$$

and the tracking error of angular velocity is

$$e_{\Omega i} = \Omega_i - R_i^T R_{di} \Omega_{di}. \quad (27)$$

Next, we will discuss the dynamics of $e_{\Omega i}$ and e_{Ri} . By noticing $\dot{R}_{di}^T = -\dot{R}_{di} R_{di}^T R_i$ and $\Omega_{di}^T \hat{\Omega}_{di} = 0$, the derivative of $e_{\Omega i}$ is calculated as follows:

$$\begin{aligned} J_i \dot{e}_{\Omega i} &= J_i \dot{\Omega}_i - J_i \left[\frac{d}{dt} (R_i^T R_{di}) \Omega_{di} + R_i^T R_{di} \dot{\Omega}_{di} \right] \\ &= J_i \dot{\Omega}_i - J_i \left[(R_{di}^T \dot{R}_i + \dot{R}_{di}^T R_i) \Omega_{di} + R_i^T R_{di} \dot{\Omega}_{di} \right] \\ &= J_i \dot{\Omega}_i \\ &\quad - J_i \left[R_{di}^T R_i (\Omega_i - R_i^T R_{di} \Omega_{di})^\wedge \Omega_{di} + R_i^T R_{di} \dot{\Omega}_{di} \right] \quad (28) \\ &= J_i \dot{\Omega}_i - J_i \left[(\Omega_{di}^T R_{di}^T R_i \hat{e}_{\Omega i})^T + R_i^T R_{di} \dot{\Omega}_{di} \right] \\ &= J_i \dot{\Omega}_i + J_i (\hat{\Omega}_i R_i^T R_{di} \Omega_{di} - R_i^T R_{di} \dot{\Omega}_{di}) \\ &= M_i - \hat{\Omega}_i J_i \Omega_i + J_i (\hat{\Omega}_i R_i^T R_{di} \Omega_{di} - R_i^T R_{di} \dot{\Omega}_{di}). \end{aligned}$$

By (26) it can be calculated that $\dot{e}_{Ri} = C(R_{di}^T R_i) e_{\Omega i}$, where $C(R_{di}^T R_i) = (1/2)(\text{tr}[R_{di}^T R_i] I_3 - R_i^T R_{di})$ and $\|C(R_{di}^T R_i)\| \leq 1$.

4.2. Attitude Controller on SO(3) with Stability Analysis. The attitude controller is designed as

$$\begin{aligned} M_i &= -k_{Ri} e_{Ri} - k_{\Omega i} e_{\Omega i} + \Omega_i \times J_i \Omega_i \\ &\quad - J_i (\hat{\Omega}_i R_i^T R_{di} \Omega_{di} - R_i^T R_{di} \dot{\Omega}_{di}). \end{aligned} \quad (29)$$

We present the following theorem to reveal the stability of the attitude dynamics.

Theorem 1. *The attitude of the quadrotor is described in (4), and the command is given in (25). Then, for positive constant scalars k_{Ri} and $k_{\Omega i}$, the tracking error of the quadrotor is stable with controller (29).*

Proof. Consider the Lyapunov candidate as

$$V_{li} = k_{Ri} \Psi(R_i, R_{di}) + \frac{1}{2} e_{\Omega i}^T J_i e_{\Omega i} + c_i e_{Ri}^T e_{\Omega i}, \quad (30)$$

where $c_i < \min\{k_{\Omega i}, \sqrt{2\lambda_{\min}(J_i)k_{Ri}}, 4k_{Ri}k_{\Omega i}\lambda_{\max}^2(J_i)/(4k_{Ri}\lambda_{\max}^2(J_i) + c_i k_{\Omega i}^2 \lambda_{\min}(J_i))\}$, and $\Psi(R_i, R_{di}) = (1/2)\text{tr}[I_3 - R_{di}^T R_i]$.

