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Research Article

H_∞ Tracking Control of Fuzzy Dynamic Output for Nonlinear Networked System with Packet Dropouts

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The tracking control of H_∞ dynamic output feedback is proposed for the fuzzy networked systems of the same category, in which each system is discrete-time nonlinear and is missing measurable data. In other words, the loss of data packet occurs randomly in both the uplink and the downlink. The independent variables that are called the Bernoulli random variables are considered to design the loss of data packets. The method of parallel distributed compensation (PDC) in terms of the T-S fuzzy model is applied to investigate the dynamic controller of tracking control on the systems. Then, it is presented that the analytical H_∞ performance of the output error between the reference model and the fuzzy model for the closed-loop system containing dynamic output feedback controller is proven. Furthermore, the achieved sufficient conditions in terms of LMIs ensure that the closed-loop system is stochastically stable in the H_∞ sense. Finally, a numerical system is offered to show the effectiveness of the established technique.

1. Introduction

The fundamental problem of control application and its theory is the problem of control tracking [1, 2]. With the development of modern science and technology, the tracking control of network control systems (NCSs) has witnessed significant achievements in the past twenty years [3–6]. The problems of synthesis, analysis, and modeling of NCSs become more and more difficult with the introduction of communication networks. To tackle these challenges, for the singular systems of the same category in which the input and state were quantitated, the plan of an event-based H_∞ control was supposed in [7].

The tracking control of output, playing an important role in the industrial, economic, and biological control processes, which is known as the control of reference model, can approximate the reference output of a given model. In some fields, the tracking control of output is employed far and wide, such as motors [8, 9], robots [10], and flights [11]. Based on the tracking control of output, many preliminary studies have been produced [12–15]. Papers [6, 16–18] proposed some questions about stability, and some scholars proposed some thinking about H_∞ control [19] as well as the problem of designing about the filter in the literature [20]. As a result of the increased complicacies of the systems, nonlinear characteristics occur randomly in reality. Furthermore, the aforementioned methods for linear networked control systems cannot be used more directly. In the light of nonlinear characteristics, a few classes of advanced techniques containing sliding mode control [21], adaptive control [22, 23], and fuzzy control [24, 25] were applied. The tracking control of network control system involves some nonlinear factors, such as bandwidth constraints, packet dropouts, and network delays. Then, the designing and analysis of the system become more difficult and complex. In general, T-S [26] fuzzy model can approximate to the smooth nonlinear system as much as possible and in fuzzy system theories we can use the developed technology to research nonlinear systems. It can be seen that the T-S fuzzy model has been successfully applied in a large number of realistic nonlinear systems from literature [27–29]. Between the controlled output and external input, literatures [30–32] describe that the H_∞ control minimizes the gain of energy. In the tracking control of H_∞ output, the system that contains nonlinear perturbations and time-varying delay is studied by Zhang and Yu [33]. In the case of packet loss and time delay, at home and abroad, the scholars are researching the problem about the performance analysis of the network control system with H_∞ output tracking and the design issues of controller [34]. In literatures [35, 36], the faulty links of communication are often described by the Markov chain and the distribution of Bernoulli random variables. In literature [37], under the links of imperfect communication, the feedback control of H_∞ output has been studied for the systems of the same category. However, it should be pointed out that the above researches do not take fully into account the links of faulty communication and a complete message of the state vector, which is the critical shortcoming of the state-feedback controller when put into effect in reality. In [38], the design of tracking control is investigated by applying IT2 T-S method and it is employed to a classical practical application which is called mass-spring-damping system. Moreover, the authors researched the presented issue for nonlinear structures in view of the fuzzy observer as well as the influence of unknown state variables and data loss in Ethernet transmission. However, they did not take into account the dynamic output feedback controller in the systems. Therefore, our idea is that for the nonlinear tracking systems of discrete time with packet loss consisting in both the downlink and the uplink a dynamic output feedback controller is designed and the parallel distributed compensation (PDC) using T-S fuzzy model is constructed to tackle nonlinearity.

In this paper, the tracking control of H_∞ dynamic output feedback is suggested for the fuzzy networked systems with missing data. The independent variables which are called the Bernoulli random variables are considered to design the loss of data packets when it occurs randomly in both the uplink and the downlink. The method of parallel distributed compensation (PDC) in terms of the T-S fuzzy model is employed to plan the dynamic output controller of tracking control. Then, the analysis of H_∞ performance for the closed-loop system containing dynamic controller is presented. Furthermore, the sufficient conditions in terms of LMIs guarantee that the closed-loop system is stochastically stable in the sense of H_∞ performance.

The rest of this article is as follows. Under data missing, the researched problem of H_∞ tracking control for the fuzzy networked systems of the same species is formulated in Section 2. It is presented that the designing of fuzzy dynamic output controller and the performance analysis of H_∞ output tracking are the main results in Section 3. Section 4 gives a numerical example and in Section 5 we put forward the conclusion of paper.

Notation. In this paper, the notation applied is comparatively standard. The matrix transposition is stood for by superscript "T" and the space of n-dimensional Euclidean is denoted by R^n. Zero matrix and the identity matrix are signified by 0 and I, respectively. The symbol $*$ is employed to denote the symmetry term in the expressions of complex matrix and symmetric block matrices, and $P \geq 0$ stands for P being real symmetric and positive definite (semidefinite). The space of square-integrable vector over $[0,\infty)$ is suggested by $l2[0,\infty)$. $\|M\| = \sqrt{\mathrm{tr}(M^TM)}$ shows matrix norm. $|\cdot|$ shows the norm of Euclidean vector and the norm of $l2[0,\infty)$ is defined by $\|\cdot\|_2$. The notation $E\{\alpha\}$ indicates the expectation of the event α. $E\{\alpha/\beta\}$ indicates the expectation of the event α conditional on the event β. It is assumed that the matrices in this paper have compatible dimensions if the dimensions are not demonstrably prescribed.

