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Research Article

Type Synthesis of 5-DoF Parallel Mechanisms with Different Submechanisms

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The type synthesis of the 5-DoF (degree of freedom) parallel mechanisms with different submechanisms is studied by utilizing digital topology graphs (DTGs). The conditions for synthesizing the 5-DoF parallel mechanisms with different submechanisms using DTGs are determined. Many valid DTGs are derived from 17 different 5-DoF associated linkages, and the valid DTGs are transformed into revised DTGs. The subplanar and/or spatial parallel mechanisms in the 5-DoF parallel mechanisms are transformed into some simple equivalent limbs, and their equivalent relations and merits are analyzed. Using the derived valid DTGs and revised DTGs, many 5-DoF parallel mechanisms with different subserial or parallel mechanisms are synthesized, and they are simplified by replacing the complicated subparallel mechanisms with their simple equivalent limbs. Finally, their DoFs are calculated to verify the correctness and effectiveness of the proposed approach.

1. Introduction

Type synthesis of mechanisms is a well-known technology for creating and designing novel mechanisms [1–5]. In this aspect, Gogu [1–3] studied the type synthesis of parallel mechanisms (PMs) and presented morphological and evolutionary approaches. Huang conducted the type synthesis of PM by utilizing Lie group and screw-theory [4]. Yang studied the topology structure design of mechanisms [6]. Johnson derived a planar associated linkage (ALs) by utilizing a determining tree and synthesized many planar mechanisms by AL [5]. Merlet proposed a design methodology for conception of PMs [7]. Ceccarelli studied design methodology for compliant binary actuated PM with flexure hinges and design and evaluation of a discretely actuated multimodule PM [8]. A TG (topology graph) has been applied widely for the type synthesis of closed mechanisms [5, 6]. A contracted graph (CG) without any binary link is applied for deriving TG. In this aspect, Vucina and Freudenstein [9] and Tsai and Norton [10] proposed TG and CG and used them to synthesize mechanisms. Yan and Kang [11] studied the configuration synthesis of mechanismsby changing types and/or motion orientations of joints. Hervé [12] studied type synthesis of mechanisms using Lie group. Pucheta and Cardona [13, 14] synthesized planar linkages based on constrained subgraph isomorphism detection and existing mechanisms. Saxena and Ananthasuresh [15] selected the best configuration based on design specifications. Lu et al. explained conditions for deriving the valid DTGs by utilizing submechanisms, discovered the relations between AL, redundant constraint, and DoFs of PMs [16], and conducted type synthesis of 3-DoF PMs by utilizing revised DTGs [17].

Generally, the tool axis of a machine tool is required to be perpendicular to the 3D free-form surfaces during the machining of workpieces (the injection moulds, dies, models of automobile windshields, impeller blades of ships or airplanes, launches, and turbines) in order to improve the machining quality and reduce the machining forces. Therefore, the 5-DoF PMs have more industrial applications and become the best option of developing the parallel machine tools for the following reasons [18–21]:

(1) The tool axis of 5-DoF parallel machine tool formed by the 5-DoF PMs can be kept perpendicular to the 3D free-form surfaces during machining 3D free-form surfaces.

(2) Comparing with the existing serial-parallel 5-DoF machine tool which is formed by a 3-DoF PM connected

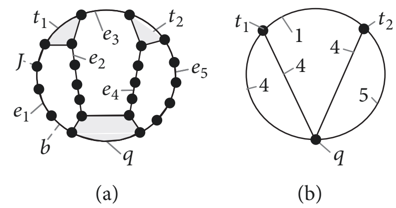


FIGURE 1: A TG with $q + 2t + 18b$ and $23J$ (a) and a DTG with $q + 2t + 18b$ and an array $\{t_1144 \quad q4445 \quad t_2541\}$ (b).

with a 2-DoF spindle actuator in series, the 5-DoF parallel machine tool formed by the 5-DoF PMs has higher rigid and precision because its actuators are close to the base and has more active limbs for supporting m.

(3) Comparing with the 6-DoF PMs, the 5-DoF PMs are simpler in structure and easier in control.

Let "R, P, C, U, and S" represent "revolute, prismatic, cylinder, universal, and spherical" pairs, respectively. In this aspect, Gao et al. developed a 5-DoF parallel machine tool with 4 PSS limbs and a composite 3UU limb [18]. Liu et al. proposed a coupling 3-PSR/PSU 5-axis compensation mechanism [19]. Fang and Tsai synthesized a class of 5-DoF overconstrained PMs with identical serial limbs [20]. Kong and Gosselin synthesized several 5-DoF PMs based on screw theory and the concept of virtual chains [21]. These studies have their merits and different focuses and have laid a theoretical foundation for this study.

Up to now, several 3- and 4-DoF PMs have been synthesized [1–6, 13]. When they are used as one submechanism of 5-DoF PMs, many novel 5-DoF PMs with high stiffness can be synthesized. Therefore, it is a significant and challenging issue to synthesize novel 5-DoF PMs with special functions and/or better characteristics (large position and orientation workspace, large capability of load bearing, high stiffness, simple structure, and easy control). For this reason, this paper focuses on the type synthesis of the 5-DoF PMs with subserial mechanisms, subplanar mechanisms, and sub-PMs or their combinations by utilizing valid DTGs (digital topology graph). The following problems are solved: (1) constructing various DTGs of 5-DoF PMs by utilizing 17 ALs; (2) determining the relationship between the valid DTGs and the equivalent limbs of the subplanar mechanisms and sub-PMs; (3) synthesizing the 5-DoF PMs with the subplanar or sub-PMs by utilizing valid DTGs.

2. Concepts and Conditions

A parallel mechanism (PM) may be composed of various links connected by several kinds of kinematic pairs such as R, P, C, U, and S (revolute, prismatic, cylinder, universal, and spherical) pairs. A Kutzbach-Grübler formula had been widely used to calculate DoF of the closed mechanisms [1–6]. By considering both the redundant constraints and the passive DoF in the PMs, Huang [4] revised the Kutzbach-Grübler formula as follows:

$$M = 6(N - n - 1) + \sum f_j + \nu - \zeta,$$
$$\sum f_j = 3n_s + 2n_u + 2n_c + n_p + n_r. \qquad (1)$$

Here, M is the prescribed DoF of the moving platform m (output link); N is the number of the links including the fixed base B; n is the number of kinematic pairs; f_j is the local DoF of the jth kinematic pair; ν is the number of all the redundant constraints; ζ is the number of passive DoFs which does not influence the moving characteristics of m; "$n_s, n_u, n_c, n_p,$ and n_r" are the numbers of "$S, U, C, P,$ and R" pairs, respectively.

Let "b, t, q, p, h" be a "binary, ternary, quaternary, pentagonal, hexagonal" link in a mechanism, respectively. Let J be a point of connection with one-DoF. Let n_i ($i = 2, \ldots, 6$) be the number of "b, t, q, p, and h," respectively. A topology graph (TG) is formed by some links connected by several J [5]; see Figure 1(a). TG is a basic tool for type synthesis of mechanisms [1–3, 5]. An associated linkage (AL) may include the acceptable group of links "$b, t, q, p, h, \ldots$." AL with given DoF is a basic tool for deriving various TGs with the same DoF [5].

