Inventory Financing with Overconfident Supplier Based on Supply Chain Contract

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Overconfidence is a universal psychological behavior. Overconfidence on demand awareness will have a significant impact on operation decisions. The supplier estimated the demand with excessive precision which influences the inventory financing decision-making deeply. We built the demand function based on the supplier's overconfidence. Then we established the retailer, supplier, and the Bank's profit function, respectively. Through the analysis of the bilevel Stackelberg game, we obtained the order quantity of the retailer with the capital constraint, the wholesale price of overconfident supplier, and the loan-to-value ratio of Bank, and we analyzed the influence of overconfidence on the decision variables. We have several findings as follows. First, the overconfidence makes the decisions of the retailer, supplier, and Bank deviate from the rational decisions. Second, the space of the market profit will affect the decision variables in the joint decision-making. Third, the financing supply chain (including the Bank and supply chain) should have a positive attitude towards the overconfidence of the supplier. Forth, in the joint decision-making, the supplier need determines the buyback price according to the capital demand; and in the decentralized decision-making, the supplier should try to use high buyback price strategy.

1. Introduction

Fund shortage and high financing cost have been a bottleneck in the development of SMEs (small and medium-sized enterprise). According to the 2014 China SMEs development report released by China Southwestern University of Finance and Economics, 25.8% of the 58 million SMEs of China have loan demand. However, only 46% of these companies received Bank loans, 11.6% of these companies were rejected, and 42.4% of these companies did not apply the loan. Due to the characteristics of the enterprise, weak solvency, small financing scale, poor financial regulation, and lack of perfect corporate governance mechanism, the SMEs' ability to resist risk is weak generally. Since banks have set up complex risk control procedures to control risks, large financial institutions lack relevant financial services solutions for SMEs generally. In this case, large amount of money in Banks has nowhere to lend. The SMEs usually obtained financing through underground Banks, private funds, guarantee companies, and investment companies.

Think of the core enterprise in the supply chain and its related upstream and downstream supporting enterprises as a whole, according to the trading relationship of enterprises in the supply chain and industry characteristics, developing a financing mode based on cargo rights and cash flow control as a whole financial solution. The supply chain financing service is different from the traditional Bank financing products. Around the upstream and downstream enterprises which have good credit, stable sales channels, and return fund sources, the supply chain financing service takes the large enterprises as the core and selects qualified upstream and downstream enterprises as the financing objects. Supply chain financing solves the difficulty about upstream and downstream enterprises financing and guarantees and eliminates the financing bottleneck of upstream and downstream also can reduce the supply chain financing costs and improve
the competitiveness of the core enterprises and related enterprises.

The flow capital of a single enterprise is mainly occupied in the form of accounts receivable and inventory. According to the difference of guarantee measures, the financial institutions divide the basic products of supply chain financing into two categories: accounts receivable financing and inventory financing.

Accounts receivable financing means a financing mode that SMEs use undue receivables committed by the core enterprises in the supply chain to obtain the fund from the financial institutions. Carrefour is a global top 500 enterprise with a steady operation, a clear term of payment for upstream suppliers, and the ability to execute the contract. Carrefour has tens of thousands of suppliers worldwide. The Bank can design a supply chain financing mode for Carrefour’s upstream suppliers. In the past several years, the Banks has been giving the supplier a credit limit after the comprehensive assessment which can be recycled after payment, and Carrefour needs to pay the Bank, which formed a closed capital chain cycle. The supply chain financing mode can alleviate the pressure of suppliers’ funds and promote the Banks to get more customers.

In many cases, there is no corresponding accounts receivable and credit guarantee for the enterprises in the supply chain besides the goods. At this time, financial institutions can give the credit to the enterprises with inventory financing mode. Inventory financing is a financing business of SMEs, which uses raw materials, semifinished products, and finished products as collateral to obtain loans from financial institutions. The traditional Bank loans are concentrated on the fixed assets mortgage or the third-party corporation guarantee. Inventory financing is that the SMEs use inventory in the trade between enterprises and upstream and downstream enterprises as collateral and then the SMEs obtain loans from financial institutions such as Banks. In the inventory financing, the financial institutions are to give credit to the enterprise after the evaluation of inventory values by the third-party logistics enterprises. For SMEs with financing needs, the lack of fixed assets makes it difficult to obtain loans from the Banks. In the developed countries, the inventory financing business has been mature. In developed countries such as the United States, 70% of the guarantee comes from accounts receivable and inventory. In China, although the inventory financing just started, its development speed was astonishing. Since the first inventory financing carried out by China Materials Storage and Transportation Group Co., Ltd (CMST), with the financial institutions in 1999, the CMST have set up inventory financing with dozens of Banks, such as Bank of China and China Construction Bank.

In the actual business process, decision-makers are overconfident. Overconfidence is a universal psychological behavior. People often think that they are superior to others in the analysis and judgment. Fischhoff et al. [1], through a study of 528 paid volunteers, show that people are always overconfident. They tend to have enough confidence in the judgment and overestimate the chances of success. Weinstein [2] tested 120 female college students and found that people always overestimated their own favorable factors, and they did not think that others had the same factors; at the same time, a survey of more than 200 college students found that people were always optimistic about the chances of success. Moore [3] through the literature review on the overconfidence found that overconfidence was always manifested in three categories: overestimation, overplacement, and overprecision.

