Hindawi
Mathematical Problems in Engineering
Volume 2018, Article ID 5615898, 15 pages
https://doi.org/10.1155/2018/5615898

Research Article

Resource Allocation with Successive Coding for OFDM-Based Cognitive System Subject to Statistical CSI

Marwa Chami, Mylene Pischella, and Didier Le Ruyet

CEDRIC/LAETITIA Laboratory, Conservatoire National des Arts et Métiers (CNAM), 292 rue Saint-Martin, 75003 Paris, France

Correspondence should be addressed to Mylene Pischella; mylene.pischella@cnam.fr

Received 17 May 2018; Accepted 30 August 2018; Published 18 September 2018

Academic Editor: Piotr Jędrzejowicz

This paper investigates the resource allocation problem for a multicarrier underlay cognitive radio system, under the assumption that only statistical Channel State Information (CSI) about the primary channels is available at the secondary user. More specifically, we maximize the system utility under primary and secondary user outage constraints and the total power constraint. The secondary user transmission is also constrained by the interference threshold imposed by the primary user. Moreover, the secondary receiver adapts its decoding strategy, which is either treating interference as noise or using successive interference cancellation or superposition coding. This leads to a nonconvex optimization problem, with either perfect or statistical CSI. Consequently, we propose a sequential-based algorithm to efficiently obtain a solution to the problem. The simulation results show that the sequential algorithm is convergent and that our global proposed scheme achieves larger secondary and sum rates than other algorithms where the decoding strategy is not adapted.

1. Introduction

The time-varying channel is one of the most challenging designs in wireless communication systems. Dynamic resource allocation algorithms have been developed for this channel type, incorporating different elements of adaptation, in order to optimally utilize the available resources. These dynamic allocation procedures present a significant improvement over constant resource allocation strategies. One of the common studied scenarios for wireless communications is the cognitive radio (CR) channel, where the unlicensed user (known as the secondary user (SU)) coexists with the licensed user (the primary user (PU)) in the same band under an interference threshold constraint.

Due to their flexibility in allocating resources among SUs, multicarrier transmissions such as Orthogonal Frequency-Division Multiplexing (OFDM) are largely used for CR networks [1]. Adaptive resource allocation for the OFDM systems has been studied extensively during the past two decades. A comprehensive survey can be found in [2] and references therein. Moreover, resource allocation for OFDM-based CR networks has attracted much attention recently. An overview of the state-of-the-art research results can be found in [3]. This issue has been studied for both single-user and multiuser cases.

Moreover, resource allocation for OFDM-based CR system has been the main issue of several research works [4–6]. The authors in [4] have improved the performance of cognitive networks by exploiting the so-called multiuser interference diversity by using the subcarrier allocation. In [5], two fast resource allocation algorithms were derived for both real-time and non-real-time services in multiuser OFDM-based CR networks. The sum capacity of a multiuser OFDM-based CR system was maximized in [6] while satisfying the SUs' proportional rate requirements.

Even though resource allocation for cognitive radio has indeed been extensively studied in the literature, combining it with advanced receiver decoding has not yet been studied widely. In our previous work [7], we have fully detailed the combination of CR and superposition coding (SC) at the secondary user and provided the multicarrier power allocation for multiuser CR systems. We have also provided a numerical solution for the nonconvex optimization problem and proposed new results on the robustness of the proposed

algorithms with imperfect CSI. The original system model presented in [7] is further studied in the present paper, with an extension to the more realistic case where the secondary transmitter and receiver only have statistical CSI on the primary channels.

In all aforementioned works, it was assumed that perfect channel knowledge is available at different nodes. However, knowing the channel perfectly requires perfect channel measurements at the receiver and a perfect feedback link to send this channel information to the transmitter, which may be impractical to implement. Thus, several research works have dealt with the resource allocation subject to partial channel knowledge for CR systems. More specifically, the impact of imperfect Channel State Information (CSI) was investigated in [8–13] with uncertainty, quantization, or estimation errors. In [8], the authors considered a cognitive underlay scenario assuming that the secondary receiver has only partial information of the link between its transmitter and the primary receiver. This paper studied different capacity regions considering average or peak interference constraints at the primary receiver and introduced the concept of interference outage allowed by the primary receiver. The same problem was considered in [9] with average received power constraint at the primary receiver in a discrete-time block-fading channel with imperfect CSI. In [10], Suraweera et al. investigated the impact of imperfect CSI of the SU-PU link on the SU mean capacity, considering the effect of CSI quantization with a finite number of quantization levels. The ergodic capacity maximization problem with quantized information about CSI available at the SU through a limited feedback link was studied in [11], while optimum power strategy and ergodic capacity were derived under outage constraints in [12]. The impact of noisy CSI on spectral efficiency of multiuser multicarrier CRs was considered in [13], where novel interference management schemes were derived based on different average-case and worst-case models of channel estimation error. Resource allocation for OFDM-based CR under primary channel uncertainty was investigated in [14–16]. The authors in [14, 15] studied the PU chance constraint which captures the PU system channel uncertainty inherent to the PU interference. In [16] Gong et al. investigated the problem of single carrier ergodic rate maximization under primary user outage constraint by assuming primary system statistical CSI.

However, the ergodic capacity which consists of the long-term achievable rate averaging over the time-varying channels is less viable for real-time applications because it requires a coding procedure over infinitely many channel realizations. On the contrary, the statistical CSI is likely to remain unchanged over a long period of time so that a lesser amount of information needs to be fed back to the transmitter. Nevertheless, this may incur outage transmission under limited delay constraints due to channel fading. Outage happens whenever the achievable rate is less than the transmission rate.

Furthermore, several works considered the effect of statistical CSI rather than instantaneous channel estimation errors [17–21]. The authors in [17] have investigated the secondary achievable rate considering statistical CSI of secondary-primary link, without considering the interference from the primary transmitter to the secondary receiver. Taking this interference into account and assuming that the SU has perfect CSI of this interfering link, the authors in [18] designed the optimal and suboptimal power allocation approaches. In [19], Smith et al. studied the impact of limited channel knowledge on the CR system capacity by comparing the effect of a statistical CSI and an erroneous instantaneous CSI of the primary channel gains. In [20], the authors investigated the effect of statistical CSI on all the channels for the primary and secondary links on the SU capacity without addressing the power allocation problem. Finally, in [21], the authors studied the power allocation problem by taking into consideration statistical CSI about both secondary-primary and primary-secondary links.

In this paper, we investigate the dynamic resource allocation problem for an underlay cognitive radio system with the assumption that only statistical CSI on both primary-primary and secondary-primary links is available at the SU. More specifically, we focus on the uplink secondary utility optimization problem under primary and secondary user outage transmission constraints. In our scenario and due to the adaptive decoding strategies applied at the secondary receiver, the secondary achievable rate depends in some cases on the primary-primary channel, which makes the problem more complex to resolve. To the best of our knowledge, there is no existing work that attempts to address the problem of adaptive decoding in CR systems under statistical CSI assumption. Compared to our previous conference paper [22], this paper studies the possibility of using all decoding strategies at the secondary receiver. In [22], the secondary receiver was only able to treat interference as noise or use SIC. In the current paper, however, it can also use SC. This mathematically complexifies the problem but provides larger gains in terms of secondary and sum rates.

The main contributions of the paper are highlighted below:

(i) We design an OFDM-based resource allocation paradigm for an uplink underlay cognitive radio system. A heuristic approach is proposed based on adaptive decoding and power optimization. This heuristic is designed first for the perfect CSI scenario by considering a cooperation between primary and secondary systems. A complete resource allocation problem is addressed in two steps:

  (a) The decoding strategies are identified per subcarrier for the SU according to several conditions that will be defined later. The SU can either treat the interference received from the PU as noise, or apply SIC or SC.

  (b) An optimization problem is then formulated, in order to maximize the secondary rate and the sum rate of the system under the constraint of power budget of each user and of a maximum allowable interference at the primary receiver.

(ii) We investigate the same problem under the assumption that only statistical CSI on the links between

both primary and secondary transmitters and the primary receiver is available at the secondary user. The problem is solved in three steps:

(a) We determine the outage probabilities corresponding to each achievable rate and constraint in the perfect CSI-based problem.
(b) We circumvent the prohibitively high computational complexity of the optimization problem by solving separable independent problems.
(c) Motivated by the alternating optimization method [23], we propose an approach that sequentially solves a feasibility problem using dual decomposition and sequential approximation algorithm.

The rest of this paper is organized as follows. Section 2 describes the system model and gives an overview on the decoding strategies at the receivers. Section 3 explains the proposed adaptive decoding algorithm to solve the resource allocation problem with full CSI. Section 4 describes the problem with statistical CSI. Section 5 evaluates the performance of the proposed algorithm with both full and statistical CSI. Finally, Section 6 concludes the paper.

Notation. Throughout this paper, we use $\mathcal{CN}(\mu, \sigma^2)$ to indicate the proper Gaussian distribution with mean μ and variance σ^2. We denote the exponential function as $\exp^{(\cdot)}$ while $\log_2(\cdot)$ and $\mathbb{P}_r$ stand for the binary logarithmic function and the probability function, respectively. The expectation of $y(x)$ over x is represented by $\mathbb{E}_x\{y(x)\}$.

2. System Model

Similar to our previous paper [7], we consider a cognitive radio system model composed of one primary cell and one secondary cell. The primary system occupies a licensed bandwidth B which is divided into L adjacent and parallel subcarriers. The secondary base station (BS) is located at a distance d_{sec} from the primary BS. All users have a single antenna. In the considered scenario, the SU needs to adapt its decoding strategy to avoid disturbing the PU as in the classic two-user G-IFC [24], with much more constraints. Consequently, the secondary transmitter has to transmit when the primary system is either off or under an interference constraint fixed by the PU. Thus, the secondary transmitter uses channel interweave when the subbands are currently left vacant by the primary system [25]; otherwise, it uses channel underlay. Firstly, we assume that the secondary receiver has perfect CSI from both primary and secondary transmitters. We study the uplink transmission in a given time slot. In the sequel, index p refers to the primary system, while index s refers to secondary system. The received primary and secondary signals in each subcarrier $k \in \{1, \ldots, L\}$ can be written as (see Figure 1)

$$\begin{aligned} y_p^k &= h_{pp}^k x_p^k + h_{ps}^k x_s^k + z_p^k \\ y_s^k &= h_{sp}^k x_p^k + h_{ss}^k x_s^k + z_s^k \end{aligned} \tag{1}$$

where y_i^k is the channel output and x_i^k is the channel input corresponding to data s_i^k with power P_i^k per subcarrier. $P_{i,\max}$ is the maximum transmit power of user i. h_{ij}^k, a zero-mean complex circular Gaussian variable with variance λ_{ij}^k denotes the channel gain between transmitter j and receiver i. The channel gains are assumed to be constant during a transmission time slot. z_i^k denotes the additive white Gaussian noise at receiver i. The noise variance $n_i^k = n_0$ is the same on each subcarrier k.