The type synthesis of PMs using TG generally is quite complicated because TG includes many b. When all links of "$t, q, p, h, \ldots$" in TG are replaced by the vertices connected by "$3, 4, 5, 6, \ldots$" edges, respectively, and each of the edges in TG is marked by a digit which represents the number of b connected in series by several J on this edge, the representation of TG can be simplified greatly. Therefore, a digital topology graph (DTG) is used to simplify the representation of TG; see Figure 1(b).

An array is an aided tool for deriving and representing DTGs and is also used to identify the isometric DTGs and the invalid DTGs. Thus, the type synthesis of PMs and their isometric identification become quite easy. Generally, a DTG includes several vertices connected by several edges. "$t, q, p, h, \ldots$" in a DTG are represented by the vertices connected by "$3, 4, 5, 6, \ldots$" edges, respectively. Each of the edges in DTG is marked by a digit. Each digit in the DTG represents the number of b connected in series by several J on this edge. For instance, a TG representing 5-DoF PMs includes $1q$, $2t$, and $18b$ which are connected by $23J$; see Figure 1(a). The complicated representation of TG can be simplified by a DTG; see Figure 1(b). In this DTG, $1q$ is represented by 1 vertex which are connected by 4 edges e_j ($j = 1, 2, 4, 5$) marked by digits $(4, 4, 4, 5)$, respectively. $2t$ are represented by 2 vertices which are connected by 5 edges e_j ($j = 1, \ldots, 5$) marked by digits $(4, 4, 1, 4, 5)$, respectively. Here, "4, 4, 1, 4, 5" represent "4, 4, 1, 4, 5" b connected in series by "5, 5, 2, 5, 6" J in e_i ($i = 1, \ldots, 5$), respectively. An array $\{t_1144 \quad q4445 \quad t_2541\}$ is used for representing this DTG. In this array, t_1144 represents the case that t_1 is connected with 3 edges e_i ($i = 1, 2, 3$). Here,

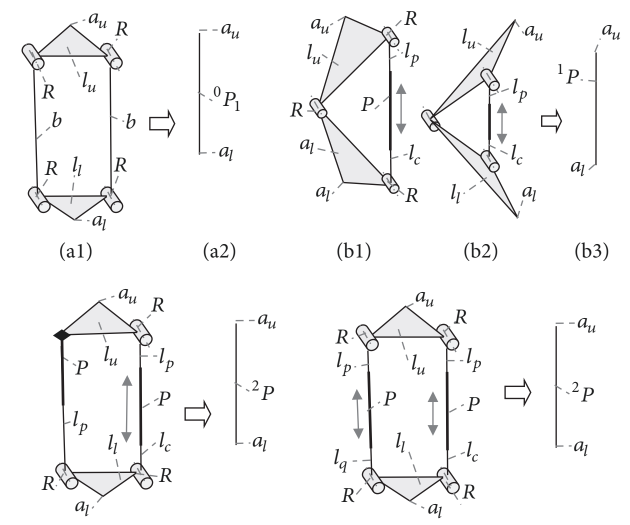


Figure 2: Four typical ${}^{i}P_M$; ${}^{0}P_1$ (a1), ${}^{1}P_1$ for increasing force (b1) and a ${}^{1}P_1$ for enlarging displacement (b2), ${}^{2}P_2$ (c1), and ${}^{2}P_3$ (d1) and their simple representations (a2), (b3), (c2), and (d2).

"e_1, e_2, and e_3" are formed by "4, 4, 1" b connected in series, respectively.

Let ${}^{i}P_M$ ($i \le 2$; $M = 1, 2, 3$) be the subplanar closed mechanisms with M DoFs and i actuators. Four typical ${}^{i}P_M$ are represented in [17]; see Figure 2. In fact, 5-DoF PMs may include some other different ${}^{i}P_M$ ($i \le 2$; $M = 1, 2, 3$).

A 5-DoF PM generally includes B, m, and several limbs. These limbs may include several links which are connected by several kinematic pairs (R, P, C, U, and S). The following 5 conditions must be satisfied in order to derive the valid DTGs and synthesize novel 5-DoF PMs:

(1) If "$\nu = 0$; $\zeta = 0$" are satisfied in a 5-DoF PM, the number of the links in any closed loop chain must be 7 or more in order to avoid any local structure.

(2) If "$\nu = 0$ and $\zeta = 0$" are satisfied in a 5-DoF PM, the number of b on any edge must be 6 or less in order to avoid any local structure.

(3) The number of J connected in series in each limb from B to m must be 5 or more.

(4) The number of the links in a closed loop chain of DTG for constructing ${}^{i}P_1$ ($i = 0, 1$) must be 7 in order to avoid any local structure.

(5) The number of the links in a closed loop chain of DTG for constructing ${}^{2}P_2$ must be 8 in order to avoid any local structure.

(6) The number of the links in a closed loop chain of DTG for constructing ${}^{2}P_3$ must be 9 in order to avoid any local structure.

The theoretical bases of the above 6 conditions have been explained in the Appendix. Another three auxiliary conditions [17] should be satisfied as follows:

(7) It is permissible to replace any one J in the TG with R or P pair in the mechanism.

(8) It is permissible to replace any $2J$ connected in series in the TG with U or C pair in the mechanism.

(9) It is permissible to replace any $3J$ connected in series in the TG with S pair in the mechanism.

In the light of the 5-DoF PMs, 17 ALk ($k = 1, \ldots, 17$) with 5-DoF and n_i ($i = 2, \ldots, 6$) links have been derived based on (1). n_i in 17 ALs with 5-DoF can be solved in [17] and listed in Table 1.

3. Valid DTGs and Revised DTGs

The valid DTG can be derived from the AL by distributing all b into the contracted graph [16]. Based on conditions (1)–(6), some DTGs with different arrays for synthesizing the 5-DoF PMs are derived and displayed in Figure 3.

The 4 DTGs with $2t + 14b$ and different arrays $\{t446\ \ t644\}$, $\{t356\ \ t653\}$, $\{t455\ \ t554\}$, and $\{t347\ \ t743\}$ are derived from AL1 in Table 1; see Figures 3(a1)–3(a4). The 2 DTGs with $2q + 19b$ and different arrays $\{q4555\ \ q5554\}$ and $\{q4546\ \ q6454\}$ are derived from AL2 in Table 1; see Figures 3(b1) and 3(b2). The 2 DTGs with $2p+24b$ and different arrays $\{p45456\ \ p65454\}$ and $\{p45555\ \ p55554\}$ are derived from AL3 in Table 1; see Figures 3(c1) and 3(c2). The 2 DTGs with $q + 2t + 18b$ and different arrays $\{t_1144\ \ q4445\ \ t_2541\}$ and $\{t_1244\ \ q4444\ \ t_1442\}$ are derived from AL6 in Table 1; see Figures 3(d1) and 3(d2). The 18 DTGs are derived from AL5 and AL7 $\rightarrow$ AL17 in Table 1; see Figures 3(e)–3(p).

If the 5-DoF PMs include ${}^{i}P_M$ ($i \le 2$; $M = 1, 2, 3$), then the DTGs must be revised into r-DTGs by utilizing approach in [17]. For instance, a DTG with $4t + 17b$ in Figure 4(a1) is revised into an r-DTG with $4T + (17 - 3)B$ in Figure 4(a2). Similarly, the other DTGs in Figures 4(b1)–4(i1) can be revised into the r-DTGs in Figures 4(b2)–4(i2). Thus, all the r-DTGs can be used to synthesize the 5-DoF PMs with ${}^{i}P_M$ ($i \le 2$; $M = 1, 2, 3$).

