Research Article

Passivity of Memristive BAM Neural Networks with Probabilistic and Mixed Time-Varying Delays

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This paper is concerned with the passivity problem of memristive bidirectional associative memory neural networks (MBAMNNs) with probabilistic and mixed time-varying delays. By applying random variables with Bernoulli distribution, the information of probability time-varying delays is taken into account. Furthermore, we consider the probability distribution of the variation and the extent of the delays; therefore, the results derived are less conservative than in the existing papers. In particular, the leakage delays as well as distributed delays are all taken into consideration. Based on appropriate Lyapunov-Krasovskii functionals (LKFs) and some useful inequalities, several conditions for passive performance are established in linear matrix inequalities (LMIs). Finally, numerical examples are given to demonstrate the feasibility of the presented theories, and the results reveal that the probabilistic and mixed time-varying delays have an unstable influence on the system and should not be ignored.

1. Introduction

Bidirectional associative memory neural networks (BAMNNs) are a class of two-layer neural systems, which were first introduced by Kosko in 1987. The neurons in the first layer are connected to another layer, and in the same layer, the neurons are not interconnected [1–3]. Owing to their special structure, BAMNNs have displayed many good features in various areas such as signal processing, image processing, and optimization problems [4–6]. In 2015, the stability of inertial BAMNNs with time-varying delay via impulsive control was discussed in [7]. Zhang et al. considered the exponential stability of BAMNNs with time-varying delays in [8]. Wang et al. addressed the global asymptotic stability of impulsive fractional-order BAMNNs with time delay in [9].

Memristor, a combination of a resistor and memory, has received increasing attention in many fields [10–13]. By applying the nonvolatile feature of the memristor, researchers were able to develop MBAMNN models. Because of the

pinched hysteresis effects, MBAMNNs have a memory function, which can be used to imitate the human brain [14, 15]. In 2015, nonfragile synchronization of MBAMNNs with random feedback gain fluctuations was investigated in [16]. Based on functional differential inclusions, Jiang et al. obtained the dynamic behaviors for MBAMNNs with time-varying delays in [17].

Moreover, passivity is a special case of a broader theory of dissipativity, which plays a significant role in the stability analysis of dynamical systems, nonlinear control, and other areas. The main innate character of passivity theory is that the passive characteristics can make the system internally stable [18–20]. In recent years, many researchers have proposed passivity analysis for memristive neural networks (MNNs). Liu and Xu investigated the passivity analysis of MNNs with different state-dependent memductance functions and mixed time-varying delays in [21]. In 2016, the passivity of MBAMNNs with uncertain delays and different memductance was investigated in [22]. Nevertheless, there are

few people to study the passivity of MBAMNNs, which encourages our idea.

In the human brain, the transmission of information in neurons is often accompanied by a time delay, so time delay is inevitable in the neural networks, which is the origin of oscillation, divergence, and so forth [23–33]. Sometimes, the value of delay may be very large, but the probability of such delay is very small. Therefore, we use the probability distribution of time delay in the interval to reflect an actual situation better. Furthermore, it is clearer to describe the probabilistic time-varying delays through introducing random variables with Bernoulli distribution. In recent years, some researchers have discussed the probabilistic time-varying delays in the neural networks [34, 35]. In 2016, Pradeep et al. investigated the robust stability analysis of stochastic neural networks with probabilistic time-varying delays in [36]. Li et al. considered passivity analysis of memristive neural networks with probabilistic time-varying delays in [37]. Hence, it is of great importance to research the passivity of MBAMNNs with probabilistic time-varying delays.

In addition, there also exist two types of time-varying delays named leakage delays (or forgetting delays) and distributed time delays. The research of leakage time delay can be traced back to the early 90s of the last century; researchers found out that, due to the delay in switching time or signal transmission, there is a time delay in the negative feedback term of the network system; this delay is named leakage time delay. As is well known, leakage delays exist in many real systems such as population dynamics and neural networks [38, 39]. Moreover, leakage delay also has a significant influence on the dynamics of neural networks because it has been shown that such kind of time delay in the leakage term has a tendency to destabilize a system. Under the influence of leakage and additive time-varying delays, robust passivity analysis for neural networks was addressed in [40]. In 2016, the robust stability analysis for discrete-time neural networks with leakage delays was studied in [41].

On the other hand, due to the presence of multiple parallel paths with a variety of neuronal synapses' lengths and sizes, there is a spatial width of the network, and then there may exist either a distribution of the transmission voltage in these parallel paths or a distribution of transmission delays over a period of time. Hence, the distribution delay is used to describe this phenomenon [42, 43]. In 2015, Du et al. investigated the passivity of neural networks with discrete and distributed time-varying delays in [44]. In 2016, Yang et al. considered finite-time stabilization of uncertain neural networks with distributed time-varying delays in [45]. However, to the best of our knowledge, there are few results on the passivity of MBAMNNs with probabilistic, leakage, and distributed time-varying delays. Thus, it is significant to study the passivity of MBAMNNs with these time-varying delays.

Motivated by the main points discussed above, the contribution of this paper lies in three aspects.

(1) This is the first attempt to discuss the passivity analysis of MBAMNNs with probabilistic and mixed time-varying delays. In particular, the leakage delays as well as distributed delays are all taken into consideration.

(2) The LKFs that we designed include double and triple integral terms, and by applying some helpful inequalities, the passivity analysis of MBAMNNs becomes less conservative than the existing results [19, 21].

(3) After using MATLAB LMI control toolbox, all the derived results are expressed in LMIs, and a feasible solution can be easily obtained.

The rest of the paper is structured as follows. In Section 2, we introduce the model of the MBAMNNs with probabilistic time-varying delays. In Section 3, the main results on passivity analysis of MBAMNNs with probabilistic and mixed time-varying delays are derived. In Section 4, some numerical simulations are provided to demonstrate the feasibility of our results. In Section 5, the conclusion is shown.

2. Model Description and Preliminaries

In this paper, we propose the MBAMNN with probabilistic time-varying delays as follows:

$$\begin{aligned}\dot{x}_i(t) &= -C_i x_i(t) + \sum_{j=1}^m a_{ji}(x_i(t)) f_j(y_j(t)) \\ &\quad + \sum_{j=1}^m b_{ji}(x_i(t)) f_j(y_j(t - \tau(t))) + u_i(t), \\ \dot{y}_j(t) &= -D_j y_j(t) + \sum_{i=1}^n e_{ij}(y_j(t)) g_i(x_i(t)) \\ &\quad + \sum_{i=1}^n h_{ij}(y_j(t)) g_i(x_i(t - d(t))) + v_j(t), \\ i &= 1, \dots, n, \quad j = 1, \dots, m,\end{aligned}\tag{1}$$

or it can be rewritten as follows:

$$\begin{aligned}\dot{x}(t) &= -Cx(t) + A(x(t)) f(y(t)) \\ &\quad + B(x(t)) f(y(t - \tau(t))) + u(t), \\ \dot{y}(t) &= -Dy(t) + E(y(t)) g(x(t)) \\ &\quad + H(y(t)) g(x(t - d(t))) + v(t),\end{aligned}\tag{2}$$

where $x_i(t)$ and $y_j(t)$ denote the state variables related to the i th and j th neurons. $A(x(t)) = (a_{ji}(x_i(t)))_{m \times n}$ and $E(y(t)) = (e_{ij}(y_j(t)))_{n \times m}$ are the connection weight matrices, $f_j(\cdot)$ and $g_i(\cdot)$ are the activation functions, and $B(x(t)) = (b_{ji}(x_i(t)))_{m \times n}$ and $H(y(t)) = (h_{ij}(y_j(t)))_{n \times m}$ are the delayed connection weight matrices. The self-feedback connection weights C_i and D_j are positive diagonal matrices. $u_i(t)$ and $v_j(t)$ represent the continuous external inputs; the nonnegative continuous variables $\tau(t)$ and $d(t)$ correspond to the time-varying delays.

Assumption 1. The functions $f_j(t)$ ($j = 1, 2, \dots, m$) and $g_i(t)$ ($i = 1, 2, \dots, n$) are bounded and continuous and satisfy the conditions as follows:

FIGURE 1: The typical current-voltage characteristic of a memristor.

$$\begin{aligned}\varsigma_j^- &\leq \frac{f_j(b) - f_j(a)}{b - a} \leq \varsigma_j^+, \\ \sigma_i^- &\leq \frac{g_i(b) - g_i(a)}{b - a} \leq \sigma_i^+, \end{aligned} \quad (3)$$

with $\varsigma^- = \text{diag}(\varsigma_1^-, \varsigma_2^-, \dots, \varsigma_m^-)$, $\varsigma^+ = \text{diag}(\varsigma_1^+, \varsigma_2^+, \dots, \varsigma_m^+)$, $\sigma^- = \text{diag}(\sigma_1^-, \sigma_2^-, \dots, \sigma_n^-)$, and $\sigma^+ = \text{diag}(\sigma_1^+, \sigma_2^+, \dots, \sigma_n^+)$, $a, b \in R$, $a \neq b$.

Based on the current-voltage characteristic and the feature of memristor, the memristive connection weights $a_{ji}(x_i(t))$, $b_{ji}(x_i(t))$, $e_{ij}(y_j(t))$, and $h_{ij}(x_i(t))$ will change with time. Then, we let

$$\begin{aligned}a_{ji}(x_i(t)) &= \begin{cases} \hat{a}_{ji}, & |x_i(t)| > \Theta_i, \\ \check{a}_{ji}, & |x_i(t)| \leq \Theta_i, \end{cases} \\ b_{ji}(x_i(t)) &= \begin{cases} \hat{b}_{ji}, & |x_i(t)| > \Theta_i, \\ \check{b}_{ji}, & |x_i(t)| \leq \Theta_i, \end{cases} \\ e_{ij}(y_j(t)) &= \begin{cases} \hat{e}_{ij}, & |y_j(t)| > \Psi_j, \\ \check{e}_{ij}, & |y_j(t)| \leq \Psi_j, \end{cases} \\ h_{ij}(y_j(t)) &= \begin{cases} \hat{h}_{ij}, & |y_j(t)| > \Psi_j, \\ \check{h}_{ij}, & |y_j(t)| \leq \Psi_j, \end{cases} \end{aligned} \quad (4)$$

in which a_{ji} , b_{ji} , e_{ij} , and h_{ij} are constants and the switching jumps $\Theta_i > 0$, $\Psi_j > 0$.

Based on Figures 1 and 2, it is clear that $a_{ji}(x_i(t))$, $b_{ji}(x_i(t))$, $e_{ij}(y_j(t))$, and $h_{ij}(y_j(t))$ are piecewise continuous functions; the solutions of the systems are indicated in Filippov's sense and the interval is represented by $[\cdot, \cdot]$. Set

FIGURE 2: The characteristic of a piecewise linear charge-controlled memristor.

$$\begin{aligned}\bar{a}_{ji} &= \max \{ \hat{a}_{ji}, \check{a}_{ji} \}, \\ \underline{a}_{ji} &= \min \{ \hat{a}_{ji}, \check{a}_{ji} \}, \\ \bar{b}_{ji} &= \max \{ \hat{b}_{ji}, \check{b}_{ji} \}, \\ \underline{b}_{ji} &= \min \{ \hat{b}_{ji}, \check{b}_{ji} \}, \\ \bar{e}_{ij} &= \max \{ \hat{e}_{ij}, \check{e}_{ij} \}, \\ \underline{e}_{ij} &= \min \{ \hat{e}_{ij}, \check{e}_{ij} \}, \\ \bar{h}_{ij} &= \max \{ \hat{h}_{ij}, \check{h}_{ij} \}, \\ \underline{h}_{ij} &= \min \{ \hat{h}_{ij}, \check{h}_{ij} \}, \end{aligned} \quad (5)$$

for $i = 1, 2, \dots, n$, $j = 1, 2, \dots, m$. $\text{co}[\bar{r}, r]$ indicates the convex closure of $[\bar{r}, r]$. Obviously, the set-valued maps are defined as

$$\begin{aligned}\text{co}[a_{ji}(x_i(t))] &= \begin{cases} \hat{a}_{ji}, & |x_i(t)| > \Theta_i, \\ [\underline{a}_{ji}, \bar{a}_{ji}], & |x_i(t)| = \Theta_i, \\ \check{a}_{ji}, & |x_i(t)| < \Theta_i, \end{cases} \\ \text{co}[b_{ji}(x_i(t))] &= \begin{cases} \hat{b}_{ji}, & |x_i(t)| > \Theta_i, \\ [\underline{b}_{ji}, \bar{b}_{ji}], & |x_i(t)| = \Theta_i, \\ \check{b}_{ji}, & |x_i(t)| < \Theta_i, \end{cases} \\ \text{co}[e_{ij}(y_j(t))] &= \begin{cases} \hat{e}_{ij}, & |y_j(t)| > \Psi_j, \\ [\underline{e}_{ij}, \bar{e}_{ij}], & |y_j(t)| = \Psi_j, \\ \check{e}_{ij}, & |y_j(t)| < \Psi_j, \end{cases} \end{aligned}$$

$$\text{co} [h_{ij}(y_j(t))] = \begin{cases} \hat{h}_{ij}, & |y_j(t)| > \Psi_j, \\ [h_{ij}, \bar{h}_{ij}], & |y_j(t)| = \Psi_j, \\ \check{h}_{ij}, & |y_j(t)| < \Psi_j. \end{cases} \quad (6)$$

As a matter of convenience, we make the following assumptions.

Assumption 2. We define the probability distribution of time delays $\tau(t)$ and $d(t)$ as follows:

$$\begin{aligned} \text{Prob} \{ \tau(t) \in [0, \tau_1] \} &= \mu_0, \\ \text{Prob} \{ \tau(t) \in [\tau_1, \tau_2] \} &= 1 - \mu_0, \\ \text{Prob} \{ d(t) \in [0, d_1] \} &= \omega_0, \\ \text{Prob} \{ d(t) \in [d_1, d_2] \} &= 1 - \omega_0, \end{aligned} \quad (7)$$

where $0 \leq \mu_0 \leq 1$ and $0 \leq \omega_0 \leq 1$ are constants.

