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*Research Article*

# MHD Peristaltic Flow of Fractional Jeffrey Model through Porous Medium

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The magnetohydrodynamic (MHD) peristaltic flow of the fractional Jeffrey fluid through porous medium in a nonuniform channel is presented. The fractional calculus is considered in Darcy's law and the constitutive relationship which included the relaxation and retardation behavior. Under the assumptions of long wavelength and low Reynolds number, the analysis solutions of velocity distribution, pressure gradient, and pressure rise are investigated. The effects of fractional viscoelastic parameters of the generalized Jeffrey fluid on the peristaltic flow and the influence of magnetic field, porous medium, and geometric parameter of the nonuniform channel are presented through graphical illustration. The results of the analogous flow for the generalized second grade fluid, the fractional Maxwell fluid, are also deduced as special cases. The comparison among them is presented graphically.

## 1. Introduction

Peristaltic flow is generated by means of contraction and expansion of channel walls, which has wide applications in many physiological processes and industries. Peristalsis or the mechanism of peristalsis is used to propel the biological fluid from one organ to another, for instance, the transport of blood in vessels and the movement of the chyme in the digestive system. Due to the important role of peristaltic flow, many investigations of peristalsis for Newtonian and non-Newtonian fluids have been carried out theoretically and experimentally since Latham [1]. In view of the fact that most biofluids show the characteristics of non-Newtonian fluid, more and more researchers focus on non-Newtonian model of peristalsis. Srivastava [2] and Siddiqui [3] studied peristaltic transport of a couple-stress fluid and second-order fluid, respectively. Hayat [4] discussed the peristaltic mechanism of a Maxwell fluid in an asymmetric channel. In another aspect, the effect of the imposed magnetic field is usually significant in peristaltic transform for its applications for conductive biological fluid and biomechanics such as blood and blood pump machines. The peristalsis through porous medium is also investigated with great interest, as an example, which could practically describe the flow through diseased biological channel. Srinivas et al. [5, 6] investigated peristalsis motion of a Jeffrey fluid under the effect of magnetic field and of Newtonian fluid with porous medium. Hayat et al. [7, 8] researched the MHD peristaltic flow of Jeffrey fluid in a channel and in a rotating system with porous medium, respectively. Some other researches about the peristalsis under the effect of magnetic field with porous medium can be found in [9–14].

As we all know, the fractional model is more flexible to describe the viscoelastic property of the non-Newtonian fluids in physics, biology, and medical engineering, because a very good fit of experimental data is achieved when the constitutive equation with fractional derivative is used [15]. The fractional Maxwell fluid such as generalized second grade fluid, fractional Oldroyd-B fluid, or generalized Burgers fluid has been considered in modern mechanics [16–20]. In recent year, peristaltic transport of fractional viscoelastic fluid in different system plays an important role through the work of Tripathi et al. [21–23]. Jamil et al. [24] discussed the magnetohydrodynamics fractional Oldroyd-B fluid. Dharmendra [25] and Hameed et al. [26] studied peristaltic flow of a fractional second grade fluid. Peristaltic transport of a fractional Burgers' fluid with variable viscosity through an inclined tube is investigated by Rachid [27].

Considering Jeffrey fluid has important applications in biological fluid mechanics and most channels in biological organs and machines are general known to be nonuniform [23, 28], we investigate the two-dimensional MHD peristalsis of fractional Jeffrey fluid through porous nonuniform channel in this paper. The fractional calculus was taken into modified Darcy's law [17] and the constitutive equation of Jeffrey fluid was introduced by Hayat et al. [7] and Bird [29], in which the time derivative is instead of the convective derivative in constitutive equation of Oldroyd-B fluid. Under the imposed magnetic field, the peristaltic flow through nonuniform channel with porous medium for an incompressible viscoelastic fluid is considered. The paper is organized as follows. Section 2 deduces the basic equations of the fluid and presents the initial and boundary value problem for the flow. In Section 3 the analysis solution of the problem is obtained. Section 4 discusses the special cases and the numerical results. Section 5 is the conclusion of the paper.

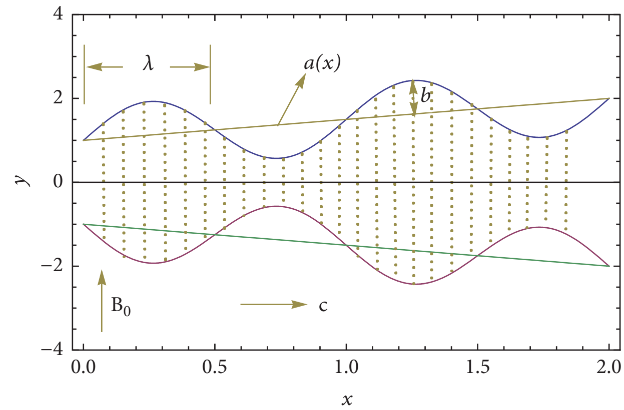


Figure 1: Geometry of the problem.

## 2. Basic Equations

The constitutive relationship of an incompressible fluid of Jeffrey model is of the form [7, 29]

$$\mathbf{T} = -p\mathbf{I} + \mathbf{S}, \tag{1}$$

$$\mathbf{S} + \lambda_1 \partial_t \mathbf{S} = \mu \left[ \mathbf{A} + \lambda_2 \partial_t \mathbf{A} \right], \tag{2}$$

where $-p\mathbf{I}$ denotes the indeterminate spherical stress due to the constraint of incompressibility, $\mathbf{S}$ is the extra-stress tensor, $\mathbf{A} = \mathbf{L} + \mathbf{L}^{\mathrm{T}}$ is the first Rivlin–Ericksen tensor, $\mathbf{L} = \nabla \mathbf{V}$ is the velocity gradient, $\mathbf{V}$ is the velocity vector, $\mu$ is the viscosity of the fluid, and $\lambda_1$ and $\lambda_2$ are constant relaxation and retardation times, respectively. According to the constitutive relationship of the classical Jeffrey fluid, the constitutive relationship of the fractional Jeffrey fluid is given by (1), and

$$\mathbf{S} + \lambda_1 D_t^{\alpha} \mathbf{S} = \mu \left[ \mathbf{A} + \lambda_2 D_t^{\beta} \mathbf{A} \right], \tag{3}$$

where $D_t^{\alpha}$ and $D_t^{\beta}$ are fractional calculus of order $\alpha$ and $\beta$ with respect to $t$, respectively, and may be defined as [30]

$$D_t^p f(t) = \frac{1}{\Gamma(1-p)} \frac{d}{dt} \int_0^t (t-\tau)^{-p} f(\tau)\, d\tau, \quad 0 \le p \le 1. \tag{4}$$