4. Submechanisms and Their Equivalent Limbs

4.1. Characteristics and Functions of Submechanisms. Let ${}^{i}L_s$ ($1 \le i \le 3$) be a subserial mechanism with i actuators. Let ${}^{i}L$ ($2 \le i \le 4$) be a subspatial i-DoF PM with i actuators. The submechanisms of the 5-DoF PMs may be ${}^{i}L_s$, ${}^{i}P_M$ ($i \le 2$; $M = 1, 2, 3$), and ${}^{i}L$. Each of them has different merits. These merits can bring benefits for the 5-DoF PMs as follows.

Wh300en ${}^{i}L_s$ is taken as the limb for connecting m with B in the 5-DoF PMs, and the total number of the required limbs must be reduced. Thus, the interference between the limbs and m may be avoided easily, and the workspace of the 5-DoF PMs can be enlarged. In addition, since the other active limbs of the 5-DoF PMs can be transformed into the SPU-type linear active limbs which are not sensitive to manufacturing error, not only can the tiny self-motion of the PMs be removed effectively, but also the capability of the load bearing can be increased. When a spatial PM includes one subplanar mechanism or more, it may have merits [17] such as simplicity in structure, ease in manufacturing compliant mechanism and increasing the mechanical advantage, and being larger in capability of the pulling force and rotation angle. When different ${}^{i}P_M$ ($i \le 2$; $M = 1, 2, 3$) and ${}^{i}L$ ($2 \le i \le 4$) are applied to constructing one limb of the 5-DoF PM, its stiffness can be increased.

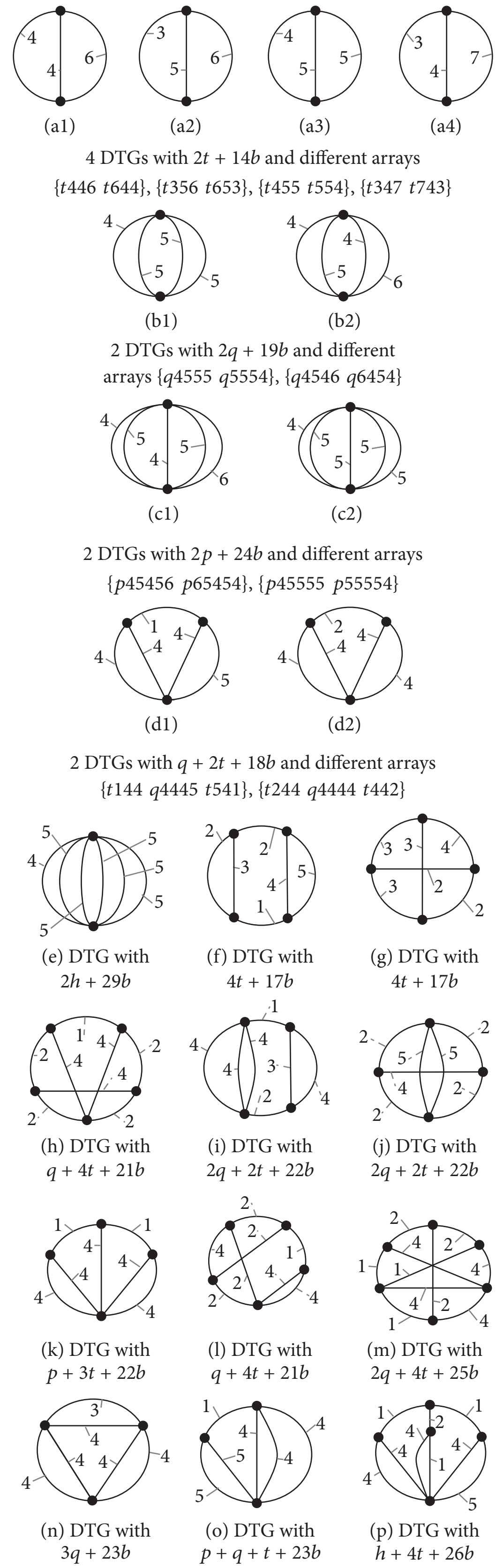
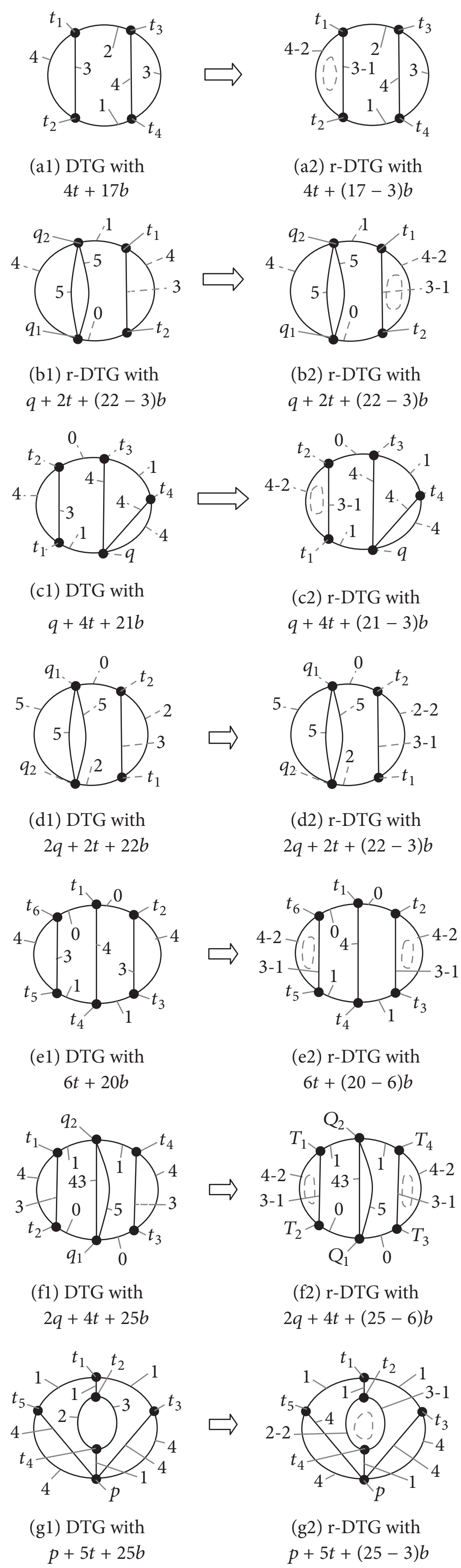


Figure 3: The 22 DTGs with their arrays.