Thus, the random variables $\mu(t)$ and $\omega(t)$ can be defined as

$$\begin{aligned} \mu(t) &= \begin{cases} 1, & \tau(t) \in [0, \tau_1], \\ 0, & \tau(t) \in [\tau_1, \tau_2], \end{cases} \\ \omega(t) &= \begin{cases} 1, & d(t) \in [0, d_1], \\ 0, & d(t) \in [d_1, d_2]. \end{cases} \end{aligned} \quad (8)$$

Then, it can be derived that $\mu(t)$ and $\omega(t)$ are Bernoulli distributed sequences with

$$\begin{aligned} \text{Prob} \{ \mu(t) = 1 \} &= \mu_0, \\ \text{Prob} \{ \mu(t) = 0 \} &= 1 - \mu_0, \\ \text{Prob} \{ \omega(t) = 1 \} &= \omega_0, \\ \text{Prob} \{ \omega(t) = 0 \} &= 1 - \omega_0. \end{aligned} \quad (9)$$

According to Assumption 2, it is easy to see that

$$\begin{aligned} \mathbb{E} \{ \mu(t) - \mu_0 \} &= 0, \\ \mathbb{E} \{ (\mu(t) - \mu_0)^2 \} &= \mu_0(1 - \mu_0), \\ \mathbb{E} \{ \omega(t) - \omega_0 \} &= 0, \\ \mathbb{E} \{ (\omega(t) - \omega_0)^2 \} &= \omega_0(1 - \omega_0). \end{aligned} \quad (10)$$

Now, time-varying delays $\tau_1(t)$, $\tau_2(t)$, $d_1(t)$, and $d_2(t)$ are introduced as

$$\begin{aligned} \tau(t) &= \begin{cases} \tau_1(t), & \tau(t) \in [0, \tau_1], \\ \tau_2(t), & \tau(t) \in [\tau_1, \tau_2], \end{cases} \\ d(t) &= \begin{cases} d_1(t), & d(t) \in [0, d_1], \\ d_2(t), & d(t) \in [d_1, d_2]. \end{cases} \end{aligned} \quad (11)$$

Assumption 3. Here, constants τ_1 , τ_2 , d_1 , d_2 , μ_1 , μ_2 , ω_1 , and ω_2 exist, such that

$$\begin{aligned} 0 &\leq \tau_1(t) \leq \tau_1, \\ \dot{\tau}_1(t) &\leq \mu_1, \\ 0 &\leq \tau_2(t) \leq \tau_2, \\ \dot{\tau}_2(t) &\leq \mu_2, \\ 0 &\leq d_1(t) \leq d_1, \\ \dot{d}_1(t) &\leq \omega_1, \\ 0 &\leq d_2(t) \leq d_2, \\ \dot{d}_2(t) &\leq \omega_2. \end{aligned} \quad (12)$$

Assumption 4. The leakage delays $\delta(t)$ and $\rho(t)$ and distributed delays $\alpha(t)$ and $\beta(t)$ satisfy

$$\begin{aligned} 0 &\leq \delta(t) \leq \delta, \\ \dot{\delta}(t) &\leq \delta_0, \\ 0 &\leq \rho(t) \leq \rho, \\ \dot{\rho}(t) &\leq \rho_0, \\ 0 &\leq \alpha(t) \leq \alpha, \\ 0 &\leq \beta(t) \leq \beta. \end{aligned} \quad (13)$$

By employing the theories of set-valued maps, differential inclusions, stochastic variables μ_0 , ω_0 , and new functions $\tau_1(t)$, $\tau_2(t)$, $d_1(t)$, and $d_2(t)$, system (1) becomes

$$\begin{aligned} \dot{x}_i(t) &\in -C_i x_i(t) + \sum_{j=1}^m \text{co} [a_{ji}(x_i(t))] f_j(y_j(t)) \\ &\quad + \mu(t) \sum_{j=1}^m \text{co} [b_{ji}(x_i(t))] f_j(y_j(t - \tau_1(t))) \\ &\quad + (1 - \mu(t)) \sum_{j=1}^m \text{co} [b_{ji}(x_i(t))] f_j(y_j(t - \tau_2(t))) \\ &\quad + u_i(t), \\ \dot{y}_j(t) &\in -D_j y_j(t) + \sum_{i=1}^n \text{co} [e_{ij}(y_j(t))] g_i(x_i(t)) \\ &\quad + \omega(t) \sum_{i=1}^n \text{co} [h_{ij}(y_j(t))] g_i(x_i(t - d_1(t))) \\ &\quad + (1 - \omega(t)) \sum_{i=1}^n \text{co} [h_{ij}(y_j(t))] g_i(x_i(t - d_2(t))) \\ &\quad + v_j(t). \end{aligned} \quad (14)$$

Or it can be rewritten as follows:

$$\begin{aligned}
 \dot{x}(t) = & -Cx(t) + A(x(t))f(y(t)) \\
 & + \mu(t)B(x(t))f(y(t - \tau_1(t))) \\
 & + (1 - \mu(t))B(x(t))f(y(t - \tau_2(t))) \\
 & + u(t), \\
 \dot{y}(t) = & -Dy(t) + E(y(t))g(x(t)) \\
 & + \omega(t)H(y(t))g(x(t - d_1(t))) \\
 & + (1 - \omega(t))H(y(t))g(x(t - d_2(t))) \\
 & + v(t);
 \end{aligned} \tag{15}$$

equivalently,

$$\begin{aligned}
 \dot{x}(t) = & -Cx(t) + A(x(t))f(y(t)) + \mu_0 B(x(t)) \\
 & \cdot f(y(t - \tau_1(t))) + (1 - \mu_0)B(x(t)) \\
 & \cdot f(y(t - \tau_2(t))) + (\mu(t) - \mu_0)B(x(t)) \\
 & \cdot (f(y(t - \tau_1(t))) - f(y(t - \tau_2(t)))) + u(t), \\
 \dot{y}(t) = & -Dy(t) + E(y(t))g(x(t)) + \omega_0 H(y(t)) \\
 & \cdot g(x(t - d_1(t))) + (1 - \omega_0)H(y(t)) \\
 & \cdot g(x(t - d_2(t))) + (\omega(t) - \omega_0)H(y(t)) \\
 & \cdot (g(x(t - d_1(t))) - g(x(t - d_2(t)))) + v(t).
 \end{aligned} \tag{16}$$

Definition 5. System (15) is called passive if there exists a scalar $\gamma > 0$ such that

$$\begin{aligned}
 & 2 \int_0^{t_z} \begin{bmatrix} f^T(y(s)) & g^T(x(s)) \end{bmatrix} \begin{bmatrix} u(s) \\ v(s) \end{bmatrix} ds \\
 & \geq -\gamma \int_0^{t_z} \begin{bmatrix} u^T(s) & v^T(s) \end{bmatrix} \begin{bmatrix} u(s) \\ v(s) \end{bmatrix} ds,
 \end{aligned} \tag{17}$$

for all solutions of (15) with $x(0) = 0$ and $y(0) = 0$ and for all $t_z \geq 0$.

Remark 6. Passivity analysis originates from circuit theory and it uses the input-output description method based on energy to design and analyze a system. The physical meaning of passivity is reflected in Definition 5 where the energy growth of the system is always less than or equal to the total energy of the external inputs; this means that the passive system is always accompanied by the loss of energy. In fact, the storage function of the passive system can be used as a Lyapunov function under certain conditions. Thus, both Lyapunov stability theory and passivity theory can be used to research the stability of the system.

Lemma 7. For any scalars $p_2 > p_1 > 0$, matrix $R \in \mathfrak{R}^{n \times n}$, $R = R^T > 0$, and vector function $\omega : [p_1, p_2] \mapsto \mathbb{R}^n$, the inequalities hold as follows:

$$\begin{aligned}
 & (p_2 - p_1) \int_{p_1}^{p_2} \omega^T(s) R \omega(s) ds \\
 & \geq \left(\int_{p_1}^{p_2} \omega(s) ds \right)^T R \left(\int_{p_1}^{p_2} \omega(s) ds \right),
 \end{aligned} \tag{18}$$

$$\begin{aligned}
 & \frac{(p_2 - p_1)^2}{2} \int_{p_1}^{p_2} \int_{t+\theta}^t \omega^T(s) R \omega(s) ds d\theta \\
 & \geq \left(\int_{p_1}^{p_2} \int_{t+\theta}^t \omega(s) ds \right)^T R \left(\int_{p_1}^{p_2} \int_{t+\theta}^t \omega(s) ds \right).
 \end{aligned} \tag{19}$$

3. Main Results

For derivation convenience, we denote

$$\begin{aligned}
 A &= \max \{ |a_{ji}|, |\bar{a}_{ji}| \}, \\
 B &= \max \{ |b_{ji}|, |\bar{b}_{ji}| \}, \\
 E &= \max \{ |e_{ij}|, |\bar{e}_{ij}| \}, \\
 H &= \max \{ |h_{ij}|, |\bar{h}_{ij}| \}.
 \end{aligned} \tag{20}$$

Theorem 8. Under Assumptions 1–3, system (16) is passive, if there exist any appropriately dimensional matrices M_m ($m = 1, 2, \dots, 4$), W_w ($w = 1, 2, \dots, 6$), a scalar $\gamma > 0$, and symmetric positive definite matrices R_i ($i = 1, 2$), Q_j ($j = 3, 4, \dots, 6$), O_o ($o = 3, 4, \dots, 6$), P_k ($k = 2, 3, 6, 7$), Z_l ($l = 1, 2, \dots, 4$), and N_n ($n = 1, 2$), such that the following LMIs hold:

$$\begin{aligned}
 \Omega_{i,j} &= (\Omega_{i,j})_{14 \times 14} < 0, \\
 \Phi_{i,j} &= (\Phi_{i,j})_{14 \times 14} < 0,
 \end{aligned} \tag{21}$$

where

$$\begin{aligned}
 \Omega_{1,1} &= -R_1 C - C^T R_1 + Q_3 + Q_4 - P_2 - 2Z_1 - 2Z_2 \\
 &\quad - M_1 C - C^T M_1 - 2\zeta^- W_1 \zeta^+, \\
 \Omega_{1,4} &= P_2, \\
 \Omega_{1,6} &= -M_1 - M_2 C, \\
 \Omega_{1,7} &= R_1 A + M_1 A + W_1 (\zeta^- + \zeta^+), \\
 \Omega_{1,8} &= \mu_0 (R_1 + M_1) B, \\
 \Omega_{1,9} &= (1 - \mu_0) (M_1 + R_1) B, \\
 \Omega_{1,12} &= \frac{2}{\tau_1} Z_1,
 \end{aligned}$$

$$\begin{aligned}
\Omega_{1,13} &= \frac{2}{\tau_2 - \tau_1} Z_2, \\
\Omega_{1,14} &= R_1 + M_1, \\
\Omega_{2,2} &= -(1 - \mu_1) Q_3 - 2\zeta^- W_2 \zeta^+, \\
\Omega_{2,8} &= -(1 - \mu_1) Q_3 + W_2 (\zeta^- + \zeta^+), \\
\Omega_{3,3} &= -(1 - \mu_2) Q_5 - 2\zeta^- W_3 \zeta^+, \\
\Omega_{3,9} &= -(1 - \mu_2) Q_5 + W_3 (\zeta^- + \zeta^+), \\
\Omega_{4,4} &= Q_5 + Q_6 - Q_4 - P_2 - P_3, \\
\Omega_{4,5} &= P_3, \\
\Omega_{4,10} &= Q_5 + Q_6 - Q_4, \\
\Omega_{5,5} &= -Q_6 - P_3, \\
\Omega_{5,11} &= -Q_6, \\
\Omega_{6,6} &= \tau_1^2 P_2 + (\tau_2 - \tau_1)^2 P_3 + \frac{\tau_1^2}{2} Z_1 + \frac{\tau_2^2 - \tau_1^2}{2} Z_2 \\
&\quad - 2M_2, \\
\Omega_{6,7} &= M_2 A, \\
\Omega_{6,8} &= \mu_0 M_2 B, \\
\Omega_{6,9} &= (1 - \mu_0) M_2 B, \\
\Omega_{6,14} &= M_2, \\
\Omega_{7,7} &= -2W_1, \\
\Omega_{7,14} &= -I, \\
\Omega_{8,8} &= -(1 - \mu_1) Q_3 - 2W_2, \\
\Omega_{9,9} &= -(1 - \mu_2) Q_5 - 2W_3, \\
\Omega_{10,10} &= Q_5 + Q_6 - Q_4, \\
\Omega_{11,11} &= -Q_6, \\
\Omega_{12,12} &= -\frac{2}{\tau_1^2} Z_1, \\
\Omega_{13,13} &= -\frac{2}{(\tau_2 - \tau_1)^2} Z_2, \\
\Omega_{14,14} &= -\gamma I, \\
\Phi_{1,1} &= -R_2 D - D^T R_2 + O_3 + O_4 - P_6 - 2Z_3 - 2Z_4 \\
&\quad - M_3 D - D^T M_3 - 2\sigma^- W_4 \sigma^+, \\
\Phi_{1,4} &= P_6, \\
\Phi_{1,6} &= -M_3 - M_4 D, \\
\Phi_{1,7} &= R_2 E + M_3 E + W_4 (\sigma^- + \sigma^+),
\end{aligned} \tag{22}$$

$$\begin{aligned}
\Phi_{1,8} &= \omega_0 (R_2 + M_3) H, \\
\Phi_{1,9} &= (1 - \omega_0) (M_3 + R_2) H, \\
\Phi_{1,12} &= \frac{2}{d_1} Z_3, \\
\Phi_{1,13} &= \frac{2}{d_2 - d_1} Z_4, \\
\Phi_{1,14} &= R_2 + M_3, \\
\Phi_{2,2} &= -(1 - \omega_1) O_3 - 2\sigma^- W_5 \sigma^+, \\
\Phi_{2,8} &= -(1 - \omega_1) O_3 + W_5 (\sigma^- + \sigma^+), \\
\Phi_{3,3} &= -(1 - \omega_2) O_5 - 2\sigma^- W_6 \sigma^+, \\
\Phi_{3,9} &= -(1 - \omega_2) O_5 + W_6 (\sigma^- + \sigma^+), \\
\Phi_{4,4} &= O_5 + O_6 - O_4 - P_6 - P_7, \\
\Phi_{4,5} &= P_7, \\
\Phi_{4,10} &= O_5 + O_6 - O_4, \\
\Phi_{5,5} &= -O_6 - P_7, \\
\Phi_{5,11} &= -O_6, \\
\Phi_{6,6} &= d_1^2 P_6 + (d_2 - d_1)^2 P_7 + \frac{d_1^2}{2} Z_3 + \frac{d_2^2 - d_1^2}{2} Z_4 \\
&\quad - 2M_4, \\
\Phi_{6,7} &= M_4 E, \\
\Phi_{6,8} &= \omega_0 M_4 H, \\
\Phi_{6,9} &= (1 - \omega_0) M_4 H, \\
\Phi_{6,14} &= M_4, \\
\Phi_{7,7} &= -2W_4, \\
\Phi_{7,14} &= -I, \\
\Phi_{8,8} &= -(1 - \omega_1) O_3 - 2W_5, \\
\Phi_{9,9} &= -(1 - \omega_2) O_5 - 2W_6, \\
\Phi_{10,10} &= O_5 + O_6 - O_4, \\
\Phi_{11,11} &= -O_6, \\
\Phi_{12,12} &= -\frac{2}{d_1^2} Z_3, \\
\Phi_{13,13} &= -\frac{2}{(d_2 - d_1)^2} Z_4, \\
\Phi_{14,14} &= -\gamma I.
\end{aligned} \tag{23}$$

Proof. Consider the following LKF candidate:

$$V(t) = \sum_{i=1}^8 V_i(t), \quad (24)$$

where

$$V_1(t) = x^T(t) R_1 x(t),$$

$$V_2(t) = y^T(t) R_2 y(t),$$

$$\begin{aligned} V_3(t) = & \int_{t-\tau_1(t)}^t \begin{bmatrix} x(s) \\ f(y(s)) \end{bmatrix}^T Q_3 \begin{bmatrix} x(s) \\ f(y(s)) \end{bmatrix} ds \\ & + \int_{t-\tau_1}^t \begin{bmatrix} x(s) \\ f(y(s)) \end{bmatrix}^T Q_4 \begin{bmatrix} x(s) \\ f(y(s)) \end{bmatrix} ds \\ & + \int_{t-\tau_2(t)}^{t-\tau_1} \begin{bmatrix} x(s) \\ f(y(s)) \end{bmatrix}^T Q_5 \begin{bmatrix} x(s) \\ f(y(s)) \end{bmatrix} ds \\ & + \int_{t-\tau_2}^{t-\tau_1} \begin{bmatrix} x(s) \\ f(y(s)) \end{bmatrix}^T Q_6 \begin{bmatrix} x(s) \\ f(y(s)) \end{bmatrix} ds, \end{aligned}$$

$$\begin{aligned} V_4(t) = & \int_{t-d_1(t)}^t \begin{bmatrix} y(s) \\ g(x(s)) \end{bmatrix}^T O_3 \begin{bmatrix} y(s) \\ g(x(s)) \end{bmatrix} ds \\ & + \int_{t-d_1}^t \begin{bmatrix} y(s) \\ g(x(s)) \end{bmatrix}^T O_4 \begin{bmatrix} y(s) \\ g(x(s)) \end{bmatrix} ds \\ & + \int_{t-d_2(t)}^{t-d_1} \begin{bmatrix} y(s) \\ g(x(s)) \end{bmatrix}^T O_5 \begin{bmatrix} y(s) \\ g(x(s)) \end{bmatrix} ds \\ & + \int_{t-d_2}^{t-d_1} \begin{bmatrix} y(s) \\ g(x(s)) \end{bmatrix}^T O_6 \begin{bmatrix} y(s) \\ g(x(s)) \end{bmatrix} ds, \end{aligned} \quad (25)$$

$$\begin{aligned} V_5(t) = & \tau_1 \int_{-\tau_1}^0 \int_{t+\theta}^t \dot{x}^T(s) P_2 \dot{x}(s) ds d\theta \\ & + (\tau_2 - \tau_1) \int_{-\tau_2}^{-\tau_1} \int_{t+\theta}^t \dot{x}^T(s) P_3 \dot{x}(s) ds d\theta, \end{aligned}$$

$$\begin{aligned} V_6(t) = & d_1 \int_{-d_1}^0 \int_{t+\theta}^t \dot{y}^T(s) P_6 \dot{y}(s) ds d\theta \\ & + (d_2 - d_1) \int_{-d_2}^{-d_1} \int_{t+\theta}^t \dot{y}^T(s) P_7 \dot{y}(s) ds d\theta, \end{aligned}$$

$$\begin{aligned} V_7(t) = & \int_{-\tau_1}^0 \int_{\theta}^0 \int_{t+\lambda}^t \dot{x}^T(s) Z_1 \dot{x}(s) ds d\lambda d\theta \\ & + \int_{-\tau_2}^{-\tau_1} \int_{\theta}^0 \int_{t+\lambda}^t \dot{x}^T(s) Z_2 \dot{x}(s) ds d\lambda d\theta, \end{aligned}$$

$$\begin{aligned} V_8(t) = & \int_{-d_1}^0 \int_{\theta}^0 \int_{t+\lambda}^t \dot{y}^T(s) Z_3 \dot{y}(s) ds d\lambda d\theta \\ & + \int_{-d_2}^{-d_1} \int_{\theta}^0 \int_{t+\lambda}^t \dot{y}^T(s) Z_4 \dot{y}(s) ds d\lambda d\theta. \end{aligned}$$