And the new material constants $\lambda_1$ and $\lambda_2$ have the dimensions of $t^{\alpha}$ and $t^{\beta}$, respectively. Some of the papers use $\lambda_1^{\alpha}$ and $\lambda_2^{\beta}$ instead of $\lambda_1$ and $\lambda_2$. However, for the sake of simplicity we keep the same notations as in the ordinary case. This model includes the ordinary Jeffrey fluid as a special case for $\alpha = \beta = 1$, in which $\lambda_1$ and $\lambda_2$ are relaxation and retardation time. This model also can be simplified to be the generalized second grade fluid when $\alpha = 0, \lambda_1 \longrightarrow 0$, to be fractional Maxwell fluid when $\beta = 0, \lambda_2 \longrightarrow 0$.

We consider the peristalsis of the electrically conducting fractional Jeffrey fluid flow through the two-dimensional nonuniform tube. In a suitable Cartesian coordinate system, we suppose that the fluid goes through a porous channel with the $x$-axis along the center line and $y$-axis normal to it (Figure 1). The geometry of the tube walls along which waves propagating is given by [23, 28]

$$y = \pm h = \pm \left[ a(x) + b \sin \frac{2\pi}{\lambda}(x - ct) \right], \tag{5}$$

where $a(x) = a_0 + mx$, $(m \ll 1)$ is the half width of the channel at any axial coordinate point $x$, $a_0$ is half width of the inlet of the channel, $m$ is constant whose magnitude depends on the length of the channel and the dimensions of the inlet and exit, and $b$, $\lambda$, $c$, and $t$ are the amplitude, wavelength, wave velocity, and time variation.

The uniform magnetic field of strength $B_0$ is applied in the transverse direction to flow, while induced magnetic field subject to low magnetic Reynolds number is neglected. So, the governing equations of the flow of MHD viscoelastic fluid in a porous medium are

$$\operatorname{div} \mathbf{V} = 0, \tag{6}$$

$$\rho \frac{d\mathbf{V}}{dt} = -\nabla p + \operatorname{div} \mathbf{S} - \sigma B_0^2 \mathbf{V} + \mathbf{r}, \tag{7}$$

where $d/dt$ is the material time derivative, $\rho$ is the density, $p$ is the pressure, and $\sigma$ is electrical conductivity. $\mathbf{r}$ is Darcy resistance and can be inferred from (3) for a generalized Jeffrey fluid in the porous medium satisfying the following equation [17]:

$$\left(1 + \lambda_1 D_t^{\alpha}\right) \mathbf{r} = -\frac{\mu \phi}{K} \left(1 + \lambda_2 D_t^{\beta}\right) \mathbf{V}, \tag{8}$$

where $K(>0)$ is permeability and $\phi(0 < \phi < 1)$ is porosity of the porous medium.

Substitute (3), (8), and $\mathbf{V} = (u, v)$ into (7), we obtain

$$\begin{aligned}(1+\lambda_1 D_t^{\alpha})\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right) &= -\frac{1}{\rho}(1+\lambda_1 D_t^{\alpha})\frac{\partial p}{\partial x}+\gamma\left(1+\lambda_2 D_t^{\beta}\right)\nabla u \\ &\quad -\frac{\sigma B_0^2}{\rho}(1+\lambda_1 D_t^{\alpha})u-\frac{\gamma\phi}{K}\left(1+\lambda_2 D_t^{\beta}\right)u,\end{aligned} \tag{9}$$

$$\begin{aligned}(1+\lambda_1 D_t^{\alpha})\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right) &= -\frac{1}{\rho}(1+\lambda_1 D_t^{\alpha})\frac{\partial p}{\partial y}+\gamma\left(1+\lambda_2 D_t^{\beta}\right)\nabla v \\ &\quad -\frac{\gamma\phi}{K}\left(1+\lambda_2 D_t^{\beta}\right)v,\end{aligned} \tag{10}$$

in which $\gamma = \mu/\rho$ is the kinematic viscosity and $u, v$ are the velocity components along $x$ and $y$ directions, respectively.

We introduce dimensionless variables as follows:

$$\begin{aligned}\hat{x} &= \frac{x}{\lambda}, \\ \hat{y} &= \frac{y}{a_0}, \\ \hat{t} &= \frac{tc}{\lambda}, \\ \hat{u} &= \frac{u}{c}, \\ \hat{v} &= \frac{v}{c\delta}, \\ \hat{\lambda}_1 &= \frac{c\lambda_1}{\lambda}, \\ \hat{\lambda}_2 &= \frac{c\lambda_2}{\lambda}, \\ \hat{p} &= \frac{pa_0^2}{\mu c\lambda}, \\ \hat{K} &= \frac{K}{\phi a_0^2}, \\ \hat{h} &= \frac{h}{a_0} = 1+\frac{m}{\delta}\hat{x}+\varphi\sin 2\pi(\hat{x}-\hat{t}), \\ \varphi &= \frac{b}{a_0}, \\ \delta &= \frac{a_0}{\lambda}, \\ \mathrm{Re} &= \frac{\rho c a_0\delta}{\mu}, \\ M &= \sqrt{\frac{\sigma}{\mu}}B_0 a_0,\end{aligned} \tag{11}$$

where $\delta, \varphi, \mathrm{Re}, M$ are wave number, amplitude ratio, Reynolds number, and Hartmann number, respectively. Then under the approximations of the long wavelength and low Reynolds number, we obtain the dimensionless equations (for simplicity, the dimensionless mark "∧" will be neglected from here on):