At present, most of the researches on inventory financing decisions are based on the assumption that decision-makers are rational. Decisions made by the overconfident decision-maker often deviate from traditional decisions. This paper mainly analyzed enterprise decision-making under inventory financing environment. This paper quantified the decision variables of all parties through the bilevel Stackelberg game, analyzed the influence of the degree of overconfidence on the decision-making of all parties, and then discussed the influence of the different supplier’s buyback strategies on the inventory financing and sought a strategy that can benefit the three parties in the inventory financing.

In Section 8, we concluded the conclusion and operation suggestions in Section 7. We concluded the conclusion and operation suggestions in Section 8.

2. Literature Review

Our work is related to two streams of literature, one deal with inventory financing and the other dealing with overconfidence.


The inventory financing business is a new financial service mode. The Bank had developed the inventory financing business actively. It is necessary to set the loan cycle, the mark-to-market frequency, the warning value, and the interest rate, so as to control the risk effectively. Jokivuolle
and Peura [10] studied the loan-to-value ratio of the inventory financing when the price fluctuated with the Merton structural method. Cossin and Hricko [11] considered the credit risk in the pledge management, constructed the credit risk pricing model which is different from the newsboy model, determined the discount rate of the inventory financing business, and obtained the loan-to-value ratio by 1 minus the discount rate. He et al. [12] established a model with the formula AR (1)-GARCH (1,1)-GED using the database of spot steel, forecasted the VaR of steel during the different risk windows in the impawn period through a method of out-of-sample, and got the impawn rate according with the risk exposure of Banks. Li [13] adopted risk evaluation method of “corporate and debt,” considered the impact of time to capture and liquidity risk, established loan-to-value ratio decision model of inventory financing with randomly-fluctuate price, then took copper for an example, confirmed all model parameters, and analyzed the impact of different factors on loan-to-value ratio. He et al. [14] proposed a new long-term extreme price risk (value at risk and conditional value at risk) measure method for inventory portfolio and an application to dynamic impawn rate interval. Zhang et al. [15] assumed that the short-term prices of the collateral follow a geometric Brownian motion and use a set of equivalent martingale measures to build the models about a Bank's maximum and minimum levels of risk tolerance in an environment with Knightian uncertainty. However, these researches only considered the loan decision of the Bank alone and did not consider the game between Bank and the capital-shortage enterprise which will influence the decision of the Bank.

As the loan enterprise of inventory financing, its financing decision will directly affect the operation performance of the enterprise. On the operation decision based on the inventory financing, Buzacott and Zhang [16] studied assets financing from the perspective of the enterprise, first combined the loan risk management of financial institutions with the inventory control of the loan enterprises, and analyzed the choice of the loan interest rate and the loan value ratio and their impact on the profits of the financial institutions and the loan enterprises. The problem of discussion is that the enterprise purchased the products by the loan and all the inventory is used as a pledge, which belongs to a stage of decision-making process. Wang et al. [17] studied the robust inventory financing model under partial information, that is, where the demand distribution is partly known. In this setting, the robust method that maximizes the worst-case profit and minimizes the firm's maximum possible regret of not acting optimally would be used to formulate the optimal sales quantity. SMEs credit risk is significant for guaranteeing supply chain financing in smooth operation. Li et al. [18] adopt the logistic regression tool to measure the default probability of the risk model and made an empirical analysis of the model based on 177 sample data from China's commercial Banks concerning automobile and steel industries. Zhu et al. [19] applied six methods to predict the SMEs credit risk.

Inventory financing business cannot be carried out without the participation of third-party logistics. Diercks [20] illustrated the necessity of the third-party logistics enterprises to participate in the inventory financing business. In inventory financing, asymmetric information between the Banks and the third-party logistics enterprises may incur moral risks and often causes economic losses of the Banks. To effectively solve this problem, Sun et al. [21] designed a pure incentive scheme and a regulatory incentive scheme with the principal-agent theory. Song et al. [22] proposed a pricing model of inventory financing that can maximize the cash flow of the 3PL enterprise, when the default rate of the SMEs is affected by the pledge price. These researches do not consider the impact of supply chain contract on financing.

The decision of inventory financing enterprises is not isolated. They are influenced by the business related parties. The pricing strategy of the supplier and the lending policy of the Bank will all affect the financing decision of the enterprise. On the supply chain management based on the inventory financing, Lee and Rhee [23] discussed coordination mechanisms by explicitly assuming capital-constrained agents and positive inventory financing costs. Yan and Sun [24] designed a capital-constrained supply chain system with a manufacturer and a capital-constrained retailer who can obtain short-term financing from the commercial Bank through inventory financing according to the value of the pledged warehouse receipt. Using the system dynamics methodology, they modelled the stock and flow diagrams and simulated the system characteristics for nonfinancing scheme and inventory financing scheme, respectively. Yan et al. [25] designed a supply chain financing system with a manufacturer, a retailer, and a commercial Bank where both the retailer and the manufacturer are capital constrained under demand uncertainties. They formulated a bilevel Stackelberg game for the supply chain financing system in which the Bank acts as the leader and the manufacturer as the subleader. Considering the bankruptcy risks of the manufacturer and the retailer, they analyzed the optimal financing interest rate for the commercial Bank, the optimal order for the retailer, and the optimal wholesale price for the manufacturer, respectively. On the basis, Yan et al. [26] performed a comparative analysis of the optimal strategies among the various financing scenarios. Lin et al. [27] found that the vender should provide the complete buyback when the demand is uncertain and affected by retailers' sales efforts, so they put forward three modified modes of inventory financing to achieve Pareto-Improvement. Based on the analytical formulation of the benefits of three relevant supply chain finance schemes (Reverse Factoring, Inventory Financing, and Dynamic Discounting), Gelsomino et al. [28] formalized a model that investigated the benefits that a buyer can achieve by onboarding suppliers onto these three schemes.