Then, we define the infinitesimal generator $\mathcal{L}$ of $V(t)$ as

$$\mathcal{L}V(t) = \lim_{\Delta \rightarrow 0^+} \frac{1}{\Delta} \left\{ \mathbb{E} \left(\frac{V(t+\Delta)}{t} \right) - V(t) \right\}. \quad (26)$$

Taking the mathematical expectation of $V(t)$, we get

$$\begin{aligned} \mathbb{E} \{ \mathcal{L}V_1(t) \} = & \mathbb{E} \left\{ 2x^T(t) R_1 (-Cx(t) + A(x(t)) \right. \\ & \cdot f(y(t)) + \mu_0 B(x(t)) f(y(t - \tau_1(t))) \\ & + (1 - \mu_0) B(x(t)) f(y(t - \tau_2(t))) + (\mu(t) - \mu_0) \\ & \cdot B(x(t)) (f(y(t - \tau_1(t))) - f(y(t - \tau_2(t)))) \\ & + u(t) \} \leq \mathbb{E} \left\{ 2x^T(t) R_1 (-Cx(t) + Af(y(t)) \right. \\ & + \mu_0 Bf(y(t - \tau_1(t))) + (1 - \mu_0) \\ & \cdot Bf(y(t - \tau_2(t))) + u(t) \} \}, \end{aligned} \quad (27)$$

$$\begin{aligned} \mathbb{E} \{ \mathcal{L}V_2(t) \} = & \mathbb{E} \left\{ 2y^T(t) R_2 (-Dy(t) + E(y(t)) \right. \\ & \cdot g(x(t)) + \omega_0 H(y(t)) g(x(t - d_1(t))) \\ & + (1 - \omega_0) H(y(t)) g(x(t - d_2(t))) + (\omega(t) \\ & - \omega_0) H(y(t)) (g(x(t - d_1(t))) \\ & - g(x(t - d_2(t)))) + v(t) \} \leq \mathbb{E} \left\{ 2y^T(t) \right. \\ & \cdot R_2 (-Dy(t) + Eg(x(t)) + \omega_0 Hg(x(t - d_1(t))) \\ & + (1 - \omega_0) Hg(x(t - d_2(t))) + v(t) \} \}. \end{aligned}$$

According to Assumption 3, we obtain

$$\begin{aligned} \mathbb{E} \{ \mathcal{L}V_3(t) \} \leq & \mathbb{E} \left\{ x^T(t) (Q_3 + Q_4) x(t) - (1 - \mu_1) \right. \\ & \cdot \begin{bmatrix} x(t - \tau_1(t)) \\ f(y(t - \tau_1(t))) \end{bmatrix}^T Q_3 \begin{bmatrix} x(t - \tau_1(t)) \\ f(y(t - \tau_1(t))) \end{bmatrix} \\ & + \begin{bmatrix} x(t - \tau_1) \\ f(y(t - \tau_1)) \end{bmatrix}^T (Q_5 + Q_6 - Q_4) \\ & \cdot \begin{bmatrix} x(t - \tau_1) \\ f(y(t - \tau_1)) \end{bmatrix} - (1 - \mu_2) \begin{bmatrix} x(t - \tau_2(t)) \\ f(y(t - \tau_2(t))) \end{bmatrix}^T \\ & \cdot Q_5 \begin{bmatrix} x(t - \tau_2(t)) \\ f(y(t - \tau_2(t))) \end{bmatrix} - \begin{bmatrix} x(t - \tau_2) \\ f(y(t - \tau_2)) \end{bmatrix}^T \\ & \cdot Q_6 \begin{bmatrix} x(t - \tau_2) \\ f(y(t - \tau_2)) \end{bmatrix} \} \}, \end{aligned}$$

$$\begin{aligned}
\mathbb{E} \{ \mathcal{L}V_4(t) \} \leq & \mathbb{E} \left\{ y^T(t) (O_3 + O_4) y(t) - (1 - \omega_1) \right. \\
& \cdot \left[\begin{array}{c} y(t - d_1(t)) \\ g(x(t - d_1(t))) \end{array} \right]^T O_3 \left[\begin{array}{c} y(t - d_1(t)) \\ g(x(t - d_1(t))) \end{array} \right] \\
& + \left[\begin{array}{c} y(t - d_1) \\ g(x(t - d_1)) \end{array} \right]^T (O_5 + O_6 - O_4) \\
& \cdot \left[\begin{array}{c} y(t - d_1) \\ g(x(t - d_1)) \end{array} \right] - (1 - \omega_2) \left[\begin{array}{c} y(t - d_2(t)) \\ g(x(t - d_2(t))) \end{array} \right]^T \\
& \cdot O_5 \left[\begin{array}{c} y(t - d_2(t)) \\ g(x(t - d_2(t))) \end{array} \right] - \left[\begin{array}{c} y(t - \tau_2) \\ g(x(t - \tau_2)) \end{array} \right]^T \\
& \cdot O_6 \left[\begin{array}{c} y(t - \tau_2) \\ g(x(t - \tau_2)) \end{array} \right] \left. \right\}. \tag{28}
\end{aligned}$$

It is obvious that

$$\begin{aligned}
\mathbb{E} \{ \mathcal{L}V_5(t) \} = & \mathbb{E} \left\{ \dot{x}^T(t) [\tau_1^2 P_2 + (\tau_2 - \tau_1)^2 P_3] \dot{x}(t) \right. \\
& - \tau_1 \int_{t-\tau_1}^t \dot{x}^T(s) P_2 \dot{x}(s) ds \\
& \left. - (\tau_2 - \tau_1) \int_{t-\tau_2}^{t-\tau_1} \dot{x}^T(s) P_3 \dot{x}(s) ds \right\}, \tag{29} \\
\mathbb{E} \{ \mathcal{L}V_6(t) \} = & \mathbb{E} \left\{ \dot{y}^T(t) [d_1^2 P_6 + (d_2 - d_1)^2 P_7] \dot{y}(t) \right. \\
& - d_1 \int_{t-d_1}^t \dot{y}^T(s) P_6 \dot{y}(s) ds \\
& \left. - (d_2 - d_1) \int_{t-d_2}^{t-d_1} \dot{y}^T(s) P_7 \dot{y}(s) ds \right\}.
\end{aligned}$$

Using Lemma 7, we have

$$\begin{aligned}
& - \tau_1 \int_{t-\tau_1}^t \dot{x}^T(s) P_2 \dot{x}(s) ds \leq - \left(\int_{t-\tau_1}^t \dot{x}(s) ds \right)^T \\
& \cdot P_2 \left(\int_{t-\tau_1}^t \dot{x}(s) ds \right) = - [x(t) - x(t - \tau_1)]^T \\
& \cdot P_2 [x(t) - x(t - \tau_1)], \\
& - (\tau_2 - \tau_1) \int_{t-\tau_2}^{t-\tau_1} \dot{x}^T(s) P_3 \dot{x}(s) ds \\
& \leq - \left(\int_{t-\tau_2}^{t-\tau_1} \dot{x}(s) ds \right)^T P_3 \left(\int_{t-\tau_2}^{t-\tau_1} \dot{x}(s) ds \right)
\end{aligned}$$

$$\begin{aligned}
& = - [x(t - \tau_1) - x(t - \tau_2)]^T \\
& \cdot P_3 [x(t - \tau_1) - x(t - \tau_2)]. \tag{30}
\end{aligned}$$

Similarly,

$$\begin{aligned}
& - d_1 \int_{t-d_1}^t \dot{y}^T(s) P_6 \dot{y}(s) ds \leq - \left(\int_{t-d_1}^t \dot{y}(s) ds \right)^T \\
& \cdot P_6 \left(\int_{t-d_1}^t \dot{y}(s) ds \right) = - [y(t) - y(t - d_1)]^T \\
& \cdot P_6 [y(t) - y(t - d_1)], \\
& - (d_2 - d_1) \int_{t-d_2}^{t-d_1} \dot{y}^T(s) P_7 \dot{y}(s) ds \\
& \leq - \left(\int_{t-d_2}^{t-d_1} \dot{y}(s) ds \right)^T P_7 \left(\int_{t-d_2}^{t-d_1} \dot{y}(s) ds \right) \tag{31} \\
& = - [y(t - d_1) - y(t - d_2)]^T \\
& \cdot P_7 [y(t - d_1) - y(t - d_2)].
\end{aligned}$$

Moreover, we obtain

$$\begin{aligned}
\mathbb{E} \{ \mathcal{L}V_7(t) \} = & \mathbb{E} \left\{ \dot{x}^T(t) \left(\frac{\tau_1^2}{2} Z_1 + \frac{\tau_2^2 - \tau_1^2}{2} Z_2 \right) \dot{x}(t) \right. \\
& - \int_{-\tau_1}^0 \int_{t+\theta}^t \dot{x}^T(s) Z_1 \dot{x}(s) ds d\theta \\
& \left. - \int_{-\tau_2}^{-\tau_1} \int_{t+\theta}^t \dot{x}^T(s) Z_2 \dot{x}(s) ds d\theta \right\}, \tag{32} \\
\mathbb{E} \{ \mathcal{L}V_8(t) \} = & \mathbb{E} \left\{ \dot{y}^T(t) \left(\frac{d_1^2}{2} Z_3 + \frac{d_2^2 - d_1^2}{2} Z_4 \right) \dot{y}(t) \right. \\
& - \int_{-d_1}^0 \int_{t+\theta}^t \dot{y}^T(s) Z_3 \dot{y}(s) ds d\theta \\
& \left. - \int_{-d_2}^{-d_1} \int_{t+\theta}^t \dot{y}^T(s) Z_4 \dot{y}(s) ds d\theta \right\}.
\end{aligned}$$

Using (18), we have

$$\begin{aligned}
& - \int_{-\tau_1}^0 \int_{t+\theta}^t \dot{x}^T(s) Z_1 \dot{x}(s) ds d\theta \\
& \leq - \frac{2}{\tau_1^2} \left(\int_{-\tau_1}^0 \int_{t+\theta}^t \dot{x}(s) ds d\theta \right)^T \\
& \cdot Z_1 \left(\int_{-\tau_1}^0 \int_{t+\theta}^t \dot{x}(s) ds d\theta \right) = - 2x^T(t) Z_1 x(t) \\
& + \frac{4}{\tau_1} x^T(t) Z_1 \left(\int_{t-\tau_1}^t x(s) ds \right) \\
& - \frac{2}{\tau_1^2} \left(\int_{t-\tau_1}^t x(s) ds \right)^T Z_1 \left(\int_{t-\tau_1}^t x(s) ds \right),
\end{aligned}$$

$$\begin{aligned}
& - \int_{-\tau_2}^{-\tau_1} \int_{t+\theta}^t \dot{x}^T(s) Z_2 \dot{x}(s) ds d\theta \\
& \leq - \frac{2}{(\tau_2 - \tau_1)^2} \left(\int_{-\tau_2}^{-\tau_1} \int_{t+\theta}^t \dot{x}(s) ds d\theta \right)^T \\
& \times Z_2 \left(\int_{-\tau_2}^{-\tau_1} \int_{t+\theta}^t \dot{x}(s) ds d\theta \right) = -2x^T(t) Z_2 x(t) \\
& + \frac{4}{\tau_2 - \tau_1} x^T(t) Z_2 \left(\int_{t-\tau_2}^{t-\tau_1} x(s) ds \right) \\
& - \frac{2}{(\tau_2 - \tau_1)^2} \left(\int_{t-\tau_2}^{t-\tau_1} x(s) ds \right)^T Z_2 \left(\int_{t-\tau_2}^{t-\tau_1} x(s) ds \right). \quad (34)
\end{aligned}$$

Similarly,

$$\begin{aligned}
& - \int_{-d_1}^0 \int_{t+\theta}^t \dot{y}^T(s) Z_3 \dot{y}(s) ds d\theta \\
& \leq - \frac{2}{d_1^2} \left(\int_{-d_1}^0 \int_{t+\theta}^t \dot{y}(s) ds d\theta \right)^T \\
& \cdot Z_3 \left(\int_{-d_1}^0 \int_{t+\theta}^t \dot{y}(s) ds d\theta \right) = -2y^T(t) Z_3 y(t) \quad (35) \\
& + \frac{4}{d_1} y^T(t) Z_3 \left(\int_{t-d_1}^t y(s) ds \right) \\
& - \frac{2}{d_1^2} \left(\int_{t-d_1}^t y(s) ds \right)^T Z_3 \left(\int_{t-d_1}^t y(s) ds \right), \\
& - \int_{-d_2}^{-d_1} \int_{t+\theta}^t y^T(s) Z_4 y(s) ds d\theta \\
& \leq - \frac{2}{(d_2 - d_1)^2} \left(\int_{-d_2}^{-d_1} \int_{t+\theta}^t y(s) ds d\theta \right)^T \\
& \times Z_4 \left(\int_{-d_2}^{-d_1} \int_{t+\theta}^t y(s) ds d\theta \right) = -2y^T(t) Z_4 y(t) \\
& + \frac{4}{d_2 - d_1} y^T(t) Z_4 \left(\int_{t-d_2}^{t-d_1} y(s) ds \right) \\
& - \frac{2}{(d_2 - d_1)^2} \left(\int_{t-d_2}^{t-d_1} y(s) ds \right)^T \\
& \cdot Z_4 \left(\int_{t-d_2}^{t-d_1} y(s) ds \right). \quad (36)
\end{aligned}$$

To derive a less conservative criterion, we add the following inequations with any matrices M_1 , M_2 , M_3 , and M_4 of appropriate dimensions:

$$\begin{aligned}
0 \leq & 2 \left(x^T(t) M_1 + \dot{x}^T(t) M_2 \right) [-\dot{x} - Cx(t) \\
& + Af(y(t)) + \mu_0 Bf(y(t - \tau_1(t)))] ,
\end{aligned}$$

$$\begin{aligned}
& + (1 - \mu_0) Bf(y(t - \tau_2(t))) + u(t) , \\
0 \leq & 2 \left(y^T(t) M_3 + \dot{y}^T(t) M_4 \right) [-\dot{y} - Dy(t) \\
& + Eg(x(t)) + \omega_0 Hg(x(t - d_1(t))) \\
& + (1 - \omega_0) Hg(x(t - d_2(t))) + v(t)] . \quad (37)
\end{aligned}$$