$$\begin{aligned}(1+\lambda_1 D_t^{\alpha})\frac{\partial p}{\partial x} &= \left(1+\lambda_2 D_t^{\beta}\right)\frac{\partial^2 u}{\partial y^2}-\frac{1}{K}\left(1+\lambda_2 D_t^{\beta}\right)u \\ &\quad -M^2(1+\lambda_1 D_t^{\alpha})u,\end{aligned} \tag{12}$$

$$\frac{\partial p}{\partial y} = 0. \tag{13}$$

The boundary and initial conditions are

$$\begin{aligned}\frac{\partial u}{\partial y} &= 0, \quad as\ y = 0; \\ u &= 0, \quad as\ y = h;\end{aligned} \tag{14}$$

$$\frac{\partial p}{\partial x} = 0, \quad as\ t = 0. \tag{15}$$

## 3. Solution of the Problem

Solving (12) with condition (14), we get the velocity

$$u = \frac{s}{k^2}\frac{\partial p}{\partial x}\left[\frac{\cosh(ky)}{\cosh(kh)}-1\right], \tag{16}$$

in which $k^2 = 1/K + M^2 s, s = (1+\lambda_1 D_t^{\alpha})/(1+\lambda_2 D_t^{\beta})$.

The volumetric flow rate in the fixed frame is given by

$$Q = \int_0^h u\,dy = \frac{s}{k^3}\frac{\partial p}{\partial x}[\tanh(kh)-kh]. \tag{17}$$

The relationships of the wave frame $(X, Y), (U, V)$ moving with velocity $c$ and the fixed frame $(x, y), (u, v)$ are given by

$$\begin{aligned}X &= x - t, \\ Y &= y, \\ U &= u - 1, \\ V &= v.\end{aligned} \tag{18}$$

Then the volumetric flow rate in the wave frame is

$$q = \int_0^h U\,dY = \int_0^h (u-1)\,dy = Q - h. \tag{19}$$

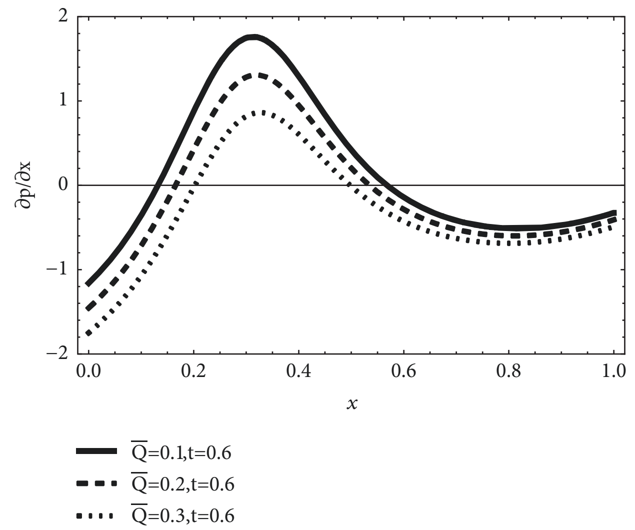


FIGURE 2: Profiles of the pressure gradient for various values of $\overline{Q}$ with fixed $m = 0.5, \delta = 0.3, \varphi = 0.5, \alpha = 0.4, \beta = 0.6, M = 1, \lambda_1 = 1, \lambda_2 = 1, t = 0.6, K = 1$.

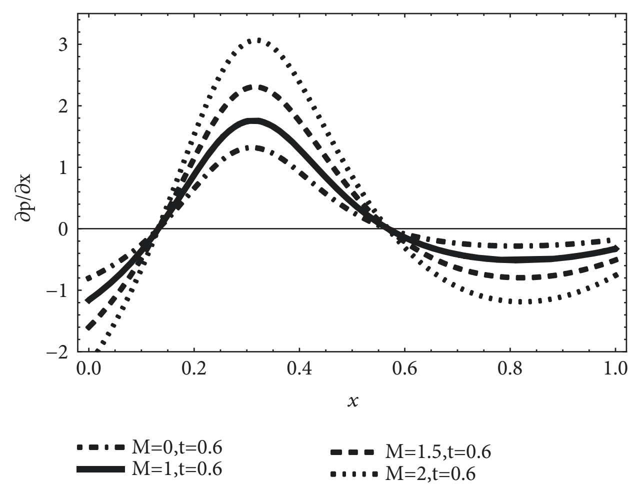


FIGURE 3: Profiles of the pressure gradient for various values of $M$ with fixed $m = 0.5, \delta = 0.3, \varphi = 0.5, \alpha = 0.4, \beta = 0.6, \lambda_1 = 1, \lambda_2 = 1, t = 0.6, K = 1, \overline{Q} = 0.1$.

And the average of the volumetric flow rate along one time period gives

$$\begin{aligned} \overline{Q} &= \int_0^1 Q dt = \int_0^1 (q+h)\, dt \\ &= \int_0^1 \left[ q + 1 + \frac{m}{\delta} x + \varphi \sin 2\pi (x - t) \right] dt \\ &= q + 1 + \frac{m}{\delta} x. \end{aligned} \tag{20}$$

From (17)-(20), we can have

$$\frac{\partial p}{\partial X} = \frac{k^3}{s} \frac{\overline{Q} + \varphi \sin 2\pi X}{\tanh (kh) - kh}, \tag{21}$$

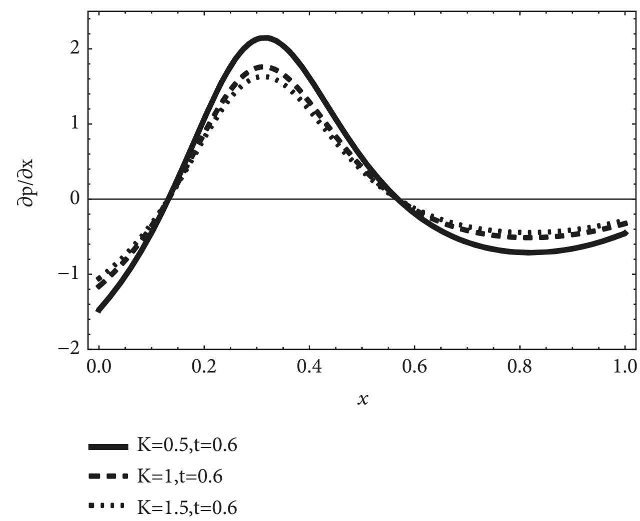


FIGURE 4: Profiles of the pressure gradient for various values of $K$ with fixed $m = 0.5, \delta = 0.3, \varphi = 0.5, \alpha = 0.4, \beta = 0.6, M = 1, \lambda_1 = 1, \lambda_2 = 1, t = 0.6, \overline{Q} = 0.1$.