Note that $e_{\Omega i}^T J_i e_{\Omega i} \geq \lambda_{\min}(J_i) \|e_{\Omega i}\|^2$, $\Psi(R_i, R_{di}) \geq (1/2) \|e_{Ri}\|^2$, and $c_i < \sqrt{2\lambda_{\min}(J_i)k_{Ri}}$. Denote $z_i = [\|e_{Ri}\| \ \|e_{\Omega i}\|]^T$ and we then have

$$V_{li} \geq \frac{1}{2} z_i^T \begin{bmatrix} k_{Ri} & -c_i \\ -c_i & 2\lambda_{\min}(J_i) \end{bmatrix} z_i > 0. \quad (31)$$

Thus, it is verified that V_{li} is positive definite. Calculate its derivative and we have

$$\begin{aligned} \dot{V}_{li} &= e_{\Omega i}^T J_i \dot{e}_{\Omega i} + k_{Ri} \frac{d}{dt} \Psi(R_i, R_{di}) + c_i \frac{d}{dt} (e_{Ri}^T e_{\Omega i}) \\ &= e_{\Omega i}^T J_i \dot{e}_{\Omega i} + \frac{1}{2} k_{Ri} \text{tr} [\hat{e}_{\Omega i} (R_{di}^T R_i - R_{di} R_i^T)] \\ &\quad - c_i k_{Ri} e_{Ri}^T J_i^{-1} e_{Ri} + c_i C(R_{di}^T R_i) e_{\Omega i}^T e_{\Omega i} \\ &\quad - c_i k_{\Omega i} e_{Ri}^T J_i^{-1} e_{\Omega i} \\ &\leq e_{\Omega i}^T J_i \dot{e}_{\Omega i} + k_{Ri} \frac{1}{2} \text{tr} [\hat{e}_{\Omega i} \hat{e}_{\Omega i}] - c_i k_{Ri} e_{Ri}^T J_i^{-1} e_{Ri} \\ &\quad + c_i e_{\Omega i}^T e_{\Omega i} - c_i k_{\Omega i} e_{Ri}^T J_i^{-1} e_{\Omega i} \\ &\leq e_{\Omega i}^T (-k_{Ri} e_{Ri} - k_{\Omega i} e_{\Omega i}) - c_i k_{Ri} \lambda_{\min}(J_i^{-1}) \|e_{Ri}\|^2 \\ &\quad + k_{Ri} e_{Ri}^T e_{\Omega i} + c_i k_{\Omega i} \|e_{Ri}\| \|e_{\Omega i}\| \\ &\quad - c_i k_{\Omega i} \lambda_{\min}(J_i^{-1}) \|e_{\Omega i}\|^2 \\ &\leq -z_i^T \begin{bmatrix} c_i k_{Ri} \lambda_{\min}(J_i^{-1}) & -c_i k_{\Omega i} \lambda_{\max}(J_i^{-1}) \\ -c_i k_{\Omega i} \lambda_{\max}(J_i^{-1}) & k_{\Omega i} - c_i \end{bmatrix} z_i \\ &< 0. \end{aligned} \quad (32)$$

Therefore, for proper value c_i that satisfies the constraint in (30), it is proved that by M_i the attitude R_i could track the desired command R_{di} and the attitude system is stable. $\square$

Remark 2. Roughly speaking, the control system of quadrotor consists of outer loop and inner loop. The outer loop refers to the trajectory/formation control, and states of neighbor quadrotors are needed to achieve formation. The inner loop of the quadrotor handles its own attitude dynamics, and it can work without other quadrotors.

5. Stability Analysis of the Event-Based Formation Control System

After designing the thrust controller (9), the trigger function (23), the attitude command R_{di} , and controller (29), in this section we discuss the stability of the closed-loop system. The following theorem is presented.