Based on Assumption 1 [20], we have

$$\begin{aligned}
& \zeta^- x(t) - f(y(t)) \leq 0, \\
& f(y(t)) - \zeta^+ x(t) \leq 0, \\
& \sigma^- y(t) - g(x(t)) \leq 0, \\
& g(x(t)) - \sigma^+ y(t) \leq 0, \quad (38)
\end{aligned}$$

and there exist positive diagonal matrices $W_i > 0$ ($i = 1, 2, \dots, 8$); the following inequalities hold:

$$\begin{aligned}
0 \leq & 2 [\zeta^- x(t) - f(y(t))]^T W_1 [f(y(t)) - \zeta^+ x(t)] \\
& \leq -2f^T(y(t)) W_1 f(y(t)) \\
& + 2x^T(t) W_1 (\zeta^- + \zeta^+) f(y(t)) \\
& - 2x^T(t) \zeta^- W_1 \zeta^+ x(t). \quad (39)
\end{aligned}$$

Similarly,

$$\begin{aligned}
0 \leq & -2f^T(y(t - \tau_1)) W_2 f(y(t - \tau_1)) \\
& + 2x(t)^T (t - \tau_1) W_2 (\zeta^- + \zeta^+) f(y(t - \tau_1)) \\
& - 2x^T(t - \tau_1) \zeta^- W_2 \zeta^+ x(t - \tau_1), \\
0 \leq & -2f^T(y(t - \tau_2)) W_3 f(y(t - \tau_2)) \\
& + 2x^T(t - \tau_2) W_3 (\zeta^- + \zeta^+) f(y(t - \tau_2)) \\
& - 2x^T(t - \tau_2) \zeta^- W_3 \zeta^+ x(t - \tau_2), \\
0 \leq & 2 [\sigma^- y(t) - g(x(t))]^T W_4 [g(x(t)) - \sigma^+ y(t)] \\
& \leq -2g^T(x(t)) W_4 g(x(t)) \\
& + 2y^T(t) W_4 (\sigma^- + \sigma^+) g(x(t)) \\
& - 2y^T(t) \sigma^- W_4 \sigma^+ y(t), \\
0 \leq & -2g^T(x(t - d_1)) W_5 g(x(t - d_1)) \\
& + 2y(t)^T (t - d_1) W_5 (\sigma^- + \sigma^+) g(x(t - d_1)) \\
& - 2y^T(t - d_1) \sigma^- W_5 \sigma^+ y(t - d_1), \\
0 \leq & -2g^T(x(t - d_2)) W_6 g(x(t - d_2)) \\
& + 2y^T(t - d_2) W_6 (\sigma^- + \sigma^+) g(x(t - d_2)) \\
& - 2y^T(t - d_2) \sigma^- W_6 \sigma^+ y(t - d_2). \quad (40)
\end{aligned}$$

Define

$$\begin{aligned} \xi(t) = & \begin{Bmatrix} x^T(t) & x^T(t - \tau_1(t)) & x^T(t - \tau_2(t)) \\ x^T(t - \tau_1) & x^T(t - \tau_2) & \dot{x}(t) & f^T(y(t)) \\ f^T(y(t - \tau_1(t))) & f^T(y(t - \tau_2(t))) & f^T(y(t - \tau_1)) \\ f^T(y(t - \tau_2)) & \int_{t-\tau_1}^t x^T(s) ds & \int_{t-\tau_2}^{t-\tau_1} x^T(s) ds & u^T(t) \end{Bmatrix}, \\ \zeta(t) = & \begin{Bmatrix} y^T(t) & y^T(t - d_1(t)) & y^T(t - d_2(t)) \\ y^T(t - d_1) & y^T(t - d_2) & \dot{y}(t) & g^T(x(t)) & g^T(x(t - d_1(t))) \\ g^T(x(t - d_2(t))) & g^T(x(t - d_1)) & g^T(x(t - d_2)) \\ \int_{t-d_1}^t y^T(s) ds & \int_{t-d_2}^{t-d_1} y^T(s) ds & v^T(t) \end{Bmatrix}. \end{aligned} \quad (41)$$

Combining (23), (27)–(37), and (40), we have

$$\begin{aligned} \mathbb{E} \left\{ \mathcal{L}V(t) - 2 \begin{bmatrix} f^T(y(s)) & g^T(x(s)) \end{bmatrix} \begin{bmatrix} u(s) \\ v(s) \end{bmatrix} \right. \\ \left. - \gamma \begin{bmatrix} u^T(t) & v^T(t) \end{bmatrix} \begin{bmatrix} u(t) \\ v(t) \end{bmatrix} ds \right\} \leq \mathbb{E} \left\{ \xi^T(t) \Omega \xi(t) \right. \\ \left. + \zeta^T(t) \Phi \zeta(t) \right\}. \end{aligned} \quad (42)$$

Thus, we conclude that if (42) holds, then

$$\begin{aligned} \mathbb{E} \left\{ \mathcal{L}V(t) - 2 \begin{bmatrix} f^T(y(s)) & g^T(x(s)) \end{bmatrix} \begin{bmatrix} u(s) \\ v(s) \end{bmatrix} \right. \\ \left. - \gamma \begin{bmatrix} u^T(t) & v^T(t) \end{bmatrix} \begin{bmatrix} u(t) \\ v(t) \end{bmatrix} ds \right\} \leq 0. \end{aligned} \quad (43)$$

By integrating (43) about t from 0 to t_z , we get

$$\begin{aligned} 2\mathbb{E} \int_0^{t_z} \begin{bmatrix} f^T(y(s)) & g^T(x(s)) \end{bmatrix} \begin{bmatrix} u(s) \\ v(s) \end{bmatrix} ds \\ \geq \mathbb{E}V(t) - \mathbb{E}V(0) \\ - \gamma \mathbb{E} \int_0^{t_z} \begin{bmatrix} u^T(s) & v^T(s) \end{bmatrix} \begin{bmatrix} u(s) \\ v(s) \end{bmatrix} ds \\ \geq -\gamma \mathbb{E} \int_0^{t_z} \begin{bmatrix} u^T(s) & v^T(s) \end{bmatrix} \begin{bmatrix} u(s) \\ v(s) \end{bmatrix} ds. \end{aligned} \quad (44)$$

Therefore, from Definition 5, we know that the MBAMNN is passive. This completes the proof. $\square$

Considering the MBAMNN with leakage, probabilistic, and distributed time-varying delays, then model (1) becomes

$$\begin{aligned} \dot{x}_i(t) = & -C_i x_i(t - \delta(t)) + \sum_{j=1}^m a_{ji}(x_i(t)) f_j(y_j(t)) \\ & + \sum_{j=1}^m b_{ji}(x_i(t)) f_j(y_j(t - \tau(t))) \end{aligned}$$

$$\begin{aligned} & + \sum_{j=1}^m k_{ji}(x_i(t)) \int_{t-\alpha(t)}^t f_j(y_j(s)) ds \\ & + u_i(t), \end{aligned}$$

$$\begin{aligned} \dot{y}_j(t) = & -D_j y_j(t - \rho(t)) + \sum_{i=1}^n e_{ij}(y_j(t)) g_i(x_i(t)) \\ & + \sum_{i=1}^n h_{ij}(y_j(t)) g_i(x_i(t - d(t))) \\ & + \sum_{i=1}^n l_{ij}(y_j(t)) \int_{t-\beta(t)}^t g_i(x_i(s)) ds \\ & + v_j(t), \end{aligned} \quad (45)$$

where the nonnegative continuous variables $\delta(t)$, $\rho(t)$ and $\alpha(t)$, $\beta(t)$ correspond to the leakage and distributed time-varying delays and $K(x(t)) = (k_{ji}(x_i(t)))_{m \times n}$ and $L(x(t)) = (l_{ij}(y_j(t)))_{n \times m}$ are the delayed connection weight matrix functions. Then, we let

$$\begin{aligned} k_{ji}(x_i(t)) = & \begin{cases} \hat{b}_{ji}, & |x_i(t)| > \Theta_i, \\ \check{b}_{ji}, & |x_i(t)| \leq \Theta_i, \end{cases} \\ l_{ij}(y_j(t)) = & \begin{cases} \hat{f}_{ij}, & |y_j(t)| > \Psi_j, \\ \check{f}_{ij}, & |y_j(t)| \leq \Psi_j, \end{cases} \end{aligned} \quad (46)$$

in which k_{ji} and l_{ij} are known constants and $k_{ji}(x_i(t))$ and $l_{ij}(y_j(t))$ are piecewise continuous functions; we set

$$\begin{aligned} \bar{k}_{ji} = & \max \{ \hat{k}_{ji}, \check{k}_{ji} \}, \\ \underline{k}_{ji} = & \min \{ \hat{k}_{ji}, \check{k}_{ji} \}, \\ \bar{l}_{ij} = & \max \{ \hat{l}_{ij}, \check{l}_{ij} \}, \\ \underline{l}_{ij} = & \min \{ \hat{l}_{ij}, \check{l}_{ij} \}. \end{aligned} \quad (47)$$

Then, the set-valued maps are defined as

$$\begin{aligned} \text{co} [k_{ji}(x_i(t))] = & \begin{cases} \hat{k}_{ji}, & |x_i(t)| > \Theta_i, \\ [\underline{k}_{ji}, \bar{k}_{ji}], & |x_i(t)| = \Theta_i, \\ \check{k}_{ji}, & |x_i(t)| < \Theta_i, \end{cases} \\ \text{co} [l_{ij}(y_j(t))] = & \begin{cases} \hat{l}_{ij}, & |y_j(t)| > \Psi_j, \\ [\underline{l}_{ij}, \bar{l}_{ij}], & |y_j(t)| = \Psi_j, \\ \check{l}_{ij}, & |y_j(t)| < \Psi_j. \end{cases} \end{aligned} \quad (48)$$

By applying the theories of set-valued maps, differential inclusions, stochastic variables, and new functions, system (44) becomes

$$\begin{aligned} \dot{x}_i(t) & \in -C_i x_i(t - \delta(t)) + \sum_{j=1}^m \text{co} [a_{ji}(x_i(t))] f_j(y_j(t)) \\ & + \mu(t) \sum_{j=1}^m \text{co} [b_{ji}(x_i(t))] f_j(y_j(t - \tau_1(t))) \\ & + (1 - \mu(t)) \sum_{j=1}^m \text{co} [b_{ji}(x_i(t))] f_j(y_j(t - \tau_2(t))) \\ & + \sum_{j=1}^m \text{co} [k_{ji}(x_i(t))] \int_{t-\alpha(t)}^t f(y(s)) ds + u_i(t), \end{aligned} \quad (49)$$

$$\begin{aligned} \dot{y}_j(t) & \in -D_j y_j(t - \rho(t)) + \sum_{i=1}^n \text{co} [e_{ij}(y_j(t))] g_i(x_i(t)) \\ & + \omega(t) \sum_{i=1}^n \text{co} [h_{ij}(y_j(t))] g_i(x_i(t - d_1(t))) \\ & + (1 - \omega(t)) \sum_{i=1}^n \text{co} [h_{ij}(y_j(t))] g_i(x_i(t - d_2(t))) \\ & + \sum_{i=1}^n \text{co} [l_{ij}(y_j(t))] \int_{t-\beta(t)}^t g(x(s)) ds + v_j(t); \end{aligned}$$

equivalently,

$$\begin{aligned} \dot{x}(t) & = -Cx(t - \delta(t)) + A(x(t)) f(y(t)) \\ & + \mu_0 B(x(t)) f(y(t - \tau_1(t))) \\ & + (1 - \mu_0) B(x(t)) f(y(t - \tau_2(t))) \\ & + (\mu(t) - \mu_0) \\ & \times B(x(t)) (f(y(t - \tau_1(t))) - f(y(t - \tau_2(t)))) \\ & + K(x(t)) \int_{t-\alpha(t)}^t f(y(s)) ds + u(t), \\ \dot{y}(t) & = -Dy(t - \rho(t)) + E(y(t)) g(x(t)) \\ & + \omega_0 H(y(t)) g(x(t - d_1(t))) \\ & + (1 - \omega_0) H(y(t)) g(x(t - d_2(t))) \\ & + (\omega(t) - \omega_0) \end{aligned}$$

$$\begin{aligned} & \times H(y(t)) (g(x(t - d_1(t))) - g(x(t - d_2(t)))) \\ & + L(y(t)) \int_{t-\beta(t)}^t g(x(s)) ds + v(t). \end{aligned} \quad (50)$$

For presentation convenience, we denote

$$\begin{aligned} K & = \max \{ |k_{ji}|, |\bar{k}_{ji}| \}, \\ L & = \max \{ |l_{ij}|, |\bar{l}_{ij}| \}. \end{aligned} \quad (51)$$

Theorem 9. Under Assumptions 1–4, system (50) is passive, if there exist any appropriately dimensional matrices M_m ($m = 1, 2, \dots, 4$), W_w ($w = 1, 2, \dots, 6$), a scalar $\gamma > 0$, and symmetric positive definite matrices R_i ($i = 1, 2$), Q_j ($j = 1, 2, \dots, 6$), O_o ($o = 1, 2, \dots, 6$), P_k ($k = 1, 2, \dots, 8$), Z_l ($l = 1, 2, \dots, 4$), and N_n ($n = 1, 2$), such that

$$\begin{aligned} \Omega_{i,j} & = (\Omega_{i,j})_{18 \times 18} < 0, \\ \Phi_{i,j} & = (\Phi_{i,j})_{18 \times 18} < 0, \end{aligned} \quad (52)$$

where

$$\begin{aligned} \Omega_{1,1} & = -R_1 C - C^T R_1 + Q_1 + Q_2 + Q_3 + Q_4 + \delta^2 P_4 \\ & \quad - P_1 - P_2 - 2Z_1 - 2Z_2 - 2\zeta^- W_1 \zeta^+, \\ \Omega_{1,2} & = -M_1 C + (1 - \delta_0)^2 P_1, \\ \Omega_{1,6} & = P_2, \\ \Omega_{1,8} & = -M_1, \\ \Omega_{1,9} & = R_1 A + M_1 A + W_1 (\zeta^- + \zeta^+), \\ \Omega_{1,10} & = \mu_0 (R_1 + M_1) B, \\ \Omega_{1,11} & = (1 - \mu_0) (M_1 + R_1) B, \\ \Omega_{1,14} & = C^T R_1 C, \\ \Omega_{1,15} & = \frac{2}{\tau_1} Z_1, \\ \Omega_{1,16} & = \frac{2}{\tau_2 - \tau_1} Z_2, \\ \Omega_{1,17} & = M_1 K + R_1 K, \\ \Omega_{1,18} & = R_1 + M_1, \\ \Omega_{2,2} & = -(1 - \delta_0) Q_1 - (1 - \delta_0)^2 P_1, \\ \Omega_{2,8} & = -M_2 C, \\ \Omega_{3,3} & = -Q_2, \\ \Omega_{4,4} & = -(1 - \mu_1) Q_3 - 2\zeta^- W_2 \zeta^+, \\ \Omega_{4,10} & = -(1 - \mu_1) Q_3 + W_2 (\zeta^- + \zeta^+), \\ \Omega_{5,5} & = -(1 - \mu_2) Q_5 - 2\zeta^- W_3 \zeta^+, \end{aligned}$$