2. Problem Formulation

Firstly, we consider the T-S fuzzy model. It is a discrete-time nonlinear system with data missing. The overall fuzzy model is described by fuzzy aggregation of the linear models.

2.1. T-S Fuzzy Model. The i-th rule of the T-S is the following:

Model Rule i:

$$\begin{aligned}
&\text{If} \quad z_1(t) \text{ is } M_{i1},\dots,z_g(t) \text{ is } M_{ig}\\
&\text{Then} \quad x(t+1) = A_ix(t) + B_iu(t) + E_i\varpi(t),\\
&\qquad r(t) = C_{1i}x(t) + D_iu(t) + F_i\varpi(t),\\
&\qquad y(t) = C_{2i}x(t),\\
&\qquad\qquad (i = 1,2,\dots,r),
\end{aligned} \tag{1}$$

where M_{id} $(d = 1,2,\dots,g)$ is the fuzzy set associated with the i-th model rule and d-th premise variable component; $x(t) \in R^{n_x\times 1}$ denotes the state vector; $u(t) \in R^{n_u\times 1}$ denotes the control input vector; $y(t) \in R^{n_y\times 1}$ denotes the vector of measured output; $r(t) \in R^{n_r\times 1}$ denotes the vector of controlled output; $\varpi(t) \in l2[0,\infty)$ are external disturbances and $\varpi(t) \in R^{n_\varpi\times 1}$. A_i, B_i, E_i, D_i, F_i, C_{1i}, and C_{2i} are local system matrices with appropriate dimensions. $z(t) = [z_1(t)\cdots z_g(t)]^T$ are known premise variables. The scalar r is the number of rules. The final fuzzy system is listed:

$$\begin{aligned}
x(t+1) &= \sum_{i=1}^{r} h_i\left[A_ix(t) + B_iu(t) + E_i\varpi(t)\right],\\
r(t) &= \sum_{i=1}^{r} h_i\left[C_{1i}x(t) + D_iu(t) + F_i\varpi(t)\right],\\
y(t) &= \sum_{i=1}^{r} h_iC_{2i}x(t),
\end{aligned} \tag{2}$$

where for all t we suppose the following: $h_i(z(t)) = \omega_i(z(t))/\sum_{i=1}^{r}\omega_i(z(t))$, $\omega_i(z(t)) = \prod_{d=1}^{g}M_{id}(z_d(t))$, $\omega_i(z(t)) \geq 0$, $\sum_{i=1}^{r}\omega_i(z(t)) > 0$, $\sum_{i=1}^{r}h_i(z(t)) = 1$, and $h_i(z(t)) \geq 0$ $(i = 1,2,\dots,r)$. In what follows, we write $h_i \triangleq h_i(z(t))$ for brevity.

The designing of fuzzy dynamic output controller is our objective. In this way, the output $r(t)$ of the controlled model can track the signal $r_r(t)$ of reference model to satisfy the performance of the required tracking. Assume the reference model as follows:

$$\begin{aligned}
x_r(t+1) &= \sum_{i=1}^{r} h_i\left[G_i^rx_r(t) + B_i^r\sigma(t)\right],\\
r_r(t) &= \sum_{i=1}^{r} h_i\left[H_i^rx_r(t) + L_i^r\sigma(t)\right],
\end{aligned}$$

$$y_r(t) = \sum_{i=1}^{r} h_i M_i^r x(t), \tag{3}$$

where $x_r(t) \in R^{n_x \times 1}$ is the state of reference model; $r_r(t) \in R^{n_r \times 1}$ is the controlled output of reference model; $y_r(t) \in R^{n_y \times 1}$ is the measured output of reference model; $\sigma(t) \in R^{n_\varpi \times 1}$ is the input of bounded reference energy; G_i^r (Hurwitz), B_i^r, H_i^r, L_i^r, and M_i^r are constant matrices with appropriate dimensions.

From (2) and (3), the augmented error system is as follows:

$$\begin{aligned} \xi(t+1) &= \sum_{i=1}^{r} h_i \left[\widetilde{A}_i \xi(t) + \widetilde{B}_i u(t) + \widetilde{E}_i \nu(t)\right], \\ e(t) &= r(t) - r_r(t) \\ &= \sum_{i=1}^{r} h_i \left[\widetilde{C}_{1i} \xi(t) + \widetilde{D}_i u(t) + \widetilde{F}_i \nu(t)\right], \\ \bar{e}(t) &= y(t) - y_r(t) = \sum_{i=1}^{r} h_i \widetilde{C}_{2i} \xi(t), \end{aligned} \tag{4}$$

where

$$\begin{aligned} \xi(t) &= \begin{bmatrix} x(t) \\ x_r(t) \end{bmatrix}, \\ \nu(t) &= \begin{bmatrix} \varpi(t) \\ \sigma(t) \end{bmatrix}, \\ \widetilde{A}_i &= \begin{bmatrix} A_i & 0 \\ 0 & G_i^r \end{bmatrix}, \\ \widetilde{B}_i &= \begin{bmatrix} B_i \\ 0 \end{bmatrix}, \\ \widetilde{E}_i &= \begin{bmatrix} E_i & 0 \\ 0 & B_i^r \end{bmatrix}, \\ \widetilde{C}_{1i} &= \begin{bmatrix} C_{1i} & -H_i^r \end{bmatrix}, \\ \widetilde{D}_i &= D_i, \\ \widetilde{F}_i &= \begin{bmatrix} F_i & -L_i^r \end{bmatrix}, \\ \widetilde{C}_{2i} &= \begin{bmatrix} C_{2i} & -M_i^r \end{bmatrix}. \end{aligned} \tag{5}$$