(a1) DTG with $4t + 17b$

(a2) r-DTG with $4t + (17 - 3)b$

(b1) r-DTG with $q + 2t + (22 - 3)b$

(b2) r-DTG with $q + 2t + (22 - 3)b$

(c1) DTG with $q + 4t + 21b$

(c2) r-DTG with $q + 4t + (21 - 3)b$

(d1) DTG with $2q + 2t + 22b$

(d2) r-DTG with $2q + 2t + (22 - 3)b$

(e1) DTG with $6t + 20b$

(e2) r-DTG with $6t + (20 - 6)b$

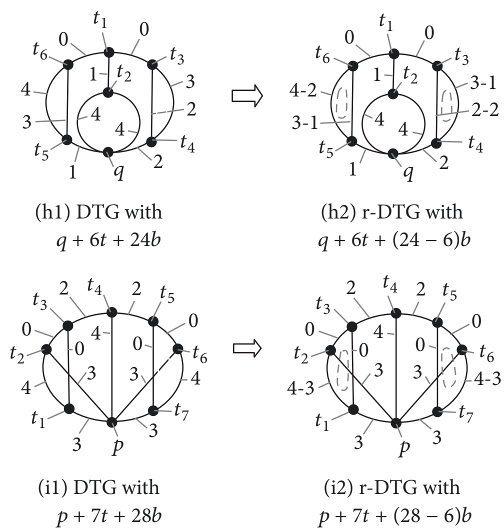
(f1) DTG with $2q + 4t + 25b$

(f2) r-DTG with $2q + 4t + (25 - 6)b$

(g1) DTG with $p + 5t + 25b$

(g2) r-DTG with $p + 5t + (25 - 3)b$

Figure 4: Continued.



(h1) DTG with $q + 6t + 24b$

(h2) r-DTG with $q + 6t + (24 - 6)b$

(i1) DTG with $p + 7t + 28b$

(i2) r-DTG with $p + 7t + (28 - 6)b$

Figure 4: The 9 DTGs with their arrays and r-DTGs.

Table 1: The number of the binary links and basic links in 17 spatial 5-DoF associated linkages (ALs).

Number	AL	$n_2, M = 5$	n_3	n_4	n_5	n_6
1.1	AL1	$5 + \zeta - \nu + 9$	2	0	0	0
2.3	AL2	$5 + \zeta - \nu + 14$	0	2	0	0
3.7	AL3	$5 + \zeta - \nu + 19$	0	0	2	0
3.8	AL4	$5 + \zeta - \nu + 18$	2	0	0	1
2.1	AL5	$5 + \zeta - \nu + 12$	4	0	0	0
2.2	AL6	$5 + \zeta - \nu + 13$	2	1	0	0
3.3	AL7	$5 + \zeta - \nu + 17$	2	2	0	0
3.2	AL8	$5 + \zeta - \nu + 16$	4	1	0	0
3.5	AL9	$5 + \zeta - \nu + 17$	3	0	1	0
3.6	AL10	$5 + \zeta - \nu + 18$	1	1	1	0
3.1	AL11	$5 + \zeta - \nu + 15$	6	0	0	0
4.3	AL12	$5 + \zeta - \nu + 20$	4	2	0	0
4.2	AL13	$5 + \zeta - \nu + 19$	6	1	0	0
4.11	AL14	$5 + \zeta - \nu + 20$	5	0	1	0
3.4	AL15	$5 + \zeta - \nu + 18$	0	3	0	0
4.6	AL16	$5 + \zeta - \nu + 21$	4	0	0	1
5.7	AL17	$5 + \zeta - \nu + 23$	7	0	1	0

Generally, the above-mentioned submechanisms can be built into different standard units with high precision by a special company. Therefore, many novel 5-DoF PMs with high precision can be built and assembled easily by including these standard units.

4.2. Equivalent Limbs of Submechanisms. Similarly, each of ${}^iL_s$ $(1 \le i \le 3)$ and ${}^iL$ $(2 \le i \le 4)$ in the 5-DoF PMs can be replaced by its equivalent limb and represented by a line marked with ${}^iL_s$ or ${}^iL$.

The equivalent symbols of various kinematic pairs are represented in Figure 5. Generally, each of ${}^iL_s$ in the 5-DoF PMs includes a lower link l_l, an upper link l_u, and i actuators which are connected in series from B to m; see Figure 5(a1). The equivalent limb of ${}^iL_s$ is represented by a line with ${}^iL_s$; see Figure 5(a2). ${}^iL_s$ can be selected from an edge with 4 or 5 in the DTG. The number of actuators can be 2 or more. The actuators may be translational ones or rotational ones. The number of ${}^iL_s$ can be one or more. For instance, a DTG with $2t + 14b$ and an array $\{t_1455\ \ t_2554\}$ is proposed to synthesize the 5-DoF PMs; see Figure 4(a3). The 6 different 5-DoF PMs with ${}^iL_s$ are synthesized using the proposed DTG; see Figure 5(b). Each of the 6 PMs has 3 limbs, the first 4 PMs include 2 ${}^2L_s$, and the rest 2 PMs include 1 ${}^3L_s$. Generally, if a DTG includes one closed loop which is formed by 4 links connected in series by $4J$, then this DTG can be revised into an r-DTG. After that, the 5-DoF PMs with ${}^0P_1$ can be synthesized by utilizing the r-DTG.

For instance, a DTG with $p + 7t + 28b$ and an array $\{t_1304\ \ t_2430\ \ t_3002\ \ t_4242\ \ t_5200\ \ t_6034\ \ t_7403\ \ p33433\}$ is

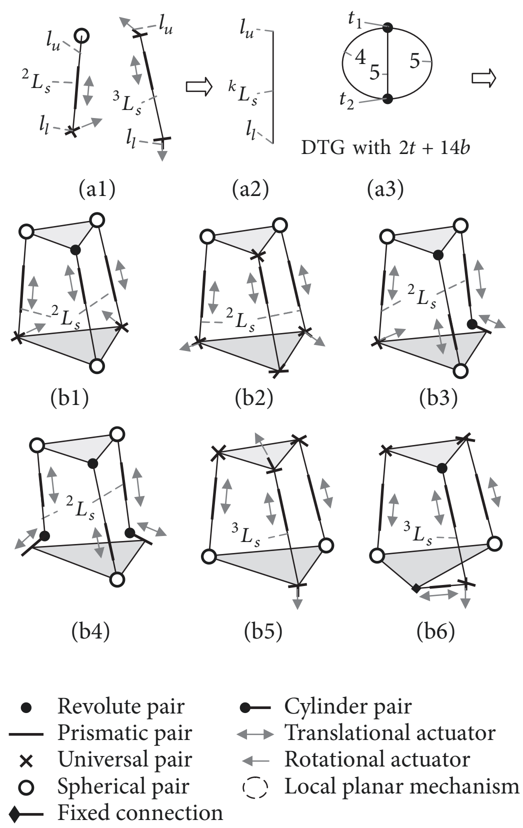


Figure 5: DTG with $2t + 14b$ and array $\{t_1455 \;\; t_2554\}$ (a) and the six 5-DoF PMs with serial mechanisms (b).

proposed for the type synthesis of the 5-DoF PMs; see Figure 6(a). Since the proposed DTG includes a closed loop which is formed by 4 links connected in series by $4J$ for constructing ${}^0P_1$, the proposed DTG can be revised into an r-DTG with $p + 7t + (28 - 6)b$ and an array $\{t_1301 \;\; t_2130 \;\; t_3002 \;\; t_4242 \;\; t_5200 \;\; t_6031 \;\; t_7103 \;\; p33433\}$; see Figure 6(b). Thus, a novel 5-DoF PM with 2 ${}^0P_1$ can be synthesized using the r-DTG; see Figure 6(c).