$$\begin{aligned}
\Omega_{5,11} &= -(1 - \mu_2) Q_5 + W_3 (\varsigma^- + \varsigma^+), \\
\Omega_{6,6} &= Q_5 + Q_6 - Q_4 - P_2 - P_3, \\
\Omega_{6,7} &= P_3, \\
\Omega_{6,12} &= Q_5 + Q_6 - Q_4, \\
\Omega_{7,7} &= -Q_6 - P_3, \\
\Omega_{7,13} &= -Q_6, \\
\Omega_{8,8} &= \delta^2 P_1 + \tau_1^2 P_2 + (\tau_2 - \tau_1)^2 P_3 + \frac{\tau_1^2}{2} Z_1 \\
&\quad + \frac{\tau_2^2 - \tau_1^2}{2} Z_2 - 2M_2, \\
\Omega_{8,9} &= M_2 A, \\
\Omega_{8,10} &= \mu_0 M_2 B, \\
\Omega_{8,11} &= (1 - \mu_0) M_2 B, \\
\Omega_{8,17} &= M_2 K, \\
\Omega_{8,18} &= M_2, \\
\Omega_{9,9} &= -2W_1 + \alpha^2 N_1, \\
\Omega_{9,14} &= -C^T R_1 A, \\
\Omega_{9,18} &= -I, \\
\Omega_{10,10} &= -(1 - \mu_1) Q_3 - 2W_2, \\
\Omega_{10,14} &= -\mu_0 C^T R_1 B, \\
\Omega_{11,11} &= -(1 - \mu_2) Q_5 - 2W_3, \\
\Omega_{11,14} &= -(1 - \mu_0) C^T R_1 B, \\
\Omega_{12,12} &= Q_5 + Q_6 - Q_4, \\
\Omega_{13,13} &= -Q_6, \\
\Omega_{14,14} &= -P_4, \\
\Omega_{14,17} &= -C^T R_1 K, \\
\Omega_{14,18} &= -C^T R_1, \\
\Omega_{15,15} &= -\frac{2}{\tau_1^2} Z_1, \\
\Omega_{16,16} &= -\frac{2}{(\tau_2 - \tau_1)^2} Z_2, \\
\Omega_{17,17} &= -N_1, \\
\Omega_{18,18} &= -\gamma I, \\
\Phi_{1,1} &= -R_2 C - C^T R_2 + O_1 + O_2 + O_3 + O_4 + \rho^2 P_8 \\
&\quad - P_5 - P_6 - 2Z_3 - 2Z_4 - 2\sigma^- W_4 \sigma^+,
\end{aligned} \tag{53}$$

$$\begin{aligned}
\Phi_{1,2} &= -M_3 C + (1 - \rho_0)^2 P_5, \\
\Phi_{1,6} &= P_6, \\
\Phi_{1,8} &= -M_3, \\
\Phi_{1,9} &= R_2 A + M_3 A + W_4 (\sigma^- + \sigma^+), \\
\Phi_{1,10} &= \omega_0 (R_2 + M_3) B, \\
\Phi_{1,11} &= (1 - \omega_0) (M_3 + R_2) B, \\
\Phi_{1,14} &= C^T R_2 C, \\
\Phi_{1,15} &= \frac{2}{d_1} Z_3, \\
\Phi_{1,16} &= \frac{2}{d_2 - d_1} Z_4, \\
\Phi_{1,17} &= M_3 K + R_2 K, \\
\Phi_{1,18} &= R_2 + M_3, \\
\Phi_{2,2} &= -(1 - \rho_0) O_1 - (1 - \rho_0)^2 P_5, \\
\Phi_{2,8} &= -M_4 C, \\
\Phi_{3,3} &= -O_2, \\
\Phi_{4,4} &= -(1 - \omega_1) O_3 - 2\sigma^- W_5 \sigma^+, \\
\Phi_{4,10} &= -(1 - \omega_1) O_3 + W_5 (\sigma^- + \sigma^+), \\
\Phi_{5,5} &= -(1 - \omega_2) O_5 - 2\sigma^- W_6 \sigma^+, \\
\Phi_{5,11} &= -(1 - \omega_2) O_5 + W_6 (\sigma^- + \sigma^+), \\
\Phi_{6,6} &= O_5 + O_6 - O_4 - P_6 - P_7, \\
\Phi_{6,7} &= P_7, \\
\Phi_{6,12} &= O_5 + O_6 - O_4, \\
\Phi_{7,7} &= -O_6 - P_7, \\
\Phi_{7,13} &= -O_6, \\
\Phi_{8,8} &= \rho^2 P_5 + d_1^2 P_6 + (d_2 - d_1)^2 P_7 + \frac{d_1^2}{2} Z_3 \\
&\quad + \frac{d_2^2 - d_1^2}{2} Z_4 - 2M_4, \\
\Phi_{8,9} &= M_4 A, \\
\Phi_{8,10} &= \omega_0 M_4 B, \\
\Phi_{8,11} &= (1 - \omega_0) M_4 B, \\
\Phi_{8,17} &= M_4 K, \\
\Phi_{8,18} &= M_4, \\
\Phi_{9,9} &= -2W_4 + \alpha^2 N_2, \\
\Phi_{9,14} &= -C^T R_2 A,
\end{aligned}$$

$$\begin{aligned}
\Phi_{9,18} &= -I, \\
\Phi_{10,10} &= -(1 - \omega_1) O_3 - 2W_5, \\
\Phi_{10,14} &= -\omega_0 C^T R_2 B, \\
\Phi_{11,11} &= -(1 - \omega_2) O_5 - 2W_6, \\
\Phi_{11,14} &= -(1 - \omega_0) C^T R_2 B, \\
\Phi_{12,12} &= O_5 + O_6 - O_4, \\
\Phi_{13,13} &= -O_6, \\
\Phi_{14,14} &= -P_8, \\
\Phi_{14,17} &= -D^T R_2 L, \\
\Phi_{14,18} &= -C^T R_2, \\
\Phi_{15,15} &= -\frac{2}{d_1^2} Z_3, \\
\Phi_{16,16} &= -\frac{2}{(d_2 - d_1)^2} Z_4, \\
\Phi_{17,17} &= -N_2, \\
\Phi_{18,18} &= -\gamma I.
\end{aligned} \tag{54}$$

Proof. Consider LKF candidate as follows:

$$V(t) = \sum_{i=1}^{10} V_i(t), \tag{55}$$

where

$$\begin{aligned}
V_1(t) &= \left(x(t) - C \int_{t-\delta(t)}^t x(s) ds \right)^T \\
&\quad \cdot R_1 \left(x(t) - C \int_{t-\delta(t)}^t x(s) ds \right), \\
V_2(t) &= \left(y(t) - D \int_{t-\rho(t)}^t y(s) ds \right)^T \\
&\quad \cdot R_2 \left(y(t) - D \int_{t-\rho(t)}^t y(s) ds \right), \\
V_3(t) &= \int_{t-\delta(t)}^t x^T(s) Q_1 x(s) ds \\
&\quad + \int_{t-\delta}^t x^T(s) Q_2 x(s) ds \\
&\quad + \int_{t-\tau_1(t)}^t \begin{bmatrix} x(s) \\ f(y(s)) \end{bmatrix}^T Q_3 \begin{bmatrix} x(s) \\ f(y(s)) \end{bmatrix} ds \\
&\quad + \int_{t-\tau_1}^t \begin{bmatrix} x(s) \\ f(y(s)) \end{bmatrix}^T Q_4 \begin{bmatrix} x(s) \\ f(y(s)) \end{bmatrix} ds
\end{aligned}$$

$$\begin{aligned}
&\quad + \int_{t-\tau_2(t)}^{t-\tau_1} \begin{bmatrix} x(s) \\ f(y(s)) \end{bmatrix}^T Q_5 \begin{bmatrix} x(s) \\ f(y(s)) \end{bmatrix} ds \\
&\quad + \int_{t-\tau_2}^{t-\tau_1} \begin{bmatrix} x(s) \\ f(y(s)) \end{bmatrix}^T Q_6 \begin{bmatrix} x(s) \\ f(y(s)) \end{bmatrix} ds, \\
V_4(t) &= \int_{t-\rho(t)}^t y^T(s) O_1 y(s) ds \\
&\quad + \int_{t-\rho}^t y^T(s) O_2 y(s) ds \\
&\quad + \int_{t-d_1(t)}^t \begin{bmatrix} y(s) \\ g(x(s)) \end{bmatrix}^T O_3 \begin{bmatrix} y(s) \\ g(x(s)) \end{bmatrix} ds \\
&\quad + \int_{t-d_1}^t \begin{bmatrix} y(s) \\ g(x(s)) \end{bmatrix}^T O_4 \begin{bmatrix} y(s) \\ g(x(s)) \end{bmatrix} ds \\
&\quad + \int_{t-d_2(t)}^{t-d_1} \begin{bmatrix} y(s) \\ g(x(s)) \end{bmatrix}^T O_5 \begin{bmatrix} y(s) \\ g(x(s)) \end{bmatrix} ds \\
&\quad + \int_{t-d_2}^{t-d_1} \begin{bmatrix} y(s) \\ g(x(s)) \end{bmatrix}^T O_6 \begin{bmatrix} y(s) \\ g(x(s)) \end{bmatrix} ds, \\
V_5(t) &= \delta \int_{-\delta}^0 \int_{t+\theta}^t \dot{x}^T(s) P_1 \dot{x}(s) ds d\theta \\
&\quad + \tau_1 \int_{-\tau_1}^0 \int_{t+\theta}^t \dot{x}^T(s) P_2 \dot{x}(s) ds d\theta + (\tau_2 - \tau_1) \\
&\quad \cdot \int_{-\tau_2}^{-\tau_1} \int_{t+\theta}^t \dot{x}^T(s) P_3 \dot{x}(s) ds d\theta \\
&\quad + \delta \int_{-\delta}^0 \int_{t+\theta}^t x^T(s) P_4 x(s) ds d\theta, \\
V_6(t) &= \rho \int_{-\rho}^0 \int_{t+\theta}^t \dot{y}^T(s) P_5 \dot{y}(s) ds d\theta \\
&\quad + d_1 \int_{-d_1}^0 \int_{t+\theta}^t \dot{y}^T(s) P_6 \dot{y}(s) ds d\theta + (d_2 - d_1) \\
&\quad \cdot \int_{-d_2}^{-d_1} \int_{t+\theta}^t \dot{y}^T(s) P_7 \dot{y}(s) ds d\theta \\
&\quad + \rho \int_{-\delta}^0 \int_{t+\theta}^t y^T(s) P_8 y(s) ds d\theta, \\
V_7(t) &= \int_{-\tau_1}^0 \int_{\theta}^0 \int_{t+\lambda}^t \dot{x}^T(s) Z_1 \dot{x}(s) ds d\lambda d\theta \\
&\quad + \int_{-\tau_2}^{-\tau_1} \int_{\theta}^0 \int_{t+\lambda}^t \dot{x}^T(s) Z_2 \dot{x}(s) ds d\lambda d\theta, \\
V_8(t) &= \int_{-d_1}^0 \int_{\theta}^0 \int_{t+\lambda}^t \dot{y}^T(s) Z_3 \dot{y}(s) ds d\lambda d\theta \\
&\quad + \int_{-d_2}^{-d_1} \int_{\theta}^0 \int_{t+\lambda}^t \dot{y}^T(s) Z_4 \dot{y}(s) ds d\lambda d\theta,
\end{aligned}$$

$$\begin{aligned}
V_9(t) &= \alpha \int_{-\alpha}^0 \int_{t+\theta}^t f^T(y(s)) N_1 f(y(s)) ds d\theta, \\
V_{10}(t) &= \beta \int_{-\beta}^0 \int_{t+\theta}^t g^T(x(s)) N_2 g(x(s)) ds d\theta.
\end{aligned} \tag{56}$$

Then, we define the infinitesimal generator $\mathcal{L}$ of $V(t)$ as

$$\mathcal{L}V(t) = \lim_{\Delta \rightarrow 0^+} \frac{1}{\Delta} \left\{ \mathbb{E} \left(\frac{V(t+\Delta)}{t} \right) - V(t) \right\}. \tag{57}$$

Taking the mathematical expectation of $V(t)$, we get

$$\begin{aligned}
\mathbb{E} \{ \mathcal{L}V_1(t) \} &= \mathbb{E} \left\{ 2 \left(x(t) - C \int_{t-\delta(t)}^t x(s) ds \right)^T R_1 \right. \\
&\times \left(-Cx(t) + A(x(t))f(y(t)) + \mu_0 B(x(t)) \right. \\
&\times f(y(t-\tau_1(t))) + (1-\mu_0)B(x(t)) \\
&\cdot f(y(t-\tau_2(t))) + (\mu(t)-\mu_0)B(x(t)) \\
&\cdot (f(y(t-\tau_1(t))) - f(y(t-\tau_2(t)))) + K(x(t)) \\
&\cdot \left. \int_{t-\alpha(t)}^t f(y(s)) ds + u(t) \right) \Big\} \leq \mathbb{E} \left\{ 2 \left(x(t) \right. \right. \\
&- C \int_{t-\delta(t)}^t x(s) ds \Big)^T R_1 \times \left(-Cx(t) + Af(y(t)) \right. \\
&+ \mu_0 Bf(y(t-\tau_1(t))) + (1-\mu_0) \\
&\cdot Bf(y(t-\tau_2(t))) + K \int_{t-\alpha(t)}^t f(y(s)) ds \\
&\left. \left. + u(t) \right) \right\}, \tag{58}
\end{aligned}$$

$$\begin{aligned}
\mathbb{E} \{ \mathcal{L}V_2(t) \} &= \mathbb{E} \left\{ 2 \left(y(t) - D \int_{t-\rho(t)}^t y(s) ds \right)^T R_2 \right. \\
&\times \left(-Dy(t) + E(y(t))g(x(t)) + \omega_0 H(y(t)) \right. \\
&\times g(x(t-d_1(t))) + (1-\omega_0)H(y(t)) \\
&\cdot g(x(t-d_2(t))) + (\omega(t)-\omega_0)H(y(t)) \\
&\cdot (g(x(t-d_1(t))) - g(x(t-d_2(t)))) + L(y(t)) \\
&\cdot \left. \int_{t-\beta(t)}^t g(x(s)) ds + v(t) \right) \Big\} \leq \mathbb{E} \left\{ 2 \left(y(t) \right. \right. \\
&- D \int_{t-\rho(t)}^t y(s) ds \Big)^T R_2 \times \left(-Dy(t) + Eg(x(t)) \right. \\
&+ \omega_0 Hg(x(t-d_1(t))) + (1-\omega_0) \\
&\cdot Hg(x(t-d_2(t))) + L \int_{t-\beta(t)}^t g(x(s)) ds + v(t) \Big) \Big\}.
\end{aligned}$$

