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Research Article

Consensus of Heterogeneous Multiagent Systems with Switching Dynamics

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Heterogeneity is an important feature of multiagent systems. This paper addresses the consensus problem of heterogeneous multiagent systems composed of first-integrator and double-integrator agents. The dynamics of each agent switches between continuous-time and discrete-time dynamics. By using the graph theory and nonnegative matrix theory, we derive that the system can achieve consensus if and only if the fixed interaction topology has a directed spanning tree. For switching topologies, we get that the system can reach consensus if each interaction topology has a directed spanning tree. Simulation examples are provided to demonstrate the effectiveness of our theoretical results.

1. Introduction

Recently, a well-known fact is that the distributed coordination and control of multiagent systems have attracted a great deal of attention from scientific community. The main reason might be that it can be widely applied in the field of system control, applied mathematics, biology, social science [1–6], etc. As a kind of classical multiagent systems, the analysis and control of cyber-physical systems are also considered by scientists and engineers. In general, the study of multiagent system pertains to consensus problems [7–12], containment control [13–15], controllability analysis [16, 17], formation control [18], tracking control [19–21], etc.

Among the above topics, consensus problem is a basic but critical problem for the multiagent system. Consensus means that a group of agents reach an agreement. In 1995, Vicsek et al. simulated that the moving direction of a group of self-driven particles can reach consensus by using local communication [1]. By virtue of graph theory, Jadbabaie et al. explained this observed consensus behavior of discrete-time multiagent systems from the perspective of the algebraic graph theory [7]. In [22], Olfati-Saber and Murray presented a theoretical framework for consensus problems of continuous-time multiagent systems based on graph theory and obtained the necessary and sufficient conditions for solving the average consensus. In [3], Ren and Beard further investigated the consensus problem for multiagent systems with directed topologies. It is shown in [3] that consensus problem can be solved if the union of the dynamically changing interaction graphs has a directed spanning tree frequently enough as the system evolves. After that, researchers investigated consensus problem from different perspectives. As a result, the fruits for solving consensus problem became more and more. In [23], the author presented the necessary and sufficient conditions of achieving consensus for multiagent systems with noises. Wang et al. studied the state consensus problem for multiagent systems with the switching topologies and bounded time delays [24]. Considering that agent dynamics is adopted as a typical point mass model based on the Newton's law, consensus problem for multiagent systems with double-integrators was also investigated [25–28]. Moreover, Guaranteed-cost consensus and group consensus were also studied in [6, 29].

Heterogeneity is an important feature of multiagent systems; i.e., the dynamics of agents might be different or changeable. On the one hand, different agents might have different dynamics. For instance, autonomous robots were used to control self-organized behavioral patterns in group-living cockroaches [30]. Therefore, many researches began to focus on the study of heterogeneous multiagent systems with multiple dynamics. Zheng et al. studied consensus problem for heterogeneous multiagent systems consisting of single- and double-integrators under different circumstances [31, 32]. Liu et al. investigated consensus problem for multiagent systems with heterogeneous nonlinear dynamics in [33]. Qin et al. [34] considered group consensus for heterogeneous multiagent systems. On the other hand, the dynamics of agents might change dynamically over time. For example, the dynamics of a motor vehicle might switch between automatic shift gear and manual shift gear. Considering this fact, Zhai studied the stability of switched systems which composed of the continuous-time (CT) and discrete-time (DT) dynamics subsystem and offered some algebraic conditions to solve the stability problem under arbitrary switching in [35]. In a CT multiagent system, if we sometimes use a computer to activate all the agents in a DT manner, then the multiagent system switches both CT and DT dynamics. As a result, multiagent systems with switching dynamics were also studied. In [36], Zheng and Wang supposed that the dynamics of agents switch between CT and DT subsystems and derive some sufficient conditions of solving consensus problems. Finite-time consensus problem of switched multiagent system wass also considered in [37].

Motivated by the above results, we consider consensus problems for the heterogeneous multiagent systems with switching dynamics. We assume that the system is consisting of single- and double-integrator agents and each agent can switch its dynamics between CT subsystem and DT subsystem. The main contribution of this paper is twofold. Firstly, we prove the system can reach consensus if the fixed interaction topology has a spanning tree. Secondly, for the system with switching topologies, we obtain that the system can solve consensus if each interaction topology has a spanning tree.

The structure of this paper is organized as follows. In Section 2, we introduce some basic notions about graph theory and present our system model. In Section 3, we give the main results of this paper. Section 4 offers some numerical simulations to illustrate the effectiveness of our theoretical results. Finally, conclusions are given in Section 5.

In this paper, $\mathbb{R}$, $\mathbb{Z}$, and $\mathbb{C}$ denote the sets of real number, integer number, and complex number. $\mathbb{R}^n$ denotes the n dimensional real vector space and $\mathbb{R}^{n\times n}$ denotes the set of $n\times n$ matrix. $\mathbf{1}_n$ ($\mathbf{0}_n$) denotes the column vector with all entries equal to one (zero), I_n denotes an n dimensional identity matrix, and $\mathbf{0}$ denotes any dimensional zeros matrix with compatible dimension. $\mathrm{diag}\{a_1, a_2, \ldots, a_n\}$ denotes a diagonal matrix with diagonal elements being $a_1, a_2, \ldots, a_n$. $\mathcal{I}_m = \{1, 2, \ldots, m\}$, $\mathcal{I}_n/\mathcal{I}_m = \{m+1, m+2, \ldots, n\}$. For a matrix A, A^T denotes its transpose. Given a complex number $\lambda \in \mathbb{C}$, $\mathrm{Re}(\lambda)$ and $|\lambda|$ denote the real part and the modulus of λ, respectively. Matrix $A = [a_{ij}]_{n\times n}$ is said to be nonnegative (nonpositive) if all entries a_{ij} are nonnegative (nonpositive), denoted by $A \geq 0$ ($A \leq 0$), respectively. Furthermore, if all its row sums are 1, A is said to be a stochastic matrix; a stochastic matrix A is called indecomposable and aperiodic (SIA) if $\lim_{n\longrightarrow\infty} A^n = \mathbf{1}_n y^T$, where y is a column vector.