Based on condition (4) in Section 2, ${}^1P_1$ can be selected from a DTG with a closed loop which is formed by $2t$ and $5b$, and the equivalent limb of ${}^1P_1$ is represented by a line with ${}^1P_1$; see Figure 7(b). In addition, the DTG for synthesizing the 5-DoF PMs with ${}^1P_1$ must be transformed into a revised DTG (r-DTG) by reducing $\nu = 3$ from the number of b in the closed chain for ${}^1P_1$. For instance, a DTG with $4t + 17b$ and an array $\{t_1132 \;\; t_2232 \;\; t_3245 \;\; t_4541\}$ is proposed for the type synthesis of 5-DoF PMs; see Figure 7(a). Since the proposed DTG includes a closed loop which is formed by $2t + 5b$ for constructing ${}^1P_1$, the proposed DTG must be revised into an r-DTG with $4t + (17 - 3)b$ and an array $\{t_1120 \;\; t_2022 \;\; t_3245 \;\; t_4541\}$; see Figure 7(b). Thus, two novel 5-DoF PMs with ${}^1P_1$ and ${}^2L_s$ are synthesized using the r-DTG; see Figure 7(c).

Based on condition (5) in Section 2, the equivalent limb of ${}^2P_2$ can be constructed by utilizing a DTG with a closed loop which is formed by $2t + 6b$. The DTG for synthesizing

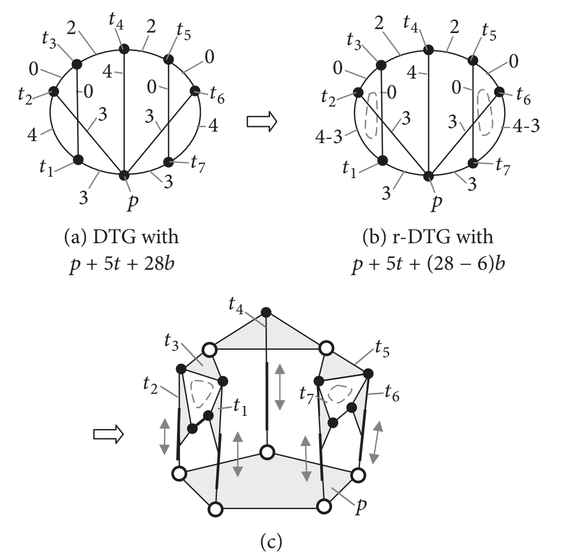


Figure 6: A DTG with $p+7t+28b$ (a), an r-DTG with $p+7t+(28-6)b$ (b), and a 5-DoF PMs with 2 ${}^0P_1$ (c).

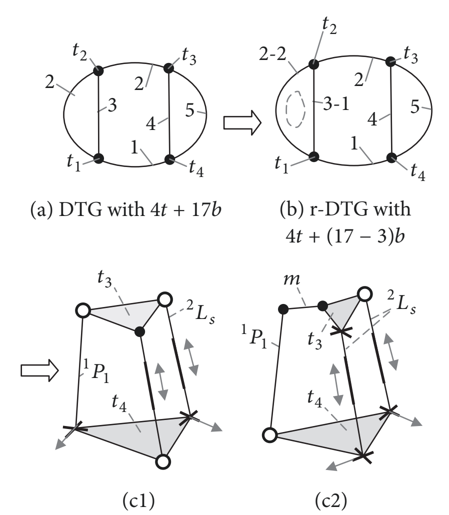


Figure 7: DTG with $4t + 17b$ (a), r-DTG with $4t + (17 - 3)b$ (b), and two novel 5-DoF PMs with ${}^1P_1$ and ${}^2L_s$ (c1, c2).

5-DoF PMs with ${}^2P_2$ must be revised into an r-DTG by reducing $\nu = 3$ from the number of b in the closed loop for constructing ${}^2P_2$. For example, a DTG with $4t + 17b$ and an array $\{t_1233 \;\; t_2331 \;\; t_3144 \;\; t_4442\}$ is proposed for the type synthesis of 5-DoF PMs; see Figure 8(a). Since the proposed DTG includes a closed loop formed by $2t$ and $7b$, it must be revised into an r-DTG with $4t + (17 - 3)b$ and an array $\{t_1221 \;\; t_2121 \;\; t_3144 \;\; t_4442\}$; see Figure 8(b). Thus, 4 novel 5-DoF hybrid PMs with ${}^2P_2$ and ${}^2L_s$ are synthesized by utilizing the r-DTG; see Figures 8(c1)–8(c4). Here, ${}^2P_2$ and ${}^2L_s$ can be represented by the lines marked by ${}^2P_2$ and ${}^2L_s$, respectively.

It is known that the number of the links in a spatial closed 3-DoF PM must be 9 or more. Based on condition (6) in Section 2, the equivalent limb of ${}^2P_3$ can be constructed by utilizing a DTG with a closed loop which is formed by

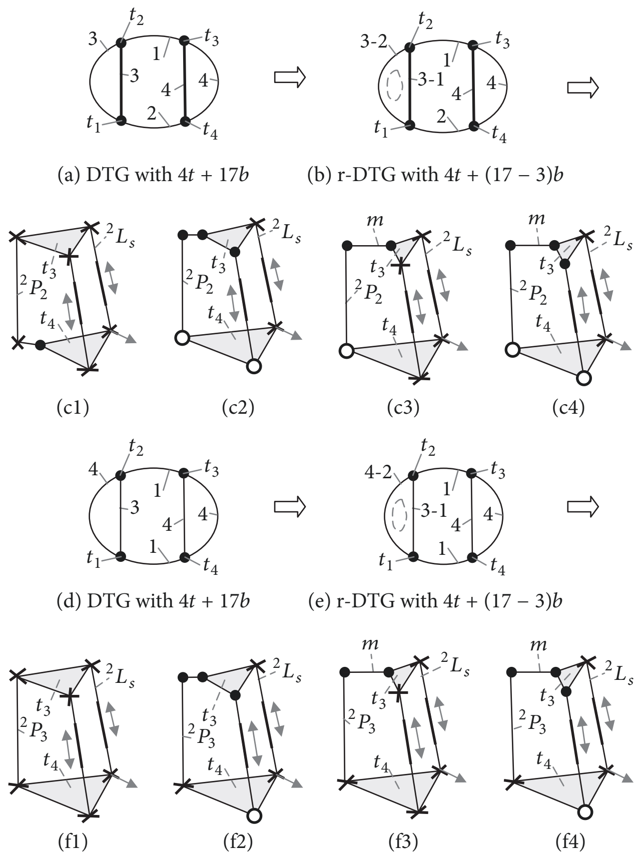


Figure 8: DTG with $4t + 17b$ (a), an r-DTG with $4t + (17-3)b$ (b), four 5-DoF hybrid PMs with 1 $^2P_2$ and 1 ^2L_s (c1–c4), a DTG with $4t+17b$ (d), an r-DTG with $4t+(17-3)b$ (e), and four 5-DoF hybrid PMs with 1 $^2P_3$ and 1 ^2L_s (f1–f4).

$2t$ and $7b$ and is represented by a line marked by $^2P_3$; see Figure 8(g). In addition, the DTG for synthesizing 5-DoF PMs with $^2P_3$ must be revised into an r-DTG by reducing $\nu = 3$ from the number of b in the closed loop for constructing $^2P_3$. For example, a DTG with $4t + 17b$ and an array $\{t_1134\ \ t_2431\ \ t_3144\ \ t_4441\}$ is proposed for the type synthesis of 5-DoF PMs; see Figure 8(d). Since the proposed DTG includes a closed loop formed by $2t$ and $7b$, it must be revised into an r-DTG with $4t + (17-3)b$ and an array $\{t_1122\ \ t_2221\ \ t_3144\ \ t_4441\}$; see Figure 8(e). Thus, the 4 novel 5-DoF hybrid PMs with $^2P_3$ and $^sL_2$ are synthesized by utilizing the r-DTG with $4t+(17-3)b$; see Figures 8(f1)–8(f4).