The traditional operations researches are usually based on the rational hypothesis and ignore the influence of psychological factors on the decision-making behavior, which leads to the deviation of the research results and the actual situation. In order to accurately reflect managers' decision-making behavior, many scholars have introduced human behavior factors into supply chain management. In the field of operation and management, Croson et al. [29] introduced overconfidence behavior into the supply chain model and
studied the newsboy model with overconfidence behavior. Ren and Croson [30] studied decision-makers in the case of overprecision estimation and described the overconfidence behavior of decision-makers through the newsboy model. It is proved that overconfidence is the cause of the mean bias effect in the newsboy decision-making experiment. Based on these research results, Ren et al. [31] further demonstrated that order bias is linear in the level of overconfidence and is increasing with the variance of the demand distribution. Order bias also has a U-shaped relationship with market profitability, and the costs of overconfidence are convex. Lu et al. [32] studied inventory decision-making and performance evaluation under various overconfidence forms, respectively. Ma et al. [33] studied advertising and pricing strategies of overconfident manufacturers in dual channel competition environment. Li et al. [34] studied the effects and implications of overconfidence in a competitive newsvendor setting. They found that when the product’s profit margin is high, overconfidence can lead to a first-best outcome. In a similar vein, they found that the more biased of two competing newsvendors is not necessarily destined to a smaller expected profit than its less biased competitor. Xu et al. [35] analyzed the impact of the retailer’s overconfidence on the supply chain performance. They considered a duopolistic market with uncertain demand where one overconfident retailer and one rational retailer compete in selling the same product. These literatures have studied the incentive contract in the context of overconfidence, but it is mainly about the production and sales decisions of supply chain enterprises and the coordination between upstream and downstream. Enterprises have capital constraints in reality, so it is necessary to study the inventory financing based on the overconfidence, design the corresponding incentive mechanism, and eliminate the negative effects of overconfidence.

There are overconfidence behaviors in supply chain member enterprises. Overconfidence has an important influence on the inventory financing decision and operation strategy of supply chain. Our study differs from the existing literature in that it analyzes the inventory financing decisions of supplier, retailer, and Bank in the context of overconfidence. The supplier is overconfident on demand, and the retailer and the Bank are rational. We went through Stackelberg game to quantify the decision variables, analyze the effect of supplier overconfidence degree on the decision, and discuss the impact of different supplier buyback policy on inventory financing.

3. Notation and Assumptions

The supplier in this article is overconfident in the accuracy of market demand forecasting, and the retailer is rational about demand. The retailer is a small and medium-sized enterprise with limited funds, and the retailer gets loans from the Bank through inventory financing. In the inventory pledge business, the supplier sets the wholesale price, the retailer decides the order quantity, and the Bank decides the loan-to-value ratio. Inventory financing business processes are as follows. First of all, the supplier and the retailer sign the purchase contract, then the retailer with the purchase contract gets the inventory pledge loan from the Bank, and at the same time the supplier signs the buyback agreement with the bank. According to the loan-to-value ratio, the bank draws the acceptance bill for the supplier. The retailer pays the remaining payment with their own money to the supplier. The supplier sends the goods to the third-party logistics enterprise which is designated by the Bank for supervision; the Bank instructs the third-party logistics enterprise to deliver the goods according to the amount of money of the specified supervision account, which is from the sale of the retailer. At the end of the sale term, if the products remain, the supplier will buy back the products according to the agreement signed with the Bank. The inventory pledge business is shown in Figure 1.

3.1. Symbolic Description. In order to facilitate the description of the model, various variables are given in Table 1.

3.2. Assumptions. Assumptions about the model are as follows:

1. We assume that the market demand is \( D \), which is a nonnegative random variable. The distribution and density function are \( F(\cdot) \) and \( f(\cdot) \), respectively. Other notations for the probability distribution include complementary CDF \( \overline{F} = 1 - F(\cdot) \) and failure rate \( h(\cdot) = f(\cdot)/F(\cdot) \). The demand distribution is assumed to have an increasing failure rate (IFR), a mild condition that is satisfied by most commonly used distributions. We set the mean of \( D \) as \( \mu \) and the variance of \( D \) as \( \sigma^2 \).

2. We assume that the overconfident supplier deem the demand is \( D_\alpha = (1-\beta)D + \beta \mu \) [29, 30, 34]. The distribution and density function are \( F_\alpha(\cdot) \) and \( f_\alpha(\cdot) \) respectively. We can know the mean of \( D_\alpha \) is \( \mu \), the variance of \( D_\alpha \) is \( (1-\beta)^2 \sigma^2 \). The supplier’s overconfidence is about the accuracy of demand forecasting, \( 0 \leq \beta \leq 1 \). The larger the \( \beta \) value, the more overconfident the supplier is. As the value of \( \beta \) approaches 1, the supplier considers the market demand to be a constant.
Table 1: Decision variables and model parameters.