Theorem 2. *Consider the quadrotor system described by (4), and the formation is given by r_{ij} . If the communication topology contains a spanning tree, then by the event-based controller (9) and attitude controller (29) on $SO(3)$, the quadrotors will converge to the desired formation; that is, $\lim_{t \rightarrow \infty} \|\varepsilon\| = \Delta$, where $\Delta = 2\sqrt{2} \|BK\| \delta_1 / (\lambda_0 - 2\sqrt{2} c_0 n_{\max} \|BK\|)$.*

Proof. Invoking the trigger function (23), we have

$$\begin{aligned} \eta_i \|e_i(t)\| + \mu_i \left(e^{\|A\|(t-t_k^i)} - e^{\rho(t-t_k^i)} \right) \\ \leq \frac{1}{\sqrt{2}} \left(\sqrt{c_0 \sum_{j \in \mathcal{N}_i} \|y_{ij}(t) - y_i(t)\|^2} + \delta_i(t) \right). \end{aligned} \quad (33)$$

Then, we are to prove that

$$\sqrt{a_0^2 + b_0} \|e_{ij}(t)\| \leq \mu_i \left(e^{\|A\|(t-t_k^i)} - e^{\rho(t-t_k^i)} \right). \quad (34)$$

Denote the Dini derivative of $\|e_{ij}(t)\|$ as $D^+ \|e_{ij}(t)\|$, and it can be verified that

$$\begin{aligned} D^+ \|e_{ij}(t)\| &\leq \|\dot{e}_{ij}(t)\| \\ &\leq \|A\| \|e_{ij}(t)\| + \|Bu_j(t_k^i)\| \|e^{A(t-t_k^i)}\|. \end{aligned} \quad (35)$$

For $\|e^{A(t-t_k^i)}\|$, note that there exist $A_0 \geq 1$ and $\rho \leq \|A\|$ so that $\|e^{A(t-t_k^i)}\| \leq A_0 e^{\rho(t-t_k^i)}$. Then, (35) can be rewritten as

$$\begin{aligned} D^+ z &= -\|A\| e^{-\|A\|(t-t_k^i)} \|e_{ij}(t)\| \\ &\quad + e^{-\|A\|(t-t_k^i)} D^+ \|e_{ij}(t)\| \\ &\leq (\|A\| - \rho) z + A_0 \|BKu_j(t_k^i)\|, \end{aligned} \quad (36)$$

where $z = e^{-\rho(t-t_k^i)} \|e_{ij}(t)\|$. Then, (34) can be verified by solving inequality (36). Then, from (33) and (34) we have

$$\begin{aligned} \eta_i^2 \|e_i(t)\|^2 + (a_0^2 + b_0) \|e_{ij}(t)\|^2 \\ \leq c_0 \sum_{j \in \mathcal{N}_i} \|y_{ij}(t) - y_i(t)\|^2 + \delta_i^2(t). \end{aligned} \quad (37)$$

Note that $\|y_{ij}(t) - y_i(t)\|^2 \leq \|e_i(t)\|^2 + \|e_{ij}(t)\|^2 + \|\tilde{s}_j(t) - \tilde{s}_i(t)\|^2$; the sum term in (37) is bounded as

$$\begin{aligned} \sum_{j \in \mathcal{N}_i} \|y_{ij}(t) - y_i(t)\|^2 &\leq |\mathcal{N}_i| \|e_i(t)\|^2 + \sum_{j \in \mathcal{N}_i} \|e_{ij}(t)\|^2 \\ &\quad + \sum_{j \in \mathcal{N}_i} \|\tilde{s}_j(t) - \tilde{s}_i(t)\|^2 \end{aligned} \quad (38)$$

and it implies that

$$\begin{aligned} & \sum_{i=1}^N \sum_{j \in \mathcal{N}_i} \|\tilde{s}_j(t) - \tilde{s}_i(t)\|^2 \\ & \leq \sum_{i=1}^N \sum_{j \in \mathcal{N}_i} \left(\|\tilde{s}_j(t) - \tilde{s}_1(t)\|^2 + \|\tilde{s}_i(t) - \tilde{s}_1(t)\|^2 \right) \\ & \leq n_{\max} \|\varepsilon(t)\|^2. \end{aligned} \quad (39)$$