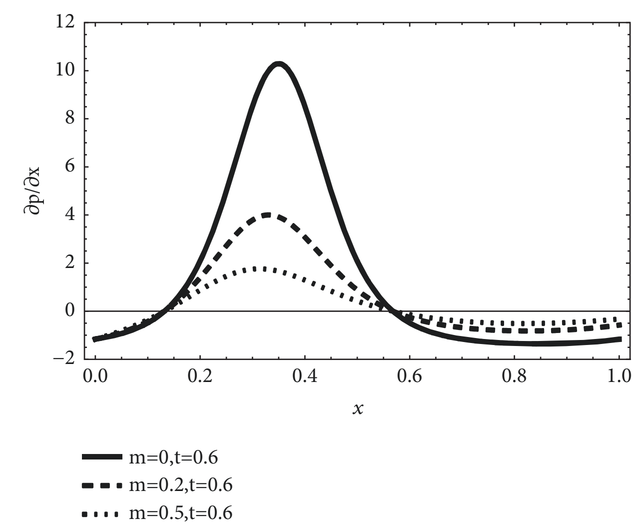


FIGURE 5: Profiles of the pressure gradient for various values of $m$ with fixed $\delta = 0.3, \varphi = 0.5, \alpha = 0.4, \beta = 0.6, M = 1, \lambda_1 = 1, \lambda_2 = 1, t = 0.6, K = 1, \overline{Q} = 0.1$.

where $h = 1 + (m/\delta)(X + t) + \varphi \sin 2\pi X$, then

$$U = k \frac{\overline{Q} + \varphi \sin 2\pi X}{\tanh (kh) - kh} \left[ \frac{\cosh (kY)}{\cosh (kh)} - 1 \right] - 1. \tag{22}$$

Because in wave frame the velocity $U = \partial \psi / \partial Y$, in which $\psi$ is the stream function, we can give

$$\psi = \frac{\overline{Q} + \varphi \sin 2\pi X}{\tanh (kh) - kh} \left[ \frac{\sinh (kY)}{\cosh (kh)} - kY \right] - Y. \tag{23}$$

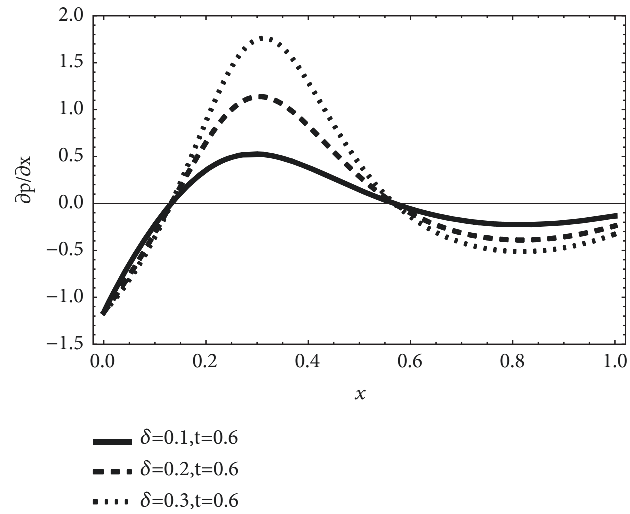


FIGURE 6: Profiles of the pressure gradient for various values of wave number $\delta$ with fixed $m = 0.5, \varphi = 0.5, \alpha = 0.4, \beta = 0.6, M = 1, \lambda_1 = 1, \lambda_2 = 1, t = 0.6, K = 1, \overline{Q} = 0.1$.

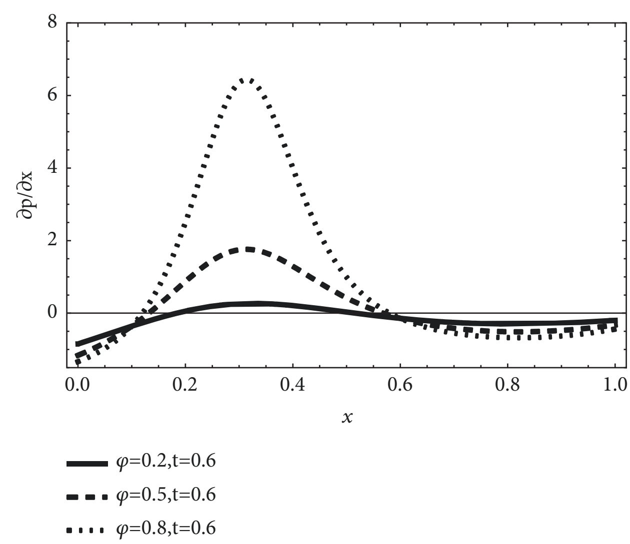


FIGURE 7: Profiles of the pressure gradient for various values of amplitude ratio $\varphi$ with fixed $m = 0.5, \delta = 0.3, \alpha = 0.4, \beta = 0.6, M = 1, \lambda_1 = 1, \lambda_2 = 1, t = 0.6, K = 1, \overline{Q} = 0.1$.

And the dimensionless pressure rise and friction can be obtained as follows:

$$\Delta p = \int_0^1 \frac{\partial p}{\partial X} dX, \qquad (24)$$

$$F = \int_0^1 -h^2 \frac{\partial p}{\partial X} dX. \qquad (25)$$

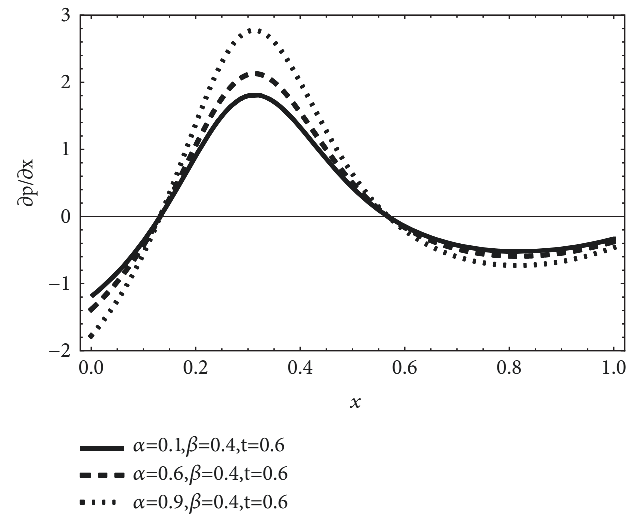


FIGURE 8: Profiles of the pressure gradient for various values of $\alpha$ with fixed $m = 0.5, \delta = 0.3, \varphi = 0.5, M = 1, \lambda_1 = 1, \lambda_2 = 1, K = 1, \overline{Q} = 0.1$.