3. Resource Allocation with Perfect CSI

In this section, we will figure out the expressions of the achievable rates when perfect CSI is available at the SU, in order to compare these results with the achievable rates considering only statistical channel information.

In the studied underlay cognitive scenario, the primary receiver is unaware of the presence of the secondary signal and thus cannot adapt its decoding strategy. Consequently, it always considers interference as noise. The rate of the primary system is given by

$$R_p^k = \frac{B}{L}\log_2\left(1 + \frac{\left|h_{pp}^k\right|^2 P_p^k}{n_0 + \left|h_{ps}^k\right|^2 P_s^k}\right) \tag{2}$$

On the other hand, the achievable secondary rate depends on the chosen decoding strategy at the SU. Our proposed algorithm is executed alternatively between the primary and the secondary systems. More specifically, given a starting power allocation $\mathbf{P}_{p,0}$ and $\mathbf{P}_{s,0}$, the SU applies a per subcarrier decoding strategy which depends on the primary and SU power allocation obtained from the iteration $n-1$. If $P_{p,(n-1)}^k \neq 0$, different decoding strategies for the SU are identified based on the interference level. The decoding strategy on each subcarrier k at the nth iteration is defined according to both primary and secondary powers in the previous iteration. These power levels are optimized using waterfilling and Lagrangian methods for the PU and the SU, respectively, as will be detailed in the next section. The different decoding strategies that can be applied at the SU are obtained from the G-IFC capacity region [24] to differentiate between weak and strong interference. We derived the additional constraint for strong interference at the secondary receiver in a cognitive scenario in our previous paper [26]. In the following, we present a quick description of these strategies along with their different achievable rates.

(1) **Strategy 1:** if $P_{p,(n-1)}^k = 0$, the secondary receiver decodes its message error-free and the secondary rate is defined by

$$R_{s,n}^k = \frac{B}{L}\log_2\left(1 + \frac{\left|h_{ss}^k\right|^2 P_{s,n}^k}{n_0}\right) \tag{3}$$

In the rest of the paper, this strategy is called 'interweave'

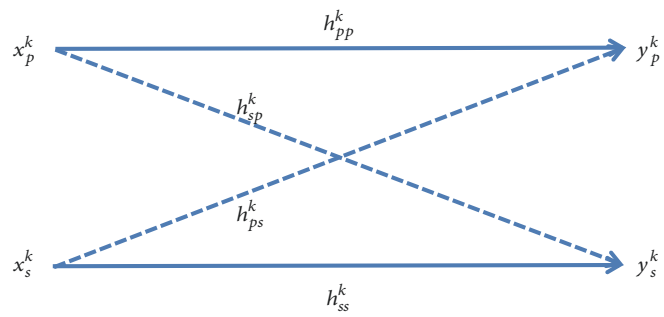

Figure 1: Underlay cognitive channels on subcarrier $k \in \{1, \ldots, L\}$.

(2) **Strategy 2**: if $P_{p,(n-1)} \neq 0$ and $|h_{sp}^k|^2 < |h_{ss}^k|^2$, the interference to the SU is weak and is treated as noise. In this case, the achievable secondary rate is given by

$$R_{s,n}^k = \frac{B}{L}\log_2\left(1 + \frac{\left|h_{ss}^k\right|^2 P_{s,n}^k}{\left|h_{sp}^k\right|^2 P_{p,n-1}^k + n_0}\right) \tag{4}$$

(3) **Strategy 3:** if $P_{p,(n-1)} \neq 0$ and $|h_{sp}^k|^2 \geqslant |h_{ss}^k|^2$, the interference on the SU is strong. Strategy 3 corresponds to the case where the interference channel is larger than the direct channel and interference should be decoded at the secondary receiver. However, this strategy is optimal in terms of sum data rate on the Gaussian interference channel according to [24], but not necessarily on the considered cognitive channel. Another constraint must be added in this case, since the quantity of information transmitted by the primary transmitter cannot be adapted so that the secondary receiver can decode it, as the primary transmitter is totally unaware of the secondary transmission. The additional constraint is the following: the quantity of information sent by the primary transmitter that only depends on the primary channel capacity (from the primary transmitter to the primary receiver) must be lower than the channel capacity between the primary transmitter and the secondary receiver. This constraint provides a new condition on the channel and power values that has been obtained in [26]. It is given by

$$a^k P_s^k \geqslant c^k \tag{5}$$

where

$$\begin{aligned} a^k &= \left|h_{sp}^k\right|^2 \left|h_{ps}^k\right|^2 - \left|h_{pp}^k\right|^2 \left|h_{ss}^k\right|^2 \\ c^k &= n_0\left(\left|h_{pp}^k\right|^2 - \left|h_{sp}^k\right|^2\right) \end{aligned} \tag{6}$$

If constraint (5) is verified, SIC can be applied. Then, the achievable secondary rate is

$$R_{s,n}^k = \frac{B}{L}\log_2\left(1 + \frac{\left|h_{ss}^k\right|^2 P_{s,n}^k}{n_0}\right) \tag{7}$$

This rate will be referred as $R_{s,n}^{k,SIC}$ in the rest of the paper

(4) **Strategy 4:** if $P_{p,(n-1)} \neq 0$ and $|h_{sp}^k|^2 \geqslant |h_{ss}^k|^2$ but (5) is not verified, the ability to apply superposition coding (SC) at the SU is tested subject to the validation of the following set of inequalities, derived in our previous paper [7]:

$$\frac{\left(\left|h_{pp}^k\right|^2 - \left|h_{sp}^k\right|^2\right)}{\left|h_{ps}^k\right|^2 \left|h_{sp}^k\right|^2} < \frac{P_{s,n}^k}{n_0} \tag{8a}$$

$$P_s^k a^k < c^k \tag{8b}$$

Thus, the secondary achievable rate can be given by

$$\begin{aligned} R_{s,n}^k = {} & \frac{B}{L}\log_2\left(1 + \frac{\alpha^k \left|h_{ss}^k\right|^2 P_{s,n}^k}{n_0}\right) \\ & + \frac{B}{L}\log_2\left(1 + \frac{\left(1-\alpha^k\right)\left|h_{ss}^k\right|^2 P_{s,n}^k}{\alpha^k \left|h_{ss}^k\right|^2 P_{s,n}^k + \left|h_{sp}^k\right|^2 P_{p,n}^k + n_0}\right) \end{aligned} \tag{9}$$

with

$$\alpha^k = \frac{\left(\left|h_{sp}^k\right|^2 - \left|h_{pp}^k\right|^2\right) n_0 + \left|h_{ps}^k\right|^2 \left|h_{sp}^k\right|^2 P_{s,n}^k}{\left|h_{pp}^k\right|^2 \left|h_{ss}^k\right|^2 P_{s,n}^k} \tag{10}$$

(5) **Strategy 5:** if $P_{p,(n-1)} \neq 0$ and $|h_{sp}^k|^2 \geqslant |h_{ss}^k|^2$ but neither SIC nor SC can be applied, the SU is turned off.

These different decoding strategies lead to four different expressions for the secondary rate defined in different domains and described in (11) to (14). $R_{s,n}^{k,int}$ is the achievable rate when the interference is treated as noise. $R_{s,n}^{k,SIC}$ represents both SIC and interweave cases and $R_{s,n}^{k,SC}$ the achievable rate

when SC is applied. $R_{s,n}^{k,off}$ denotes the null rate when the secondary transmitter is turned off.

$$R_{s,n}^{k,int} = \frac{B}{L}\log_2\left(1+\frac{\left|h_{ss}^k\right|^2 P_{s,n}^k}{\left|h_{sp}^k\right|^2 P_{p,n-1}^k + n_0}\right), \quad k \in \mathcal{S}_2 \tag{11}$$

$$R_{s,n}^{k,SIC} = \frac{B}{L}\log_2\left(1+\frac{\left|h_{ss}^k\right|^2 P_{s,n}^k}{n_0}\right), \quad k \in \mathcal{S}_1 \text{ and } \mathcal{S}_3 \tag{12}$$

$$R_{s,n}^{k,SC} = \frac{B}{L}\log_2\left(1+\frac{\alpha^k\left|h_{ss}^k\right|^2 P_{s,n}^k}{n_0}\right) + \frac{B}{L} \cdot \log_2\left(1+\frac{\left(1-\alpha^k\right)\left|h_{ss}^k\right|^2 P_{s,n}^k}{\alpha^k\left|h_{ss}^k\right|^2 P_{s,n}^k + \left|h_{sp}^k\right|^2 P_{p,n}^k + n_0}\right), \quad k \in \mathcal{S}_4 \tag{13}$$

$$R_{s,n}^{k,off} = 0, \quad k \in \mathcal{S}_5 \tag{14}$$

$$\begin{aligned}
\mathcal{S}_1 &= \{k \in \{1,\ldots,L\} \text{ for “Interweave”}\}\\
\mathcal{S}_2 &= \{k \in \{1,\ldots,L\} \\
&\quad \text{when interference is treated as a noise}\}\\
\mathcal{S}_3 &= \{k \in \{1,\ldots,L\} \text{ for “SIC”}\}\\
\mathcal{S}_4 &= \{k \in \{1,\ldots,L\} \text{ for “SC”}\}\\
\mathcal{S}_5 &= \{k \in \{1,\ldots,L\} \\
&\quad \text{when the secondary is turned off}\}
\end{aligned} \tag{15}$$

To optimally allocate the available power on different subcarriers, the power optimization problem is written, at the nth iteration, as

$$\max_{\mathbf{P}_{s,n}} \quad R_{s,n} \tag{16a}$$

$$\text{s.t.} \quad \sum_{k=1}^{L} P_{s,n}^k \leqslant P_{s,max} \tag{16b}$$

$$\text{s.t.} \quad P_{s,n}^k \geqslant 0 \quad \forall k \in \{1,\ldots,L\} \tag{16c}$$

$$\text{s.t.} \quad \left|h_{ps}^k\right|^2 P_{s,n}^k \leqslant I_{th}^k \quad \forall k \notin \mathcal{S}_1 \tag{16d}$$

$$\text{s.t.} \quad (5), \quad \forall k \in \mathcal{S}_3 \tag{16e}$$

$$\text{s.t.} \quad (8a) \text{ and } (8b), \quad \forall k \in \mathcal{S}_4 \tag{16f}$$

where

$$R_{s,n} = \sum_{k\in\mathcal{S}_2} R_{s,n}^{k,int} + \sum_{k\in\mathcal{S}_1\cap\mathcal{S}_3} R_{s,n}^{k,SIC} + \sum_{k\in\mathcal{S}_4} R_{s,n}^{k,SC} \tag{17}$$

and I_{th}^k is the interference threshold. The solution of problem (16a), (16b), (16c), (16d), (16e), and (16f) is detailed and analyzed in our previous paper [7].