2. Preliminaries

In this section, we present some basic concepts and results of algebra graph theory [38] and give the system model.

2.1. Graph Theory. Let $G = \{V, E, A\}$ be a weighted directed graph with a vertex set $V = \{1, 2, \ldots, n\}$, an edge set $E = \{(i, j) \in V \times V\}$. The adjacency matrix $A = [a_{ij}]$ is defined as $a_{ij} > 0$ if and only if $(j, i) \in E$, and $a_{ij} = 0$ otherwise. $(i, j) \in E$ is an edge of the graph G, where i is called the parent vertex and j is called the child vertex. A directed tree is the directed graph, where every vertex has exactly one parent except one special vertex called the root vertex. A directed spanning tree is a directed tree, which consists of all vertices and some edges of G. The degree matrix $D = \mathrm{diag}\{d_1, d_2, \ldots, d_n\}$ is the diagonal matrix with $d_i = \sum_{j=1}^{n} a_{ij}$. The Laplacian matrix of the graph G is defined as $L = [l_{ii}]_{n\times n} = D - A$. Denote by $|\lambda_1| \leq |\lambda_2| \leq \cdots \leq |\lambda_n|$ the eigenvalues of L. Apparently, $\lambda_1 = 0$ and $L\mathbf{1}_n = \mathbf{0}_n$.

Lemma 1 (see [3]). *The directed graph G associated with the Laplacian matrix L has a directed spanning tree if and only if L has only one zero eigenvalue with algebraic multiplicity* 1.

Lemma 2 (see [36]). *Let* $h \in (0, \min_{i=2,\ldots,n}(2\,\mathrm{Re}(\lambda_i)/|\lambda_i|^2))$. *Suppose that the directed graph G associated with the Laplacian matrix L has a directed spanning tree. Then,*

$$\lim_{t\longrightarrow\infty} e^{-Lt} = \mathbf{1}_n\omega^T,$$
$$\lim_{t\longrightarrow\infty} \left(I_n - hL\right)^t = \mathbf{1}_n\omega^T, \quad (1)$$

where ω^T *is the left eigenvector of L associated with eigenvalue* 0. *Moreover,* ω *is a nonnegative vector and* $\omega^T\mathbf{1}_n = 1$.

Lemma 3. *Let* $B = \mathrm{diag}\{b_1, \ldots, b_m\}$ *and* $\Theta = \mathrm{diag}\{\theta_1, \ldots, \theta_m\}$, *where* $b_i > 0$ *and* $0 < \theta_i < b_i/\max_{i\in\mathcal{I}_m} d_{ii}$. *Let* $L = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix}$, *where* $L_{11} \in \mathbb{R}^{m\times m}$, $L_{12} \in \mathbb{R}^{m\times(n-m)}$, $L_{21} \in \mathbb{R}^{(n-m)\times m}$, *and* $L_{22} \in \mathbb{R}^{(n-m)\times(n-m)}$. *Suppose that L is the Laplacian matrix of the directed graph G. Then,*

(1)

$$L' = \begin{bmatrix} \Theta^{-1} & -\Theta^{-1} & \mathbf{0} \\ \Theta L_{11} - B & B & \Theta L_{12} \\ L_{21} & \mathbf{0} & L_{22} \end{bmatrix} \quad (2)$$

is a Laplacian matrix of a directed graph G' *with* $n+m$ *vertices.*

(2) G' *has a directed spanning tree if and only if G has a directed spanning tree.*

Proof. (1) Since L is the Laplacian matrix of the directed graph G, we know that $L\mathbf{1}_n = \mathbf{0}_n$, $L_{12} \leq 0$, $L_{21} \leq 0$, and $L_{22} > 0$. Then, we have $L'\mathbf{1}_{n+m} = \mathbf{0}_{n+m}$. Since $b_i > 0$ and $0 < \theta_i < b_i/\max_{i\in\mathcal{I}_m} d_{ii}$, we have $B > 0$, $\Theta^{-1} > 0$, $\Theta L_{12} \leq 0$, and $\Theta L_{11} - B < 0$. Then, L' is a Laplacian matrix of a directed graph G' with $n+m$ vertices.

(2) (Sufficiency) If the graph G has a directed spanning tree, then matrix L has exactly one zero eigenvalue; i.e., rank(L)=$n-1$. Let

$$P = \begin{bmatrix} I_m & \mathbf{0} & \mathbf{0} \\ -B\Theta & I_m & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & I_{n+m} \end{bmatrix} \tag{3}$$

and

$$Q = \begin{bmatrix} I_m & -I_m & \mathbf{0} \\ I_m & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & I_{n+m} \end{bmatrix}, \tag{4}$$

and we have $L' = PMQ$, where

$$M = \begin{bmatrix} \Theta^{-1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \Theta L_{11} & \Theta L_{12} \\ \mathbf{0} & L_{21} & L_{22} \end{bmatrix}. \tag{5}$$

Since P and Q are the nonsingular matrix, we have $rank(L') = rank(M) = m + rank(L) = n + m - 1$, which implies that the matrix L' has exactly one zero eigenvalue. Therefore, the directed graph G' associated with L' has a directed spanning tree.