A subspatial 3-DoF PM ^3L generally includes an upper platform l_u, a lower platform l_l, and 3 active limbs. Since l_l and l_u are required to have 4 connection points: three connection points for connecting with three active limbs and 1 for connection point (a_l and a_u) at the two ends of ^3L for connecting with 5-DoF PMs, both l_l and l_u must be quaternary links. Therefore, the equivalent limb of ^3L can be constructed using a DTG with $2q$ which are connected by 3 edges in parallel with $12b$. The 8 different DTGs for the type synthesis of various 3-DoF PMs can be obtained; see Figure 9(a). Their equivalent limb is represented by a line with ^3L; see Figure 9(b). If ^3L has symmetry in structure, the first

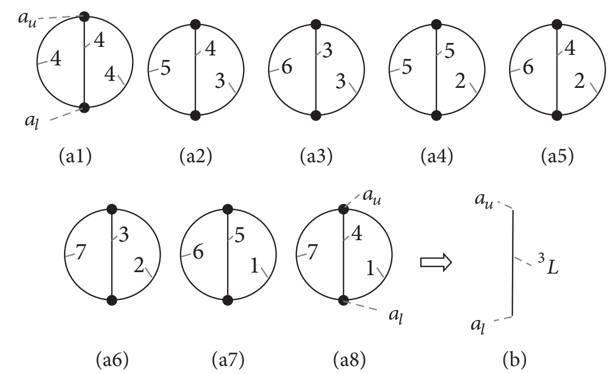


Figure 9: The 8 different DTGs of ^3L (a1–a8) and their equivalent limb (b).

DTG in Figure 9(a) can be used for synthesizing symmetry ^3L, and the 26 different symmetry kinematic chains [16] can be applied to the construction of the 26 different symmetry ^3L. When ^3L is asymmetry in structure, the other 7 DTGs in Figure 9(a) can be used for synthesizing asymmetry ^3L, and more asymmetry kinematic chains can be obtained. Thus, many novel 5-DoF PMs can be synthesized.

A sub-4-DoF PM ^4L generally includes an upper platform l_u, a lower platform l_l, and 4 active limbs; see Figure 10. Since l_l and l_u are required to provide 5 connection points: 4 connection points are used for connecting with 4 active limbs and 1 is used as the connection point (a_l and a_u) at the 2 ends of ^4L for connecting with 5-DoF PMs, both l_l and l_u must be pentagonal links. Therefore, the equivalent limb of ^4L can be constructed using a DTG with $2p$ which are connected by 4 edges in parallel with $18b$. The 14 different DTGs with $2q+18b$ for synthesizing various 4-DoF spatial PMs can be obtained. Three different promised DTGs are proposed for synthesizing 7 different subspatial 4-DoF PMs; see Figures 10(a)–10(c). Their equivalent limb is represented by a line with ^4L; see Figure 10(d). Thus, many novel 5-DoF PMs can be synthesized.

For instance, a DTG with $2p + 24b$ and an array $\{p45456\ \ p65454\}$ is proposed for the type synthesis of the 5-DoF PMs with ^4L; see Figure 11(a). Two equivalent mechanisms I and II with ^4L are derived from the proposed DTG; see Figures 11(b) and 11(d). Three novel 5-DoF PMs with the p-type base are synthesized from the equivalent mechanism I; see Figures 11(c1)–11(c3). Three novel 5-DoF PMs with the p-type moving platform are synthesized from the equivalent mechanism II; see Figures 11(e1)–11(e3). In fact, some novel 5-DoF PMs with ^iP_M and ^iL or their combinations can be synthesized. Some examples are illustrated as follows.

Example 1. A DTG with $2q + 2t + 22b$ and an array $\{t_1134\ \ t_2432\ \ q_12444\ \ q_24441\}$ is proposed for the type synthesis of the 5-DoF PMs with ^3L and $^2P_3$; see Figure 12(a). Since the proposed DTG includes both one parallel closed loop formed by $2q+12b$ and one closed loop formed by $2t+9b$, it must be revised into an r-DTG with $2q + 2t + (22-3)b$ and an array $\{t_1122\ \ t_2222\ \ q_12444\ \ q_24441\}$; see Figure 12(b). An equivalent mechanism with 1 ^3L and 1 $^2P_3$ is constructed

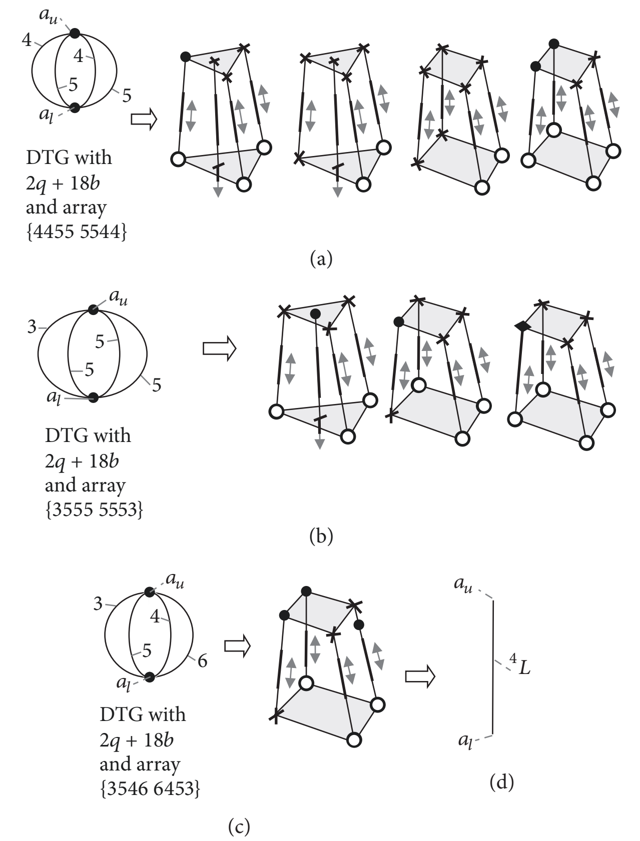


Figure 10: The 3 DTGs of ^4L ((a), (b), and (c)) and the 9 subspatial 4-DoF PM ^4L and their equivalent limb (d).

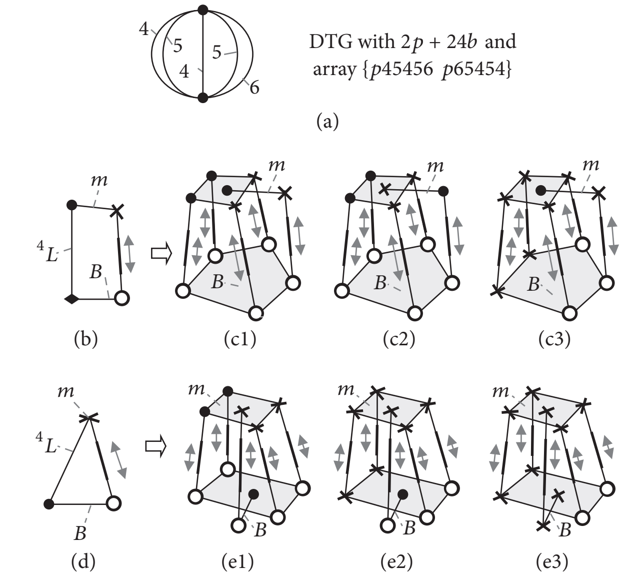


Figure 11: A DTG with $2p + 24b$ (a), equivalent mechanism I with ^4L, (b), 3 novel 5-DoF PMs with p-type base (c1–c3), equivalent mechanism II with ^4L (d), and 3 novel 5-DoF PMs with p-type moving platform (e1–e3).