<table>
<thead>
<tr>
<th>Decision variable of the Bank</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega$</td>
<td>the loan-to-value ratio</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Decision variable of the supplier</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega$</td>
<td>the wholesale price</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Decision variable of the retailer</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q$</td>
<td>the order quantity of the retailer, $q_o$ is order quantity with overconfident supplier, $q_r$ is order quantity with rational supplier</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_1$</td>
<td>the deposit interest rate</td>
</tr>
<tr>
<td>$r_2$</td>
<td>the loan interest rate which include all expenses incurred during the pledge period</td>
</tr>
<tr>
<td>$p$</td>
<td>the sale price of the product</td>
</tr>
<tr>
<td>$c$</td>
<td>the cost of the product</td>
</tr>
<tr>
<td>$v$</td>
<td>the buyback price of the product</td>
</tr>
<tr>
<td>$s$</td>
<td>the salvage value of the product</td>
</tr>
<tr>
<td>$T$</td>
<td>the period of the inventory pledge loan contract, the unit is year, value range is $0 &lt; T \leq 1$</td>
</tr>
</tbody>
</table>

rather than a random variable. $\beta = 0$ shows that the supplier is rational in the demand forecasting precision.

(3) Without regard to the loss of sale, the unsold products are repurchased by the supplier with price $v$.

(4) The Bank evaluates the product with the sale price.

(5) The retailer is short of money, is more cautious with their decisions, and has a better view of the market, so we assume that the retailer is rational, and the Bank makes the decision based on the loan application, so the judgment on demand of the Bank is the same as that of retailer’s.

4. Model of Bank Profit

In the inventory pledge business, the Bank provides loans to the retailer with the loan interest rate of $r_2$, the loan amount is $B = \omega qp$, and the retailer pays the principal and interest $\omega q p (1 + T r_2)$ when the loan is due. When the retailer fails to repay the loan, the products will be repurchased by the supplier at the price of $v$, and the Bank loan will be repaid with the buyback money.

At the end of the sale, the retailer will default if the retailer’s sales and buyback value of the remaining products cannot repay the Bank’s principal and interest. Let the probability of default be $\varepsilon$. When the retailer defaults, the Bank returns are $p \min (D, q) + v (q - D)^+$, otherwise the Bank’s profit is $\omega q p (1 + T r_2) - \omega q p (1 + T r_1)$.

Let the critical value of product demand be $\delta$. When the actual demand is the critical value, the retailer’s sales and buyback value of the remaining products are equal to the Bank’s principal and interest. Below this threshold, the retailer will default.

**Proposition 1.** The default probability of retailer is $\varepsilon = F((\omega q p (1 + T r_2) - v q) / (p - v))$.

**Proof.** The default condition of the retailer is

$$p \min (\delta, q) + v (q - \delta)^+ \leq \omega q p (1 + T r_2)$$

When default occurs, the critical demand is $\delta \leq q$, so $\min (\delta, q) = \delta$.

We can get $p \delta + v (q - \delta) \leq \omega q p (1 + T r_2)$.

When $\delta = (\omega q p (1 + T r_2) - v q) / (p - v)$, the retailer chooses to default.

So we can get the default probability of the retailer: $P(D \leq \delta) = F((\omega q p (1 + T r_2) - v q) / (p - v))$. \[
\]

The critical value of product demand is $\delta = (\omega q p (1 + T r_2) - v q) / (p - v)$, and the Bank’s profit is as follows:

$$\pi^b = \begin{cases} 
\omega q p T (r_2 - r_1), & D \geq \delta \\
p D + v (q - D) - \omega q p (1 + T r_1), & D < \delta 
\end{cases}$$

We can find that the expected profit of the Bank is

$$E \pi^b = \omega q p T (r_2 - r_1) - (p - v) \int_{0}^{\delta} F(x) \, dx$$

By taking the first derivatives of $E \pi^b$ with respect to $\omega$, we have

$$d E \pi^b / d \omega = q p T (r_2 - r_1) - q p (1 + T r_2) F(\delta)$$

Because $d^2 E \pi^b / d \omega^2 = -((q p (1 + T r_2))^2 / (p - v)) f(\delta) < 0$, let $d E \pi^b / d \omega = 0$ and set $G = T (r_2 - r_1) / (1 + T r_2)$; we find that the optimal loan-to-value ratio will satisfy

$$\omega = \frac{(p - v) \cdot F^{-1} (G) + v q}{q p (1 + T r_2)}$$

5. Joint Decision of Supply Chain

The overconfident supplier is the core enterprise which makes the decision in the joint decision-making. Let $q_o$ be the order quantity in the overconfident condition. The overall profit of the supply chain is as follows:

$$\pi^c = p \min (q_o, D_o) + s (q_o - D_o)^+ - c q_o - \omega q_o p T r_2$$


We can obtain the whole supply chain expects profit as follows:

$$E \pi^c = (p - c)q_o - (p - s) \int_0^{q_o} F_o(x_o) \, dx_o - \omega q_o p T r_2$$

(7)

**Proposition 2.** In the joint decision-making, the optimal order quantity and the loan-to-value ratio are

1. $q^*_o = F_o^{-1}((p - c - v T r_2)/(1 + T r_2))/(p - s)$;
2. $\omega = ((p - v) \cdot F^{-1}(G) + v q_o^*)/q_o^* p(1 + T r_2)$.

Proof. Using Stackelberg game, the supply chain first determines the order quantity, and then the Bank decides the loan-to-value ratio. Using the inverse method, the loan-to-value ratio of the Bank is determined first which is (5).