4. Resource Allocation with Statistical CSI

In this section, we suppose that the SU has only statistical knowledge about the channel gains of the links between both primary and secondary transmitters and the primary receiver. Since $h_{ij}^k \sim \mathcal{CN}(0,\lambda_{ij}^k)$ with $(i,j) \in \{s,p\}^2$, $|h_{ij}^k|^2$ is exponentially distributed and its probability density function is expressed by $(1/\lambda_{ij}^k)\exp^{-|h_{ij}^k|^2/\lambda_{ij}^k}$. We now assume that only λ_{pp}^k and λ_{ps}^k are known at the secondary receiver. Then, in this section, we first study the convexity of the power optimization problem. Different outage terms are investigated. The first one represents the outage on the secondary rate when SC is applied, since this rate depends on $|h_{pp}^k|^2$ and $|h_{ps}^k|^2$. The second one, which is called ”interference outage probability,” is related to (16d). The other outage probabilities result from the SIC and SC constraints in (5) and (8a) and (8b). Only the interference outage probability is shown to be convex; consequently, the optimization problem is nonconvex. In the next section, a sequential approximation algorithm is proposed to solve the problem.

4.1. Determination of the Outage Probabilities. In this section, we detail the outage probabilities on different constraints. The index n is dropped to simplify equations writing.

4.1.1. Interference Constraint with Statistical CSI. The outage probability on constraint (16d) is

$$\mathbb{P}_r\left\{\left|h_{ps}^k\right|^2 P_s^k > I_{th}^k\right\} \leqslant \theta^k \tag{18}$$

where θ^k is the authorized interference outage. Therefore, the interference outage constraint can be written as

$$\exp^{-I_{th}^k/P_s^k\lambda_{ps}^k} \leqslant \theta^k \tag{19}$$

which is equivalent to

$$P_s^k \leqslant \frac{I_{th}^k}{\lambda_{ps}^k\log_2\left(1/\theta^k\right)} \tag{20}$$

4.1.2. SIC Constraint with Statistical CSI. The outage probability on constraint (5) is calculated with

$$\mathbb{P}_r\left\{\frac{\left|h_{pp}^k\right|^2 P_p^k}{n_0+\left|h_{ps}^k\right|^2 P_s^k} > \frac{\left|h_{sp}^k\right|^2 P_p^k}{n_0+\left|h_{ss}^k\right|^2 P_s^k}\right\} \leqslant \mu^k \tag{21}$$

and it is equal to

$$\exp^{-\nu^k n_0/\lambda_{pp}^k P_p^k}\frac{1}{1+\nu^k\left(P_s^k/P_p^k\right)\left(\lambda_{ps}^k/\lambda_{pp}^k\right)} \leqslant \mu^k \tag{22}$$

where

$$\nu^k = \frac{\left|h_{sp}^k\right|^2 P_p^k}{n_0+\left|h_{ss}^k\right|^2 P_s^k} \tag{23}$$

4.1.3. SC Constraints with Statistical CSI. If statistical CSI on $|h_{pp}^k|^2$ and $|h_{ps}^k|^2$ is available at the SU, outage probability has to be calculated on α^k and on both constraints (8a) and (8b).

We recall the equation with which α^k has been calculated with perfect CSI

$$\frac{\left|h_{pp}^k\right|^2}{\left|h_{ps}^k\right|^2 P_s^k + n_0} = \frac{\left|h_{sp}^k\right|^2}{\alpha^k \left|h_{ss}^k\right|^2 P_s^k + n_0} \tag{24}$$

which can be equivalently written as

$$\frac{\left|h_{pp}^k\right|^2}{\left|h_{sp}^k\right|^2} = \frac{\left|h_{ps}^k\right|^2 P_s^k + n_0}{\alpha^k \left|h_{ss}^k\right|^2 P_s^k + n_0} \tag{25}$$

With statistical CSI, constraint (24) becomes

$$\Pr\left\{\frac{\left|h_{pp}^k\right|^2}{\left|h_{sp}^k\right|^2} = \frac{\left|h_{ps}^k\right|^2 P_s^k + n_0}{\alpha^k \left|h_{ss}^k\right|^2 P_s^k + n_0}\right\} = 1 \tag{26}$$

This constraint can be written as

$$\frac{\lambda_{pp}^k}{\left|h_{sp}^k\right|^2} + \frac{\lambda_{ps}^k P_s^k + n_0}{\alpha^k \left|h_{ss}^k\right|^2 P_s^k + n_0} = 1 \tag{27}$$

which is equivalent to

$$\alpha^k = \frac{\left|h_{sp}^k\right|^2 \lambda_{ps}^k P_s^k + \lambda_{pp}^k n_0}{\left|h_{ss}^k\right|^2 P_s^k \left(\left|h_{sp}^k\right|^2 - \lambda_{pp}^k\right)} \tag{28}$$

The proof is given in the appendix.

The outage probability on constraint (8a) is calculated as

$$\mathbb{P}_r\left\{\frac{\left(\left|h_{pp}^k\right|^2 - \left|h_{sp}^k\right|^2\right)}{\left|h_{ps}^k\right|^2 \left|h_{sp}^k\right|^2} \geqslant \frac{P_s^k}{n_0}\right\} \leqslant \rho^k \tag{29}$$

and it is equivalent to

$$\exp^{-|h_{sp}^k|^2/\lambda_{pp}^k} \frac{1}{1 + \left|h_{sp}^k\right|^2 P_s^k \lambda_{ps}^k / n_0} \leqslant \rho^k \tag{30}$$

The outage probability on constraint (8b) is calculated as

$$\mathbb{P}_r\left\{\frac{\left|h_{pp}^k\right|^2 P_p^k}{n_0 + \left|h_{ps}^k\right|^2 P_s^k} \leqslant \frac{\left|h_{sp}^k\right|^2 P_p^k}{n_0 + \left|h_{ss}^k\right|^2 P_s^k}\right\} \leqslant \gamma^k \tag{31}$$

which can be equivalently formulated as

$$1 - \mathbb{P}_r\left\{\frac{\left|h_{pp}^k\right|^2 P_p^k}{n_0 + \left|h_{ps}^k\right|^2 P_s^k} > \frac{\left|h_{sp}^k\right|^2 P_p^k}{n_0 + \left|h_{ss}^k\right|^2 P_s^k}\right\} \leqslant \gamma^k \tag{32}$$

The outage probability on (8b) can be then deduced from (22) and it is equal to

$$\exp^{-\nu^k n_0/\lambda_{pp}^k P_p^k} \frac{1}{1 + \nu^k \left(P_s^k / P_p^k\right)\left(\lambda_{ps}^k / \lambda_{pp}^k\right)} \geqslant 1 - \gamma^k \tag{33}$$

Consequently, the power allocation problem with statistical CSI and outage limits μ^k, θ^k, ρ^k, and γ^k is formulated as

$$\max_{\mathbf{P}_s} \quad \widetilde{R}_s \tag{34a}$$

$$\text{s.t.} \quad \exp^{-\nu^k n_0/\lambda_{pp}^k P_p^k} \frac{1}{1 + \nu^k \left(P_s^k / P_p^k\right)\left(\lambda_{ps}^k / \lambda_{pp}^k\right)} \leqslant \mu^k, \quad \forall k \in \mathcal{S}_3 \tag{34b}$$

$$\text{s.t.} \quad \exp^{-I_{th}^k / P_s^k \lambda_{ps}^k} \leqslant \theta^k, \quad \forall k \notin \mathcal{S}_1 \tag{34c}$$

$$\text{s.t.} \quad \exp^{-|h_{sp}^k|^2/\lambda_{pp}^k} \frac{1}{1 + \left|h_{sp}^k\right|^2 P_s^k \lambda_{ps}^k / n_0} \leqslant \rho^k, \quad \forall k \in \mathcal{S}_4 \tag{34d}$$

$$\text{s.t.} \quad \exp^{-\nu^k n_0/\lambda_{pp}^k P_p^k} \frac{1}{1 + \nu^k \left(P_s^k / P_p^k\right)\left(\lambda_{ps}^k / \lambda_{pp}^k\right)} \geqslant 1 - \gamma^k, \quad \forall k \in \mathcal{S}_4 \tag{34e}$$

$$\text{s.t.} \quad \sum_{k=1}^{L} P_s^k \leqslant P_{s,\max} \tag{34f}$$

$$\text{s.t.} \quad P_{s,n}^k \geqslant 0, \quad \forall k \in \{1, \ldots, L\} \tag{34g}$$

where

$$\widetilde{R}_s = \sum_{k \in \mathcal{S}_2} R_{s,n}^{k,int} + \sum_{k \in \mathcal{S}_1 \cap \mathcal{S}_3} R_{s,n}^{k,SIC} + \sum_{k \in \mathcal{S}_4} R_{s,n}^{k,SC} \tag{35}$$

The presented optimization problem is nonconvex, due to the nonconvexity of constraints (34b) and (34e). We aim to maximize these constraints using the first Taylor approximation in order to transform the optimization problem into a convex problem.

4.2. Optimization Problem with Taylor Approximation. In this section, the first-order Taylor approximation is used on (34b) and (34e) in order to obtain convex constraints. The first-order Taylor approximation allows turning a nonconvex constraint function into an affine function and consequently turning the original nonconvex problem into a convex one. Using it iteratively corresponds to solving the problem through a series of approximation by convex optimization problems, called 'sequential convex approximation'. The final solution of the iterative algorithm is a lower bound on the optimal data rate. Since the first-order Taylor approximation is applied on the constraints, it is not theoretically possible to prove that the algorithm converges, but numerical convergence on the data rate at $\sigma = 2\%$ is obtained in less than 10 iterations in average. Due to the large complexity of this problem with several nonconvex constraint functions, we have decided to use the first-order Taylor approximation for its simplicity. However, other sequential convex approximation techniques such as the second-order Taylor approximation (providing convex functions instead of affine ones) or particle

methods that fit data with either an affine or a quadratic convex function could have been used [27, 28].