Next, let $\delta(t) = \delta_0 e^{-\beta t} + \delta_1$, where $\delta_0 = \sqrt{\sum_{i=1}^N (\delta_0^i)^2}$, $\delta_1 = \sqrt{\sum_{i=1}^N (\delta_1^i)^2}$, and $\beta = \min_{1 \leq i \leq N} \{\beta_i\}$. Invoking Cauchy-Schwarz inequality, we have

$$\sum_{i=1}^N \left(\delta_0^i e^{-\beta_i t} + \delta_1^i \right)^2 \leq \delta^2(t). \quad (40)$$

The sum from i to N of (37) can be expressed as

$$\begin{aligned} & \sum_{i=1}^N \left(\eta_i^2 \|e_i(t)\|^2 + (a_0^2 + b_0) \|e_{ij}(t)\|^2 \right) \\ & \leq c_0 \sum_{i=1}^N \sum_{j \in \mathcal{N}_i} \|y_{ij}(t) - y_i(t)\|^2 + \sum_{i=1}^N \delta_i^2(t). \end{aligned} \quad (41)$$

By combining (38)–(40), rewrite (41) as

$$\begin{aligned} & (d_0^2 + c_0 |\mathcal{N}_i|) \|e(t)\|^2 + (a_0^2 + b_0) \|\tilde{e}(t)\|^2 \\ & \leq c_0 |\mathcal{N}_i| \|e(t)\|^2 + c_0 \|\tilde{e}(t)\|^2 + c_0 n_{\max} \|\varepsilon(t)\|^2 \\ & + \delta^2(t). \end{aligned} \quad (42)$$

Then, we obtain the fact that

$$d_0 \|e(t)\| + a_0 \|\tilde{e}(t)\| \leq \sqrt{2c_0 n_{\max}} \|\varepsilon(t)\| + \sqrt{2} \delta(t). \quad (43)$$

Choose a Lyapunov function as $V_2 = (1/2)\varepsilon^T(t)\varepsilon(t)$ and calculate its derivative. From (22), it is obtained that

$$\begin{aligned} \dot{V}_2 &= \varepsilon(t) \dot{\varepsilon}(t) \\ & \leq -\lambda_0 \|\varepsilon(t)\|^2 \\ & + \|BK\| \|\varepsilon(t)\| (d_0 \|e(t)\| + a_0 \|\tilde{e}(t)\|) \\ & + \|\varepsilon(t)\| \|\varepsilon_p(t)\|. \end{aligned} \quad (44)$$

By substituting (43) into (44), $\dot{V}_2$ is rewritten as

$$\begin{aligned} \dot{V}_2 & \leq -(\lambda_0 - \sqrt{2c_0 n_{\max}} \|BK\|) \|\varepsilon(t)\|^2 \\ & + \sqrt{2} \|BK\| \|\varepsilon(t)\| \delta(t) + \|\varepsilon(t)\| \|\varepsilon_p(t)\|. \end{aligned} \quad (45)$$

To further estimate the upper bound of $\dot{V}_2$, we would begin with $\|\varepsilon_p(t)\|$. From the definition of $\varepsilon_p(t)$, it is learned that

$$\|\varepsilon_{pi}(t)\| = \left\| \frac{B\sigma_i}{m_i} - \frac{B\sigma_1}{m_1} \right\| \leq \frac{\|B\sigma_i\|}{m_i} + \frac{\|B\sigma_1\|}{m_1}. \quad (46)$$

It is noted by (12) that $f_i = \|F_i\| e_3^T R_{di}^T R_i e_3$, and we have

$$\begin{aligned} \|B\sigma_i\| &= \|BF_i\| \|(e_3^T R_{di}^T R_i e_3) R_i e_3 - R_{di} e_3\| \\ &\leq \|BF_i\| \|e_{Ri}\|. \end{aligned} \quad (47)$$

Theorem 1 implies that the attitude subsystem is stable, $\|e_{Ri}\| = \sqrt{\Psi(2 - \Psi)} \leq 1$. Denote $E_R = \max\{\|e_{Ri}\|\}$, and thus we have $E_R \leq 1$.