According to Assumptions 3 and 4, we get

$$\begin{aligned}
\mathbb{E} \{ \mathcal{L}V_3(t) \} &= \mathbb{E} \left\{ x^T(t) (Q_1 + Q_2 + Q_3 + Q_4) x(t) \right. \\
&- (1-\delta_0) x^T(t-\delta(t)) Q_1 x(t-\delta(t)) \\
&- x^T(t-\delta) Q_2 x(t-\delta) - (1-\mu_1) \\
&\cdot \left[\begin{array}{c} x(t-\tau_1(t)) \\ f(y(t-\tau_1(t))) \end{array} \right]^T Q_3 \left[\begin{array}{c} x(t-\tau_1(t)) \\ f(y(t-\tau_1(t))) \end{array} \right] \\
&- (1-\mu_2) \left[\begin{array}{c} x(t-\tau_2(t)) \\ f(y(t-\tau_2(t))) \end{array} \right]^T \\
&\cdot Q_5 \left[\begin{array}{c} x(t-\tau_2(t)) \\ f(y(t-\tau_2(t))) \end{array} \right] + \left[\begin{array}{c} x(t-\tau_1) \\ f(y(t-\tau_1)) \end{array} \right]^T \\
&\cdot (Q_5 + Q_6 - Q_4) \left[\begin{array}{c} x(t-\tau_1) \\ f(y(t-\tau_1)) \end{array} \right] \\
&\left. + \left[\begin{array}{c} x(t-\tau_2) \\ f(y(t-\tau_2)) \end{array} \right]^T Q_6 \left[\begin{array}{c} x(t-\tau_2) \\ f(y(t-\tau_2)) \end{array} \right] \right\}, \tag{59}
\end{aligned}$$

$$\begin{aligned}
\mathbb{E} \{ \mathcal{L}V_4(t) \} &= \mathbb{E} \left\{ y^T(t) (O_1 + O_2 + O_3 + O_4) y(t) \right. \\
&- (1-\rho_0) y^T(t-\rho(t)) O_1 y(t-\rho(t)) \\
&- y^T(t-\rho) O_2 y(t-\rho) - (1-\omega_1) \\
&\cdot \left[\begin{array}{c} y(t-d_1(t)) \\ g(x(t-d_1(t))) \end{array} \right]^T O_3 \left[\begin{array}{c} y(t-d_1(t)) \\ g(x(t-d_1(t))) \end{array} \right] \\
&- (1-\omega_2) \left[\begin{array}{c} y(t-d_2(t)) \\ g(x(t-d_2(t))) \end{array} \right]^T \\
&\cdot O_5 \left[\begin{array}{c} y(t-d_2(t)) \\ g(x(t-d_2(t))) \end{array} \right] + \left[\begin{array}{c} y(t-d_1) \\ g(x(t-d_1)) \end{array} \right]^T \\
&\cdot (O_5 + O_6 - O_4) \left[\begin{array}{c} y(t-d_1) \\ g(x(t-d_1)) \end{array} \right] \\
&\left. + \left[\begin{array}{c} y(t-d_2) \\ g(x(t-d_2)) \end{array} \right]^T O_6 \left[\begin{array}{c} y(t-d_2) \\ g(x(t-d_2)) \end{array} \right] \right\}.
\end{aligned}$$

It is obvious that

$$\begin{aligned}
\mathbb{E} \{ \mathcal{L}V_5(t) \} &= \mathbb{E} \left\{ \dot{x}^T(t) [\delta^2 P_1 + \tau_1^2 P_2 + (\tau_2 - \tau_1)^2 P_3] \dot{x}(t) \right. \\
&+ \delta^2 x^T(t) P_4 x(t) - \delta \int_{t-\delta}^t \dot{x}^T(s) P_1 \dot{x}(s) ds
\end{aligned}$$

$$\begin{aligned}
& -\tau_1 \int_{t-\tau_1}^t \dot{x}^T(s) P_2 \dot{x}(s) ds \\
& -(\tau_2 - \tau_1) \int_{t-\tau_2}^{t-\tau_1} \dot{x}^T(s) P_3 \dot{x}(s) ds \\
& -\delta \int_{t-\delta}^t x^T(s) P_4 x(s) ds \Big\}, \\
\mathbb{E}\{\mathcal{L}V_6(t)\} & = \mathbb{E}\left\{\dot{y}^T(t) [\rho^2 P_5 + d_1^2 P_6 + (d_2 - d_1)^2 P_7] \dot{y}(t) \right. \\
& + \rho^2 y^T(t) P_8 y(t) - \rho \int_{t-\rho}^t \dot{y}^T(s) P_5 \dot{y}(s) ds \\
& - d_1 \int_{t-d_1}^t \dot{y}^T(s) P_6 \dot{y}(s) ds \\
& - (d_2 - d_1) \int_{t-d_2}^{t-d_1} \dot{y}^T(s) P_7 \dot{y}(s) ds \\
& \left. - \rho \int_{t-\rho}^t y^T(s) P_8 y(s) ds \right\}.
\end{aligned} \tag{60}$$

Using Lemma 7, we have

$$\begin{aligned}
& -\delta \int_{t-\delta}^t \dot{x}^T(s) P_1 \dot{x}(s) ds \\
& \leq -\left(\int_{t-\delta(t)}^t \dot{x}(s) ds\right)^T P_1 \left(\int_{t-\delta(t)}^t \dot{x}(s) ds\right) \\
& = -[x(t) - (1 - \delta_0)x(t - \delta)]^T P_1 \\
& \quad \times [x(t) - (1 - \delta_0)x(t - \delta)], \\
& -\delta \int_{t-\delta}^t x^T(s) P_4 x(s) ds \\
& \leq -\left(\int_{t-\delta(t)}^t x(s) ds\right)^T P_4 \left(\int_{t-\delta(t)}^t x(s) ds\right).
\end{aligned} \tag{61}$$

Similarly, we have

$$\begin{aligned}
& -\rho \int_{t-\rho}^t \dot{y}^T(s) P_4 \dot{y}(s) ds \\
& \leq -\left(\int_{t-\rho(t)}^t \dot{y}(s) ds\right)^T P_4 \left(\int_{t-\rho(t)}^t \dot{y}(s) ds\right) \\
& = -[y(t) - (1 - \rho_0)y(t - \rho)]^T P_4 \\
& \quad \times [y(t) - (1 - \rho_0)y(t - \rho)], \\
& -\rho \int_{t-\rho}^t y^T(s) P_8 y(s) ds \\
& \leq -\left(\int_{t-\rho(t)}^t y(s) ds\right)^T P_8 \left(\int_{t-\rho(t)}^t y(s) ds\right).
\end{aligned} \tag{62}$$

Then, the same as (32)–(35), we can obtain $\mathbb{E}\{\mathcal{L}V_7(t)\}$ and $\mathbb{E}\{\mathcal{L}V_8(t)\}$.

Moreover,

$$\begin{aligned}
\mathbb{E}\{\mathcal{L}V_9(t)\} & = \mathbb{E}\left\{\alpha^2 f^T(y(t)) N_1 f(y(t)) \right. \\
& \left. - \alpha \int_{t-\alpha}^t f^T(y(s)) N_1 f(y(s)) ds \right\},
\end{aligned} \tag{63}$$

$$\begin{aligned}
\mathbb{E}\{\mathcal{L}V_{10}(t)\} & = \mathbb{E}\left\{\beta^2 g^T(x(t)) N_2 g(x(t)) \right. \\
& \left. - \beta \int_{t-\beta}^t g^T(x(s)) N_2 g(x(s)) ds \right\}.
\end{aligned} \tag{64}$$

Also, we get

$$\begin{aligned}
& -\alpha \int_{t-\alpha}^t f^T(y(s)) N_1 f(y(s)) ds \\
& \leq -\left(\int_{t-\alpha(t)}^t f(y(s)) ds\right)^T N_1 \left(\int_{t-\alpha(t)}^t f(y(s)) ds\right), \\
& -\beta \int_{t-\beta}^t g^T(x(s)) N_2 g(x(s)) ds \\
& \leq -\left(\int_{t-\beta(t)}^t g(x(s)) ds\right)^T N_2 \left(\int_{t-\beta(t)}^t g(x(s)) ds\right).
\end{aligned} \tag{65}$$

Now, we add the following inequations with any matrices M_1 , M_2 , M_3 , and M_4 of appropriate dimensions to derive a less conservative criterion:

$$\begin{aligned}
0 & \leq 2 \left(x^T(t) M_1 + \dot{x}^T(t) M_2 \right) \left[-\dot{x} - Cx(t - \delta(t)) \right. \\
& \quad + Af(y(t)) + \mu_0 Bf(y(t - \tau_1(t))) \\
& \quad + (1 - \mu_0) Bf(y(t - \tau_2(t))) \\
& \quad \left. + K \int_{t-\alpha(t)}^t f(y(s)) ds + u(t) \right], \\
0 & \leq 2 \left(y^T(t) M_3 + \dot{y}^T(t) M_4 \right) \left[-\dot{y} - Dy(t - \rho(t)) \right. \\
& \quad + Eg(x(t)) + \omega_0 Hg(x(t - d_1(t))) \\
& \quad + (1 - \omega_0) Hg(x(t - d_2(t))) \\
& \quad \left. + L \int_{t-\beta(t)}^t g(x(s)) ds + v(t) \right].
\end{aligned} \tag{66}$$

Define

$$\begin{aligned}
\xi(t) & = \begin{bmatrix} x^T(t) & x^T(t - \delta(t)) & x^T(t - \delta) & x^T(t - \tau_1(t)) \\ x^T(t - \tau_2(t)) & x^T(t - \tau_1) & x^T(t - \tau_2) & \dot{x}(t) \end{bmatrix}
\end{aligned}$$

$$\begin{aligned}
& f^T(y(t)) \quad f^T(y(t - \tau_1(t))) \quad f^T(y(t - \tau_2(t))) \\
& f^T(y(t - \tau_1)) \quad f^T(y(t - \tau_2)) \quad \int_{t-\delta(t)}^t x^T(s) ds \\
& \int_{t-\tau_1}^t x^T(s) ds \quad \int_{t-\tau_2}^{t-\tau_1} x^T(s) ds \quad \int_{t-\alpha(t)}^t f^T(y(s)) ds \quad u^T(t) \Big\}, \\
\zeta(t) = & \left\{ y^T(t) \quad y^T(t - \rho(t)) \quad y^T(t - \rho) \quad y^T(t - d_1(t)) \right. \\
& y^T(t - d_2(t)) \quad y^T(t - d_1) \quad y^T(t - d_2) \quad \dot{y}(t) \\
& g^T(x(t)) \quad g^T(x(t - d_1(t))) \quad g^T(x(t - d_2(t))) \\
& g^T(x(t - d_1)) \quad g^T(x(t - d_2)) \quad \int_{t-\rho(t)}^t y^T(s) ds \\
& \left. \int_{t-d_1}^t y^T(s) ds \quad \int_{t-d_2}^{t-d_1} y^T(s) ds \quad \int_{t-\beta(t)}^t g^T(x(s)) ds \quad v^T(t) \right\}.
\end{aligned} \tag{67}$$

Combining (40) and (58)–(66), we have

$$\begin{aligned}
& \mathbb{E} \left\{ \mathcal{L}V(t) - 2 \left[f^T(y(s)) \quad g^T(x(s)) \right] \begin{bmatrix} u(s) \\ v(s) \end{bmatrix} \right. \\
& \left. - \gamma \left[u^T(t) \quad v^T(t) \right] \begin{bmatrix} u(t) \\ v(t) \end{bmatrix} ds \right\} \leq \mathbb{E} \left\{ \xi^T(t) \Omega \xi(t) \right. \\
& \left. + \zeta^T(t) \Phi \zeta(t) \right\}.
\end{aligned} \tag{68}$$

Thus, we conclude that if (68) holds, then

$$\begin{aligned}
& \mathbb{E} \left\{ \mathcal{L}V(t) - 2 \left[f^T(y(s)) \quad g^T(x(s)) \right] \begin{bmatrix} u(s) \\ v(s) \end{bmatrix} \right. \\
& \left. - \gamma \left[u^T(t) \quad v^T(t) \right] \begin{bmatrix} u(t) \\ v(t) \end{bmatrix} ds \right\} \leq 0.
\end{aligned} \tag{69}$$

By integrating (69) about t from 0 to t_z , we have

$$\begin{aligned}
& 2\mathbb{E} \int_0^{t_z} \left[f^T(y(s)) \quad g^T(x(s)) \right] \begin{bmatrix} u(s) \\ v(s) \end{bmatrix} ds \\
& \geq \mathbb{E}V(t) - \mathbb{E}V(0) \\
& - \gamma \mathbb{E} \int_0^{t_z} \left[u^T(s) \quad v^T(s) \right] \begin{bmatrix} u(s) \\ v(s) \end{bmatrix} ds \\
& \geq -\gamma \mathbb{E} \int_0^{t_z} \left[u^T(s) \quad v^T(s) \right] \begin{bmatrix} u(s) \\ v(s) \end{bmatrix} ds.
\end{aligned} \tag{70}$$

Therefore, from Definition 5, we know that the MBAMNN is passive. This completes the proof. $\square$

Remark 10. In this paper, model (50) contains the leakage, probabilistic, and distributed time-varying delays. In particular, we treat some of these delays as a special case of some existing results.

Remark 11. In the corollary, we define leakage delays δ and ρ as constant delays. According to the preceding conditions, we consider the MBAMNN in the following:

$$\begin{aligned}
& \dot{x}(t) \\
& = -Cx(t - \delta) + A(x(t)) f(y(t)) \\
& \quad + \mu_0 B(x(t)) f(y(t - \tau_1(t))) \\
& \quad + (1 - \mu_0) B(x(t)) f(y(t - \tau_2(t))) \\
& \quad + (\mu(t) - \mu_0) \\
& \quad \times B(x(t)) (f(y(t - \tau_1(t))) - f(y(t - \tau_2(t)))) \\
& \quad + u(t),
\end{aligned} \tag{71}$$

$$\begin{aligned}
& \dot{y}(t) \\
& = -Dy(t - \rho) + E(y(t)) g(x(t)) \\
& \quad + \omega_0 H(y(t)) g(x(t - d_1(t))) \\
& \quad + (1 - \omega_0) H(y(t)) g(x(t - d_2(t))) \\
& \quad + (\omega(t) - \omega_0) \\
& \quad \times H(y(t)) (g(x(t - d_1(t))) - g(x(t - d_2(t)))) \\
& \quad + v(t).
\end{aligned}$$

Corollary 12. When Assumptions 1–3 hold, system (71) is passive, if there exist any appropriately dimensional matrices M_m ($m = 1, 2, \dots, 4$), W_w ($w = 1, 2, \dots, 6$), a scalar $\gamma > 0$, and symmetric positive definite matrices R_i ($i = 1, 2$), Q_j ($j = 2, \dots, 6$), O_o ($o = 2, \dots, 6$), P_k ($k = 1, 2, \dots, 8$), and Z_l ($l = 1, 2, \dots, 4$), such that

$$\begin{aligned}
& \Omega_{i,j} = (\Omega_{i,j})_{17 \times 17} < 0, \\
& \Phi_{i,j} = (\Phi_{i,j})_{17 \times 17} < 0,
\end{aligned} \tag{72}$$

where

$$\begin{aligned}
& \Omega_{1,1} = -R_1 C - C^T R_1 + Q_2 + Q_3 + Q_4 + \delta^2 P_4 - P_1 \\
& \quad - P_2 - 2Z_1 - 2Z_2 - 2\zeta^- W_1 \zeta^+, \\
& \Omega_{1,2} = P_1 - M_1 C, \\
& \Omega_{1,5} = P_2, \\
& \Omega_{1,7} = -M_1, \\
& \Omega_{1,8} = R_1 A + M_1 A + W_1 (\zeta^- + \zeta^+), \\
& \Omega_{1,9} = \mu_0 (R_1 + M_1) B, \\
& \Omega_{1,10} = (1 - \mu_0) (M_1 + R_1) B, \\
& \Omega_{1,13} = C^T R_1 C, \\
& \Omega_{1,14} = \frac{2}{\tau_1} Z_1,
\end{aligned}$$