2.2. Fuzzy Dynamic Output Feedback Controller Design. Based on the T-S fuzzy model (4), in this paper, we construct the following dynamical output feedback controller:

Rule C_i:

$$\begin{aligned} &\text{If} \quad z_1(t) \text{ is } M_{i1}, \ldots, z_g(t) \text{ is } M_{ig} \\ &\text{Then} \quad \eta_c(t+1) = A_i^c \eta_c(t) + B_i^c e^c(t), \\ &\qquad\quad u^c(t) = C_i^c \eta_c(t), \end{aligned} \tag{6}$$

where $\eta_c(k) \in R^{n_\eta \times 1}$ is the state vector of the controller; $e^c(t) \in R^{n_e \times 1}$ is the input vector of the controller; $u^c(t) \in R^{n_u \times 1}$ is the output vector of the controller; A_i^c, B_i^c, and C_i^c are matrices with appropriate dimensions. Then

$$\begin{aligned} \eta_c(t+1) &= \sum_{i=1}^{r} h_i \left[A_i^c \eta_c(t) + B_i^c e^c(t)\right], \\ u^c(t) &= \sum_{i=1}^{r} h_i C_i^c \eta_c(t). \end{aligned} \tag{7}$$

2.3. Unreliable Communication Links. It can be seen that the model is with the links of communication network from Figure 1. In this paper, we consider that a few elements are introduced via network and the loss of data packets occurs randomly in both the uplink and the downlink. Thus $\bar{e}(t) \neq e^c(t)$ and $u^c(t) \neq u(t)$. We represent the above phenomenon by applying a stochastic method and it is described as follows:

$$\begin{aligned} e^c(t) &= \alpha(t) \bar{e}(t), \\ u(t) &= \beta(t) u^c(t), \end{aligned} \tag{8}$$

where $\{\alpha(t)\}$ and $\{\beta(t)\}$ satisfy the process of Bernoulli random distribution. $\{\alpha(t)\}$ presents the downlink of unreliable communication and $\{\beta(t)\}$ describes the uplink. Assume $\{\alpha(t)\}$ and $\{\beta(t)\}$ as follows:

$$\begin{aligned} &\text{prob}\{\alpha(t) = 1\} = E\{\alpha(t)\} = \bar{\alpha}, \\ &\text{prob}\{\alpha(t) = 0\} = 1 - \bar{\alpha}, \\ &\text{prob}\{\beta(t) = 1\} = E\{\beta(t)\} = \bar{\beta}, \\ &\text{prob}\{\beta(t) = 0\} = 1 - \bar{\beta}. \end{aligned} \tag{9}$$

According to (8), we obtain

$$\begin{aligned} \eta_c(t+1) &= \sum_{i=1}^{r} h_i \left[A_i^c \eta_c(t) + \alpha(t) B_i^c \bar{e}(t)\right], \\ u^c(t) &= \sum_{i=1}^{r} h_i C_i^c \eta_c(t). \end{aligned} \tag{10}$$

Combining (4) and (10), one has the augmented closed-loop system:

$$\begin{aligned} \bar{\xi}(t+1) &= \sum_{i=1}^{r} \sum_{j=1}^{r} h_i h_j \left[A_{ij} \bar{\xi}(t) + \Xi_i \nu(t)\right], \\ e(t) &= \sum_{i=1}^{r} \sum_{j=1}^{r} h_i h_j \left[C_{ij} \bar{\xi}(t) + \widetilde{F}_i \nu(t)\right], \end{aligned} \tag{11}$$

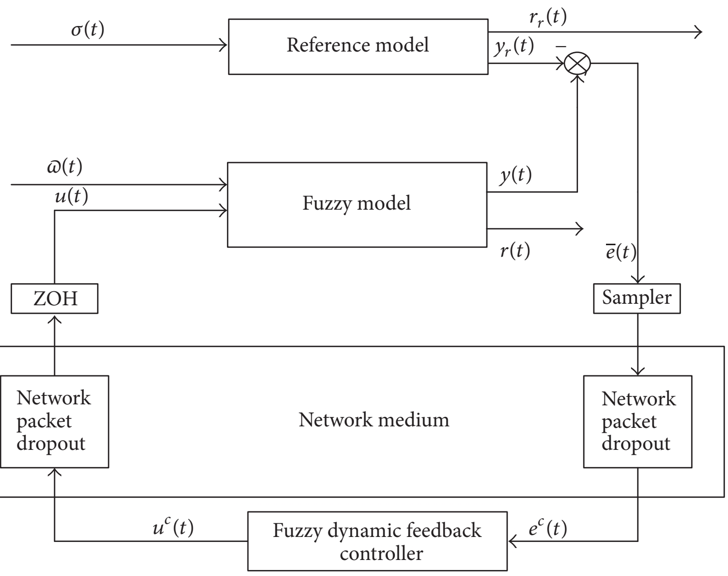

Figure 1: Plant flow chart.

where

$$
\begin{aligned}
A_{ij} &= \begin{bmatrix} \widetilde{A}_i & \beta(t)\widetilde{B}_i C_j^c \\ \alpha(t) B_j^c \widetilde{C}_{2i} & A_j^c \end{bmatrix}, \\
\Xi_i &= \begin{bmatrix} \widetilde{E}_i \\ 0 \end{bmatrix}, \\
C_{ij} &= \begin{bmatrix} \widetilde{C}_{1i} & \beta(k)\widetilde{D}_i C_j^c \end{bmatrix}, \\
\bar{\xi}(t) &= \begin{bmatrix} \xi(t) \\ \eta_c(t) \end{bmatrix}.
\end{aligned}
\tag{12}
$$

Assume that

$$
\begin{aligned}
\alpha(t) &= \bar{\alpha} + \tilde{\alpha}(t), \\
\beta(t) &= \bar{\beta} + \tilde{\beta}(t);
\end{aligned}
\tag{13}
$$

then

$$
\begin{aligned}
E\{\tilde{\alpha}(t)\tilde{\alpha}(t)\} &= \bar{\alpha}(1-\bar{\alpha}), \\
E\{\tilde{\beta}(t)\tilde{\beta}(t)\} &= \bar{\beta}\left(1-\bar{\beta}\right).
\end{aligned}
\tag{14}
$$

In this paper, we design the output feedback controller to ensure the stochastic stability of closed-loop system which fulfills the performance of external disturbance attenuation. Therefore, the definition is as follows.