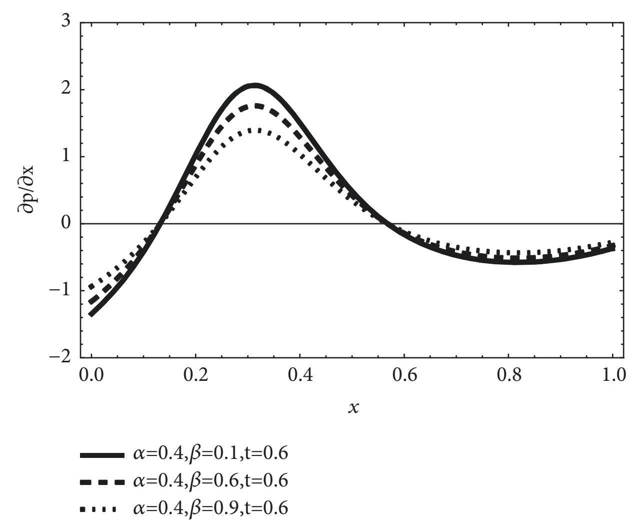


FIGURE 9: Profiles of the pressure gradient for various values of $\beta$ with fixed $m = 0.5, \delta = 0.3, \varphi = 0.5, M = 1, \lambda_1 = 1, \lambda_2 = 1, K = 1, \overline{Q} = 0.1$.

## 4. Discussion and Numerical Results

In a special case, if we consider the MHD peristaltic flow in nonporous medium, the velocity (16) reduces to

$$u = \frac{1}{M^2} \frac{\partial p}{\partial x} \left[ \frac{\cosh\left(Ms^{1/2} y\right)}{\cosh\left(Ms^{1/2} h\right)} - 1 \right]. \qquad (26)$$

If strength of the applied magnetic field $B_0 = 0$, *i.e.* $M = 0$, corresponding to the peristalsis for fractional Jeffrey fluid with porous medium, result (16) is simplified to

$$u = \frac{s}{k^2} \frac{\partial p}{\partial x} \left[ \frac{\cosh(ky)}{\cosh(kh)} - 1 \right], \qquad (27)$$

where $k^2 = 1/K$.

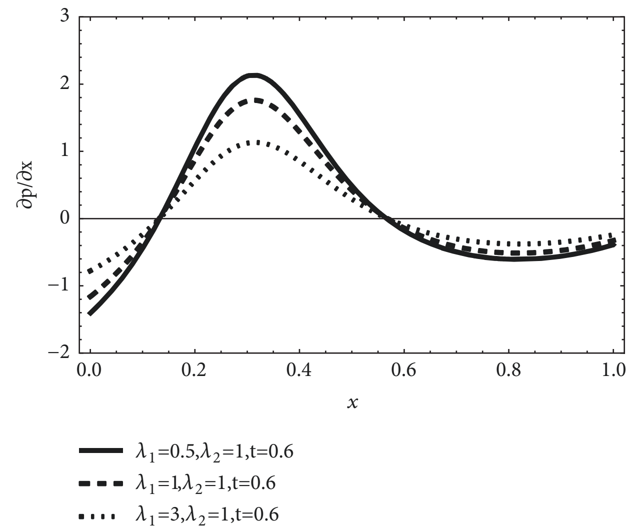


FIGURE 10: Profiles of the pressure gradient for various values of $\lambda_1$ with fixed $m = 0.5, \delta = 0.3, \varphi = 0.5, \alpha = 0.4, \beta = 0.6, M = 1, K = 1, \overline{Q} = 0.1$.

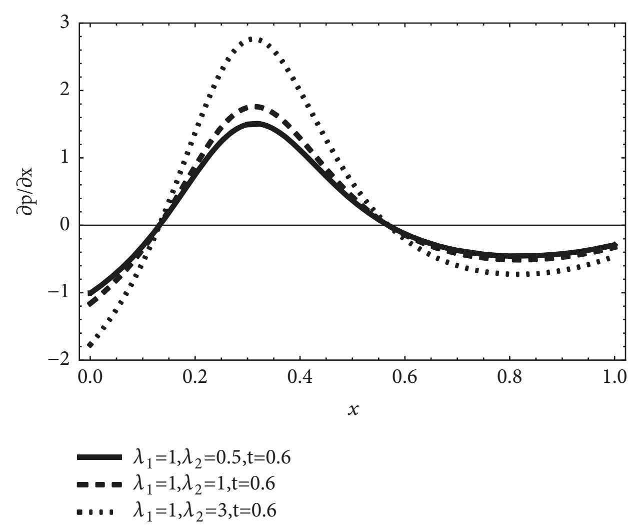


FIGURE 11: Profiles of the pressure gradient for various values of $\lambda_2$ with fixed $m = 0.5, \delta = 0.3, \varphi = 0.5, \alpha = 0.4, \beta = 0.6, M = 1, K = 1, \overline{Q} = 0.1$.