Following a similar approach in (15), (16), and (22), denote $F_B = [(BF_2)^T (BF_3)^T \dots (BF_N)^T]^T$ and $F_{B1} = [(BF_1)^T (BF_1)^T \dots (BF_1)^T]^T$, where F_i is defined in (11). Then, considering (46) and (47), $\|\varepsilon_p(t)\|$ is estimated as

$$\begin{aligned} \|\varepsilon_p(t)\| &\leq \|F_{B1}\| E_R + \|F_{B2}\| E_R \\ &\leq \|(D_2 \otimes BK) \bar{e}_2(t) + (A_2 \otimes BK) \bar{e}_{22}(t)\| E_R \\ &\quad + \|(D_1 \otimes BK) \bar{e}_1(t) - (A_1 \otimes BK) \bar{e}_{11}(t)\| E_R \\ &\leq E_R \|BK\| (d_0 \|e(t)\| + a_0 \|\tilde{e}(t)\|) \\ &\leq E_R \sqrt{2c_0 l_{\max}} \|BK\| \|\varepsilon(t)\| + \sqrt{2} E_R \|BK\| \delta(t). \end{aligned} \quad (48)$$

By combining (45) with (48), $\dot{V}_2$ can be written as

$$\begin{aligned} \dot{V}_2 &\leq -(\lambda_0 - \sqrt{2c_0 n_{\max}} \|BK\|) \|\varepsilon(t)\|^2 \\ &\quad + \sqrt{2} \|BK\| \|\varepsilon(t)\| \delta(t) \\ &\quad + E_R \sqrt{2c_0 n_{\max}} \|BK\| \|\varepsilon(t)\|^2 \\ &\quad + \sqrt{2} E_R \|BK\| \|\varepsilon(t)\| \delta(t) \\ &\leq -(\lambda_0 - 2\sqrt{2c_0 n_{\max}} \|BK\|) \|\varepsilon(t)\|^2 \\ &\quad + 2\sqrt{2} \|BK\| \|\varepsilon(t)\| \delta(t). \end{aligned} \quad (49)$$

Therefore, from (49) we know that $\|\varepsilon(t)\|$ is stable in the sense of ultimately uniformly bounded (UUB) and $\|\varepsilon(t)\| \leq 2\sqrt{2} \|BK\| \delta(t) / (\lambda_0 - 2\sqrt{2c_0 n_{\max}} \|BK\|)$. Then, $\lim_{t \rightarrow \infty} \|\varepsilon\| \leq \Delta$. This proves that the quadrotors could achieve the desired formation. $\square$

Remark 3. It is noted that, at nontrigger times $t \in (t_k^i, t_{k+1}^i)$, the i th quadrotor could only use $s_j(t_k^i)$, namely, the value of its neighbors at the last trigger time $t = t_k^i$. A typical approach in previous works [23] is to use the zero-order-hold model of $s_j(t_k^i)$. Thus, the controller is formulated with the static value of $s_j(t_k^i)$ during (t_k^i, t_{k+1}^i) . Instead, the proposed event-based controller (9) generates the states $y_{ij}(t)$ based on $s_j(t_k^i)$ to use it dynamically.

6. Simulation

To verify the effectiveness of the proposed method, in this section simulation is conducted with five quadrotors. The

TABLE 1: Initial conditions of the quadrotors.

Quadrotor	Position/m	Velocity/(m/s)	Pitch Angle/deg	Yaw Angle/deg	Roll Angle/deg
1	(2, -2, 0)	(0.60, 0.20, 0.29)	5	30	1
2	(-2, 2, 3.1)	(0.70, 0.07, 0.38)	1.5	10	2
3	(-2, -2, 2)	(0.75, -0.20, 0.45)	0	-10	4
4	(2, 2, 3)	(0.82, -0.28, 0.49)	0.5	-8	1
5	(0, 0, 1)	(0.94, -0.03, 0.56)	-1	0	2

FIGURE 3: Topology of the quadrotors.