(Necessity) If the graph G does not have a spanning tree, then the Laplacian matrix L of the graph G has at least two zero eigenvalues. Thus, we know that $rank(L') < n + m - 1$. It follows from Lemma 1 that the graph G' does not have a spanning tree. □

Lemma 4 (see [3]). *Let $S_1, S_2, \ldots, S_k$ be a finite set of SIA matrices with the property that for each sequences for each $S_{i_j}, S_{i_{j-1}}, \ldots, S_{i_1}$ is SIA. Then, for each infinite sequence $S_{i_1}, S_{i_2}, \cdots$, there exists a column vector ω such that*

$$\lim_{j \longrightarrow \infty} S_{i_j}, S_{i_{j-1}}, \ldots, S_{i_1} = \mathbf{1}\omega^T. \tag{6}$$

2.2. System Model. In this subsection, we propose the heterogeneous multiagent system composed of first-integrator agents and double-integrator agents with CT and DT dynamics.

Suppose that the heterogeneous multiagent system is composed of $n-m$ first-integrator agents and m double-integrator agents. The double-integrator agent are indicated by $1, \ldots, m$ and the first-integrator agents agent are indicated by $m+1, \ldots, n$. Denote by $G = \{V, E\}$ the interaction graph of all agents, where $V = \{1, \ldots, n\}$ and $(i, j) \in E$ if agent j sends information to agent i. Therefore, the Laplacian matrix of the graph G can be written as $L = \begin{bmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{bmatrix}$, where $L_{11} \in \mathbb{R}^{m \times m}$, $L_{12} \in \mathbb{R}^{m \times (n-m)}$, $L_{21} \in \mathbb{R}^{(n-m) \times m}$, and $L_{22} \in \mathbb{R}^{(n-m) \times (n-m)}$.

The dynamics of agent $i \in \mathcal{I}_n$ switch between a CT model and a DT model. Specifically, the dynamics of the double-integrator agent $i \in \mathcal{I}_m$ can be modeled as

$$\begin{aligned} &\dot{x}_i(t) = v_i(t) \\ &\dot{v}_i(t) = u_i(t), \quad \text{for CT subsystem}, \\ &x_i(t+1) = x_i(t) + hv_i(t) \\ &v_i(t+1) = v_i(t) + u_i(t), \quad \text{for DT subsystem}, \end{aligned} \tag{7}$$

and the dynamics of the first-integrator agent $i \in \mathcal{I}_n / \mathcal{I}_m$ can be described as

$$\begin{aligned} &\dot{x}_i(t) = u_i(t), \quad \text{for CT subsystem}, \\ &x_i(t+1) = x_i(t) + u_i(t), \quad \text{for DT subsystem}, \end{aligned} \tag{8}$$

where $x_i \in \mathbb{R}$, $v_i \in \mathbb{R}$, and $u_i \in \mathbb{R}$ are the position, the velocity, and the control input of the agent i, respectively. The initial states are $x_i(0) = x_{i0}$, $i \in \mathcal{I}_n$, and $v_i(0) = v_{i0}$, $i \in \mathcal{I}_m$. Let $x(0) = [x_{10}, x_{20}, \ldots, x_{n0}]^T$ and $v(0) = [v_{10}, v_{20}, \ldots, v_{m0}]^T$.

Definition 5. The heterogeneous multiagent system (7)-(8) is said to reach consensus if

$$\begin{aligned} &\lim_{t \longrightarrow \infty} \left\| x_i(t) - x_j(t) \right\| = 0, \quad \text{for } i, j \in \mathcal{I}_n, \\ &\lim_{t \longrightarrow \infty} \left\| v_i(t) - v_j(t) \right\| = 0, \quad \text{for } i, j \in \mathcal{I}_m \end{aligned} \tag{9}$$

holds for any initial states $x(0) \in \mathbb{R}^n$, $v(0) \in \mathbb{R}^m$.

3. Main Results

3.1. Consensus with Fixed Topology. In this subsection, we discuss the consensus of the heterogeneous multiagent with switched dynamics (7)-(8) under directed fixed topology. At the time t, we assume the switching rule is arbitrary.

First, we present the linear consensus protocol for system (7)-(8) with the fixed topology. For the first-integrator agent $i \in \mathcal{I}_n / \mathcal{I}_m$, we let

$$u_i(t) = \begin{cases} \displaystyle\sum_{j=1}^{n} a_{ij}\left(x_j - x_i\right), & \text{for CT subsystem}, \\ \displaystyle h\sum_{j=1}^{n} a_{ij}\left(x_j - x_i\right), & \text{for DT subsystem}, \end{cases} \tag{10}$$

and for the double-integrator agent $i \in \mathcal{I}_m$, we let

$$u_i(t) = \begin{cases} \displaystyle\sum_{j=1}^{n} a_{ij}\left(x_j - x_i\right) - kv_i, & \text{for CT subsystem}, \\ \displaystyle h\sum_{j=1}^{n} a_{ij}\left(x_j - x_i\right) - hkv_i, & \text{for DT subsystem}, \end{cases} \tag{11}$$

where a_{ij} is the (i, j) entry of the adjacency matrix A of G, $h > 0$ is the sampling period, and $k > 0$ is the feedback gain.

Let $y_i = \theta_i v_i + x_i$ $(i \in \mathcal{I}_m)$, where

$$\theta_i = \begin{cases} \dfrac{k}{4d_{ii}}, & \text{for } d_{ii} \neq 0 \\ \dfrac{2}{k}, & \text{for } d_{ii} = 0. \end{cases} \tag{12}$$

Then, we have

$$\dot{y}_i = \theta_i \sum_{j=1}^{n} a_{ij} \left(x_j - y_i \right) + b_i \left(x_i - y_i \right), \tag{13}$$

where $b_i = k - 1/\theta_i - \theta_i d_{ii}$ for $i \in \mathcal{I}_m$.