4.2.1. Applying Taylor Approximation on (34b). The outage probability on the SIC constraint (34b) is equivalent to

$$\frac{n_0\left|h_{sp}^k\right|^2}{\lambda_{pp}^k\left(n_0+\left|h_{ss}^k\right|^2P_s^k\right)}+\log_2\left(1+\frac{\lambda_{ps}^k\left|h_{sp}^k\right|^2P_s^k}{\lambda_{pp}^k\left(n_0+\left|h_{ss}^k\right|^2P_s^k\right)}\right)\geqslant\log_2\left(\frac{1}{\mu^k}\right) \tag{36}$$

The first ratio in (36) is convex in P_s^k. We can write it as follows:

$$\frac{n_0\left|h_{sp}^k\right|^2}{\lambda_{pp}^k\left(n_0+\left|h_{ss}^k\right|^2P_s^k\right)}=\frac{\left|h_{sp}^k\right|^2}{\lambda_{pp}^k}\frac{1}{1+\left|h_{ss}^k\right|^2P_s^k/n_0} \tag{37}$$

Using Taylor approximation at feasible point $\overline{P_s^k}$, we obtain

$$\frac{1}{1+\left|h_{ss}^k\right|^2P_s^k/n_0}=\frac{1}{1+\left(\left|h_{ss}^k\right|^2/n_0\right)\overline{P_s^k}}-\frac{\left|h_{ss}^k\right|^2}{n_0\left(1+\left(\left|h_{ss}^k\right|^2/n_0\right)\overline{P_s^k}\right)^2}\left(P_s^k-\overline{P_s^k}\right) \tag{38}$$

The SIC outage probability is then formulated as

$$\frac{\left|h_{sp}^k\right|^2}{\lambda_{pp}^k}\left(\frac{1}{1+\left(\left|h_{ss}^k\right|^2/n_0\right)\overline{P_s^k}}-\frac{\left|h_{ss}^k\right|^2}{n_0\left(1+\left(\left|h_{ss}^k\right|^2/n_0\right)\overline{P_s^k}\right)^2}\left(P_{s,n}^k-\overline{P_s^k}\right)\right)+\log_2\left(1+\frac{\lambda_{ps}^k\left|h_{sp}^k\right|^2P_{s,n}^k}{\lambda_{pp}^k\left(n_0+\left|h_{ss}^k\right|^2P_{s,n}^k\right)}\right)\geqslant\log_2\left(\frac{1}{\mu^k}\right),\quad\forall k\in\mathcal{S}_3 \tag{39}$$

which is a convex constraint.

4.2.2. Applying Taylor Approximation on (34e). The outage probability on the SC constraint in (34e) is equivalent to

$$\frac{n_0\left|h_{sp}^k\right|^2}{\lambda_{pp}^k\left(n_0+\left|h_{ss}^k\right|^2P_s^k\right)}+\log_2\left(1+\frac{\lambda_{ps}^k\left|h_{sp}^k\right|^2P_s^k}{\lambda_{pp}^k\left(n_0+\left|h_{ss}^k\right|^2P_s^k\right)}\right)\leqslant\log_2\left(\frac{1}{1-\gamma^k}\right) \tag{40}$$

and it is equal to

$$\frac{n_0\left|h_{sp}^k\right|^2}{\lambda_{pp}^k\left(n_0+\left|h_{ss}^k\right|^2P_s^k\right)}+\log_2\left(\left(\lambda_{ps}^k\left|h_{sp}^k\right|^2+\lambda_{pp}^k\left|h_{ss}^k\right|^2\right)P_s^k+\lambda_{pp}^kn_0\right)-\log_2\left(\lambda_{pp}^k\left(\left|h_{ss}^k\right|^2P_s^k+n_0\right)\right)\leqslant\log_2\left(\frac{1}{1-\gamma^k}\right) \tag{41}$$

The only concave function in this constraint $\log_2((\lambda_{ps}^k|h_{sp}^k|^2+\lambda_{pp}^k|h_{ss}^k|^2)P_s^k+\lambda_{pp}^kn_0)$ can be approximated using first Taylor approximation at feasible point $\overline{P_s^k}$ with

$$\log_2\left(\left(\lambda_{ps}^k\left|h_{sp}^k\right|^2+\lambda_{pp}^k\left|h_{ss}^k\right|^2\right)P_s^k+\lambda_{pp}^kn_0\right)=\log_2\left(\left(\lambda_{ps}^k\left|h_{sp}^k\right|^2+\lambda_{pp}^k\left|h_{ss}^k\right|^2\right)\overline{P_s^k}+\lambda_{pp}^kn_0\right)+\frac{1}{\ln 2}\cdot\frac{\lambda_{ps}^k\left|h_{sp}^k\right|^2+\lambda_{pp}^k\left|h_{ss}^k\right|^2}{\left(\lambda_{ps}^k\left|h_{sp}^k\right|^2+\lambda_{pp}^k\left|h_{ss}^k\right|^2\right)\overline{P_s^k}+\lambda_{pp}^kn_0}\left(P_s^k-\overline{P_s^k}\right) \tag{42}$$

Thus, the outage probability on (34e) can be formulated as

$$\frac{n_0\left|h_{sp}^k\right|^2}{\lambda_{pp}^k\left(n_0+\left|h_{ss}^k\right|^2P_s^k\right)}-\log_2\left(\lambda_{pp}^k\left(\left|h_{ss}^k\right|^2P_s^k+n_0\right)\right)+\log_2\left(\left(\lambda_{ps}^k\left|h_{sp}^k\right|^2+\lambda_{pp}^k\left|h_{ss}^k\right|^2\right)\overline{P_s^k}+\lambda_{pp}^kn_0\right)+\frac{1}{\ln 2}\cdot\frac{\lambda_{ps}^k\left|h_{sp}^k\right|^2+\lambda_{pp}^k\left|h_{ss}^k\right|^2}{\left(\lambda_{ps}^k\left|h_{sp}^k\right|^2+\lambda_{pp}^k\left|h_{ss}^k\right|^2\right)\overline{P_s^k}+\lambda_{pp}^kn_0}\left(P_s^k-\overline{P_s^k}\right)\leqslant\log_2\left(\frac{1}{1-\gamma^k}\right) \tag{43}$$

which is a convex constraint.

4.3. Solving the Optimization Problem by Decomposition. The investigated optimization problem can be efficiently solved using the Lagrange dual decomposition method, since it is separable, by decomposing the original problem into three subproblems depending on the decoding strategies applied at the SU. Dual decomposition [29] is optimal for convex problems and leads to the Karush-Kuhn-Tucker equations. For nonconvex problems, a duality gap remains between the solution obtained in the dual space and the solution of the primary problem. However, it was shown in [30] that if the problem concerns a large enough number of parallel subcarriers, the duality gap tends to zero. This very important result justifies why dual decomposition is often used to solve multicarrier resource allocation optimization problems, as, for instance, in papers [31, 32].

In this section, we firstly present the decomposed optimization problem and then we calculate the first-order Taylor approximation of each nonconvex constraint. Finally, we propose a general solution of the investigated problem using dual decomposition. As mentioned in the previous section, our algorithm is iterative and index n is reintroduced in this section to denote the iteration.

First of all, we write the Lagrangian of problem (34a), (34b), (34c), (34d), (34e), (34f), and (34g) taking into account constraints (34f) and (34g) as

$$\mathbb{L}\left(P_{s,n}^{k},\lambda\right) = -R_{s,n} + \lambda\left(\sum_{k=1}^{L} P_{s,n}^{k} - P_{s,\max}\right) - \sum_{k=1}^{L}\mu^{k}P_{s,n}^{k} \quad (44)$$

with $\lambda, \mu^{k} \geqslant 0$ being Lagrange multipliers. μ^{k} can be dropped since it is a slack variable. Let D be the set specified by the remaining constraints in (34c), (34b), (34d), and (34e). Consequently, problem (34a), (34b), (34c), (34d), (34e), (34f), and (34g) can be given by three subproblems C_{int}, C_{SIC}, and C_{SC} defined over $\mathcal{S}_2$, $\mathcal{S}_3$, and $\mathcal{S}_4$, respectively, as

$$\begin{aligned} C_{int}: &\max_{\{P_{s,n}^{k}\}_{k\in\mathcal{S}_2}} \sum_{k\in\mathcal{S}_2} R_{s,n}^{k,int} - \mu_{s,n}\sum_{k\in\mathcal{S}_2} P_{s,n}^{k} \\ &P_{s,n}^{k} \leqslant \frac{I_{th}^{k}}{\lambda_{ps}^{k}\log\left(1/\theta^{k}\right)}, \quad \forall k\in\mathcal{S}_2 \end{aligned} \quad (45a)$$

$$\begin{aligned} C_{SIC}: &\max_{\{P_{s,n}^{k}\}_{k\in\mathcal{S}_3}} \sum_{k\in\mathcal{S}_3} R_{s,n}^{k,SIC} - \mu_{s,n}\sum_{k\in\mathcal{S}_3} P_{s,n}^{k} \\ &P_{s,n}^{k} \leqslant \frac{I_{th}^{k}}{\lambda_{ps}^{k}\log\left(1/\theta^{k}\right)}, \quad \forall k\in\mathcal{S}_3 \\ &E^{k}P_{s,n}^{k} \geqslant F^{k}, \quad \forall k\in\mathcal{S}_3 \end{aligned} \quad (45b)$$

$$\begin{aligned} C_{SC}: &\max_{\{P_{s,n}^{k}\}_{k\in\mathcal{S}_4}} \sum_{k\in\mathcal{S}_4} R_{s,n}^{k,SC} - \mu_{s,n}\sum_{k\in\mathcal{S}_4} P_{s,n}^{k} \\ &P_{s,n}^{k} \leqslant \frac{I_{th}^{k}}{\lambda_{ps}^{k}\log\left(1/\theta^{k}\right)}, \quad \forall k\in\mathcal{S}_4 \\ &G^{k}P_{s,n}^{k} > H^{k}, \quad \forall k\in\mathcal{S}_4 \\ &E^{k}P_{s,n}^{k} \leqslant J^{k}, \quad \forall k\in\mathcal{S}_4 \end{aligned} \quad (45c)$$

where the subtracted term in the objective function represents the total power constraint which is common to all subproblems. Problem (45a), (45b), and (45c) reformulates the aforementioned problem introducing the results obtained in the previous section. In this section, we proceed to the resolution of three subproblems in order to find a general solution for problem (45a), (45b), and (45c). Note that the Lagrange dual function associated with (45a), (45b), and (45c) can be defined as

$$g(\lambda) = \max_{\mathbf{p}_{s,n}} \mathbb{L}\left(P_{s,n}^{k},\lambda\right) - \lambda\left(\sum_{k=1}^{L} P_{s,n}^{k} - P_{s,\max}\right) \quad (46)$$

where $p_{s,n}$ is constant.