By taking the first derivatives of $E \pi^c$ with respect to $q_o$, we have

$$\frac{dE \pi^c}{dq_o} = (p - c) - (p - s) \cdot F_o(q_o) - \omega p T r_2$$

$$- \frac{d \omega}{dq_o} q_o p T r_2$$

(8)

Because $d \omega/dq_o = -(p - v) F^{-1}(G)/q_o^* p(1 + T r_2)$, we can simplify (8) to

$$\frac{dE \pi^c}{dq_o} = (p - c) - (p - s) F_o(q_o) - \frac{v T r_2}{1 + T r_2}$$

(9)

Because $d^2E \pi^c/dq_o^2 = -(p - s) F_o(q_o) < 0$, let $dE \pi^c/dq_o = 0$, and we find that the optimal-order quantity will satisfy

$$q^*_o = F_o^{-1}\left(\frac{p - c - v T r_2}{1 + T r_2}\right)$$

(10)

We substitute (10) into (5):

$$\omega = \frac{(p - v) \cdot F^{-1}(G) + v q_o^*}{q_o^* p (1 + T r_2)}$$

(11)

**Proposition 3.** In the rational joint decision, the order quantity is $q^*_o$. In the overconfident joint decision, the order quantity is $q^*_o^\star$. We can know that $q^*_o = (1 - \beta)q^*_o + \beta \mu$.

Proof. The optimal decision in a rational situation conforms to

$$q^*_o = F_o^{-1}\left(\frac{p - c - v T r_2}{1 + T r_2}\right)$$

$$\eta = P\left(q^*_o \leq q^*_o^\star\right) = P\left(\frac{(1 - \beta) q^*_o + \beta \mu \leq q^*_o^\star\right)$$

(13)

We can get $q^*_o = (1 - \beta)q^*_o + \beta \mu$.

**Proposition 4.** When demand follows normal distributed, in the high-profit market environment, i.e., $p - c - v T r_2/(1 + T r_2) > (1/2)(p - s)$, we have $q^*_o > \mu$, and the greater the degree of overconfidence in the accuracy of the prediction, the lower the order quantity and the greater the loan-to-value ratio; in the low profit market environment, i.e., $p - c - v T r_2/(1 + T r_2) < (1/2)(p - s)$, we have $q^*_o < \mu$, and the greater the degree of overconfidence in the accuracy of the prediction, the higher the order quantity and the lower the loan-to-value ratio; while $p - c - v T r_2/(1 + T r_2) = (1/2)(p - s)$, we have $q^*_o = \mu$, and the degree of overconfidence in the accuracy of the prediction has no relationship with the order quantity and the loan-to-value ratio.

Proof. Standardizing (10), we have

$$q_o = F_o^{-1}\left(\eta\right) = (1 - \beta) \sigma \cdot \Phi^{-1}(\eta) + \mu$$

(14)

In (14), $\eta$ represents $(p - c - v T r_2/(1 + T r_2))/(p - s)$.

In the high-profit market environment, i.e., $\eta > 0.5$ and $\Phi^{-1}(\eta) > 0$, we can know that $q_o > \mu$, and the greater the value of $\beta$, the lower the value of $q_o$. From (11), we can know that the greater $\beta$ value, the greater the loan-to-value ratio $\omega$.

In the low profit market environment, i.e., $\eta < 0.5$, $\Phi^{-1}(\eta) < 0$, we can know that $q_o < \mu$, and the greater the value of $\beta$, the greater the value of $q_o$. From (11), we can know that the greater the $\beta$ value, the lower the loan-to-value ratio $\omega$.

While $p - c - v T r_2/(1 + T r_2) = (1/2)(p - s)$, we can know that $\Phi^{-1}(\eta) = 0$, so we have $q_o = \mu$.

In order to better explain Proposition 4, we provide numerical examples. $r_1$ is the annual deposit interest rate, and the annual deposit interest rate is 2-3% in China. $r_2$ is the annual loan interest rate. The annual loan interest rate of the Bank is usually 6-8% in China; plus management fees of the inventory pledged products and other expenses, we took $r_2$ at 10%. So we assume that the parameters are as follows: $r_1 = 0.03, r_2 = 0.1, c = 5, v = 4, s = 2, T = 1,$ and $T = N(100,30)$.

We set $p = 9$ as high-profit market environment and set $p = 8$ as low profit market environment. We can know the change trend about order quantity and loan-to-value ratio visually from Figures 2 and 3.
The difference between the optimal expected profit \( \mathcal{E}_\pi \) be greater. That is to say, the larger the degree of overconfidence increases, the profit deviation will be greater. That is to say, the larger the \( \beta \) value is, the greater the difference between the optimal expected profit \( \mathcal{E}_\pi^* \) and the corresponding real profit \( \mathcal{E}_r^* \).

**Proof.** The optimal expected profit of the supply chain in the context of overconfidence is

\[
\mathcal{E}_\pi^* = (p - c) q_o^* - (p - s) \int_{0}^{q_o^*} F_o(\theta_o) d\theta_o - \omega q_o^* pT_r
\]

The real profit of the supply chain is

\[
\mathcal{E}_r = (p - c) q_o^* - (p - s) \int_{0}^{q_o^*} F(\theta) d\theta - \omega q_o^* pT_r
\]

The profit deviation is \( \mathcal{E}_\pi^{*} - \mathcal{E}_r = (p - s) \int_{0}^{q_o^*} F(x) dx - \int_{0}^{q_o^*} F_o(x_o) dx_o \).