Next, by introducing y_i $(i \in \mathcal{I}_m)$, we separate the double-integrator agent i into two first-integrator agents: agent i with the state x_i and agent i' with the state y_i. Thus, we get a first-integrator multiagent system which consists of the agent $1, \ldots, m, 1', \ldots, m', m+1, \ldots, n$ with the CT subsystem:

$$\begin{aligned}
\dot{x}_i &= \frac{1}{\theta_i} \left(y_i - x_i \right), \quad \textit{for agent } i \in \{1, \ldots, m\}, \\
\dot{y}_i &= \theta_i \sum_{j=1}^{n} a_{ij} \left(x_j - y_i \right) + b_i \left(x_i - y_i \right), \\
&\qquad\qquad \textit{for agent } i \in \left\{1', \ldots, m'\right\}, \\
\dot{x}_i &= \sum_{j=1}^{n} a_{ij} \left(x_j - x_i \right), \quad \textit{for agent } i \in \{m+1, \ldots, n\},
\end{aligned} \tag{14}$$

and the DT subsystem:

$$\begin{aligned}
x_i(t+1) &= \left(1 - \frac{h}{\theta_i} \right) x_i(t) + \frac{h}{\theta_i} y_i(t), \\
&\qquad\qquad \textit{for agent } i \in \{1, \ldots, m\}, \\
y_i(t+1) &= h\theta_i \sum_{j=1}^{n} a_{ij} \left(x_j - y_i \right) + hb_i \left(x_i - y_i \right) + y_i(t), \\
&\qquad\qquad \textit{for agent } i \in \left\{1', \ldots, m'\right\}, \\
x_i(t+1) &= x_i(t) + h \sum_{j=1}^{n} a_{ij} \left(x_j - x_i \right), \\
&\qquad\qquad \textit{for agent } i \in \{m+1, \ldots, n\}.
\end{aligned} \tag{15}$$

Let $Z = [x_1, \ldots, x_m, y_1, \ldots, y_m, x_{m+1}, \ldots, x_n]^T \in \mathbb{R}^{n+m}$ and $Z(0) = [x_1(0), \ldots, x_m(0), y_1(0), \ldots, y_m(0), x_{m+1}(0), \ldots, x_n(0)]^T$. Then, the first-integrator multiagent system with the CT subsystem (14) and the DT subsystem (15) can be rewritten as

$$\begin{aligned}
\dot{Z}(t) &= -L' Z(t) \quad \text{for CT subsystem}, \\
Z(t+1) &= \left(I_{n+m} - hL' \right) Z(t) \quad \text{for DT subsystem},
\end{aligned} \tag{16}$$

where

$$L' = \begin{bmatrix} \Theta^{-1} & -\Theta^{-1} & \mathbf{0} \\ \Theta L_{11} - B & B & \Theta L_{12} \\ L_{21} & \mathbf{0} & L_{22} \end{bmatrix},$$

$$I_{n+m} - hL' = \begin{bmatrix} I_m - h\Theta^{-1} & h\Theta^{-1} & \mathbf{0} \\ -\Theta h L_{11} + hB & I_m - hB & -\Theta L_{12} \\ -hL_{21} & \mathbf{0} & I_{n-m} - hL_{22} \end{bmatrix}, \tag{17}$$

$B = \text{diag}\{b_1, \ldots, b_m\}$ and $\Theta = \text{diag}\{\theta_1, \ldots, \theta_m\}$. Let $|\lambda_1| < |\lambda_2| \leq \cdots \leq |\lambda_{n+m}|$ be the eigenvalues of the matrix L'. Therefore, we have the following.

Theorem 6. *Let the sampling period* $h \in (0, \min_{i=2,\ldots,n+m}(2\,\text{Re}(\lambda_i)/|\lambda_i|^2))$ *and the feedback gain* $k > 1 + 2\max_{i\in\mathcal{I}_m} d_{ii}$. *Protocols (10)-(11) can solve consensus problem of system (7)-(8) under fixed topology if and only if the interaction graph G has a directed spanning tree.*

Proof.

(Sufficiency). Since $k > 1 + 2\max_{i\in\mathcal{I}_m} d_{ii}$ and

$$\theta_i = \begin{cases} \dfrac{k}{4d_{ii}}, & \text{for } d_{ii} \neq 0 \\ \dfrac{2}{k}, & \text{for } d_{ii} = 0, \end{cases} \tag{18}$$

we have $b_i > 0$ and

$$\theta_i d_{ii} - b_i = \begin{cases} \dfrac{8d_{ii} - k^2}{2k} < 0, & \text{for } d_{ii} \neq 0 \\ -\dfrac{2}{k} < 0, & \text{for } d_{ii} = 0. \end{cases} \tag{19}$$

It follows that $B > 0$ and $\Theta L_{11} - B < 0$. By Lemma 3, it is easy to find that L' is the Laplacian matrix of the directed graph G'. Recalling (16), we know that G' is the interaction graph of the first-integrator multiagent system (14)-(15). G has a spanning tree, we have that G' has a spanning tree by Lemma 3. Moreover, it follows from Lemma 2 that

$$\lim_{t\to\infty} e^{-L't} = \mathbf{1}_{n+m}\omega^T \tag{20}$$

and

$$\lim_{t\to\infty} \left(I_{n+m} - hL' \right)^t = \mathbf{1}_{n+m}\omega^T. \tag{21}$$

For any $t > 0$, we have $t = t_c + t_d$, where $t_c \in \mathbb{R}$ is the total duration time on CT subsystem and $t_d \in \mathbb{Z}$ is the total duration time on DT subsystem. Due to

$$e^{-L't} \left(I_{n+m} - hL' \right) = \left(I_{n+m} - hL' \right) e^{-L't}, \tag{22}$$

we have

$$Z(t) = e^{-L't_c} \left(I_{n+m} - hL' \right)^{t_d} Z(0). \tag{23}$$

When $t \longrightarrow \infty$, there are three cases as follows.