Definition 1 (see [39]). Any initial condition is considered $\bar{\xi}(0)$. Under $\nu(t) \equiv 0$, if there exists a matrix $W > 0$, then $E\{\sum_{t=0}^{\infty} |\bar{\xi}(t)|^2|_{\bar{\xi}(0)}\} < \bar{\xi}^T(0)W\bar{\xi}(0)$, and the closed-loop system (11) is stochastically stable.

Then, in this paper, the problem that is dealt with is drawn up as follows: the augmented system in (11) is asymptotically stable with $\nu(t) \equiv 0$ and satisfies

$$
E\left\{\sum_{t=0}^{\infty} |e(t)|^2\right\} \le \gamma^2 \|\nu\|_2^2, \tag{15}
$$

where $\gamma > 0$; then the H_∞ performance γ of output tracking is obtained.

3. Main Results

Two parts of results are included in this section, containing the conditions of designing controller and the criteria of sufficient stability.

Theorem 2. *Consider the model of the augmented fuzzy system (11). With the supposed matrices A_j^c, B_j^c, and C_j^c ($j = 1, \ldots, r$) of the controller gain, the system of closed-loop fuzzy model in (11) is stochastically stable and fulfills H_∞ performance γ of external disturbance attenuation, if there exist matrices p_l ($l = 1, \ldots, r$), fulfilling the following inequality:*

$$
\left(A_{ij}^1\right)^T P_l A_{ij}^1 + \left(A_{ij}^2\right)^T P_l A_{ij}^2 - P_i < 0, \tag{16}
$$

$$
\begin{bmatrix}
-I & 0 & 0 & 0 & C_{ij}^1 & \widetilde{F}_i \\
0 & -I & 0 & 0 & C_{ij}^2 & 0 \\
0 & 0 & -P_l^{-1} & 0 & A_{ij}^1 & \Xi_i \\
0 & 0 & 0 & -P_l^{-1} & A_{ij}^2 & 0 \\
* & * & * & * & -P_i & 0 \\
* & * & * & * & 0 & -\gamma^2 I
\end{bmatrix} < 0, \tag{17}
$$

where

$$A_{ij}^1 = \begin{bmatrix} \widetilde{A}_i & \overline{\beta}\widetilde{B}_i C_j^c \\ \overline{\alpha} B_j^c \widetilde{C}_{2i} & A_j^c \end{bmatrix},$$

$$A_{ij}^2 = \begin{bmatrix} 0 & \sqrt{\overline{\beta}\left(1-\overline{\beta}\right)}\widetilde{B}_i C_j^c \\ \sqrt{\overline{\alpha}\left(1-\overline{\alpha}\right)} B_j^c \widetilde{C}_{2i} & 0 \end{bmatrix}, \tag{18}$$

$$C_{ij}^1 = \left[\widetilde{C}_{1i} \quad \overline{\beta}\widetilde{D}_i C_j^c\right],$$

$$C_{ij}^2 = \left[0 \quad \sqrt{\overline{\beta}\left(1-\overline{\beta}\right)}\widetilde{D}_i C_j^c\right].$$

Proof. Defining $\nu(t) \equiv 0$, the stochastic stability of system (11) is proven. For system (11), the following Lyapunov function is chosen:

$$V(t) = \overline{\xi}^T(t)\left[\sum_{i=1}^r h_i P_i\right]\overline{\xi}(t), \tag{19}$$

where $P_i > 0$; supposing that $h_l^+ = h_l^+(z(t+1))$, we have

$$\begin{aligned} E\{\Delta V(t)\} &= E\left\{V(t+1)\,|\,\overline{\xi}(t)\right\} - V(t) = E\left\{\overline{\xi}^T(t)\right. \\ &\quad \cdot \sum_{l=1}^r h_l^+\left[\sum_{i=1}^r\sum_{j=1}^r\sum_{s=1}^r\sum_{t=1}^r h_i h_j h_s h_t \left(A_{ij}{}^T P_l A_{st}\right)\right]\left.\overline{\xi}(t)\right\} \\ &\quad - \overline{\xi}^T(t)\left[\sum_{i=1}^r h_i P_i\right]\overline{\xi}(t) \le \overline{\xi}^T(t)\sum_{l=1}^r h_l^+ \\ &\quad \cdot \sum_{i=1}^r\sum_{j=1}^r h_i h_j\left[\left(A_{ij}^1\right)^T P_l A_{ij}^1 + \left(A_{ij}^2\right)^T P_l A_{ij}^2\right]\overline{\xi}(t) \\ &\quad - \overline{\xi}^T(t)\left[\sum_{i=1}^r h_i P_i\right]\overline{\xi}(t) = \overline{\xi}^T(t)\sum_{l=1}^r h_l^+ \\ &\quad \cdot \sum_{i=1}^r\sum_{j=1}^r h_i h_j\left[\left(A_{ij}^1\right)^T P_l A_{ij}^1 + \left(A_{ij}^2\right)^T P_l A_{ij}^2 - P_i\right]\overline{\xi}(t). \end{aligned} \tag{20}$$