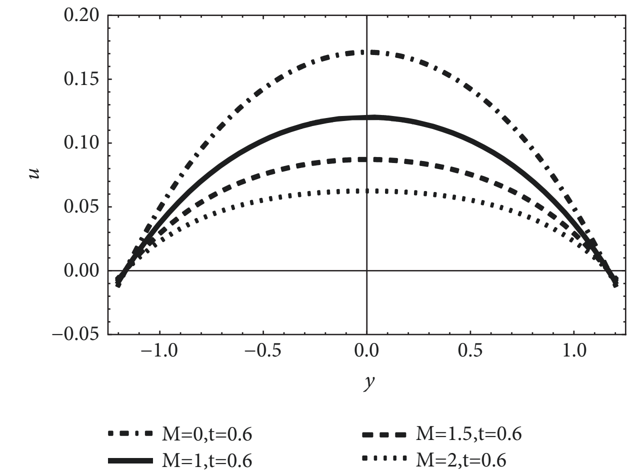


FIGURE 12: Profiles of the velocity $u$ for various values of $M$ with fixed $\partial p/\partial x = -0.35, m = 0.5, \delta = 0.3, \varphi = 0.5, \alpha = 0.4, \beta = 0.6, \lambda_1 = 1, \lambda_2 = 1, x = 0.1, K = 1$.

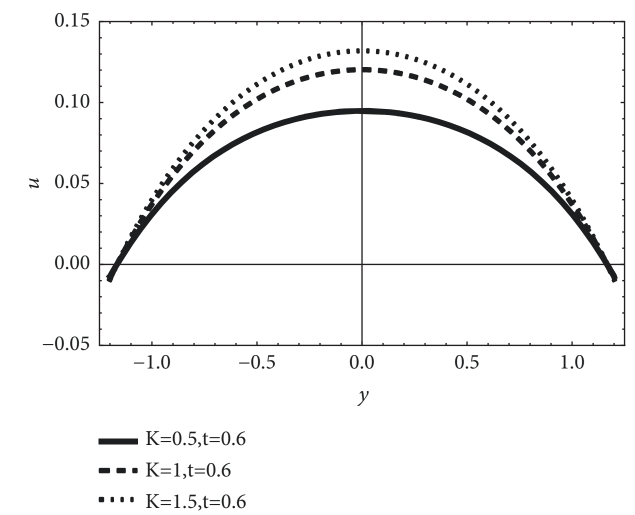


FIGURE 13: Profiles of the velocity $u$ for various values of $K$ with fixed $\partial p/\partial x = -0.35, m = 0.5, \delta = 0.3, \varphi = 0.5, \alpha = 0.4, \beta = 0.6, M = 1, \lambda_1 = 1, \lambda_2 = 1, x = 0.1$.

When $\alpha = 0, \lambda_1 \longrightarrow 0, s = (1 + \lambda_2 D_t^{\beta})^{-1}$ and the above results reduce to the solutions of the MHD peristalsis in porous medium for the generalized second grade fluid (GSF).

While $\beta = 0, \lambda_2 \longrightarrow 0, s = 1 + \lambda_1 D_t^{\alpha}$ and the solutions corresponding to the peristaltic flow of fractional Maxwell fluid (FMF) are obtained.

In addition, the influences of the parameters of magnetic field, medium, tube, and viscoelastic fluid on the flow motion are discussed through graphical illustrations. From Figures 2–11, the pressure gradient of the fixed frame as the function of the axial coordinate point$x$is profiled. Figure 2 reveals pressure gradient decreases with the average of volumetric flow rate. The influences of Hartmann number $M$, wave number $\delta$, and amplitude ratio $\varphi$ on the pressure gradient are shown in Figures 3, 6, and 7, respectively. The pressure gradient is shown to be the increase function with regard to strength of the magnetic field, wave number, and amplitude when pressure gradient is positive, while it is shown to the decrease function when pressure gradient is negative. From Figures 4 and 5, we notice that the pressure gradient decreases with increased porous parameter $K$ and channel parameter $m$, when pressure gradient is positive and inverse when pressure gradient is negative. Through Figures 8, 9, 10, and 11, it is clear that the fractional parameters $\alpha$, $\beta$ and the material parameters $\lambda_1$, $\lambda_2$ have effect on the pressure gradient. When pressure gradient is positive, the pressure gradient is direct proportion to fractional parameters $\alpha$ and retardation time

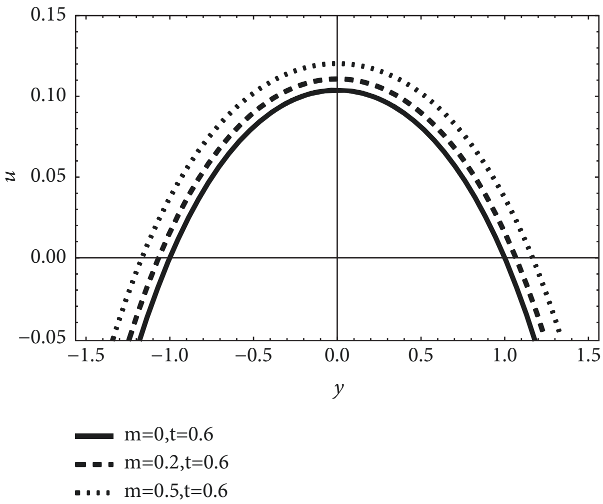


FIGURE 14: Profiles of the velocity $u$ for various values of $m$ with fixed $\partial p/\partial x = -0.35, \delta = 0.3, \varphi = 0.5, \alpha = 0.4, \beta = 0.6, M = 1, \lambda_1 = 1, \lambda_2 = 1, x = 0.1, K = 1$.

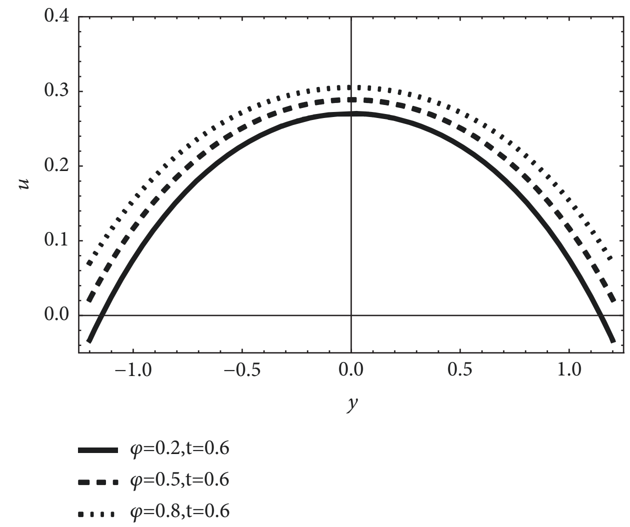


FIGURE 16: Profiles of the velocity $u$ for various values of $\varphi$ with fixed $\partial p/\partial x = -0.35, m = 0.5, \delta = 0.3, \alpha = 0.4, \beta = 0.6, M = 1, \lambda_1 = 1, \lambda_2 = 1, x = 0.1, K = 1$.