The subproblem when interference is treated as noise can be formulated as C_{int}:

$$\begin{aligned} &\max_{\{P_{s,n}^{k}\}_{k\in\mathcal{S}_2}} \sum_{k\in\mathcal{S}_2} R_{s,n}^{k,int} - \mu_{s,n}\sum_{k\in\mathcal{S}_2} P_{s,n}^{k} \\ &P_{s,n}^{k} \leqslant \frac{I_{th}^{k}}{\lambda_{ps}^{k}\log\left(1/\theta^{k}\right)}, \quad \forall k\in\mathcal{S}_2 \\ &P_{s,n}^{k} \geqslant 0, \quad \forall k\in\mathcal{S}_2 \end{aligned} \quad (47)$$

To solve subproblem C_{int}, let us define the unconstrained subproblem

$$\widehat{C}_{int}: \max_{\{P_{s,n}^{k}\}_{k\in\mathcal{S}_2}} \sum_{k\in\mathcal{S}_2} R_{s,n}^{k,int} - \mu_{s,n}\sum_{k\in\mathcal{S}_2} P_{s,n}^{k} \quad (48)$$

with Lagrangian

$$\widehat{\mathbb{L}}_{int}\left(P_{s,n}^{k},\mu_{s,n}\right) = -\sum_{k\in\mathcal{S}_2} R_{s,n}^{k,int} + \mu_{s,n}\sum_{k\in\mathcal{S}_2} P_{s,n}^{k} \quad (49)$$

Let $\widehat{f}_{int}$ be the partial derivative of $\widehat{\mathbb{L}}_{int}$ with respect to $P_{s,n}^{k}$.

$$\widehat{f}\left(P_{s,n}^{k},\mu_{s,n}\right) = -\frac{\left|h_{ss}^{k}\right|^{2}}{\left|h_{sp}^{k}\right|^{2}P_{p,n-1}^{k} + n_{0}} + \mu_{s,n} = 0 \quad (50)$$

Thus, the solution of subproblem $\widehat{C}_{int}$ is given by

$$\widehat{P}_{s,n}^{k,int} = \left[\frac{1}{\mu_{s,n}} - \frac{\left|h_{sp}^{k}\right|^{2}P_{p,n-1}^{k} + n_{0}}{\left|h_{ss}^{k}\right|^{2}}\right]^{+} \quad (51)$$

Taking into consideration the constraint of subproblem C_{int}, its general solution $P_{s,n}^{k,int}$ has to be

$$P_{s,n}^{k,int} = \min\left\{\widehat{P}_{s,n}^{k,int}, \frac{I_{th}^{k}}{\lambda_{ps}^{k}\log\left(1/\theta^{k}\right)}\right\} \quad (52)$$

When SIC is applied, the optimization subproblem is formulated as C_{SIC} :

$$\begin{aligned} &\max_{\{P_{s,n}^k\}_{k\in\mathcal{S}_3}} \sum_{k\in\mathcal{S}_3} R_{s,n}^{k,SIC} - \mu_{s,n} \sum_{k\in\mathcal{S}_3} P_{s,n}^k \\ &P_{s,n}^k \leqslant \frac{I_{th}^k}{\lambda_{ps}^k \log\left(1/\theta^k\right)}, \quad \forall k \in \mathcal{S}_3 \\ &E^k P_{s,n}^k \geqslant F^k, \quad \forall k \in \mathcal{S}_3 \\ &P_{s,n}^k \geqslant 0 \end{aligned} \tag{53}$$

To solve subproblem C_{SIC}, let us define the unconstrained subproblem,

$$\widehat{C}_{SIC} : \max_{\{P_{s,n}^k\}_{k\in\mathcal{S}_3}} \sum_{k\in\mathcal{S}_3} R_{s,n}^{k,SIC} - \mu_{s,n} \sum_{k\in\mathcal{S}_3} P_{s,n}^k \tag{54}$$

with Lagrangian

$$\widehat{\mathbb{L}}_{SIC}\left(P_{s,n}^k, \mu_{s,n}\right) = -\sum_{k\in\mathcal{S}_3} R_{s,n}^{k,SIC} + \mu_{s,n} \sum_{k\in\mathcal{S}_3} P_{s,n}^k \tag{55}$$

Let $\widehat{f}_{SIC}$ be the partial derivative of $\widehat{\mathbb{L}}_{SIC}$ with respect to $P_{s,n}^k$.

$$\widehat{f}_{SIC}\left(P_{s,n}^k, \mu_{s,n}\right) = -\frac{\left|h_{ss}^k\right|^2}{\left|h_{ss}^k\right|^2 P_{s,n}^k + n_0} + \mu_{s,n} = 0 \tag{56}$$

Thus, the solution of subproblem $\widehat{C}_{SIC}$ is given by

$$\widehat{P}_{s,n}^{k,SIC} = \left[\frac{1}{\mu_{s,n}} - \frac{n_0}{\left|h_{ss}^k\right|^2}\right]^+ \tag{57}$$

Taking into consideration the constraint of subproblem C_{SIC}, its general solution $P_{s,n}^{k,SIC}$ has to meet the conditions in Table 1.

When SC is applied, the optimization subproblem is C_{SC}:

$$\begin{aligned} &\max_{\{P_{s,n}^k\}_{k\in\mathcal{S}_4}} \sum_{k\in\mathcal{S}_4} R_{s,n}^{k,SC} - \mu_{s,n} \sum_{k\in\mathcal{S}_4} P_{s,n}^k \\ &P_{s,n}^k \leqslant \frac{I_{th}^k}{\lambda_{ps}^k \log\left(1/\theta^k\right)}, \quad \forall k \in \mathcal{S}_4 \\ &G^k P_{s,n}^k > H^k, \quad \forall k \in \mathcal{S}_4 \\ &E^k P_{s,n}^k \leqslant J^k, \quad \forall k \in \mathcal{S}_4 \\ &P_{s,n}^k \geqslant 0 \end{aligned} \tag{58}$$

To solve subproblem C_{SC}, let us define the unconstrained subproblem

$$\widehat{C}_{SC} : \max_{\{P_{s,n}^k\}_{k\in\mathcal{S}_4}} \sum_{k\in\mathcal{S}_4} R_{s,n}^{k,SC} - \mu_{s,n} \sum_{k\in\mathcal{S}_4} P_{s,n}^k \tag{59}$$

with Lagrangian

$$\widehat{\mathbb{L}}_{SC}\left(P_{s,n}^k, \mu_{s,n}\right) = -\sum_{k\in\mathcal{S}_4} R_{s,n}^{k,SC} + \mu_{s,n} \sum_{k\in\mathcal{S}_4} P_{s,n}^k \tag{60}$$

Table 1: SIC conditions.

		E^k	
		Positive	Negative
F^k	Positive	$P_{s,n}^{k,SIC} > \frac{E^k}{F^k}$	Impossible
	Negative	No constraint	$P_{s,n}^{k,SIC} < \frac{E^k}{F^k}$

Table 2: Conditions from constraint (34d).

		E^k	
		Positive	Negative
J^k	Positive	$P_{s,n}^{k,SC} \leqslant \frac{J^k}{E^k}$	No constraint
	Negative	Impossible	$P_{s,n}^{k,SC} \geqslant \frac{J^k}{E^k}$

Table 3: Conditions from constraint (34e).

		G^k	
		Positive	Negative
H^k	Positive	$P_{s,n}^{k,SC} > \frac{H^k}{G^k}$	Impossible
	Negative	No constraint	$P_{s,n}^{k,SC} < \frac{H^k}{G^k}$

Let $\widehat{f}_{SC}$ be the partial derivative of $\widehat{\mathbb{L}}_{SC}$ with respect to $P_{s,n}^k$.

$$\widehat{f}_{SC}\left(P_{s,n}^k, \mu_{s,n}\right) = -\frac{\lambda_{ps}^k}{\lambda_{ps}^k P_{s,n}^k + n_0} + B_n^k\left(\overline{P_{s,n}^k}\right) + \mu_{s,n} = 0 \tag{61}$$

Thus, the solution of subproblem $\widehat{C}_{SC}$ is given by

$$\widehat{P}_{s,n}^{k,SC} = \frac{1}{\mu_{s,n} + B_n^k\left(\overline{P_{s,n}^k}\right)} - \frac{n_0}{\lambda_{ps}^k} \tag{62}$$

which can be written as

$$\widehat{P}_{s,n}^{k,SC} = \frac{1}{\widehat{\mu}_{s,n}} - \frac{n_0}{\lambda_{ps}^k} \tag{63}$$

with

$$\widehat{\mu}_{s,n} = \mu_{s,n} + B_n^k\left(\overline{P_{s,n}^k}\right) \tag{64}$$

Taking into consideration the constraint of subproblem C_{SC}, its general solution $P_{s,n}^{k,SC}$ has to meet the conditions in Tables 2 and 3.

Finally, the general solution of problem (34a), (34b), (34c), (34d), (34e), (34f), and (34g) is summarized in Table 4, where the values of $b_{s,(n-1)}^k$ are given in Table 5.

Consequently, we obtained $g(\lambda)$ for a given λ. Then, we can solve the dual problem which aims to minimize $g(\lambda)$

$$\begin{aligned} &\min_{\lambda} \quad g(\lambda) \\ &\text{s.t.} \quad \lambda \geqslant 0 \end{aligned} \tag{65}$$

Table 4: Optimized values of $P_{s,n}^k$.