Let

$$\Psi = \sum_{l=1}^r h_l^+ \sum_{i=1}^r\sum_{j=1}^r h_i h_j\left[\left(A_{ij}^1\right)^T P_l A_{ij}^1 + \left(A_{ij}^2\right)^T P_l A_{ij}^2 - P_i\right]. \tag{21}$$

From

$$\begin{aligned} \lambda_{\min}(-\Psi)\left|\overline{\xi}(t)\right|^2 &\le \overline{\xi}^T(t)(-\Psi)\overline{\xi}(t) \\ &\le \lambda_{\max}(-\Psi)\left|\overline{\xi}(t)\right|^2 \end{aligned} \tag{22}$$

we obtain

$$\begin{aligned} &E\left\{\overline{\xi}^T(t+1)\left[\sum_{l=1}^r h_l^+ P_l\right]\overline{\xi}(t+1)\right\} \\ &\quad - \overline{\xi}^T(t)\left[\sum_{i=1}^r h_i P_i\right]\overline{\xi}(t) \\ &\quad \le -\lambda_{\min}(-\Psi)\overline{\xi}^T(t)\overline{\xi}(t). \end{aligned} \tag{23}$$

From $t = 0, 1, \ldots, k$ and $k \ge 1$, for the above inequality, calculating and summing mathematical expectation, we can have

$$\begin{aligned} &E\left\{\overline{\xi}^T(k+1)\left[\sum_{l=1}^r h_l^+ P_l\right]\overline{\xi}(k+1)\right\} \\ &\quad - \overline{\xi}^T(0)\left[\sum_{i=1}^r h_i P_i\right]\overline{\xi}(0) \\ &\quad \le -\lambda_{\min}(-\Psi)E\left\{\sum_{t=0}^k\left|\overline{\xi}(t)\right|^2\right\}. \end{aligned} \tag{24}$$

Then, when $k = 1, \ldots, \infty$ and given $E\{\overline{\xi}^T(\infty)[\sum_{l=1}^r h_l^+ P_l]\overline{\xi}(\infty)\} \ge 0$, we obtain

$$\begin{aligned} E\left\{\sum_{t=0}^k\left|\overline{\xi}(t)\right|^2\right\} &\le \left(\lambda_{\min}(-\Psi)\right)^{-1} \\ &\quad \cdot \left\{\overline{\xi}^T(0)\left[\sum_{i=1}^r h_i P_i\right]\overline{\xi}(0)\right. \\ &\quad \left. - E\left\{\overline{\xi}^T(k+1)\left[\sum_{l=1}^r h_l^+ P_l\right]\overline{\xi}(k+1)\right\}\right\} \\ &\le \left(\lambda_{\min}(-\Psi)\right)^{-1}\overline{\xi}^T(0)\left[\sum_{i=1}^r h_i P_i\right]\overline{\xi}(0) = \overline{\xi}^T(0) \\ &\quad \cdot \left[\left(\lambda_{\min}(-\Psi)\right)^{-1}\sum_{i=1}^r h_i P_i\right]\overline{\xi}(0). \end{aligned} \tag{25}$$

Let $W \triangleq (\lambda_{\min}(-\Psi))^{-1}\sum_{i=1}^r h_i P_i$; we obtain $\Psi < 0$ and $W > 0$. Therefore, we acquire that system (11) is stochastically stable in terms of Definition 1.

The following section will introduce the H_∞ performance of external disturbance attenuation when the initial condition is zero. The H_∞ index is as follows:

$$\begin{aligned} J &\triangleq E\left\{e^T(t)e(t)\big|_{\Theta(t)}\right\} - \gamma^2\nu^T(t)\nu(t) \\ &\quad + E\left\{V(t+1)\big|_{\Theta(t)}\right\} - V(t). \end{aligned} \tag{26}$$

Let $\Theta(k) = \begin{bmatrix} \overline{\xi}(k) \\ \nu(k) \end{bmatrix}$; then, we obtain