The profit deviation is equivalent to

\[
\mathcal{E}_\pi^{*} - \mathcal{E}_r = (p - s) \left[ \int_{0}^{q_o^*} F(x) dx - \int_{0}^{q_o^*} (1 - \beta) F(x) dx \right]
\]

Because \( \int_{0}^{q_o^*} F(x) dx - \int_{0}^{q_o^*} (1 - \beta) F(x) dx = \int_{0}^{q_o^*} F(x) dx - \int_{0}^{(1 - \beta)q_o^*} F(x) dx \), we should compare \( q_o^* \) and \( (1 - \beta)q_o^* \).

From Proposition 1, we know that \( q_o^* = (1 - \beta)q_o^* + \beta \mu \). And because \( 0 \leq \beta \leq 1 \) and \( \mu > 0 \), we can know that \( q_o^* - (1 - \beta)q_o^* = \beta \mu > 0 \).

So \( \mathcal{E}_\pi^{*} - \mathcal{E}_r = (p - s) \left[ \int_{0}^{q_o^*} F(x) dx - \int_{0}^{q_o^*} (1 - \beta) F(x) dx \right] > 0 \).

The larger the value of \( \beta \), the greater the value of \( q_o^* - (1 - \beta)q_o^* \) and the larger the \( \mathcal{E}_\pi^{*} - \mathcal{E}_r \).

In order to better explain Proposition 5, we provide numerical examples. Assume that the parameters \( T_r = 0.03 \), \( r_1 = 0.1 \), \( p = 9 \), \( c = 5 \), \( v = 4 \), \( s = 2 \), \( T = 1 \), and \( D \sim N(100, 30) \).

We can know the change trend about the profit of supply chain visually from Figure 4.

![Figure 4: The profit of supply chain in different situations.](image)

6. The Wholesale Price Contract with Buyback

The retailer makes decision before the cycle begins, and the principle of decision-making is the maximization of profit. The retailer’s profit consists of four parts. The profit of the retailer is as follows:

\[
\pi^r = p \min(D, q_r) + v(q_r - D)^+ - \omega q_r - \omega q_r pT_r
\]

The expected profit of the retailer is as follows:

\[
\mathcal{E}_r = (p - w) q_r - (p - v) \int_{0}^{q_r} F(x) dx - \omega q_r pT_r
\]

The supplier makes decision before the cycle starts and the principle of decision-making is the maximization of profit. The profit of the supplier is as follows:

\[
\pi^m = w q_r - v(q_r - D)^+ + s(q_r - D)^+ - cq_r
\]
The expected profit of the supplier with overconfidence is as follows:

$$E\pi^m = (w - c)q_r - (v - s)\int_0^{q_r} F_o(x_o) \, dx_o$$  \hspace{1cm} (21)$$

The supplier, as a core enterprise, first decides its wholesale price, and then the retailer decides the order quantity, and the Bank determines the loan-to-value ratio according to the purchase contract and buyback agreement of the supplier and the retailer. The Bank's loan-to-value ratio is related to the retailer's order quantity, and the retailer's order quantity is related to the supplier's wholesale price. Therefore, the Bank's loan-to-value ratio is also related to the supplier's wholesale price through transmission. The whole game process consists of two parts, one part the game between the Bank and the retailer and another part the game between the retailer and the supplier; so it is called bilevel Stackelberg. The bilevel Stackelberg is used to solve the problem, and the retailer is first analyzed.

6.1. Retailer Decision Analysis. First, we analyze the retailer’s decision. The first and second derivatives of (19) with respect to \( q_r \) are

$$\frac{dE\pi'}{dq_r} = (p - w) - (p - v) F(q_r) - \omega p Tr_2$$
$$\frac{d\omega}{dq_r} q_r p Tr_2$$  \hspace{1cm} (22)$$

Because \( dw/dq_r = -(p - v) F^{-1}(G)/q_r^2 p(1 + Tr_2) \), we can simplify (22) to

$$\frac{dE\pi'}{dq_r} = (p - w) - (p - v) F(q_r) - \frac{v Tr_2}{(1 + Tr_2)}$$  \hspace{1cm} (23)$$

Because \( \frac{d^2E\pi'}{dq_r^2} = -(p - v) f(q_r) < 0 \), we let \( dE\pi'/dq_r = 0 \) and then we have

$$q_r = F^{-1} \left( \frac{p - w - v Tr_2}{(1 + Tr_2)} \right)$$  \hspace{1cm} (24)$$

6.2. Supplier Decision Analysis. Now we analyze the supplier’s decision about wholesale price. By taking the first derivative of (21) with respect to \( w \), we have

$$\frac{dE\pi^m}{dw} = q_r + [(w - c) - (v - s) F_o(q_r)] \frac{dq_r}{dw}$$  \hspace{1cm} (25)$$

We can take the derivative of \( q_r \) with respect to \( w \) from (24):

$$\frac{dq_r}{dw} = -\frac{1}{(p - v) f(q_r)}$$  \hspace{1cm} (26)$$

Substitute (26) into (25):

$$\frac{dE\pi^m}{dw} = q_r - \frac{(w - c) - (v - s) F_o(q_r)}{(p - v) f(q_r)}$$  \hspace{1cm} (27)$$

Because \( \frac{d^2E\pi^m}{dw^2} = -(p - v) f(q_r) + (v - s) f_o(q_r)/[(p - v) f(q_r)]^2 - ((w - c) - (v - s) F_o(q_r)) / (p - v)^2[f(q_r)] f'(q_r) < 0 \), we let \( dE\pi^m/dw = 0 \) and then we have

$$q_r = \frac{(w - c) - (v - s) F_o(q_r)}{(p - v) f(q_r)} = 0$$  \hspace{1cm} (28)$$

We can get \( q_r^* \) and \( w^* \) by solving the simultaneous equations which consist of (24) and (28).