$$\begin{aligned}
\Omega_{1,15} &= \frac{2}{\tau_2 - \tau_1} Z_2, \\
\Omega_{1,16} &= M_1 K, \\
\Omega_{1,17} &= R_1 + M_1, \\
\Omega_{2,2} &= -Q_2 - P_1, \\
\Omega_{2,7} &= -M_2 C, \\
\Omega_{3,3} &= -(1 - \mu_1) Q_3 - 2\zeta^- W_2 \zeta^+, \\
\Omega_{3,9} &= -(1 - \mu_1) Q_3 + W_2 (\zeta^- + \zeta^+), \\
\Omega_{4,4} &= -(1 - \mu_2) Q_5 - 2\zeta^- W_3 \zeta^+, \\
\Omega_{4,10} &= -(1 - \mu_2) Q_5 + W_3 (\zeta^- + \zeta^+), \\
\Omega_{5,5} &= Q_5 + Q_6 - Q_4 - P_2 - P_3, \\
\Omega_{5,6} &= P_3, \\
\Omega_{5,11} &= Q_5 + Q_6 - Q_4, \\
\Omega_{6,6} &= -Q_6 - P_3, \\
\Omega_{6,12} &= -Q_6, \\
\Omega_{7,7} &= \delta^2 P_1 + \tau_1^2 P_2 + (\tau_2 - \tau_1)^2 P_3 + \frac{\tau_1^2}{2} Z_1 \\
&\quad + \frac{\tau_2^2 - \tau_1^2}{2} Z_2 - 2M_2, \\
\Omega_{7,8} &= M_2 A, \\
\Omega_{7,9} &= \mu_0 M_2 B, \\
\Omega_{7,10} &= (1 - \mu_0) M_2 B, \\
\Omega_{7,16} &= M_2 K, \\
\Omega_{7,17} &= M_2, \\
\Omega_{8,8} &= -2W_1 + \alpha^2 N_1, \\
\Omega_{8,13} &= -C^T R_1 A, \\
\Omega_{8,17} &= -I, \\
\Omega_{9,9} &= -(1 - \mu_1) Q_3 - 2W_2, \\
\Omega_{9,13} &= -\mu_0 C^T R_1 B, \\
\Omega_{10,10} &= -(1 - \mu_2) Q_5 - 2W_3, \\
\Omega_{10,13} &= -(1 - \mu_0) C^T R_1 B, \\
\Omega_{11,11} &= Q_5 + Q_6 - Q_4, \\
\Omega_{12,12} &= -Q_6, \\
\Omega_{13,13} &= -P_4, \\
\Omega_{13,17} &= -C^T R_1, \\
\Omega_{14,14} &= -\frac{2}{\tau_1^2} Z_1, \\
\Omega_{15,15} &= -\frac{2}{(\tau_2 - \tau_1)^2} Z_2, \\
\Omega_{16,16} &= -N_1, \\
\Omega_{17,17} &= -\gamma I, \\
\Phi_{1,1} &= -R_2 C - C^T R_2 + O_2 + O_3 + O_4 + \rho^2 P_8 - P_5 \\
&\quad - P_6 - 2Z_3 - 2Z_4 - 2\sigma^- W_4 \sigma^+, \\
\Phi_{1,2} &= P_5 - M_3 D, \\
\Phi_{1,5} &= P_6, \\
\Phi_{1,7} &= -M_3, \\
\Phi_{1,8} &= R_2 A + M_3 A + W_4 (\sigma^- + \sigma^+), \\
\Phi_{1,9} &= \omega_0 (R_2 + M_3) B, \\
\Phi_{1,10} &= (1 - \omega_0) (M_3 + R_2) B, \\
\Phi_{1,13} &= C^T R_2 C, \\
\Phi_{1,14} &= \frac{2}{d_1} Z_3, \\
\Phi_{1,15} &= \frac{2}{d_2 - d_1} Z_4, \\
\Phi_{1,16} &= M_3 K, \\
\Phi_{1,17} &= R_2 + M_3, \\
\Phi_{2,2} &= -O_2 - P_5, \\
\Phi_{2,7} &= -M_4 C, \\
\Phi_{3,3} &= -(1 - \omega_1) O_3 - 2\sigma^- W_5 \sigma^+, \\
\Phi_{3,9} &= -(1 - \omega_1) O_3 + W_5 (\sigma^- + \sigma^+), \\
\Phi_{4,4} &= -(1 - \omega_2) O_5 - 2\sigma^- W_6 \sigma^+, \\
\Phi_{4,10} &= -(1 - \omega_2) O_5 + W_6 (\sigma^- + \sigma^+), \\
\Phi_{5,5} &= O_5 + O_6 - O_4 - P_6 - P_7, \\
\Phi_{5,6} &= P_7, \\
\Phi_{5,11} &= O_5 + O_6 - O_4, \\
\Phi_{6,6} &= -O_6 - P_7, \\
\Phi_{6,12} &= -O_6, \\
\Phi_{7,7} &= \rho^2 P_5 + d_1^2 P_6 + (d_2 - d_1)^2 P_7 + \frac{d_1^2}{2} Z_3 \\
&\quad + \frac{d_2^2 - d_1^2}{2} Z_4 - 2M_4,
\end{aligned} \tag{73}$$

$$\begin{aligned}
\Phi_{7,8} &= M_4 A, \\
\Phi_{7,9} &= \omega_0 M_4 B, \\
\Phi_{7,10} &= (1 - \omega_0) M_4 B, \\
\Phi_{7,16} &= M_4 K, \\
\Phi_{7,17} &= M_4, \\
\Phi_{8,8} &= -2W_4 + \alpha^2 N_2, \\
\Phi_{8,13} &= -C^T R_2 A, \\
\Phi_{8,17} &= -I, \\
\Phi_{9,9} &= -(1 - \omega_1) O_3 - 2W_5, \\
\Phi_{9,13} &= -\omega_0 C^T R_2 B, \\
\Phi_{10,10} &= -(1 - \omega_2) O_5 - 2W_6, \\
\Phi_{10,13} &= -(1 - \omega_0) C^T R_2 B, \\
\Phi_{11,11} &= O_5 + O_6 - O_4, \\
\Phi_{12,12} &= -O_6, \\
\Phi_{13,13} &= -P_8, \\
\Phi_{13,17} &= -C^T R_2, \\
\Phi_{14,14} &= -\frac{2}{d_1^2} Z_3, \\
\Phi_{15,15} &= -\frac{2}{(d_2 - d_1)^2} Z_4, \\
\Phi_{16,16} &= -N_2, \\
\Phi_{17,17} &= -\gamma I.
\end{aligned}$$

(74)

Remark 13. In numerical simulations, we use the models of MBAMNNs with mixed time-varying delays; the models that we used are novel. In the existing papers, we can see that the most of the papers are about passivity analysis of memristive single-layer neural networks and there are few results about MBAMNNs [17–19]. MBAMNNs are a class of two-layer systems which can realize heteroassociation memory and autoassociation memory; hence, the MBAMNNs can be able to imitate the human brain better, and the limitation of traditional neural networks can be broken. The LKFs that we designed include double and triple integral terms, and using some effective methods, we can obtain several conditions about MBAMNNs with leakage, probabilistic, and distributed time-varying delays. Hence, our research results are less conservative than the existing ones in the models and the treatment methods [19, 21]. In particular, these delays are all taken into consideration separately. Thus, our paper not only improves the models but also considers a variety of time-varying delays [18–21].

4. Numerical Simulation

In this section, we provide several numerical examples to illustrate the usefulness of our stability results.

Example 1. We consider a 2-dimensional MBAMNN of (16) with the parameters as follows:

$$\begin{aligned}
a_{11}(x_1(t)) &= \begin{cases} 0.3, & |x_1(t)| > \Phi_1, \\ 0.7, & |x_1(t)| \leq \Phi_1, \end{cases} \\
a_{12}(x_1(t)) &= \begin{cases} 0.4, & |x_1(t)| > \Phi_1, \\ 0.5, & |x_1(t)| \leq \Phi_1, \end{cases} \\
a_{21}(x_2(t)) &= \begin{cases} -0.5, & |x_2(t)| > \Phi_2, \\ -0.6, & |x_2(t)| \leq \Phi_2, \end{cases} \\
a_{22}(x_2(t)) &= \begin{cases} -0.76, & |x_2(t)| > \Phi_2, \\ -0.7, & |x_2(t)| \leq \Phi_2, \end{cases} \\
b_{11}(x_1(t)) &= \begin{cases} 0.33, & |x_1(t)| > \Phi_1, \\ 0.3, & |x_1(t)| \leq \Phi_1, \end{cases} \\
b_{12}(x_1(t)) &= \begin{cases} 0.25, & |x_1(t)| > \Phi_1, \\ 0.2, & |x_1(t)| \leq \Phi_1, \end{cases} \\
b_{21}(x_2(t)) &= \begin{cases} 0.6, & |x_2(t)| > \Phi_2, \\ 0.56, & |x_2(t)| \leq \Phi_2, \end{cases} \\
b_{22}(x_2(t)) &= \begin{cases} 0.63, & |x_2(t)| > \Phi_2, \\ 0.6, & |x_2(t)| \leq \Phi_2, \end{cases} \\
e_{11}(y_1(t)) &= \begin{cases} 1.3, & |y_1(t)| > \Psi_1, \\ 0.4, & |y_1(t)| \leq \Psi_1, \end{cases} \\
e_{12}(y_1(t)) &= \begin{cases} 1.28, & |y_1(t)| > \Psi_1, \\ 0.38, & |y_1(t)| \leq \Psi_1, \end{cases} \\
e_{21}(y_2(t)) &= \begin{cases} -1.05, & |y_2(t)| > \Psi_2, \\ -1.1, & |y_2(t)| \leq \Psi_2, \end{cases} \\
e_{22}(y_2(t)) &= \begin{cases} -0.88, & |y_2(t)| > \Psi_2, \\ -0.8, & |y_2(t)| \leq \Psi_2, \end{cases} \\
h_{11}(y_1(t)) &= \begin{cases} 0.38, & |y_1(t)| > \Psi_1, \\ 0.4, & |y_1(t)| \leq \Psi_1, \end{cases}
\end{aligned}$$

$$\begin{aligned}
 h_{12}(y_1(t)) &= \begin{cases} 0.25, & |y_1(t)| > \Psi_1, \\ 0.2, & |y_1(t)| \leq \Psi_1, \end{cases} \\
 h_{21}(y_2(t)) &= \begin{cases} 0.5, & |y_2(t)| > \Psi_2, \\ 0.87, & |y_2(t)| \leq \Psi_2, \end{cases} \\
 h_{22}(y_2(t)) &= \begin{cases} 0.38, & |y_2(t)| > \Psi_2, \\ -0.5, & |y_2(t)| \leq \Psi_2. \end{cases}
 \end{aligned} \tag{75}$$

Here, we take the activation functions as $f_i(s) = g_i(s) = \tanh(s)$, $i = 1, 2$. Then, let $\tau_1 = 0.5$, $\tau_2 = 0.6$, $\mu_1 = 0.3$, $\mu_2 = 0.5$, $\mu_0 = 0.4$, $d_1 = 1.5$, $d_2 = 2$, $\omega_1 = 0.6$, $\omega_2 = 0.8$, and $\omega_0 = 0.3$. Probabilistic time-varying delays $\tau_1(t) = 0.7 + 0.2 \sin(t)$, $\tau_2(t) = 0.65 + 0.6 \cos(0.5t)$, $d_1(t) = 0.4 + 0.4 \sin(t)$, and $d_2(t) = 0.3 + 0.3 \cos(t)$. According to Assumption 1, we verify that $\zeta^- = \sigma^- = 0$, and $\zeta^+ = \sigma^+ = 1$. Then, there exist the matrices as follows:

$$\begin{aligned}
 C &= \begin{bmatrix} 1.52 & 0 \\ 0 & 1.33 \end{bmatrix}, \\
 D &= \begin{bmatrix} 1.21 & 0 \\ 0 & 1.76 \end{bmatrix}, \\
 A &= \begin{bmatrix} 0.7 & 0.5 \\ -0.6 & -0.76 \end{bmatrix}, \\
 B &= \begin{bmatrix} 0.33 & 0.25 \\ 0.6 & 0.63 \end{bmatrix}, \\
 E &= \begin{bmatrix} 1.3 & 1.28 \\ -1.1 & -0.88 \end{bmatrix}, \\
 H &= \begin{bmatrix} 0.4 & 0.25 \\ 0.87 & -0.5 \end{bmatrix}.
 \end{aligned} \tag{76}$$

Figure 3 shows the state trajectories of system (16); it is actually unstable. By solving the LMIs (21), we get the following feasible solutions:

$$\begin{aligned}
 R_1 &= \begin{bmatrix} 10.3478 & 2.3731 \\ 2.3731 & 9.8579 \end{bmatrix}, \\
 Q_3 &= \begin{bmatrix} 3.1634 & 0.6423 \\ 0.6423 & 3.7317 \end{bmatrix}, \\
 Q_4 &= \begin{bmatrix} 7.1504 & 1.4750 \\ 1.4750 & 8.3565 \end{bmatrix}, \\
 Q_5 &= \begin{bmatrix} 4.0425 & 1.1230 \\ 1.1230 & 4.5925 \end{bmatrix}, \\
 Q_6 &= \begin{bmatrix} 1.9672 & 0.2093 \\ 0.2093 & 2.3559 \end{bmatrix},
 \end{aligned}$$

FIGURE 3: State trajectories of system (16).

$$\begin{aligned}
 P_2 &= \begin{bmatrix} 4.3159 & 0.1023 \\ 0.1023 & 4.0132 \end{bmatrix}, \\
 P_3 &= \begin{bmatrix} 4.7189 & -0.0015 \\ -0.0015 & 4.6924 \end{bmatrix}, \\
 Z_1 &= \begin{bmatrix} 0.3110 & 0.0541 \\ 0.0541 & 0.3985 \end{bmatrix}, \\
 Z_2 &= \begin{bmatrix} 0.0457 & -0.0004 \\ -0.0004 & 0.0450 \end{bmatrix}, \\
 R_2 &= \begin{bmatrix} 14.0110 & 3.7932 \\ 3.7932 & 14.2218 \end{bmatrix}, \\
 O_3 &= \begin{bmatrix} 3.6595 & 0.9958 \\ 0.9958 & 4.4985 \end{bmatrix}, \\
 O_4 &= \begin{bmatrix} 8.0128 & 2.0885 \\ 2.0885 & 9.9776 \end{bmatrix}, \\
 O_5 &= \begin{bmatrix} 5.3912 & 2.2361 \\ 2.2361 & 6.2646 \end{bmatrix}, \\
 O_6 &= \begin{bmatrix} 1.6776 & -0.0893 \\ -0.0893 & 2.3607 \end{bmatrix}, \\
 P_6 &= \begin{bmatrix} 7.4849 & 0.1076 \\ 0.1076 & 7.7770 \end{bmatrix}, \\
 P_7 &= \begin{bmatrix} 8.7529 & -0.0353 \\ -0.0353 & 8.8750 \end{bmatrix},
 \end{aligned}$$

$$Z_3 = \begin{bmatrix} 0.3312 & -0.0060 \\ -0.0060 & 0.4389 \end{bmatrix},$$

$$Z_4 = \begin{bmatrix} 0.2262 & 0.0004 \\ 0.0004 & 0.2189 \end{bmatrix},$$

$$M_1 = \begin{bmatrix} 1.7488 & 0.1600 \\ 0.1600 & 1.7944 \end{bmatrix},$$

$$M_2 = \begin{bmatrix} 4.3667 & 0.4139 \\ 0.4139 & 4.0400 \end{bmatrix},$$

$$W_1 = \begin{bmatrix} 5.4082 & 3.0213 \\ 3.0213 & 6.4308 \end{bmatrix},$$

$$W_2 = \begin{bmatrix} 3.5446 & 0.5053 \\ 0.5053 & 3.7780 \end{bmatrix},$$

$$W_3 = \begin{bmatrix} 3.5391 & 0.8050 \\ 0.8050 & 3.7611 \end{bmatrix},$$

$$M_3 = \begin{bmatrix} 2.0055 & 0.2579 \\ 0.2579 & 2.2772 \end{bmatrix},$$

$$M_4 = \begin{bmatrix} 8.1716 & 1.6020 \\ 1.6020 & 7.4034 \end{bmatrix},$$

$$W_4 = \begin{bmatrix} 7.9401 & 4.1641 \\ 4.1641 & 9.2674 \end{bmatrix},$$

$$W_5 = \begin{bmatrix} 5.0707 & 1.2361 \\ 1.2361 & 5.9315 \end{bmatrix},$$

$$W_6 = \begin{bmatrix} 5.3825 & 2.0405 \\ 2.0405 & 6.1309 \end{bmatrix},$$

$$\gamma = 20.4722.$$

(77)

Using the feasible solution, we obtain the state trajectories which are described in Figure 4, and we can see that both $x(t)$ and $y(t)$ are converging to the zero point; therefore, the proved MBAMNN is internally stable. Thus, it is globally passive.