Conditions	Decoding strategies	$P_{s,n}^k$
$P_{p,(n-1)}^k = 0$	Interweave	$\left[\frac{1}{\mu_{s,n}} - \frac{1}{b_{s,(n-1)}^k}\right]^+$
$P_{p,(n-1)}^k \neq 0$	Int = Noise	$\min\left\{\left[\frac{1}{\mu_{s,n}} - \frac{1}{b_{s,(n-1)}^k}\right]^+; \frac{I_{th}^k}{\lambda_{ps}^k \log(1/\theta^k)}\right\}$
$P_{p,(n-1)}^k \neq 0$ $\{E^k < 0; F^k > 0\}$	SIC	0
$P_{p,(n-1)}^k \neq 0$ $\{E^k > 0; F^k < 0\}$	SIC	$\min\left\{\left[\frac{1}{\mu_{s,n}} - \frac{1}{b_{s,(n-1)}^k}\right]^+; \frac{I_{th}^k}{\lambda_{ps}^k \log(1/\theta^k)}\right\}$
$P_{p,(n-1)}^k \neq 0$ $\{E^k < 0; F^k < 0\}$	SIC	$\min\left\{\left[\frac{1}{\mu_{s,n}} - \frac{1}{b_{s,(n-1)}^k}\right]^+; \frac{I_{th}^k}{\lambda_{ps}^k \log(1/\theta^k)}; \frac{F^k}{E^k}\right\}$
$P_{p,(n-1)}^k \neq 0$ $\{E^k > 0; F^k > 0\}$	SIC if $\frac{F^k}{E^k} \leqslant \frac{I_{th}^k}{\lambda_{ps}^k \log(1/\theta^k)}$	$\max\left\{\min\left\{\left[\frac{1}{\mu_{s,n}} - \frac{1}{b_{s,(n-1)}^k}\right]^+; \frac{I_{th}^k}{\lambda_{ps}^k \log(1/\theta^k)}\right\}; \frac{E^k}{F^k}\right\}$
	otherwise	0
$P_{p,(n-1)}^k \neq 0$ $\{G^k < 0; H^k > 0\}$ or $\{E^k > 0; J^k < 0\}$	SC	0
$P_{p,(n-1)}^k \neq 0$ $\{G^k > 0; H^k > 0\}$ and $\{E^k > 0; J^k > 0\}$	SC	$\max\left\{\min\left\{\left[\frac{1}{\widehat{\mu}_{s,n}} - \frac{1}{b_{s,(n-1)}^k}\right]^+; \frac{I_{th}^k}{\lambda_{ps}^k \log(1/\theta^k)}; \frac{J^k}{E^k}\right\}; \frac{H^k}{G^k}\right\}$
$P_{p,(n-1)}^k \neq 0$ $\{G^k > 0; H^k > 0\}$ and $\{E^k < 0; J^k > 0\}$	SC	$\max\left\{\min\left\{\left[\frac{1}{\widehat{\mu}_{s,n}} - \frac{1}{b_{s,(n-1)}^k}\right]^+; \frac{I_{th}^k}{\lambda_{ps}^k \log(1/\theta^k)}\right\}; \frac{H^k}{G^k}\right\}$
$P_{p,(n-1)}^k \neq 0$ $\{G^k > 0; H^k > 0\}$ and $\{E^k < 0; J^k < 0\}$	SC	$\max\left\{\min\left\{\left[\frac{1}{\widehat{\mu}_{s,n}} - \frac{1}{b_{s,(n-1)}^k}\right]^+; \frac{I_{th}^k}{\lambda_{ps}^k \log(1/\theta^k)}\right\}; \frac{H^k}{G^k}; \frac{J^k}{E^k}\right\}$
$P_{p,(n-1)}^k \neq 0$ $\{G^k > 0; H^k < 0\}$ and $\{E^k > 0; J^k > 0\}$	SC	$\min\left\{\left[\frac{1}{\widehat{\mu}_{s,n}} - \frac{1}{b_{s,(n-1)}^k}\right]^+; \frac{I_{th}^k}{\lambda_{ps}^k \log(1/\theta^k)}; \frac{J^k}{E^k}\right\}$
$P_{p,(n-1)}^k \neq 0$ $\{G^k > 0; H^k < 0\}$ and $\{E^k < 0; J^k > 0\}$	SC	$\min\left\{\left[\frac{1}{\widehat{\mu}_{s,n}} - \frac{1}{b_{s,(n-1)}^k}\right]^+; \frac{I_{th}^k}{\lambda_{ps}^k \log(1/\theta^k)}\right\}$
$P_{p,(n-1)}^k \neq 0$ $\{G^k > 0; H^k < 0\}$ and $\{E^k < 0; J^k < 0\}$	SC	$\max\left\{\min\left\{\left[\frac{1}{\widehat{\mu}_{s,n}} - \frac{1}{b_{s,(n-1)}^k}\right]^+; \frac{I_{th}^k}{\lambda_{ps}^k \log(1/\theta^k)}\right\}; \frac{J^k}{E^k}\right\}$
$P_{p,(n-1)}^k \neq 0$ $\{G^k < 0; H^k < 0\}$ and $\{E^k > 0; J^k > 0\}$	SC	$\min\left\{\left[\frac{1}{\widehat{\mu}_{s,n}} - \frac{1}{b_{s,(n-1)}^k}\right]^+; \frac{I_{th}^k}{\lambda_{ps}^k \log(1/\theta^k)}; \frac{J^k}{E^k}; \frac{H^k}{G^k}\right\}$
$P_{p,(n-1)}^k \neq 0$ $\{G^k < 0; H^k < 0\}$ and $\{E^k < 0; J^k > 0\}$	SC	$\min\left\{\left[\frac{1}{\widehat{\mu}_{s,n}} - \frac{1}{b_{s,(n-1)}^k}\right]^+; \frac{I_{th}^k}{\lambda_{ps}^k \log(1/\theta^k)}; \frac{H^k}{G^k}\right\}$
$P_{p,(n-1)}^k \neq 0$ $\{G^k < 0; H^k < 0\}$ and $\{E^k < 0; J^k < 0\}$	SC	$\max\left\{\min\left\{\left[\frac{1}{\widehat{\mu}_{s,n}} - \frac{1}{b_{s,(n-1)}^k}\right]^+; \frac{I_{th}^k}{\lambda_{ps}^k \log(1/\theta^k)}; \frac{H^k}{G^k}\right\}; \frac{J^k}{E^k}\right\}$

4.4. Sequential Convex Approximation Algorithm. Previously, we proposed an approximation method to transform the optimization problem in (34a), (34b), (34c), (34d), (34e), (34f), and (34g) into a convex problem. To improve the restrictive approximation, we propose a sequential algorithm where the optimization problem (34a), (34b), (34c), (34d), (34e), (34f), and (34g) is approximated using the optimal solution obtained at the previous iteration using dual decomposition. The proposed algorithm is summarized in Algorithm 1, where at the mth iteration, $\mathbf{p}_{s,m} = (P_{s,m}^1, \ldots, P_{s,m}^L)^T$.

(1) **Input** θ^k, μ^k, δ^k, γ^k >0, $m = 0$, and a feasible point $\overline{\mathbf{p}_{s,0}}$ for problem (34a), (34b), (34c), (34d), (34e), (34f), and (34g);
(2) **Repeat**
(3) $m = m + 1$
(4) Solve (34a), (34b), (34c), (34d), (34e), (34f), and (34g) using dual decomposition to obtain $\mathbf{p}_{s,m}$;
(5) Set $\overline{\mathbf{p}_{s,m}} = \mathbf{p}_{s,m}$;
(6) **Until** $(|\sum_k R^k_{s,m}| - |\sum_k R^k_{s,m-1}|)/|\sum_k R^k_{s,m-1}| < \sigma$;
(7) **Output** $\mathbf{p}_{s,m}$.

Algorithm 1: Sequential convex approximation algorithm.

Table 5: Values of coefficient $b^k_{s,(n-1)}$ in Table 4.

Cases	$b^k_{s,(n-1)}$
$P^k_{p,(n-1)} = 0$	$\dfrac{\lvert h^k_{ss}\rvert^2}{n_0}$
$P^k_{p,(n-1)} \neq 0$ and $\lvert h^k_{sp}\rvert^2 \leqslant \lvert h^k_{ss}\rvert^2$	$\dfrac{\lvert h^k_{ss}\rvert^2}{\lvert h^k_{sp}\rvert^2 P^k_{p,(n-1)} + n_0}$
$P^k_{p,(n-1)} \neq 0$ and $\lvert h^k_{sp}\rvert^2 > \lvert h^k_{ss}\rvert^2$ (SIC)	$\dfrac{\lvert h^k_{ss}\rvert^2}{n_0}$
$P^k_{p,(n-1)} \neq 0$ and $\lvert h^k_{sp}\rvert^2 > \lvert h^k_{ss}\rvert^2$ (SC)	$\dfrac{\lambda^k_{ps}}{n_0}$

Once $\mathbf{p}_s$ is computed for a given $\mathbf{p}_p$ using dual decomposition, an iterative process is applied to switch between the primary and secondary users in order to optimally allocate both of them taking into account the total power constraint of each user and the interference limit allowed on the primary receiver. The convergence of the proposed algorithm is in $\mathcal{O}(NML\log_2(L))$, where N is the required number of iterations needed by the sequential algorithm to converge, M is the number of iterations required to obtain the optimal value of Lagrange multiplier λ in problem (65), and $\mathcal{O}(L\log_2(L))$ is the power allocation complexity.

5. Simulation Results

The performance of the proposed algorithm is assessed using Monte Carlo simulations, where the location of both primary and secondary transmitters follows a uniform distribution. The performance is evaluated with statistical CSI and compared to several schemes with full and statistical CSI. The power constraint per transmitter is 21 dBm and the thermal noise has a spectral density $N_0 = -174$ dBm/Hz. The number of subcarriers is $L = 64$ and $B = 0.5$ MHz and $n_0 = N_0B/L$. The allowed degradation on the primary rate due to interference from the secondary transmitter is λ =0.1, which means that 90% of the interference-free rate is guaranteed. The influence of the distance between the primary and secondary BSs, d_{sec}, will be evaluated in the simulations. In all of our simulation results, all rates are normalized with respect to the bandwidth B. All outage limits θ^k, μ^k, ρ^k, and γ^k are equal to 0.01.