$$
\begin{aligned}
J &\triangleq E\left\{V(t+1)|_{\Theta(t)}\right\} - V(t) + E\left\{e^T(t)\,e(t)\big|_{\Theta(t)}\right\} - \gamma^2\nu^T(t)\,\nu(t) = E\left\{\Theta^T(t)\sum_{l=1}^{r}h_l^+\right.\\
&\cdot\left.\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{s=1}^{r}\sum_{t=1}^{r}h_ih_jh_sh_t\left(\begin{bmatrix}A_{ij} & \Xi_i\end{bmatrix}^T P_l\begin{bmatrix}A_{st} & \Xi_s\end{bmatrix}\right)\Theta(t)\right\} - \gamma^2\nu^T(t)\,\nu(t) - \bar{\xi}^T(t)\left[\sum_{i=1}^{r}h_iP_i\right]\bar{\xi}(t) + E\left\{\Theta^T(t)\right.\\
&\cdot\left.\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{s=1}^{r}\sum_{t=1}^{r}h_ih_jh_sh_t\left(\begin{bmatrix}C_{ij} & \widetilde{F}_i\end{bmatrix}^T\begin{bmatrix}C_{st} & \widetilde{F}_s\end{bmatrix}\right)\Theta(t)\right\} = \Theta^T(t)\sum_{l=1}^{r}h_l^+\\
&\cdot\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{s=1}^{r}\sum_{t=1}^{r}h_ih_jh_sh_t\begin{bmatrix}\left(A_{ij}^1\right)^T P_lA_{st}^1 + \left(A_{ij}^2\right)^T P_lA_{st}^2 & \left(A_{ij}^1\right)^T P_l\Xi_s\\ \Xi_i^TP_lA_{st}^1 & \Xi_i^TP_l\Xi_s\end{bmatrix}\Theta(t) - \Theta^T(t)\sum_{i=1}^{r}h_i\begin{bmatrix}P_i & 0\\ 0 & \gamma^2I\end{bmatrix}\Theta(t) + \Theta^T(t)\sum_{l=1}^{r}h_l^+\\
&\cdot\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{s=1}^{r}\sum_{t=1}^{r}h_ih_jh_sh_t\begin{bmatrix}\left(C_{ij}^1\right)^T C_{st}^1 + \left(C_{ij}^2\right)^T C_{st}^2 & \left(C_{ij}^1\right)^T \widetilde{F}_s\\ \widetilde{F}_i^TC_{st}^1 & \widetilde{F}_i^T\widetilde{F}_s\end{bmatrix}\Theta(t) \le \Theta^T(t)\sum_{l=1}^{r}h_l^+\\
&\cdot\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\begin{bmatrix}\left(A_{ij}^1\right)^T P_lA_{ij}^1 + \left(A_{ij}^2\right)^T P_lA_{ij}^2 & \left(A_{ij}^1\right)^T P_l\Xi_i\\ \Xi_i^TP_lA_{ij}^1 & \Xi_i^TP_l\Xi_i\end{bmatrix}\Theta(t) - \Theta^T(t)\sum_{i=1}^{r}h_i\begin{bmatrix}P_i & 0\\ 0 & \gamma^2I\end{bmatrix}\Theta(t) + \Theta^T(t)\sum_{l=1}^{r}h_l^+\\
&\cdot\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\begin{bmatrix}\left(C_{ij}^1\right)^T C_{ij}^1 + \left(C_{ij}^2\right)^T C_{ij}^2 & \left(C_{ij}^1\right)^T \widetilde{F}_i\\ \widetilde{F}_i^TC_{ij}^1 & \widetilde{F}_i^T\widetilde{F}_i\end{bmatrix}\Theta(t) = \Theta^T(t)\sum_{l=1}^{r}h_l^+\\
&\cdot\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\left\{\left[\begin{bmatrix}\left(A_{ij}^1\right)^T P_lA_{ij}^1 + \left(A_{ij}^2\right)^T P_lA_{ij}^2 & \left(A_{ij}^1\right)^T P_l\Xi_i\\ \Xi_i^TP_lA_{ij}^1 & \Xi_i^TP_l\Xi_i\end{bmatrix} + \begin{bmatrix}\left(C_{ij}^1\right)^T C_{ij}^1 + \left(C_{ij}^2\right)^T C_{ij}^2 & \left(C_{ij}^1\right)^T \widetilde{F}_i\\ \widetilde{F}_i^TC_{ij}^1 & \widetilde{F}_i^T\widetilde{F}_i\end{bmatrix} - \begin{bmatrix}P_i & 0\\ 0 & \gamma^2I\end{bmatrix}\right]\right\}\\
&\cdot\Theta(t) = \Theta^T(t)\sum_{l=1}^{r}h_l^+\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\left\{\begin{bmatrix}C_{ij}^1 & \widetilde{F}_i\\ C_{ij}^2 & 0\end{bmatrix}^T\begin{bmatrix}C_{ij}^1 & \widetilde{F}_i\\ C_{ij}^2 & 0\end{bmatrix} + \begin{bmatrix}A_{ij}^1 & \Xi_i\\ A_{ij}^2 & 0\end{bmatrix}^T\begin{bmatrix}P_l & 0\\ 0 & P_l\end{bmatrix}\begin{bmatrix}A_{ij}^1 & \Xi_i\\ A_{ij}^2 & 0\end{bmatrix} - \begin{bmatrix}P_i & 0\\ 0 & \gamma^2I\end{bmatrix}\right\}\Theta(t).
\end{aligned}
\tag{27}
$$

From Schur complement, according to (17), we can obtain $J \le 0$ and $E\{\sum_{t=0}^{\infty}|e(t)|^2\} \le \gamma^2\|\nu\|_2^2$.

The proof is finished. □

4. Simulation Results

A numerical example is applied to explain the validity of the proposed method in the following section of this paper. Consider the following systems:

Model Rule i:

If $x_i(t)$ is $h_i(x_i(t))$ $i = 1, 2$.

$$
\begin{aligned}
\text{Then}\quad x(t+1) &= A_ix(t) + B_iu(t) + E_i\varpi(t),\\
r(t) &= C_{1i}x(t) + D_iu(t) + F_i\varpi(t),\\
y(t) &= C_{2i}x(t),
\end{aligned}
\tag{28}
$$

where

$$
\begin{aligned}
A_1 &= \begin{bmatrix}-0.4618 & 0.8913; & -0.6137 & 1.2153\end{bmatrix};\\
A_2 &= \begin{bmatrix}-0.0146 & -1.1552; & 1.0101 & 0.3005\end{bmatrix};\\
B_1 &= \begin{bmatrix}0.5857 & 1.5251\end{bmatrix};\\
B_2 &= \begin{bmatrix}2.2415 & 2.9953\end{bmatrix};\\
E_1 &= \begin{bmatrix}-0.0099 & 0.0258\end{bmatrix};\\
E_2 &= \begin{bmatrix}-0.0623 & 0.0774\end{bmatrix};\\
F_1 &= 0.0023;\\
F_2 &= 0.0290;\\
C_{11} &= \begin{bmatrix}-1.0789 & 0\end{bmatrix};\\
C_{21} &= \begin{bmatrix}0.9962 & 0\end{bmatrix};\\
D_1 &= -1;\\
C_{12} &= \begin{bmatrix}0.0585 & 0\end{bmatrix};\\
C_{22} &= \begin{bmatrix}0.4795 & 0\end{bmatrix};\\
D_2 &= 1.
\end{aligned}
\tag{29}
$$