6.3. Analysis of Each Decision Variable

Proposition 6. In decentralized decision-making, the loan-to-value ratio of the Bank is negatively correlated with the retailer’s order quantity and the loan-to-value ratio of the Bank is positively correlated with the buyback price and wholesale price set by the supplier.

Proof. We can take the derivative of \( \omega \) with respect to \( q_r \) from (5):

$$\frac{\partial \omega}{\partial q_r} = \frac{(p - v) F^{-1}(G)}{q_r^2 p (1 + Tr_2)} < 0$$  \hspace{1cm} (29)$$

From (29), we can know that the loan-to-value ratio of the Bank is negatively correlated with the retailer’s order quantity. That is to say, the loan-to-value ratio of the Bank decreases with the increase of the retailer’s order quantity.

We can take the derivative of \( \omega \) with respect to \( v \) from (5):

$$\frac{\partial \omega}{\partial v} = \frac{q_r - F^{-1}(G)}{q_r p (1 + Tr_2)} > 0$$  \hspace{1cm} (30)$$

From (30), we can know that the loan-to-value ratio of the Bank is positively correlated with the supplier’s the buyback price. That is to say, the loan-to-value ratio of the Bank increases with the increase of the supplier’s buyback price.

Through (26) and (29), it is known that

$$\frac{\partial \omega}{\partial w} = \frac{\partial \omega}{\partial q_o} \frac{\partial q_o}{\partial w} = \frac{(p - v) F^{-1}(G)}{q_o^2 p (1 + Tr_2)} \left[ -\frac{1}{(p - v) f(q_r)} \right]$$  \hspace{1cm} (31)$$

From (31), we can know that the loan-to-value ratio of the Bank is positively correlated with the supplier’s wholesale price. That is to say, the loan-to-value ratio of the Bank increases with the increase of the supplier’s wholesale price.

In order to find the relationship between order quantity and decision-making modes, we provided numerical examples. Assume that the parameters \( r_1 = 0.03 \), \( r_2 = 0.1 \), \( c = 5 \), \( v = 4 \), \( s = 2 \), \( T = 1 \), and \( D \sim N(100, 30) \).
From Figure 5, we can know that the order quantity of joint decision is higher than the order quantity of decentralized decision. In the same decision-making mode, the higher sale price, the higher order quantity. In the decentralized decision-making, the order quantity increases with the increase of the supplier’s overconfidence. When supplier’s overconfidence increases to a certain value, the retailer’s order quantity is not related to the supplier’s overconfidence.

In order to find the relationship between wholesale price and the degree of overconfidence, we provided numerical examples. Assume that the parameters \( r_1 = 0.03, r_2 = 0.1, c = 5, v = 4, s = 2, T = 1, \) and \( D \sim N(100,30) \).

From Figure 6, we can know that the wholesale price of high-profit market is higher than the wholesale price of low profit market and the wholesale price of supplier decreases with the increase of overconfidence. When overconfidence degree increases to a certain value, the wholesale price does not change with the degree of overconfidence.

In order to find the relationship between the loan-to-value ratio and the degree of overconfidence, we provided numerical examples. Assume that the parameters \( r_1 = 0.03, r_2 = 0.1, c = 5, v = 4, s = 2, T = 1, \) and \( D \sim N(100,30) \).

From Figure 7, we can know that the loan-to-value ratio decreases with the increase of overconfidence in the decentralized decision-making, but when the overconfidence level reaches a certain level, the loan-to-value ratio does not change with the degree of overconfidence. A comparison of Figure 3 shows that the loan-to-value ratio of joint decision-making is lower than that of decentralized decision-making.

Detailed analysis of decision variables is shown in Table 2.

7. Profit Analysis
In order to analyze the impact of decisions on the profit, we now conduct numerical experiments and graphically illustrate the influence of the overconfidence on the managerial decisions. The input parameters of the example are given as follows: \( r_1 = 0.03, r_2 = 0.1, p = 9, c = 5, s = 2, T = 1, \) and \( D \sim N(100,30) \).

7.1. The Profit of the Bank. From Figure 8, we can know that the Bank’s profit in joint decision-making is greater than the profit in decentralized decision-making. The profit of the Bank increases with the increase of the buyback price. In joint decision-making, the profit of the Bank has
Table 2: Analysis of the each decision variable.