Example 2. Consider a 2-dimensional MBAMNN of (50) with the parameters as follows.

Here, we take the activation functions as $f_i(s) = g_i(s) = \tanh(s)$, $i = 1, 2$. Then, let $\delta = 0.2$, $\delta_0 = 0.2$, $\tau_1 = 0.6$, $\tau_2 = 1$, $\mu_1 = 0.3$, $\mu_2 = 0.5$, $\mu_0 = 0.4$, $\alpha = 0.1$, $\rho = 0.18$, $\rho_0 = 0.15$, $d_1 = 1.5$, $d_2 = 2$, $\omega_1 = 0.6$, $\omega_2 = 0.8$, $\omega_0 = 0.3$, and $\beta = 0.2$. Leakage time-varying delays $\delta(t) = 0.5 + 0.3 \sin(t)$ and $\rho(t) = 0.4 + 0.5 \cos(t)$; probabilistic time-varying delays $\tau_1(t) = 0.4 + 0.3 \sin(t)$, $\tau_2(t) = 0.5 + 0.5 \cos(0.5t)$, $d_1(t) = 0.3 + 0.3 \sin(t)$, and $d_2(t) = 0.1 + 0.5 \cos(t)$; distributed

FIGURE 4: State trajectories of system (16).

time-varying delays $\alpha(t) = \beta(t) = 0.5 + 0.5 \cos(t)$. After changing parameters, there exist the matrices

$$C = \begin{bmatrix} 1.52 & 0 \\ 0 & 1.33 \end{bmatrix},$$

$$D = \begin{bmatrix} 1.21 & 0 \\ 0 & 1.76 \end{bmatrix},$$

$$A = \begin{bmatrix} 0.3 & 0.4 \\ -0.6 & -0.76 \end{bmatrix},$$

$$B = \begin{bmatrix} 0.33 & 0.25 \\ 0.6 & 0.63 \end{bmatrix},$$

$$E = \begin{bmatrix} 1.3 & 1.28 \\ -1.1 & -0.88 \end{bmatrix},$$

$$H = \begin{bmatrix} 0.4 & 0.25 \\ 0.87 & -0.5 \end{bmatrix},$$

$$K = \begin{bmatrix} 0.12 & 0 \\ 0 & -0.1 \end{bmatrix},$$

$$L = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.06 \end{bmatrix}.$$

(78)

From the state trajectories which are depicted in Figure 5, we find that system (50) is actually unstable. By solving the LMIs (52), we have the following feasible solutions:

$$R_1 = \begin{bmatrix} 7.0452 & 2.9361 \\ 2.9361 & 11.6063 \end{bmatrix},$$

FIGURE 5: State trajectories of system (50).

$$Q_1 = \begin{bmatrix} 6.5971 & -1.6049 \\ -1.6049 & 9.2078 \end{bmatrix},$$

$$Q_2 = \begin{bmatrix} 0.5551 & 0.4601 \\ 0.4601 & 3.5493 \end{bmatrix},$$

$$Q_3 = \begin{bmatrix} 1.7184 & 1.0171 \\ 1.0171 & 5.9298 \end{bmatrix},$$

$$Q_4 = \begin{bmatrix} 3.5521 & 2.1946 \\ 2.1946 & 13.1443 \end{bmatrix},$$

$$Q_5 = \begin{bmatrix} 2.6799 & 1.5175 \\ 1.5175 & 7.6941 \end{bmatrix},$$

$$Q_6 = \begin{bmatrix} 0.5770 & 0.4402 \\ 0.4402 & 3.5446 \end{bmatrix},$$

$$P_1 = \begin{bmatrix} 5.6725 & -11.6662 \\ -11.6662 & 76.6622 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 0.0858 & 0.0272 \\ 0.0272 & 1.0798 \end{bmatrix},$$

$$P_3 = \begin{bmatrix} 0.1919 & 0.0517 \\ 0.0517 & 2.2235 \end{bmatrix},$$

$$P_4 = \begin{bmatrix} 478.9177 & 72.1940 \\ 72.1940 & 122.7031 \end{bmatrix},$$

$$Z_1 = \begin{bmatrix} 0.0835 & 0.0516 \\ 0.0516 & 0.6378 \end{bmatrix},$$

$$Z_2 = \begin{bmatrix} 0.1446 & -0.0612 \\ -0.0612 & 1.8780 \end{bmatrix},$$

$$M_1 = \begin{bmatrix} 1.1710 & 1.1366 \\ 1.1366 & 2.7731 \end{bmatrix},$$

$$M_2 = \begin{bmatrix} 0.5725 & 0.0760 \\ 0.0760 & 5.8435 \end{bmatrix},$$

$$W_1 = \begin{bmatrix} 2.9231 & 3.9208 \\ 3.9208 & 15.5396 \end{bmatrix},$$

$$W_2 = \begin{bmatrix} 2.5516 & 1.3400 \\ 1.3400 & 8.4107 \end{bmatrix},$$

$$W_3 = \begin{bmatrix} 2.7557 & 1.4738 \\ 1.4738 & 7.8878 \end{bmatrix},$$

$$N_1 = \begin{bmatrix} 20.6455 & 0.1674 \\ 0.1674 & 25.5954 \end{bmatrix},$$

$$R_2 = \begin{bmatrix} 22.1627 & 10.5483 \\ 10.5483 & 21.6007 \end{bmatrix},$$

$$O_1 = \begin{bmatrix} 29.6665 & -0.0928 \\ -0.0928 & 17.2449 \end{bmatrix},$$

$$O_2 = \begin{bmatrix} 2.1311 & 0.8588 \\ 0.8588 & 3.0198 \end{bmatrix},$$

$$O_3 = \begin{bmatrix} 6.9160 & 1.2013 \\ 1.2013 & 7.3828 \end{bmatrix},$$

$$O_4 = \begin{bmatrix} 20.7004 & 2.6570 \\ 2.6570 & 20.3705 \end{bmatrix},$$

$$O_5 = \begin{bmatrix} 17.4326 & 1.3252 \\ 1.3252 & 15.6910 \end{bmatrix},$$

$$O_6 = \begin{bmatrix} 2.1603 & 0.8746 \\ 0.8746 & 3.0879 \end{bmatrix},$$

$$P_5 = \begin{bmatrix} 77.8766 & -12.7230 \\ -12.7230 & 176.5013 \end{bmatrix},$$

$$P_6 = \begin{bmatrix} 0.1260 & 0.0457 \\ 0.0457 & 0.1924 \end{bmatrix},$$

$$P_7 = \begin{bmatrix} 0.8705 & 0.3558 \\ 0.3558 & 1.3869 \end{bmatrix},$$

$$P_8 = \begin{bmatrix} 860.5842 & 235.3961 \\ 235.3961 & 447.6840 \end{bmatrix},$$

$$Z_3 = \begin{bmatrix} 0.2328 & 0.0901 \\ 0.0901 & 0.3687 \end{bmatrix},$$

FIGURE 6: State trajectories of system (50).

FIGURE 7: State trajectories of system (71).

$$\begin{aligned}
 Z_4 &= \begin{bmatrix} 0.3230 & 0.1292 \\ 0.1292 & 0.5292 \end{bmatrix}, \\
 M_3 &= \begin{bmatrix} 5.2095 & 2.3987 \\ 2.3987 & 3.2962 \end{bmatrix}, \\
 M_4 &= \begin{bmatrix} 5.1434 & 1.3833 \\ 1.3833 & 8.8319 \end{bmatrix}, \\
 W_4 &= \begin{bmatrix} 25.3297 & 14.2601 \\ 14.2601 & 34.2731 \end{bmatrix}, \\
 W_5 &= \begin{bmatrix} 5.9791 & 1.1879 \\ 1.1879 & 6.6651 \end{bmatrix}, \\
 W_6 &= \begin{bmatrix} 7.1161 & 0.6066 \\ 0.6066 & 6.5388 \end{bmatrix}, \\
 N_2 &= \begin{bmatrix} 40.5252 & 2.2784 \\ 2.2784 & 38.2262 \end{bmatrix}, \\
 \gamma &= 50.2752.
 \end{aligned} \tag{79}$$

Using the feasible solution, we obtain the state trajectories in Figure 6, and we find that both $x(t)$ and $y(t)$ are converging to the zero point; thus, the proved MBAMNN is internally stable. Therefore, it is globally passive.

Example 3. Consider a 2-dimensional MBAMNN of (71) with the parameters as follows.

Here, we take the activation functions as $f_i(s) = g_i(s) = \tanh(s)$, $i = 1, 2$. And let $\delta = 0.3$, $\tau_1 = 0.2$, $\tau_2 = 0.3$, $\mu_1 = 0.3$, $\mu_2 = 0.4$, $\mu_0 = 0.4$, $d_1 = 1.3$, $d_2 = 1.5$, $\omega_1 = 0.6$, $\omega_2 =$

0.8 , and $\omega_0 = 0.25$. Probabilistic time-varying delays $\tau_1(t) = 0.6 + 0.4 \sin(t)$, $\tau_2(t) = 0.8 + 0.5 \cos(0.5t)$, $d_1(t) = 0.5 + 0.3 \sin(t)$, and $d_2(t) = 0.6 + 0.5 \cos(t)$. After changing parameters, there exist the matrices

$$\begin{aligned}
 C &= \begin{bmatrix} 1.52 & 0 \\ 0 & 1.33 \end{bmatrix}, \\
 D &= \begin{bmatrix} 1.21 & 0 \\ 0 & 1.76 \end{bmatrix}, \\
 A &= \begin{bmatrix} 0.7 & 0.5 \\ -0.6 & -0.76 \end{bmatrix}, \\
 B &= \begin{bmatrix} 0.33 & 0.25 \\ 0.6 & 0.63 \end{bmatrix}, \\
 E &= \begin{bmatrix} 1.3 & 1.28 \\ -1.1 & -0.88 \end{bmatrix}, \\
 H &= \begin{bmatrix} 0.4 & 0.25 \\ 0.87 & -0.5 \end{bmatrix}.
 \end{aligned} \tag{80}$$

Figure 7 shows the state trajectories of system (71); it is actually unstable. By solving the LMIs in Corollary 12, we obtain the following feasible solutions:

$$\begin{aligned}
 R_1 &= \begin{bmatrix} 59.3283 & 30.3806 \\ 30.3806 & 56.1253 \end{bmatrix}, \\
 Q_2 &= \begin{bmatrix} 11.2692 & 4.8104 \\ 4.8104 & 13.1800 \end{bmatrix},
 \end{aligned}$$

$$\begin{aligned}
Q_3 &= \begin{bmatrix} 14.8695 & 8.8717 \\ 8.8717 & 14.8525 \end{bmatrix}, \\
Q_4 &= \begin{bmatrix} 29.6331 & 17.8695 \\ 17.8695 & 30.7251 \end{bmatrix}, \\
Q_5 &= \begin{bmatrix} 21.2565 & 12.7666 \\ 12.7666 & 19.9407 \end{bmatrix}, \\
Q_6 &= \begin{bmatrix} 5.4645 & 3.2592 \\ 3.2592 & 6.9941 \end{bmatrix}, \\
P_1 &= \begin{bmatrix} 36.1361 & 6.0248 \\ 6.0248 & 31.4429 \end{bmatrix}, \\
P_2 &= \begin{bmatrix} 25.8085 & 3.8441 \\ 3.8441 & 27.0796 \end{bmatrix}, \\
P_3 &= \begin{bmatrix} 33.1166 & 1.3714 \\ 1.3714 & 34.0596 \end{bmatrix}, \\
P_4 &= \begin{bmatrix} 296.1210 & 93.8823 \\ 93.8823 & 252.4336 \end{bmatrix}, \\
Z_1 &= \begin{bmatrix} 0.1137 & 0.0919 \\ 0.0919 & 0.1535 \end{bmatrix}, \\
Z_2 &= \begin{bmatrix} 0.7613 & -0.1194 \\ -0.1194 & 0.7099 \end{bmatrix}, \\
M_1 &= \begin{bmatrix} 8.3925 & 0.6621 \\ 0.6621 & 10.4709 \end{bmatrix}, \\
M_2 &= \begin{bmatrix} 12.4316 & 0.3775 \\ 0.3775 & 13.7920 \end{bmatrix}, \\
W_1 &= \begin{bmatrix} 27.1266 & 17.2502 \\ 17.2502 & 33.8015 \end{bmatrix}, \\
W_2 &= \begin{bmatrix} 19.6366 & 10.3508 \\ 10.3508 & 19.7140 \end{bmatrix}, \\
W_3 &= \begin{bmatrix} 23.9080 & 13.1438 \\ 13.1438 & 22.5256 \end{bmatrix}, \\
R_2 &= \begin{bmatrix} 25.5707 & 20.3266 \\ 20.3266 & 41.5693 \end{bmatrix}, \\
O_2 &= \begin{bmatrix} 15.1779 & 4.9012 \\ 4.9012 & 43.6250 \end{bmatrix}, \\
O_3 &= \begin{bmatrix} 13.7722 & 0.9440 \\ 0.9440 & 8.8489 \end{bmatrix}, \\
M_4 &= \begin{bmatrix} 17.9661 & 1.5273 \\ 1.5273 & 15.4807 \end{bmatrix}, \\
W_4 &= \begin{bmatrix} 55.1410 & 44.6653 \\ 44.6653 & 57.6623 \end{bmatrix}, \\
O_6 &= \begin{bmatrix} 4.9903 & 1.5755 \\ 1.5755 & 3.5372 \end{bmatrix}, \\
P_5 &= \begin{bmatrix} 104.0967 & 34.3385 \\ 34.3385 & 87.8965 \end{bmatrix}, \\
P_6 &= \begin{bmatrix} 0.8159 & 0.3667 \\ 0.3667 & 0.4678 \end{bmatrix}, \\
P_7 &= \begin{bmatrix} 24.1326 & 8.8218 \\ 8.8218 & 15.3127 \end{bmatrix}, \\
P_8 &= \begin{bmatrix} 220.2420 & 251.8009 \\ 251.8009 & 686.0176 \end{bmatrix}, \\
Z_3 &= \begin{bmatrix} 2.2774 & 1.0053 \\ 1.0053 & 1.2605 \end{bmatrix}, \\
Z_4 &= \begin{bmatrix} 38.7115 & 4.7302 \\ 4.7302 & 32.7023 \end{bmatrix}, \\
M_3 &= \begin{bmatrix} 2.8959 & 0.6752 \\ 0.6752 & 8.0316 \end{bmatrix}, \\
W_5 &= \begin{bmatrix} 12.3093 & 1.3710 \\ 1.3710 & 8.1296 \end{bmatrix}, \\
W_6 &= \begin{bmatrix} 15.9416 & -1.5426 \\ -1.5426 & 9.4538 \end{bmatrix}, \\
O_4 &= \begin{bmatrix} 46.4425 & -1.9760 \\ -1.9760 & 28.1517 \end{bmatrix}, \\
O_5 &= \begin{bmatrix} 38.9043 & -4.3696 \\ -4.3696 & 22.8177 \end{bmatrix}, \\
\gamma &= 183.3912.
\end{aligned} \tag{81}$$

Using the feasible solution, we obtain the state trajectories in Figure 8, and we can see that both $x(t)$ and $y(t)$ are converging to the zero point; therefore, the proved MBAMNN is internally stable. Thus, it is globally passive.

5. Conclusion

We have studied the passivity problem of MBAMNNs with probabilistic and mixed time-varying delays in this paper. By introducing random variables with Bernoulli distribution, using some useful inequalities, and constructing appropriate

FIGURE 8: State trajectories of system (71).

LKFs, we have got new delay-dependent conditions in LMIs, which ensure the passivity criteria. Future work will focus on the passivity analysis of MBAMNNs with different types of time delays.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Authors' Contributions

Weiping Wang and Xiong Luo contributed equally to this work.

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