FIGURE 4: The desired formation of quadrotors.

parameters of the quadrotors are as follows: $m_i = 3$ kg, $J_i = \text{diag}\{0.004, 0.004, 0.008\}$ kg·m², $i = 1, 2, \dots, 5$. The initial conditions are given in Table 1.

The topology between quadrotors is shown in Figure 3, and the weights of the edges in the graph are $a_{14} = 0.56$, $a_{23} = 0.33$, $a_{35} = 0.24$, $a_{41} = 0.41$, $a_{43} = 0.20$, and $a_{51} = 0.37$.

The desired formation is described by $r_{14} = [1 \ 1.5 \ -1]^T$, $r_{23} = [2 \ -1.5 \ 0]^T$, $r_{35} = [-2 \ 0 \ 1]^T$, $r_{41} = [-1 \ -1.5 \ 1]^T$, $r_{43} = [2 \ 0 \ 1]^T$, and $r_{53} = [2 \ 0 \ -1]^T$, as shown in Figure 4.

Choose $Q = 0.08 \times \text{diag}(1 \ 1 \ 1 \ 0.1 \ 0.1 \ 0.1)$. By solving the Riccati equation (18), the control gain K in (9) is obtained as follows:

$$K = \begin{bmatrix} 0.68 & 0 & 0 & 1.82 & 0 & 0 \\ 0 & 0.68 & 0 & 0 & 1.82 & 0 \\ 0 & 0 & 0.68 & 0 & 0 & 1.82 \end{bmatrix}. \quad (50)$$

The parameters of attitude controller (29) are selected as $k_{Ri} = 0.04$ and $k_{\Omega i} = 0.08$. Then, the simulation is conducted from 0 s to 20 s. Simulation results of trajectories of the quadrotors are shown in Figures 5–7. The trajectories in the x - y plane and y - z plane are given in Figures 6 and 7, respectively. Symbol “ Δ ” describes the initial position of each quadrotor. The figures illustrate that the five quadrotors achieve the desired formation stably.

The thrusts of quadrotors are illustrated in Figure 8. As a characteristic of the event-based control method, the vertices of the curves indicate the triggering time instants. Note that each quadrotor only communicates with its neighbors at triggering time instants, and then the controller forms its value according to the updated states. This shows that the updates of the thrust controller are driven by the events, not

FIGURE 5: Trajectories of the quadrotors.

by the pass of time. From Figure 8 it is also learned that the thrust converges to a constant value for equilibrium.

Note that $R \in \text{SO}(3)$ is used to model the attitude during the formation control design and simulation. To better present the results, R is transformed as attitude angles as shown in Figure 9. From (25) it is known that as the thrusts converge to a constant, the attitude commands R_{di} of the quadrotors would also be the same. It is found that the pitch, yaw, and roll angles of quadrotors reach consensus and the attitudes are stable with the proposed controllers on $\text{SO}(3)$.

The design of $\delta_i(t)$ in trigger function (23) affects the number of triggered events and control accuracy. To further

TABLE 2: Comparative results with different parameters.

Case	Parameters $(\delta_0^i, \delta_1^i, \beta_i)$	Numbers of triggering in Quadrotors 1–5	Convergence time
1	(20, 1.5, -0.03)	(63, 64, 63, 62, 63)	19.12 s
2	(20, 5, -0.03)	(54, 54, 54, 55, 54)	19.24 s
3	(10, 1.5, -0.03)	(104, 105, 104, 103, 104)	18.70 s
4	(30, 1.5, -0.03)	(47, 48, 47, 46, 47)	19.29 s
5	(20, 1.5, -0.05)	(74, 75, 74, 72, 74)	19.09 s

FIGURE 6: Trajectories from top view.