Case 1. $t_c \longrightarrow \infty$ and $t_d \in \mathbb{Z}$ is a constant. Thus, we have

$$\begin{aligned}
\lim_{t\to\infty} Z(t) &= \lim_{t_c\to\infty} e^{-L't_c} \left(I_{n+m} - hL' \right)^{t_d} Z(0) \\
&= \mathbf{1}_{n+m}\omega^T \left(I_{n+m} - hL' \right)^{t_d} Z(0)
\end{aligned}$$

$$= \mathbf{1}_{n+m}\omega^T Z(0). \tag{24}$$

Case 2. $t_c \in \mathbb{R}$ is a constant and $t_d \longrightarrow \infty$. It follows that

$$\begin{aligned}\lim_{t\longrightarrow\infty} Z(t) &= \lim_{t_d\longrightarrow\infty} e^{-L't_c}\left(I_{n+m} - hL'\right)^{t_d} Z(0) \\ &= e^{-L't_c}\mathbf{1}_{n+m}\omega^T Z(0) \\ &= \left(I_{n+m} + \left(-L't_c\right) + \frac{1}{2!}\left(-L't_c\right)^2 + \cdots\right) \\ &\quad \cdot \mathbf{1}_{n+m}\omega^T Z(0) = \mathbf{1}_{n+m}\omega^T Z(0).\end{aligned} \tag{25}$$

Case 3. $t_c \longrightarrow \infty$ and $t_d \longrightarrow \infty$. Thus

$$\lim_{t\longrightarrow\infty} Z(t) = \mathbf{1}_{n+m}\omega^T \mathbf{1}_{n+m}\omega^T Z(0) = \mathbf{1}_{n+m}\omega^T Z(0). \tag{26}$$

From (24)-(26), we have

$$\begin{aligned}&\lim_{t\longrightarrow\infty}\left\|x_i(t) - x_j(t)\right\| = 0, \quad \text{for } i, j \in \mathcal{I}_n \\ &\lim_{t\longrightarrow\infty}\left\|y_i(t) - x_i(t)\right\| = 0, \quad \text{for } i \in \mathcal{I}_m.\end{aligned} \tag{27}$$

Therefore, we get $\lim_{t\longrightarrow\infty} v_i(t) = 0$, $i \in \mathcal{I}_m$, which means that

$$\lim_{t\longrightarrow\infty}\left\|v_i(t) - v_j(t)\right\| = 0, \quad \text{for } i, j \in \mathcal{I}_m. \tag{28}$$

From Definition 5, we can conclude that protocols (10)-(11) can solve consensus problem for the heterogeneous multiagent system (7)-(8).

(Necessity). When the heterogeneous multiagent system (7)-(8) can achieve the consensus under the linear protocol (10)-(11), we know that the first-integrator multiagent system (16) can solve the consensus problem and consensus states are $\mathbf{1}_{n+m}\omega^T Z(0)$. But if the interaction graph G associated with the matrix L does not have a directed spanning tree, neither does graph G' associated with the matrix L' by Lemma 3, which implies that $\lim_{t\longrightarrow\infty} Z(t) \neq \mathbf{1}_{n+m}\omega^T Z(0)$ by Lemma 2, the first-integrator multiagent system (16) can not achieve consensus from [36, 39]. Thus, protocols (10)-(11) for the heterogeneous multiagent system (7)-(8) can not solve consensus problem. □

3.2. Consensus with Switching Topology. In this subsection, we discuss the consensus problem of system (7)-(8) under directed switching topology. The interaction topology is dynamically changing; we assume that the switching rule is followed CT and DT subsystems.

We let $G = \{G_1, \ldots, G_N\}$ be the set of all possible directed interaction graphs of agents $1, 2, \ldots, n+m$. Let $L^{(l)} = \begin{bmatrix} L_{11}^{(l)} & L_{12}^{(l)} \\ L_{21}^{(l)} & L_{22}^{(l)} \end{bmatrix}$ be the Laplacian matrix of G_l, $(l \in \{1, \ldots, N\})$, where $L_{11}^{(l)} \in \mathbb{R}^{m\times m}$, $L_{12}^{(l)} \in \mathbb{R}^{m\times(n-m)}$, $L_{21}^{(l)} \in \mathbb{R}^{(n-m)\times m}$, and $L_{22}^{(l)} \in \mathbb{R}^{(n-m)\times(n-m)}$. At the time interval $(t_k, t_{k+1}]$, the interaction graph of this system is $G(t) \in G$.

Assumption 7. Suppose that $G_i (i \in \{1, \ldots, N\})$ has a spanning tree.

Assumption 8. There exists a $\tau_0 > 0$, for any $k \in \mathcal{N}^+$, we have $|t_{k+1} - t_k| \geq \tau_0 > 0$.