<table>
<thead>
<tr>
<th></th>
<th>joint decision-making</th>
<th>comparison</th>
<th>decentralized decision-making</th>
</tr>
</thead>
<tbody>
<tr>
<td>loan-to-value ratio</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>low profit market</td>
<td>$\beta \uparrow, \omega \downarrow$</td>
<td>$&lt;$</td>
<td>$\beta \uparrow, \omega \downarrow; \beta \uparrow\uparrow, \omega \downarrow \rightarrow$</td>
</tr>
<tr>
<td>high profit market</td>
<td>$\beta \uparrow, \omega \uparrow$</td>
<td>$&lt;$</td>
<td>$\beta \uparrow, \omega \downarrow; \beta \uparrow\uparrow, \omega \downarrow \rightarrow$</td>
</tr>
<tr>
<td>order quantity</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>low profit market</td>
<td>$q &lt; \mu; \beta \uparrow, q \uparrow$</td>
<td>$&gt;$</td>
<td>$\beta \uparrow, q \uparrow; \beta \uparrow\uparrow, q \downarrow \rightarrow$</td>
</tr>
<tr>
<td>high profit market</td>
<td>$q &gt; \mu; \beta \uparrow, q \downarrow$</td>
<td>$&gt;$</td>
<td>$\beta \uparrow, q \uparrow; \beta \uparrow\uparrow, q \downarrow \rightarrow$</td>
</tr>
<tr>
<td>wholesale price</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>low profit market</td>
<td>$\beta \uparrow, w \downarrow$</td>
<td></td>
<td>$\beta \uparrow, w \downarrow; \beta \uparrow\uparrow, w \rightarrow$</td>
</tr>
<tr>
<td>high profit market</td>
<td>$\beta \uparrow, w \downarrow$</td>
<td></td>
<td>$\beta \uparrow, w \downarrow; \beta \uparrow\uparrow, w \rightarrow$</td>
</tr>
</tbody>
</table>

Figure 9: The profit of the retailer with respect to overconfidence.

7.2. The Profit of the Retailer in Decentralized Decision-Making. From Figure 9, we can know that the profit of the retailer increases with the increase of the buyback price. In decentralized decision-making, the profit of the retailer increases with the increase of overconfidence; when the degree of overconfidence reaches a certain value, the profit of the Bank is not affected by the degree of overconfidence.

7.3. The Profit of the Supplier in Decentralized Decision-Making. From Figure 10, we can know that expected profit of the supplier is bigger than the real profit. The profit of the supplier increases with the increase of the buyback price. In decentralized decision-making, the expected profit of the supplier increases with the increase of overconfidence; when the degree of overconfidence reaches a certain value, the expected profit of the supplier is not affected by the degree of overconfidence. In decentralized decision-making, the real profit of the supplier decreases with the increase of overconfidence; when the degree of overconfidence reaches a certain value, the real profit of the supplier is not affected by the degree of overconfidence. The lower the buyback price, the lower the impact of overconfidence on the supplier’s real profit.

7.4. The Profit of the Supply Chain. From Figure 11, we can get some conclusions as follows. The real profit of the supply chain of joint decision-making is greater than that of decentralized decision-making. In the joint decision-making, the real profit of the supply chain decreases with the increase of overconfidence, but the change is very small and it can almost be considered that it has no effect, and the higher the buyback price, the lower the real profit of the supply chain. In decentralized decision-making, the real profit of the supply chain increases with the increase of overconfidence, and when overconfidence comes to a certain value, it has little impact on the profit, and the higher the buyback price, the higher the real profit of the supply chain.

Detailed analysis is shown in Table 3 (where $\pi_m^e$ means the expected profit of the supplier and $\pi_m^r$ means the real profit of the supplier).

8. Conclusion and Operation Suggestions

Inventory financing is the combination of logistics and finance. Inventory financing is an effective operation for small and medium-sized enterprises with the limited funds.
This paper focuses on the financing supply chain which consists of the retailer, the overconfident supplier, and the Bank and discusses the coordination problems between the supplier, the retailer, and the Bank under the wholesale price contract. We took into account the overconfidence about the demand forecasting accuracy and then constructed the model of inventory financing which contains order quantity, wholesale price, and the loan-to-value ratio. We used bilevel Stackelberg game to analyze the effect of the overconfidence of the supplier on three parties and then discussed the impact of buyback strategy of the supplier on decision-making. Through the previous analysis, the following conclusions and operation suggestions are given:

(1) In the joint decision-making, the variation trend of each decision variable is affected by the market profit space, but it is not affected by the market profit space in the decentralized decision-making. The real profit of the supply chain of joint decision-making is greater than that of decentralized decision-making.

(2) The financing supply chain (including the Bank and supply chain) should have a positive attitude towards the overconfidence of the supplier. When the supply chain adopts joint decision-making, the Bank profit and supply chain profit are hardly affected by overconfidence. When the supply chain adopts decentralized decision-making, the profits of the Bank, the retailer, and the supply chain increase with the increase of overconfidence of the supplier. But when overconfidence increases to a certain value, the profits do not change with overconfidence. The effect of overconfidence is convergent.

(3) In the joint decision-making, the higher the buyback price, the lower the supply chain profit. So the supplier will reduce the buyback price. When the supplier reduces the buyback price, the Bank will reduce the loan-to-value ratio correspondingly. The supply chain can only get a small amount of loans from the Bank, which may not meet the needs of capital. So the supplier need determines the buyback price according to the capital demand.

(4) In the decentralized decision-making, the higher the buyback price, the higher the profits of all parties. So the supplier should try to increase the buyback price. With the overconfidence psychological behavior of the supplier, the entire inventory financing chain real total profits of three parties increase with the increase of the buyback price, so the supplier should try to use high buyback price strategy.

This paper took the overconfidence into the inventory financing decision and put forward corresponding management strategies for the inventory pledge business in the supply chain. However, this paper only discussed the financing of supply chain inventory in the case of information symmetry, and the inventory financing decision under asymmetric information will be the direction of the next research.

**Data Availability**

The data used to support the findings of this study are available from the first author upon request.

**Conflicts of Interest**

The authors declare that they have no conflicts of interest.

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