The corresponding reference model is assumed as follows:

$$\begin{aligned} x_r(t+1) &= G_i^r x_r(t) + B_i^r \sigma(t), \\ r_r(t) &= H_i^r x_r(t) + L_i^r \sigma(t), \\ y_r(t) &= M_i^r x(t), \end{aligned} \tag{30}$$

where

$$\begin{aligned} G_1^r &= -0.02; \\ B_1^r &= 0.5; \\ H_1^r &= -0.3; \\ L_1^r &= 0.5; \\ M_1^r &= -0.15; \\ G_2^r &= 0.1; \\ B_2^r &= 0.2; \\ H_2^r &= 0.03; \\ L_2^r &= -0.35; \\ M_2^r &= 0.135. \end{aligned} \tag{31}$$

The corresponding form of controller is expressed as follows:

$$\begin{aligned} \eta_c(t+1) &= A_i^c \eta_c(t) + B_i^c e^c(t), \\ u^c(t) &= C_i^c \eta_c(t). \end{aligned} \tag{32}$$

When the minimum value of γ is $\gamma = 0.6039$, $\text{prob}\{\alpha(t) = 1\} = 0.95$, and $\text{prob}\{\beta(t) = 1\} = 0.85$, the gains of controller are as follows:

$$\begin{aligned} A_1^C &= \begin{bmatrix} 0.0144 & 0.0452 & -0.0090 \\ 0.0461 & 0.1613 & -0.0242 \\ 0.0066 & -0.0245 & -0.0159 \end{bmatrix}, \\ A_2^C &= \begin{bmatrix} -0.0023 & -0.0180 & 0.0019 \\ 0.0022 & -0.0423 & -0.0004 \\ -0.0039 & 0.0156 & 0.0021 \end{bmatrix}, \\ B_1^C &= \begin{bmatrix} 0.0518 \\ 0.2082 \\ -0.0842 \end{bmatrix}, \\ B_2^C &= \begin{bmatrix} 0.1035 \\ 0.2613 \\ 0.0835 \end{bmatrix}, \\ C_1^C &= \begin{bmatrix} 0.0045 & 0.0133 & -0.0037 \end{bmatrix}, \\ C_2^C &= \begin{bmatrix} -0.0016 & -0.0043 & 0.0014 \end{bmatrix}. \end{aligned} \tag{33}$$

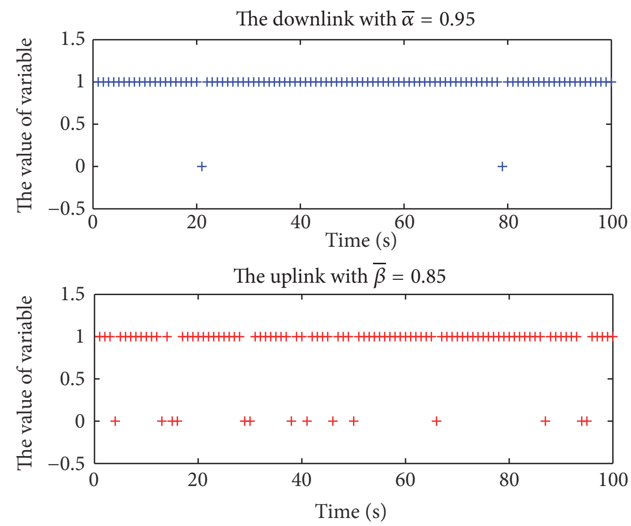

FIGURE 2: Random packet dropouts.

The Membership functions are as follows:

$$\begin{aligned} h_1(x_1(t)) &= \begin{cases} \dfrac{(x_1(t)+3)}{3}, & -3 \le x_1(t) \le 0, \\ \dfrac{(3-x_1(t))}{3}, & 0 \le x_1(t) \le 3, \\ 0, & \text{else}, \end{cases} \\ h_2(x_1(t)) &= 1 - h_1(x_1(t)). \end{aligned} \tag{34}$$

In Figure 2, the dropouts of random data packet are displayed, in which it contains both the uplink and the downlink. Furthermore, it is assumed that the initial value of the model is $[-1 \ \ 4]^T$ and the initial value of the reference system is 1. The external disturbance of the model is assumed as follows: $\varpi(t) = 1/(2+t)$; and the disturbance term of the reference system is $\sigma(t) = \sin(1/0.5 * (t))$; in Figure 3, output $r_r(t)$ of the reference model and the output $r(t)$ of the system are opened up before our eyes, in which $r_r(t)$ is tracked well by the output $r(t)$, despite the fact that the initial values are not equal to zero. The initial value of the controller is $[-2 \ \ 1 \ \ 0]^T$. Figure 4 draws up the state of dynamic output feedback controller. Figure 5 illustrates that the dynamical output feedback controller can make the error system tend towards stability. In this simulation, the external disturbances are added to characterize the fluctuation of system. In Figure 6, under the action of the dynamical output feedback controller, the controlled output result of the closed-loop system is shown. At each sampling time, as a result of the controller, the error $e(t)$ is gradually approaching 0 and the steady states of the closed-loop system are finally reached by continuously adjusting. Therefore, in this paper, the plan of dynamic output feedback controller can ensure that H_∞ performance is satisfied and the required stochastic stability is obtained.