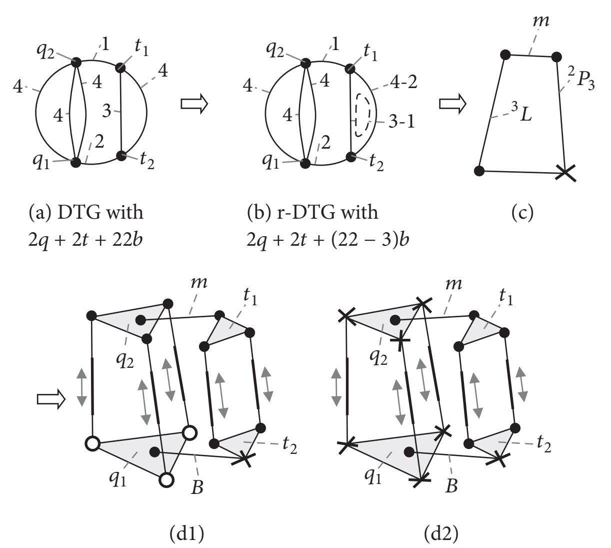


Figure 12: A DTG with $2q + 2t + 22b$ (a), an r-DTG with $2q + 2t + (22 - 3)b$ (b), equivalent mechanism with both ^3L and $^2P_3$ (c), two novel 5-DoF PMs (d1, d2).

from the r-DTG; see Figure 12(c). Finally, two 5-DoF PMs are synthesized from equivalent mechanism; see Figures 12(d1) and 12(d2).

Example 2. A DTG with $2q + 2t + 25b$ and an array $\{t_1034\ t_2431\ q_11451\ t_3123\ t_4321\ q_21540\}$ is proposed for synthesizing the 5-DoF PMs with $^1P_1$ and $^2P_3$; see Figure 13(a). Since the proposed DTG includes both one closed loop chain formed by $2t$ and $9b$ and one closed loop chain formed by $2t$ and $7b$, it must be revised into an r-DTG with $2q + 2t + (25 - 6)b$ and an array $\{t_1022\ t_2211\ q_11451\ t_3102\ t_4201\ q_21540\}$; see Figure 13(b). Thus, the equivalent mechanism of a novel 5-DoF PM with 1 $^1P_1$ and 1 $^2P_3$ (see Figure 13(c)) can be constructed from the r-DTG in Figure 13(b). Similarly, a DTG with $q + 6t + 24b$ and an array $\{t_1011\ t_2144\ t_3123\ t_4321\ t_5134\ t_6430\ q1441\}$ is proposed for synthesizing the 5-DoF PMs with $^1P_1$ and $^2P_3$; see Figure 13(d). It can be revised into an r-DTG with $q + 6t + (24 - 6)b$ and an array $\{t_1011\ t_2144\ t_3102\ t_4201\ t_5122\ t_6220\ q1441\}$; see Figure 13(e). Thus, the equivalent mechanism of a novel 5-DoF PM with 1 $^1P_1$ and 1 $^2P_3$ is constructed from the r-DTG in Figure 13(e); see Figure 13(f) ($q + 6t + (24 - 6)b$ (e), equivalent mechanism with $^1P_1$ and $^2P_3$ (f)).

Example 3. A DTG with $6t + 20b$ and an array $\{t_1040\ t_2034\ t_3431\ t_4141\ t_5134\ t_6430\}$ is proposed for synthesizing the 5-DoF PMs with 2 $^2P_3$; see Figure 14(a). Since the proposed DTG includes 2 closed loop chains formed by $2t$ and $9b$, it must be revised into an r-DTG with $6t + (20 - 6)b$ and an array $\{t_1040\ t_2022\ t_3221\ t_4141\ t_5122\ t_6220\}$; see Figure 14(b). Finally, the equivalent mechanism of a novel 5-DoF PM with 2 $^2P_3$ is constructed from the r-DTG in Figure 14(b); see Figure 14(c). Similarly, a DTG with $2q + 4t + 25b$ and an array $\{t_1134\ t_2430\ q_10450\ t_3034\ t_4431\ q_21541\}$

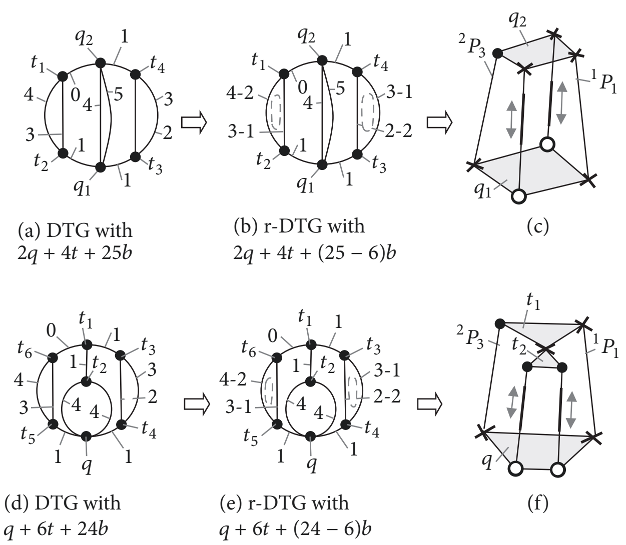


Figure 13: A DTG with $2q + 2t + 25b$ (a), an r-DTG with $2q + 2t + (25 - 6)b$ (b), equivalent mechanism with ${}^1P_1$ and ${}^2P_3$ (c), DTG with $q + 6t + 24b$ (d), and r-DTG with ${}^1P_1$ and ${}^2P_3$ (f).

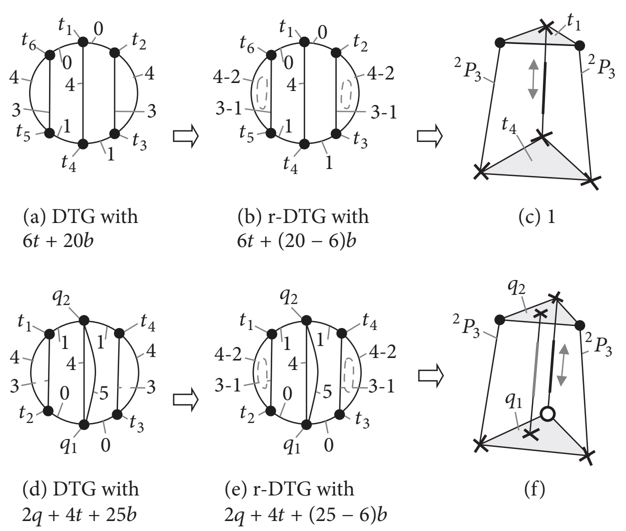


Figure 14: A DTG with $6t + 20b$ (a), an r-DTG with $6t + (20 - 6)b$ (b), equivalent mechanism with 2 ${}^2P_3$ (c), a DTG with $2q + 4t + 25b$ (d), an r-DTG with $2q + 4t + (25 - 6)b$ (b), and equivalent mechanism with 2 ${}^2P_3$ (f).

is proposed for synthesizing the 5-DoF PMs with 2 ${}^2P_3$; see Figure 14(d). Since the proposed DTG includes 2 closed loop chains formed by $2t$ and $9b$, it must be revised into an r-DTG with $2q + 4t + (25 - 6)b$ and an array $\{t_1 122\ \ t_2 220\ \ q_1 0450\ \ t_3 022\ \ t_4 221\ \ q_2 1541\}$; see Figure 14(e). Finally, the equivalent mechanism of a novel 5-DoF PM with 2 ${}^2P_3$ is constructed from the r-DTG in Figure 14(e); see Figure 14(f).