Assumption 9. At the time $t_k (k \in \mathcal{Z}^+)$, the switching rule of the interaction topology or the dynamic is changing.

we present the linear consensus protocol for system (7)-(8) with the switching topology. For the first-integrator agent $i \in \mathcal{I}_n/\mathcal{I}_m$, we let

$$u_i(t) = \begin{cases} \displaystyle\sum_{j=1}^{n} a_{ij}(t)\left(x_j - x_i\right), & \text{for CT subsystem}, \\ \displaystyle h\sum_{j=1}^{n} a_{ij}(t)\left(x_j - x_i\right), & \text{for DT subsystem}, \end{cases} \tag{29}$$

where $a_{ij}(t)$ is the (i, j) entry of $G(t)$, $h > 0$ is the sampling period, and $k > 0$ is the feedback gain. Likewise, for the double-integrator agent $i \in \mathcal{I}_m$, we let

$$u_i(t) = \begin{cases} \displaystyle\sum_{j=1}^{n} a_{ij}(t)\left(x_j - x_i\right) - kv_i, & \text{for CT subsystem}, \\ \displaystyle h\sum_{j=1}^{n} a_{ij}(t)\left(x_j - x_i\right) - hkv_i, & \text{for DT subsystem}, \end{cases} \tag{30}$$

Let $y_i = \theta_i(t)v_i + x_i (i \in \mathcal{I}_m)$, where

$$\theta_i(t) = \begin{cases} \dfrac{k}{4d_{ii}(t)}, & \text{for } d_{ii}(t) \neq 0 \\ \dfrac{2}{k}, & \text{for } d_{ii}(t) = 0, \end{cases} \tag{31}$$

and $d_{ii}(t)$ is the degree of vertex i in $G(t)$.

Denote $Z(t) = [x_1(t), \ldots, x_m(t), y_1(t), \ldots, y_m(t), x_{m+1}(t), \ldots, x_n(t)]^T \in \mathbb{R}^{n+m}$, and $Z(0) = [x_1(0), \ldots, x_m(0), y_1(0), \ldots, y_m(0), x_{m+1}(0), \ldots, x_n(0)]^T$. Thus, we obtain

$$\begin{aligned}\dot{Z}(t) &= -\widetilde{L}(t) Z(t), \\ &\qquad t \in \left[t_k, t_{k+1}\right), \text{ for CT subsystem}, \\ Z(t+1) &= \left(I_{n+m} - h\widetilde{L}(t)\right) Z(t), \\ &\qquad t \in \left[t_k, t_{k+1}\right), \text{ for DT subsystem},\end{aligned} \tag{32}$$

where

$$\begin{aligned}\widetilde{L}(t) &= \begin{bmatrix} \Theta(t)^{-1} & -\Theta(t)^{-1} & \mathbf{0} \\ \Theta(t) L_{11}(t) - B(t) & B(t) & \Theta(t) L_{12}(t) \\ L_{21}(t) & \mathbf{0} & L_{22}(t) \end{bmatrix}, \\ I_{n+m} - h\widetilde{L}(t) &= \begin{bmatrix} I_m - h\Theta(t)^{-1} & h\Theta(t)^{-1} & \mathbf{0} \\ -\Theta(t) hL_{11}(t) + hB(t) & I_m - hB(t) & -\Theta(t) L_{12}(t) \\ -hL_{21}(t) & \mathbf{0} & I_{n-m} - hL_{22}(t) \end{bmatrix}\end{aligned} \tag{33}$$

and $\Theta(t) = \operatorname{diag}\{\theta_1(t), \ldots, \theta_m(t)\}$, $B(t) = \operatorname{diag}\{b_1(t), \ldots, b_m(t)\}$. Denote $G' = \{\widetilde{G_1}, \ldots, \widetilde{G_N}\}$, where $\widetilde{G_l}$ is the graph associated with the Laplacian matrix:

$$\widetilde{L}^{(l)} = \begin{bmatrix} \Theta_l^{-1} & -\Theta_l^{-1} & \mathbf{0} \\ \Theta L_{11}^{(l)} - B_l & B_l & \Theta L_{12}^{(l)} \\ L_{21}^{(l)} & \mathbf{0} & L_{22}^{(l)} \end{bmatrix}. \tag{34}$$

Denote $|\lambda_1^{(l)}| \leq |\lambda_2^{(l)}| \leq \cdots \leq |\lambda_{n+m}^{(l)}|$ as the eigenvalues of the matrix $\widetilde{L}^{(l)}$. Let $d = \max_{l=1,\ldots,N}\{\max_{i=1,\ldots,m}\{d_i^{(l)}\}\}$ and $\rho = \min_{l=1,\ldots,N}\{\min_{s=2,\ldots,n+m}\{2\operatorname{Re}(\lambda_s^{(l)})/|\lambda_s^{(l)}|^2\}\}$, where $d_i^{(l)}$ is the degree of vertex i in G_l ($l \in \{1, \ldots, N\}$). We have the following.

Theorem 10. *Suppose Assumption 7-Assumption 9 hold. Assume that the sampling period $h \in (0, \rho)$ and $k > 1 + 2d$. Then, protocols (7)-(8) can solve consensus problem for system (7)-(8) under switching topology.*

Proof. For any $t \in (t_k, t_{k+1}]$, let $t = \Delta t + \tau_k + \tau_{k-1} + \cdots + \tau_1$. We have $Z(t) = F(\Delta t)\prod_{s=1}^{k} F_s Z(0)$, where

$$F(\Delta t) = \begin{cases} e^{-\widetilde{L(t)}\Delta t}, & \text{for CT subsystem} \\ \left(I_{n+m} - h\widetilde{L(t)}\right)^{\Delta t}, & \text{for DT subsystem} \end{cases} \tag{35}$$

and

$$F_s = \begin{cases} e^{-\widetilde{L}_s \tau_s}, & \text{for CT subsystem} \\ \left(I_{n+m} - h\widetilde{L}_s\right)^{\tau_s}, & \text{for DT subsystem.} \end{cases} \tag{36}$$

From Assumption 7 and Lemma 3, we know that $\widetilde{G_l}$ has a spanning tree. Therefore, by Lemma 2, $F(\Delta t)$ and F_s are SIA.