FIGURE 7: Trajectories from side view.

discuss this, define Q_p as the errors between current state and desired formation:

$$Q_p = \sum_{i=2}^5 \|\varepsilon_i\|. \quad (51)$$

Then, different parameters of the trigger function are chosen for comparative simulations. Under the same initial conditions, the convergence time and number of triggering when Q_p reaches 0.01 are recorded. The results are given in Table 2. The time responses of Q_p are given in Figure 10. It can be seen that, for relatively small $|\delta_i(t)|$, the quadrotors would converge to the desired formation quicker with more triggering times, and vice versa. Thus, the parameters of $\delta_i(t)$ in trigger function (23) can be designed to regulate the frequency of triggering and control accuracy.

To illustrate the efficiency of the event-based control method, two time-driven methods [9, 11] are conducted for comparison. A continuous-time controller is used in [9], and in [11] the controller is developed on the Euler angle model. The results of Q_p are also added in Figure 10. Roughly speaking, the methods with continuous communications

FIGURE 8: The thrusts of the quadrotors.

would converge quicker than the event-driven method. In [9] Q_p reaches 0.01 at 18.45 s. By the finite time controller in [11], Q_p converges fast at the beginning, but the convergence

FIGURE 9: The attitudes of the quadrotors.

FIGURE 10: Comparisons of Q_p with different methods.

time is 21.22 s. The comparative results demonstrate that the proposed method is effective, and the formation can be achieved with proper speed of convergence and triggering times.

7. Conclusion

This paper proposes an event-based formation control method for multiple quadrotors. With communication topology considered, the formation is described by relative positions between quadrotors, and the control model is established on $SO(3)$. Then, the triggering functions are designed to generate events, and by these events the thrust controller updates its states. The attitude command and controller are developed on $SO(3)$ to avoid singularity, and the closed-loop system stability analysis is presented. In simulation the frequency of triggering events and control accuracy is discussed with different parameters in triggering functions, and this shows that, by the proposed method, the multiple quadrotors could achieve the given formation with stable attitudes. The event-based formation control method does not require continuous communication, and it would be more applicable for quadrotors formation.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

This work is supported by the National Natural Science Foundation of China (no. 61374012).