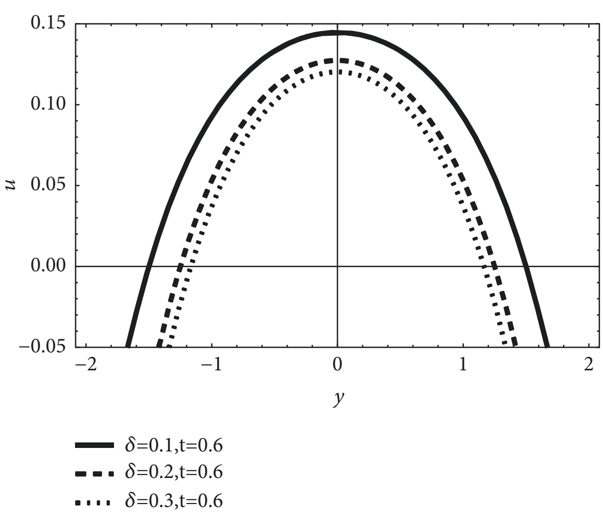


FIGURE 15: Profiles of the velocity $u$ for various values of $\delta$ with fixed $\partial p/\partial x = -0.35, m = 0.5, \varphi = 0.5, \alpha = 0.4, \beta = 0.6, M = 1, \lambda_1 = 1, \lambda_2 = 1, x = 0.1, K = 1$.

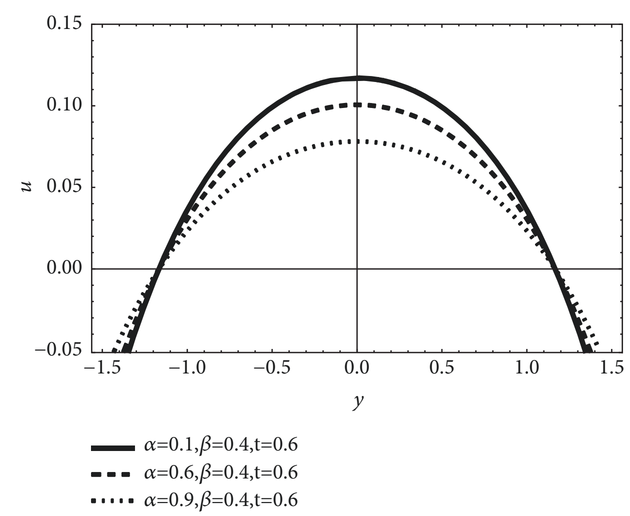


FIGURE 17: Profiles of the velocity $u$ for various values of $\alpha$ with fixed $\partial p/\partial x = -0.35, m = 0.5, \delta = 0.3, \varphi = 0.5, M = 1, \lambda_1 = 1, \lambda_2 = 1, x = 0.1, K = 1$.

$\lambda_2$ and is inverse proportion to fractional parameter $\beta$ and relaxation time $\lambda_1$.

The velocity distribution is plotted as a function of $y$ in Figures 12–20 with fixed axial coordinate and time. Figures 12 and 15 depict velocity distribution for various values of Hartmann number $M$ and wave number $\delta$, respectively. The velocity distribution is shown to be the decrease function with regard to the imposed magnetic field and wave number. Inversely, in Figures 13, 14, and 16, we notice that the pressure gradient is increased with increase of porous parameter $K$, channel parameter $m$, and amplitude ratio $\varphi$. Figures 17–20 show the influences of the fractional parameters and the material parameters on the velocity function. It is found that velocity distribution decreases with increasing fractional parameter $\alpha$ and retardation time $\lambda_2$ and increases with increasing fractional parameter $\beta$ and relaxation time $\lambda_1$, respectively.

Finally, Figures 21 and 22 are drawn, respectively, to study the difference of the pressure gradient and velocity distribution for generalized second grade fluid (GSF, with $\alpha = 0, \lambda_1 \longrightarrow 0$), fractional Maxwell fluid (FMF, with $\beta = 0, \lambda_2 \longrightarrow 0$), and fractional Jeffrey fluid (FJF). It is discovered that the pressure gradient and velocity of fractional Jeffrey fluid are just between the ones of other two fluid models.

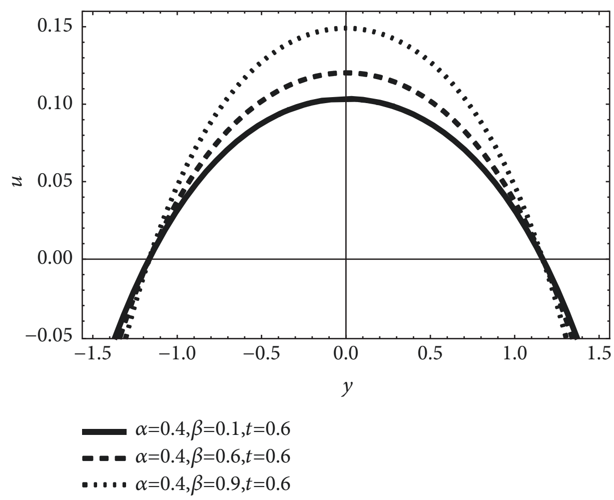


Figure 18: Profiles of the velocity $u$ for various values of $\beta$ with fixed $\partial p/\partial x = -0.35, m = 0.5, \delta = 0.3, \varphi = 0.5, M = 1, \lambda_1 = 1, \lambda_2 = 1, x = 0.1, K = 1$.