When $t \longrightarrow \infty$, there are two cases as follows.

Case 1. $\Delta t \longrightarrow \infty$ and $\tau_k + \tau_{k-1} + \cdots + \tau_1 \in \mathbb{R}$ is a constant. Thus, we have

$$\begin{aligned} \lim_{\Delta t \longrightarrow \infty} Z(t) &= \lim_{\Delta t \longrightarrow \infty} F(\Delta t) \prod_{s=1}^{k} F_s Z(0) \\ &= \mathbf{1}_{n+m}\eta^T \prod_{s=1}^{k} F_s Z(0) = \mathbf{1}_{n+m}\eta^T Z(0). \end{aligned} \tag{37}$$

Case 2. $\Delta t \in \mathbb{R}$ is a constant and $\tau_k + \tau_{k-1} + \cdots + \tau_1 \longrightarrow \infty$. It follows that

$$\begin{aligned} \lim_{k \longrightarrow \infty} Z(t) &= \lim_{k \longrightarrow \infty} F(\Delta t) \prod_{s=1}^{k} F_s Z(0) \\ &= F(\Delta t)\,\mathbf{1}_{n+m}\eta^T Z(0) = \mathbf{1}_{n+m}\eta^T Z(0). \end{aligned} \tag{38}$$

Thus, we have

$$\lim_{t \longrightarrow \infty} Z(t) = \lim_{t \longrightarrow \infty} F(\Delta t) \prod_{s=1}^{k} F_s Z(0) = \mathbf{1}_{n+m}\eta^T Z(0), \tag{39}$$

where η is a nonnegative vector and $\eta^T \mathbf{1}_{n+m} = 1$.

From (37)-(39), we have

$$\begin{aligned} &\lim_{t \longrightarrow \infty} \left\| x_i(t) - x_j(t) \right\| = 0, \quad \text{for } i, j \in \mathcal{I}_n \\ &\lim_{t \longrightarrow \infty} \left\| y_i(t) - x_i(t) \right\| = 0, \quad \text{for } i \in \mathcal{I}_m. \end{aligned} \tag{40}$$

Therefore, we get $\lim_{t \longrightarrow \infty} v_i(t) = 0$, $i \in \mathcal{I}_m$, which means that

$$\lim_{t \longrightarrow \infty} \left\| v_i(t) - v_j(t) \right\| = 0, \quad \text{for } i, j \in \mathcal{I}_m. \tag{41}$$

From Definition 5, we can conclude that protocols (29)-(30) can solve consensus problem for system (7)-(8). □

4. Simulations

In this section, some simulations are presented to verify the efficiency of the theoretical results. Let $h = 0.8$, $k = 2$, $\theta = 1$, and the other initial conditions of all the agents are generated randomly.

Example 11. The interaction graph G is shown in Figure 1; it is obvious that the interaction graph in Figure 1 has a directed spanning tree. The heterogeneous multiagent system consists of three single-integrator agents 1-3 and three double-integrator agents 4-6. The switching law of the heterogeneous multiagent system (7)-(8) and simulation results are shown in Figure 2, respectively. We can find that the linear consensus protocols (10)-(11) can solve consensus problem for the heterogeneous multiagent system (7)-(8) which is consistent with the result of Theorem 6.

Example 12. Figure 3 shows the communication topologies G_1, G_2, G_3. It is obvious that G_1, G_2, G_3 have the directed spanning tree. The heterogeneous multiagent system consists of six vertices with a directed switching topology. Vertices $1-3$ denote the single-integrator agents and vertices $4-6$ denote the double-integrator agents. The switching law of topology, system, the trajectories of agents $1-6$, and the velocity of agents $4-6$ are shown in (a), (b), (c), and (d) in Figure 4, respectively. It is shown that linear consensus protocols (29)-(30) can solve consensus problem for system (7)-(8) under the directed switching topology in Figure 4. This result is consistent with the theoretical result in Theorem 10.

5. Conclusions

This paper studied the heterogeneous multiagent system composed of single-integrators and double-integrators whose dynamics can switch between DT and CT dynamics arbitrarily. Linear consensus protocols are given to solve the

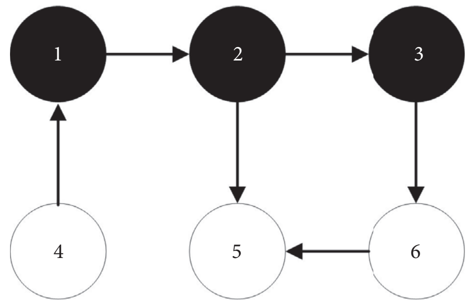

Figure 1: A directed fixed topology with 6 vertices.