References

- [1] R. Wang and J. Liu, "Adaptive formation control of quadrotor unmanned aerial vehicles with bounded control thrust," *Chinese Journal of Aeronautics*, vol. 30, no. 2, pp. 807–817, 2017.
- [2] W. Zhao and T. H. Go, "Quadcopter formation flight control combining MPC and robust feedback linearization," *Journal of The Franklin Institute*, vol. 351, no. 3, pp. 1335–1355, 2014.
- [3] X. Dong, B. Yu, Z. Shi, and Y. Zhong, "Time-varying formation control for unmanned aerial vehicles: theories and applications," *IEEE Transactions on Control Systems Technology*, vol. 23, no. 1, pp. 340–348, 2015.
- [4] B. Mu, K. Zhang, and Y. Shi, "Integral Sliding Mode Flight Controller Design for a Quadrotor and the Application in a Heterogeneous Multi-Agent System," *IEEE Transactions on Industrial Electronics*, vol. 64, no. 12, pp. 9389–9398, 2017.
- [5] N. Nigam, S. Bieniawski, I. Kroo, and J. Vian, "Control of multiple UAVs for persistent surveillance: algorithm and flight test results," *IEEE Transactions on Control Systems Technology*, vol. 20, no. 5, pp. 1236–1251, 2012.
- [6] R. C. Avram, X. Zhang, and J. Muse, "Quadrotor Actuator Fault Diagnosis and Accommodation Using Nonlinear Adaptive Estimators," *IEEE Transactions on Control Systems Technology*, vol. 25, no. 6, pp. 2219–2226, 2017.
- [7] W. Hao and B. Xian, "Nonlinear adaptive fault-tolerant control for a quadrotor UAV based on immersion and invariance methodology," *Nonlinear Dynamics*, vol. 90, no. 4, pp. 2813–2826, 2017.
- [8] K.-K. Oh, M.-C. Park, and H.-S. Ahn, "A survey of multi-agent formation control," *Automatica*, vol. 53, pp. 424–440, 2015.
- [9] X. Dong, Y. Zhou, Z. Ren, and Y. Zhong, "Time-Varying Formation Tracking for Second-Order Multi-Agent Systems Subjected to Switching Topologies With Application to Quadrotor Formation Flying," *IEEE Transactions on Industrial Electronics*, vol. 64, no. 6, pp. 5014–5024, 2017.
- [10] C.-G. Rudy and O. Nejat, "Stability of formation control using a consensus protocol under directed communications with two time delays and delay scheduling," *International Journal of Systems Science*, vol. 47, no. 2, pp. 433–449, 2016.
- [11] H. Du, W. Zhu, G. Wen, and D. Wu, "Finite-time formation control for a group of quadrotor aircraft," *Aerospace Science and Technology*, vol. 69, pp. 609–616, 2017.
- [12] Y. Wang, Q. Wu, and Y. Wang, "Distributed cooperative control for multiple quadrotor systems via dynamic surface control," *Nonlinear Dynamics*, vol. 75, no. 3, pp. 513–527, 2014.
- [13] L. Garcia-Delgado, A. Dzul, V. Santibanez, and M. Llama, "Quad-rotors formation based on potential functions with obstacle avoidance," *IET Control Theory & Applications*, vol. 6, no. 12, pp. 1787–1802, 2012.
- [14] R. Lopez-Gutierrez, A. E. Rodriguez-Mata, S. Salazar, I. Gonzalez-Hernandez, and R. Lozano, "Robust quadrotor control: attitude and altitude real-time results," *Journal of Intelligent & Robotic Systems*, vol. 88, pp. 299–312, 2017.
- [15] T. Lee, M. Leok, and N. H. McClamroch, "Geometric tracking control of a quadrotor UAV on $SE(3)$," in *Proceedings of the 2010 49th IEEE Conference on Decision and Control, CDC 2010*, pp. 5420–5425, Atlanta, GA, USA, December 2010.
- [16] T. Lee, "Global exponential attitude tracking controls on $SO(3)$," *Institute of Electrical and Electronics Engineers Transactions on Automatic Control*, vol. 60, no. 10, pp. 2837–2842, 2015.
- [17] A. P. Sandiwan, A. Cahyadi, and S. Herdjunto, "Robust proportional-derivative control on $SO(3)$ with disturbance compensation for quadrotor UAV," *International Journal of Control, Automation, and Systems*, vol. 15, no. 5, pp. 2329–2342, 2017.
- [18] S. Weng, D. Yue, and T. Yang, "Coordinated attitude motion control of multiple rigid bodies on manifold $SO(3)$," *IET Control Theory & Applications*, vol. 7, no. 16, pp. 1984–1991, 2013.
- [19] W. P. Heemels, J. H. Sandee, and P. . Van Den Bosch, "Analysis of event-driven controllers for linear systems," *International Journal of Control*, vol. 81, no. 4, pp. 571–590, 2008.
- [20] W. Liu and J. Huang, "Event-triggered cooperative robust practical output regulation for a class of linear multi-agent systems," *Automatica*, vol. 85, pp. 158–164, 2017.
- [21] D. V. Dimarogonas, E. Frazzoli, and K. . Johansson, "Distributed event-triggered control for multi-agent systems," *Institute of Electrical and Electronics Engineers Transactions on Automatic Control*, vol. 57, no. 5, pp. 1291–1297, 2012.
- [22] S. S. Kia, J. Cortés, and S. Martínez, "Distributed event-triggered communication for dynamic average consensus in networked systems," *Automatica*, vol. 59, pp. 112–119, 2015.
- [23] W. Zhu, Z.-P. Jiang, and G. Feng, "Event-based consensus of multi-agent systems with general linear models," *Automatica*, vol. 50, no. 2, pp. 552–558, 2014.
- [24] X. Meng and T. Chen, "Event based agreement protocols for multi-agent networks," *Automatica*, vol. 49, no. 7, pp. 2125–2132, 2013.