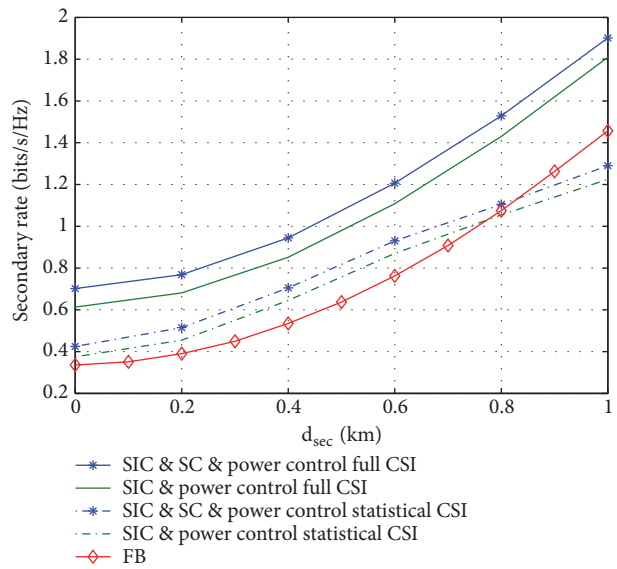

Figure 2: Secondary rates with statistical CSI.

In Figure 2, we also compare the secondary achievable rate using statistical CSI with the achievable rate when full CSI is available at the secondary transmitter. While, in Figure 3, sum rates are compared in both full and statistical CSI scenarios. With full CSI, the complex channel gains h^k_{ij} take into account path loss, log-normal shadowing, and Rayleigh fading. We suppose here that all subcarriers are subject to independent Rayleigh fading. The path loss model is COST 231 extension to Hata model at 800 MHz in dense urban environment, $L_{dB}(d) = 125.08 + 35.22 \times \log_{10}(d)$, and the shadowing standard deviation is equal to 6 dB. Both primary and secondary cells have omnidirectional antennas with the same radius $d_p = d_s = 1$ km. Due to the lack of perfect CSI of the primary channels, rigorous outage limits are imposed on the outage probabilities which lead to strict constraints and thus result in a degradation of the secondary rate and thus in the sum rate. We can notice that increasing the number of subcarriers L would lead to larger frequency diversity and consequently increase performances. We used Rayleigh independent and identically distributed fading over the subcarriers. If we had correlated fading, then frequency diversity would decrease and the data rates would also decrease. If we used Rice fading instead of Rayleigh fading, the larger correlation of channel gains across subcarriers would also lead to lower average data rates, even though, in

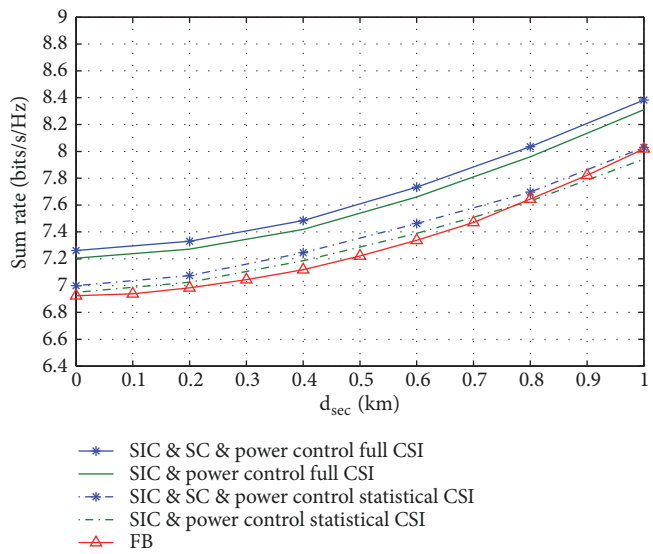

Figure 3: Sum rates with statistical CSI.

some configurations with large Line Of Sight conditions, data rates could be increased.

The proposed algorithm is also compared to the classical power allocation scheme where the secondary system can transmit on the whole bandwidth of a cognitive underlay/interweave system by considering the primary system's interference as noise in all subcarriers. This algorithm is denoted by "FB" for "Full Band" and its complexity is $2 \cdot N \cdot O(L\log_2 L)$, with N being the number of iterations. Simulation results in Figures 2 and 3 demonstrate that our algorithm outperforms 'FB' for small distances between the BSs. The secondary data rate strongly decreases when only statistical CSI is available, compared to the full CSI case. It decreases 21 to 39%, with lower loss when the distance between BS d_{sec} is medium and larger loss when it is low. This indicates that the secondary receiver is less often able to use SIC or SC with statistical CSI. Regarding the sum data rate, the rate decrease is less important since the primary rate is still large. The relative sum rate decrease is between 3.6 and 4.8% and slightly increases when d_{sec} increases.

To study the influence of outage limits on the secondary achievable rate, we present in Figure 4 the performance in terms of achievable rate versus μ^k, θ^k, ρ^k, and γ^k for a fixed d_{sec}= 0.4 km. The dashed line represents the secondary achievable rate at this distance with full CSI. The first curve labeled as μ^k is obtained for fixed θ^k, ρ^k, and γ^k=1% while varying μ^k. The same procedure is applied for the other curves, by fixing one of the outage limits and varying the others. Figure 4 establishes that the secondary achievable rate with statistical CSI approaches from the secondary achievable rate with full CSI when μ^k, θ^k, ρ^k, and γ^k increase. We can also observe that the SIC outage limit has a greater effect on the secondary rate degradation than the interference and SC outage limits.

The influence of the predefined interference level at the primary user is studied on Figures 5 and 6. For that, we plot

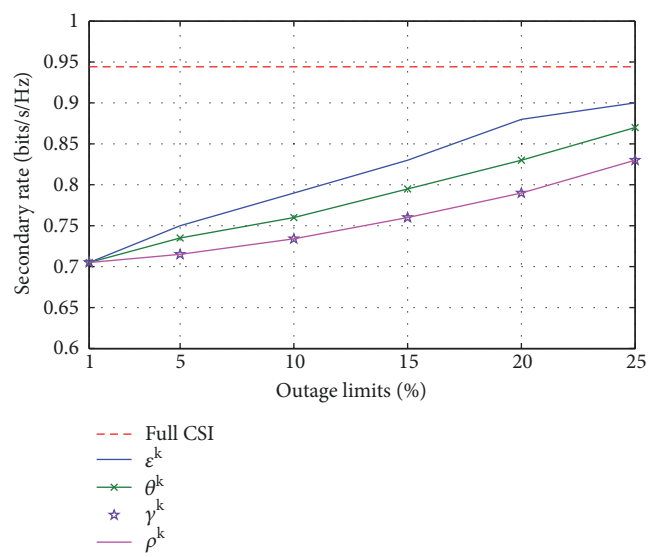

Figure 4: Achievable secondary rate depending on the outage limits at d_{sec} = 0.4 km.

the variations of both secondary and sum rates for different tolerable interference percentage, in Figures 5 and 6. More specifically, we figure out the achievable rates when 95% and 85% of interference-free is guaranteed at the PU, which corresponds to $\lambda = 0.05$ and $\lambda = 0.15$, respectively.

6. Conclusion

In this paper, we proposed a resource allocation algorithm for a multicarrier cognitive radio system subject to statistical CSI. An adaptive decoding algorithm based on both successive interference cancellation and superposition coding has been proposed in order to maximize the achievable system rate, while optimizing power allocation. Both theoretical analysis

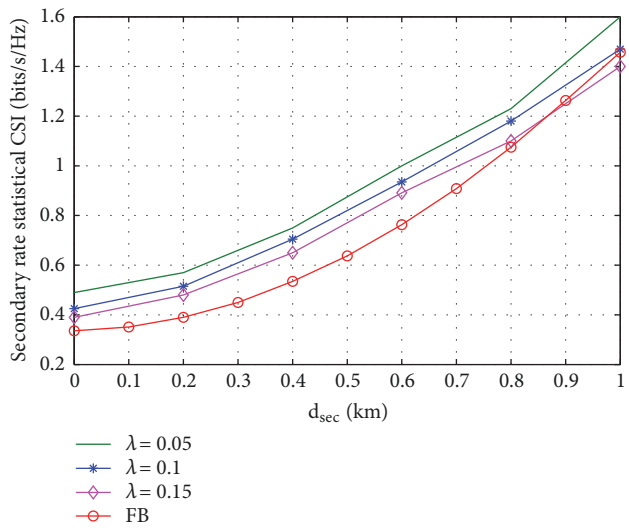

FIGURE 5: Secondary rates of proposed algorithm with statistical CSI for different I_{th}^k.

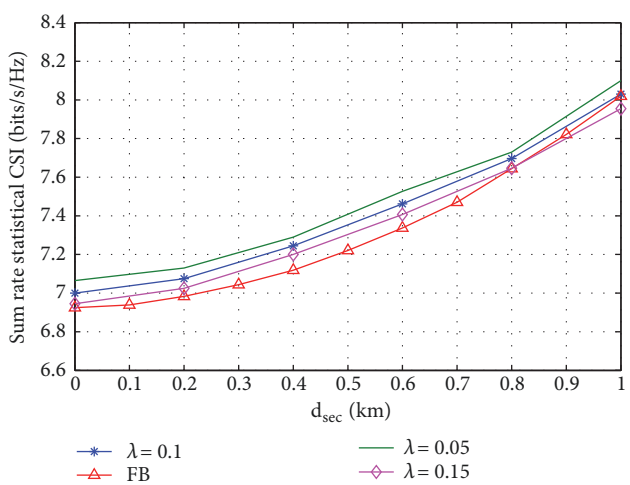

FIGURE 6: Sum rates of proposed algorithm with statistical CSI for different I_{th}^k.

and simulation results have shown that the proposed algorithm achieves higher sum rate than classical algorithms, providing high-enough data rates for the secondary system. Simulations showed that the proposed algorithm is efficient and robust with statistical CSI. For further work, the proposed cooperative underlay cognitive system could be extended to a Multiple-Input Multiple-Output model or to a CR relay network.