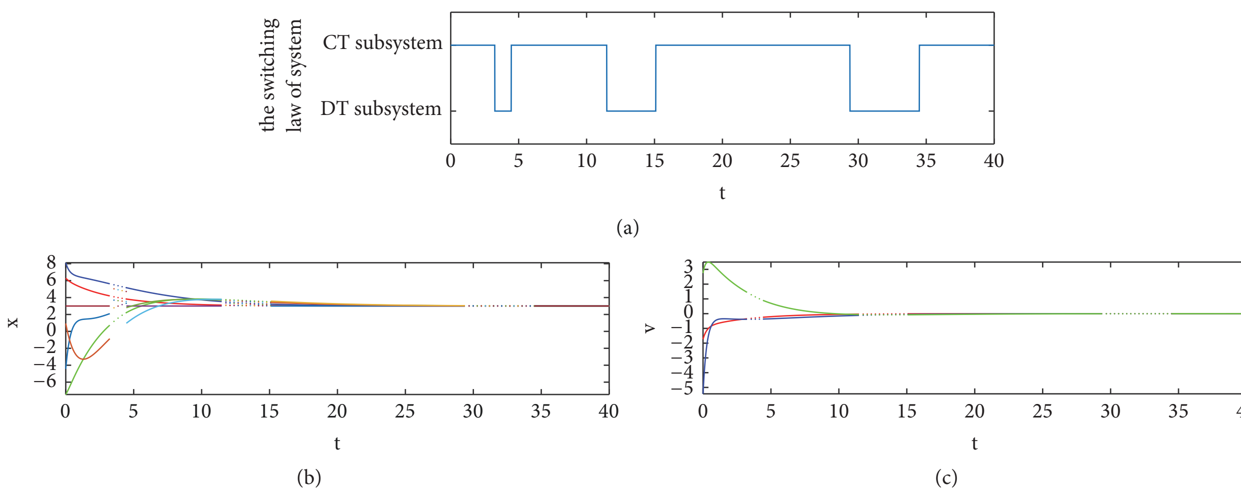

Figure 2: (a) Switching law of system; (b) the trajectories of agents 1-6; (c) the velocity of agents 4-6.

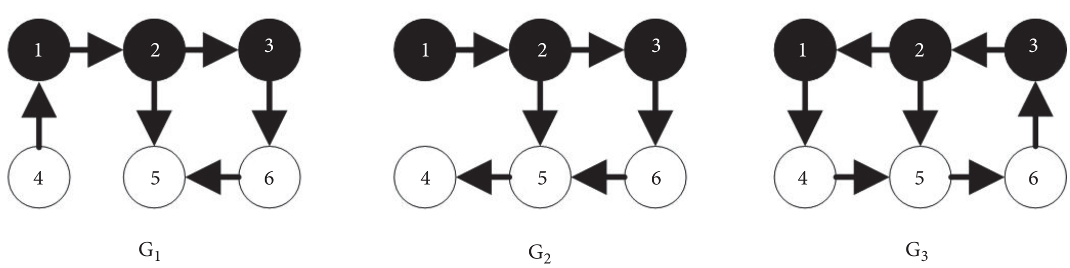

Figure 3: Three directed topologies with 6 vertices.

consensus problem of the system. The necessary and sufficient condition is established for solving the consensus of the heterogeneous multiagent system with switching dynamics. Moreover, under switching topologies, we also proved that the system can solve consensus problems when all interaction topologies have spanning tree. Finally, some examples are offered to illustrate the effectiveness of theoretical results.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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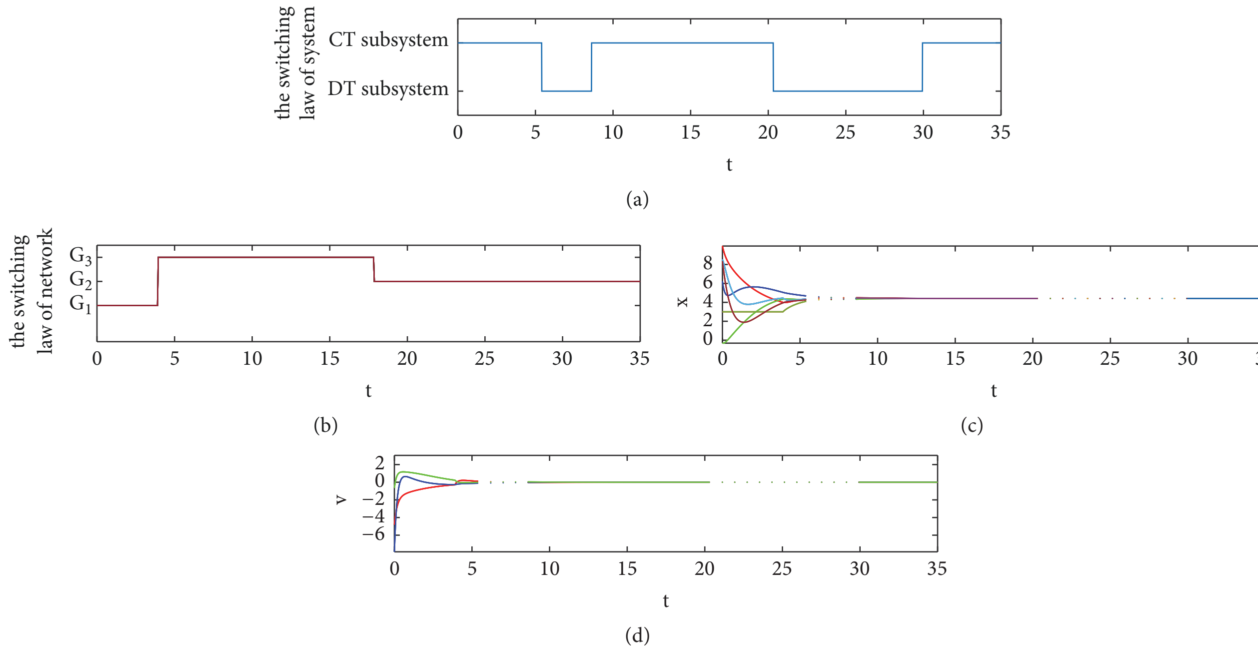

FIGURE 4: (a) Switching law of interaction topology; (b) switching law of system; (c) the trajectories of agents 1-6; (d) the velocity of agents 4-6.

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