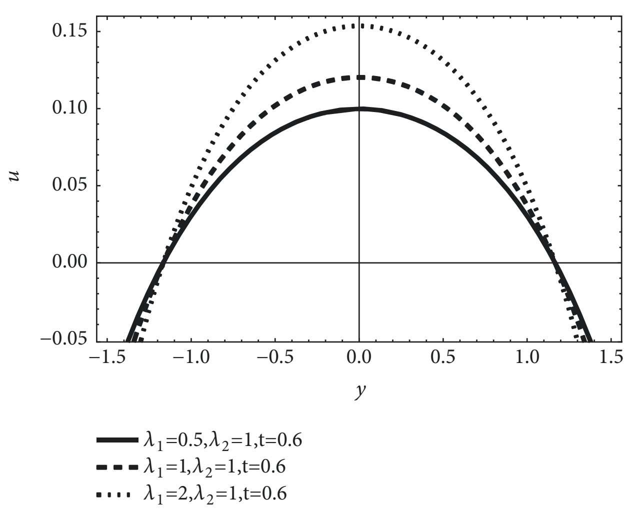


Figure 19: Profiles of the velocity $u$ for various values of $\lambda_1$ with fixed $\partial p/\partial x = -0.35, m = 0.5, \delta = 0.3, \varphi = 0.5, \alpha = 0.4, \beta = 0.6, M = 1, x = 0.1, K = 1$.

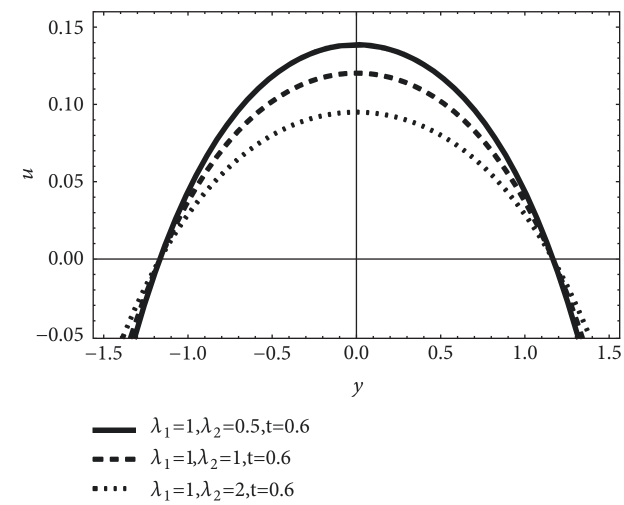


Figure 20: Profiles of the velocity $u$ for various values of $\lambda_2$ with fixed $\partial p/\partial x = -0.35, m = 0.5, \delta = 0.3, \varphi = 0.5, \alpha = 0.4, \beta = 0.6, M = 1, x = 0.1, K = 1$.

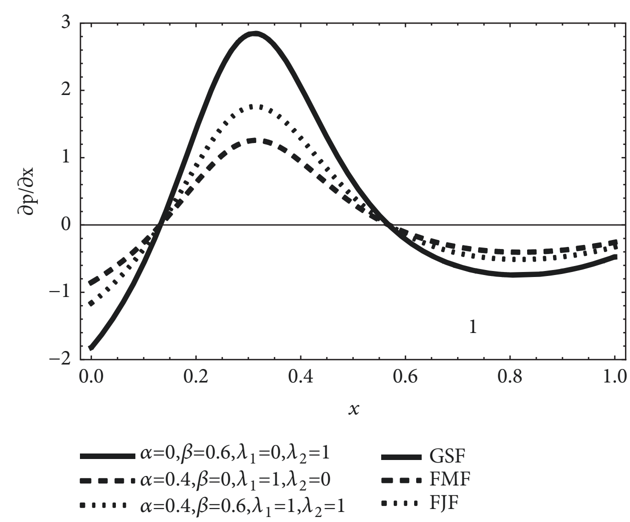


Figure 21: Profiles of the pressure gradient $\partial p/\partial X$ for GSF, FMF and FJF, with fixed $m = 0.5, \delta = 0.3, \varphi = 0.5, M = 1, t = 0.6, K = 1, \overline{Q} = 0.1$.

## 5. Conclusion

In this investigation, we established a mathematic model of the MHD peristaltic flow of fractional Jeffrey fluid through porous a nonuniform tube. Using the assumptions of long wavelength and low Reynolds number, we obtained the analysis expression of velocity component along $x$ directions, the relationship between pressure gradient and the volumetric flow rate, pressure rise, friction force, and stream function. And these results can be simplified to peristaltic flow of the generalized second grade and fractional Maxwell models when relevant parameters assume special values. The viscoelastic effects of the fractional Jeffrey fluid in porous nonuniform tube and the influence of magnetic field and porosity parameter on the flow motion are depicted through graphical illustrations. Based on the above theoretical and numerical research, the main conclusions are that pressure gradient with respect to axial coordinate is suppressed by the average of volume flow rate, porous parameter, nonuniform channel parameter, fractional parameter $\beta$, and relaxation time and inversely is accentuated by imposed magnetic field, wave number, amplitude ratio, fractional parameter $\alpha$, and retardation time, while it is inverse for velocity distribution in axial coordinate direction. The viscoelasticity of fractional Jeffrey fluid is between the fractional Maxwell and generalized second fluid.

## Data Availability

No data were used to support this study.

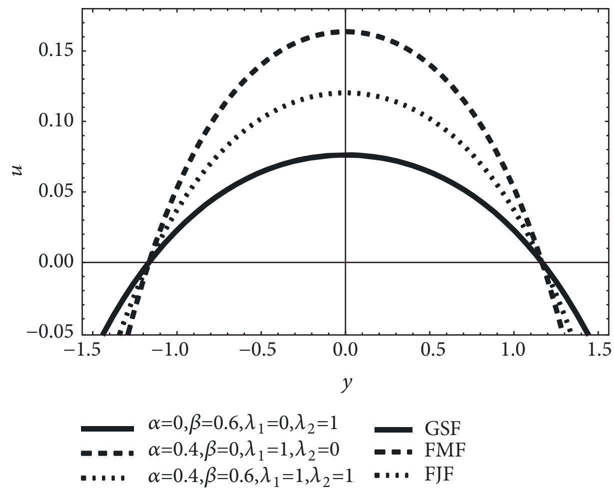


Figure 22: Profiles of the velocity $u$ for GSF, FMF and FJF, with fixed $\partial p/\partial x = -0.35, m = 0.5, \delta = 0.3, \varphi = 0.5, M = 1, t = 0.6, x = 0.1, K = 1$.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.


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