Appendix

Proof of (28). To begin with the proof, we rewrite the outage probability related to α^k

$$\Pr\left\{\frac{\left|h_{pp}^k\right|^2}{\left|h_{sp}^k\right|^2} = \frac{\left|h_{ps}^k\right|^2 P_s^k + n_0}{\alpha^k \left|h_{ss}^k\right|^2 P_s^k + n_0}\right\} = 1 \tag{A.1}$$

which is equivalent to $\Pr\{X = Y\} = 1$, where

$$X \triangleq \frac{\left|h_{pp}^k\right|^2}{\left|h_{sp}^k\right|^2}$$
$$Y \triangleq \frac{\left|h_{ps}^k\right|^2 P_s^k + n_0}{\alpha^k \left|h_{ss}^k\right|^2 P_s^k + n_0} \tag{A.2}$$

Basically for two discrete random variables

$$\Pr\{X = Y\} = \sum_{t\in\mathcal{S}} \Pr\{X = t, Y = t\} \tag{A.3}$$

where $\mathcal{S}$ is the set of all possible value of t. By extending (A.3) in the continuous time, we have

$$\Pr\{X = Y\} = \int_{t\in\mathcal{S}} f_{X,Y}(X = t, Y = t)\, dt \tag{A.4}$$

where $f_{X,Y}(X = t, Y = t)$ is the joint probability distribution function of X and Y. By independence, it follows that the joint probability density function of X and Y is

$$f_{X,Y}(X = t, Y = t) = \frac{1}{\lambda_X \lambda_Y} e^{-(1/\lambda_X + 1/\lambda_X)t}; \tag{A.5}$$

hence

$$\Pr\{X = Y\} = \int_0^\infty \frac{1}{\lambda_X \lambda_Y} e^{-(1/\lambda_X + 1/\lambda_X)t} dt = \frac{1}{\lambda_X + \lambda_Y} \tag{A.6}$$

with

$$\lambda_X = \mathbb{E}\left\{\frac{\left|h_{pp}^k\right|^2}{\left|h_{sp}^k\right|^2}\right\} = \frac{\lambda_{pp}^k}{\left|h_{sp}^k\right|^2} \tag{A.7}$$

and

$$\lambda_Y = \mathbb{E}\left\{\frac{\left|h_{ps}^k\right|^2 P_s^k + n_0}{\alpha^k \left|h_{ss}^k\right|^2 P_s^k + n_0}\right\} = \frac{\lambda_{ps}^k P_s^k + n_0}{\alpha^k \left|h_{ss}^k\right|^2 P_s^k + n_0} \tag{A.8}$$

Consequently, this constraint can be written as

$$\frac{\lambda_{pp}^k}{\left|h_{sp}^k\right|^2} + \frac{\lambda_{ps}^k P_s^k + n_0}{\alpha^k \left|h_{ss}^k\right|^2 P_s^k + n_0} = 1 \tag{A.9}$$

which is equivalent to

$$\alpha^k = \frac{\left|h_{sp}^k\right|^2 \lambda_{ps}^k P_s^k + \lambda_{pp}^k n_0}{\left|h_{ss}^k\right|^2 P_s^k \left(\left|h_{sp}^k\right|^2 - \lambda_{pp}^k\right)} \tag{A.10}$$

This concludes the proof. □

Data Availability

All data used in the manuscript were generated with standard Matlab function such as rand and randn. The authors did not use any specific data aside from that.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This work was partially funded through French National Research Agency (ANR) project ACCENT5 with grant agreement code ANR-14-CE28-0026-02.

References

[1] H. Mahmoud, T. Yücek, and H. Arslan, "OFDM for cognitive radio: merits and challenges," *IEEE Wireless Communications Magazine*, vol. 16, no. 2, pp. 6–15, 2009.

[2] S. Sadr, A. Anpalagan, and K. Raahemifar, "Radio resource allocation algorithms for the downlink of multiuser OFDM communication systems," *IEEE Communications Surveys & Tutorials*, vol. 11, no. 3, pp. 92–106, 2009.

[3] R. Zhang, Y.-C. Liang, and S. Cui, "Dynamic resource allocation in cognitive radio networks," *IEEE Signal Processing Magazine*, vol. 27, no. 3, pp. 102–114, 2010.

[4] R. Zhang and Y.-C. Liang, "Investigation on multiuser diversity in spectrum sharing based cognitive radio networks," *IEEE Communications Letters*, vol. 14, no. 2, pp. 133–135, 2010.

[5] M. Ge and S. Wang, "Fast optimal resource allocation is possible for multiuser OFDM-based cognitive radio networks with heterogeneous services," *IEEE Transactions on Wireless Communications*, vol. 11, no. 4, pp. 1500–1509, 2012.

[6] S. Wang, F. Huang, and C. Wang, "Adaptive proportional fairness resource allocation for OFDM-based cognitive radio networks," *Wireless Networks*, vol. 19, no. 3, pp. 273–284, 2013.

[7] M. Chami, M. Pischella, and D. Le Ruyet, "Resource allocation for OFDM-based multiuser cooperative underlay cognitive systems," *EURASIP Journal on Wireless Communications and Networking*, vol. 2017, no. 1, 2017.

[8] L. Musavian and S. Aïssa, "Fundamental capacity limits of cognitive radio in fading environments with imperfect channel information," *IEEE Transactions on Communications*, vol. 57, no. 11, pp. 3472–3480, 2009.

[9] X. Xie and W. Guo, "Fundamental effective capacity limits of cognitive radio in fading environments with imperfect channel information," in *Proceedings of the 2011 International Conference on Computational Problem-Solving, ICCP 2011*, pp. 232–235, China, October 2011.

[10] H. A. Suraweera, P. J. Smith, and M. Shafi, "Capacity limits and performance analysis of cognitive radio with imperfect channel knowledge," *IEEE Transactions on Vehicular Technology*, vol. 59, no. 4, pp. 1811–1822, 2010.

[11] Y. He and S. Dey, "Throughput maximization in cognitive radio under peak interference constraints with limited feedback," *IEEE Transactions on Vehicular Technology*, vol. 61, no. 3, pp. 1287–1305, 2012.

[12] Z. Rezki and M.-S. Alouini, "Ergodic capacity of cognitive radio under imperfect channel-state information," *IEEE Transactions on Vehicular Technology*, vol. 61, no. 5, pp. 2108–2119, 2012.

[13] A. Shojaeifard, H. Saki, M. M. Mahyari, and M. Shikh-Bahaei, "Resource allocation and interference management for adaptive modulation and coding-based OFDMA cognitive radio networks," in *Proceedings of the 2014 1st IEEE International Conference on Communications, ICC 2014*, pp. 5908–5913, Australia, June 2014.

[14] X. Gong, A. Ispas, G. Dartmann, and G. Ascheid, "Outage-constrained power allocation in spectrum sharing systems with partial CSI," *IEEE Transactions on Communications*, vol. 62, no. 2, pp. 452–466, 2014.

[15] S.-J. Kim, N. Y. Soltani, and G. B. Giannakis, "Resource allocation for OFDMA cognitive radios under channel uncertainty," *IEEE Transactions on Wireless Communications*, vol. 12, no. 7, pp. 3578–3587, 2013.

[16] N. Y. Soltani, S.-J. Kim, and G. B. Giannakis, "Chance-constrained optimization of OFDMA cognitive radio uplinks," *IEEE Transactions on Wireless Communications*, vol. 12, no. 3, pp. 1098–1107, 2013.

[17] S. Lim, H. Wang, H. Kim, and D. Hong, "Mean value-based poaer allocation without instantaneous CSI feedback in spectrum sharing systems," *IEEE Transactions on Wireless Communications*, vol. 11, no. 3, pp. 874–879, 2012.

[18] K. Son, B. C. Jung, S. Chong, and D. K. Sung, "Power allocation policies with full and partial inter-system channel state information for cognitive radio networks," *Wireless Networks*, vol. 19, no. 1, pp. 99–113, 2013.

[19] P. J. Smith, P. A. Dmochowski, H. A. Suraweera, and M. Shafi, "The effects of limited channel knowledge on cognitive radio system capacity," *IEEE Transactions on Vehicular Technology*, vol. 62, no. 2, pp. 927–933, 2013.

[20] P. A. Dmochowski, H. A. Suraweera, P. J. Smith, and M. Shafi, "Impact of Channel Knowledge on Cognitive Radio System Capacity," in *Proceedings of the 2010 IEEE Vehicular Technology Conference (VTC 2010-Fall)*, pp. 1–5, Ottawa, ON, Canada, September 2010.

[21] X. Gong, A. Ispas, G. Dartmann, and G. Ascheid, "Power allocation and performance analysis in spectrum sharing systems with statistical CSI," *IEEE Transactions on Wireless Communications*, vol. 12, no. 4, pp. 1819–1831, 2013.

[22] M. Chami, M. Pischella, and D. Le Ruyet, "Resource allocation with SIC under statistical CSI in multi-carrier based cognitive radio networks," in *Proceedings of the 2016 IEEE Wireless Communications and Networking Conference, WCNC 2016*, Qatar, April 2016.

[23] D. P. Bertsekas, *Nonlinear Programming*, Belmont, Mass., Athena Scientific, USA, 1995.

[24] T. S. Han and K. Kobayashi, "A new achievable rate region for the interference channel," *Institute of Electrical and Electronics Engineers Transactions on Information Theory*, vol. 27, no. 1, pp. 49–60, 1981.

[25] A. Goldsmith, S. A. Jafar, I. Maric, and S. Srinivasa, "Breaking spectrum gridlock with cognitive radios: an information theoretic perspective," *Proceedings of the IEEE*, vol. 97, no. 5, pp. 894–914, 2009.

[26] M. Pischella and D. Le Ruyet, "Adaptive resource allocation and decoding strategy for underlay multi-carrier cooperative cognitive radio systems," *Transactions on Emerging Telecommunications Technologies*, vol. 24, no. 7-8, pp. 748–761, 2013.

[27] A. R. Conn, N. I. M. Gould, and P. Toint, *Trust Region Method*, Society for Industrial and Applied Mathematics, Philadelphia, Pa, USA, 2000.

[28] R. Fletcher, *Practical Methods of Optimization*, John Wiley & Sons, New York, NY, USA, 2nd edition, 1987.

[29] S. Boyd and L. Vandenberghe, *Convex Optimization*, Cambridge University Press, 2004.

[30] W. Yu and R. Lui, "Dual methods for nonconvex spectrum optimization of multicarrier systems," *IEEE Transactions on Communications*, vol. 54, no. 7, pp. 1310–1322, 2006.

[31] D. P. Palomar and M. Chiang, "A tutorial on decomposition methods for network utility maximization," *IEEE Journal on Selected Areas in Communications*, vol. 24, no. 8, pp. 1439–1451, 2006.

[32] M. Chiang, S. H. Low, A. R. Calderbank, and J. C. Doyle, "Layering as optimization decomposition: a mathematical theory of network architectures," *Proceedings of the IEEE*, vol. 95, no. 1, pp. 255–312, 2007.