Based on the above survey, the simulation outcome may certify that the project of dynamic output feedback controller satisfies the defined requirements of designing. It can be seen

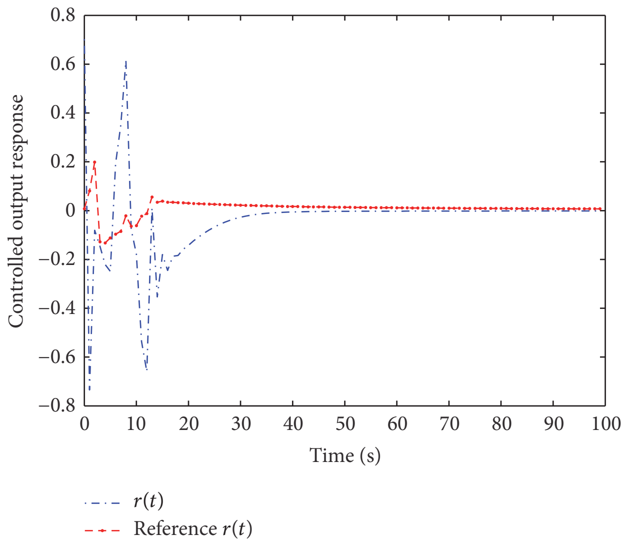

FIGURE 3: Output tracking response of the system.

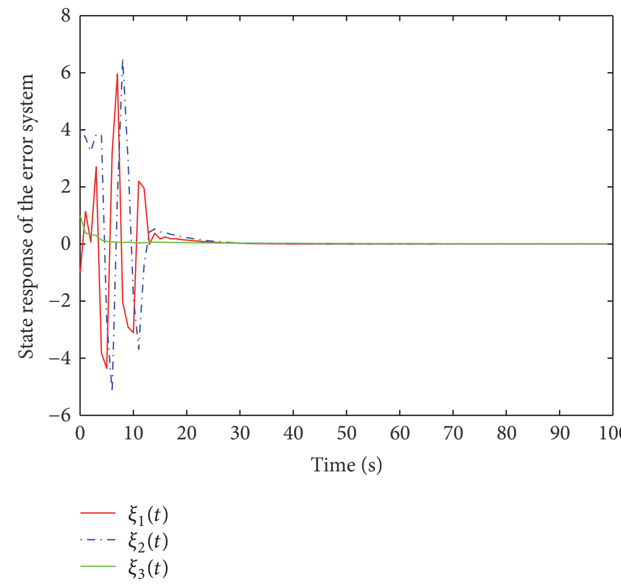

FIGURE 5: State response of the error system.

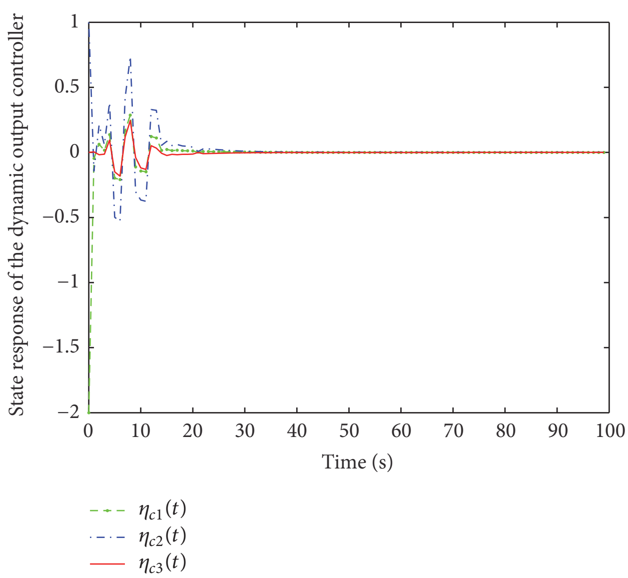

FIGURE 4: State response of the dynamic output feedback controller.

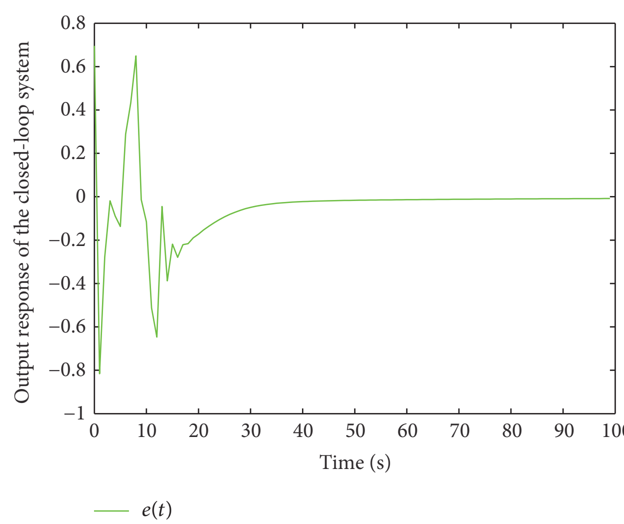

FIGURE 6: Output response of the closed-loop system.

that under the circumstances of questionable communication links the method [27] of state-feedback control is not valid for the nonlinear tracking systems when the state variables of the tracking systems are unmeasured or undiscovered. In this paper, the proposed plan results of dynamic output feedback controller can finish off the above questionable system in which the measurements are missing.

5. Conclusions

In this paper, the tracking control of H_∞ dynamic output feedback is suggested for the fuzzy networked systems with missing data. When the loss of data packets occurs randomly in both the uplink and the downlink, the Bernoulli random variables are considered to design it. The method of planning the dynamic output controller is employed to achieve the stability of tracking control systems. Then, the analysis of H_∞ performance for the closed-loop system is shown. Furthermore, the achieved sufficient conditions in terms of LMIs ensure that the closed-loop system is stochastically stable in the H_∞ sense.

We will devote ourselves to the more realistic and practical output feedback framework for tracking control. We will consider unknown state variables and data loss. In view of T-S fuzzy model, the problem of tracking control with the observer is proposed for uncertain nonlinear networked control systems. The unmeasurable state variables are estimated by the fuzzy observer rather than the model, which may bring about more precise results. The controller of fuzzy tracking control is proposed in terms of the fuzzy observer. The applied controller designing method may cut down complexity and strengthen the flexibility.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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