Example 4. A DTG with $2q + 2t + 22b$ and an array $\{t_1 043\ \ t_2 430\ \ q_1 0555\ \ q_2 5550\}$ is proposed for the type synthesis of the 5-DoF PMs with ${}^2P_3$; see Figure 15(a). Since the proposed DTG includes one closed loop formed by $2t + 7b$

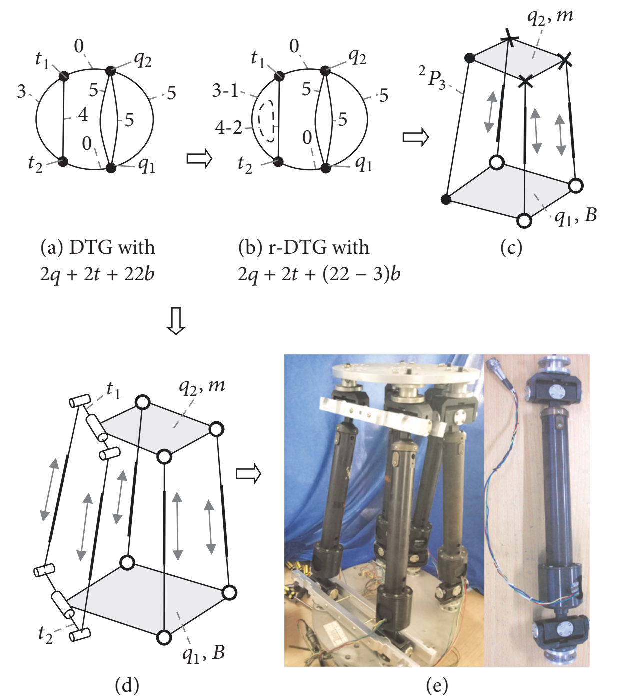


Figure 15: A DTG with $2q + 2t + 22b$ (a), an r-DTG with $2q + 2t + (22 - 3)b$ (b), a novel 5-DoF PM with ${}^2P_3$ (c), equivalent mechanism of novel 5-DoF PM with ${}^2P_3$ (d), and prototype of novel 5-DoF PM with ${}^2P_3$ (e).

and one parallel closed loop formed by $2q + 15b$, it must be revised into an r-DTG with $2q + 2t + (22 - 3)b$ and an array $\{t_1 022\ \ t_2 220\ \ q_1 0555\ \ q_2 5550\}$; see Figure 15(b). An equivalent mechanism with 1 ${}^2P_3$ is constructed from the r-DTG; see Figure 15(c). Finally, one 5-DoF PM is synthesized from equivalent mechanism; see Figure 15(d).

A prototype of the novel 5-DoF PM with ${}^2P_3$ is built up from an existing prototype of the PM with 5 SPS type limbs in Yanshan University; see Figures 15(e) and 15(f). Here, each S pair can be transformed into U pair or R pair by adding constraint. Therefore, after an upper constraint and a lower constraint between two closed SPS limbs in the prototype of PM are added by utilizing four beam-blocks, the upper $2S$ pairs of the two closed SPS limbs are transformed into $3R$ pairs, in which one R is connected with m and is crossed with another two parallel R. Similarly, the lower $2S$ pairs of the two closed SPS limbs are transformed into $3R$ pairs, in which one R is connected with B and is crossed with other two parallel R. It is verified by experiment that this novel 5-DoF PM with ${}^2P_3$ can be moved well.

The DoFs of the 30 different 5-DoF PMs with different submechanisms synthesized from the 10 valid DGTs in Figures 3(b)–15(c) are verified by utilizing (1) and listed in Table 2.

5. Conclusions

The conditions for the type synthesis of 5-DoF parallel mechanisms with subserial or subparallel mechanisms are

Table 2: DoF verifications of the 5-DoF PMs with different submechanisms synthesized from the 10 valid DGTs in Figures 3(b)–15(c).

Number	Figure	n_2	n_3	n_4	n_5	n_6	N	n	ζ	ν	${}^iL_s$, ${}^iP_M$, ${}^iL$
1	5(b1)	14	2	0	0	0	8	9	0	0	2^2L_s
2	5(b2)	14	2	0	0	0	8	9	0	0	2^2L_s
3	5(b3)	14	2	0	0	0	8	9	0	0	2^2L_s
4	5(b4)	14	2	0	0	0	8	9	0	0	2^2L_s
5	5(b5)	14	2	0	0	0	8	9	0	0	${}^3L_s$
6	5(b6)	14	2	0	0	0	9	10	0	0	${}^3L_s$
7	6(c)	28 − 6	5	0	1	0	16	21	0	6	2^0P_1
8	7(e1)	17 − 3	4	0	0	0	4 + 6	6 + 6	0	3	${}^2L_s + {}^2P_3$
9	7(e2)	17 − 3	4	0	0	0	5 + 6	7 + 6	0	3	${}^2L_s + {}^2P_3$
10	8(c1)	17 − 3	4	0	0	0	7 + 5	9 + 5	0	3	${}^2L_s + {}^2P_2$
11	8(c2)	17 − 3	4	0	0	0	7 + 5	9 + 5	0	3	${}^2L_s + {}^2P_2$
12	8(c3)	17 − 3	4	0	0	0	7 + 5	9 + 5	0	3	${}^2L_s + {}^2P_2$
13	8(c4)	17 − 3	4	0	0	0	7 + 5	9 + 5	0	3	${}^2L_s + {}^2P_2$
14	8(f1)	17 − 3	4	0	0	0	6 + 6	8 + 6	0	3	${}^2L_s + {}^2P_3$
15	8(f2)	17 − 3	4	0	0	0	7 + 6	9 + 6	0	3	${}^2L_s + {}^2P_3$
16	8(f3)	17 − 3	4	0	0	0	7 + 6	9 + 6	0	3	${}^2L_s + {}^2P_3$
17	8(f4)	17 − 3	4	0	0	0	7 + 6	9 + 6	0	3	${}^2L_s + {}^2P_3$
18	11(c1)	24	0	0	2	0	3 + 10	4 + 12	0	0	${}^4L$
19	11(c2)	24	0	0	2	0	3 + 10	4 + 12	0	0	${}^4L$
20	11(c3)	24	0	0	2	0	3 + 10	4 + 12	0	0	${}^4L$
21	11(e1)	24	0	0	2	0	3 + 10	4 + 12	0	0	${}^4L$
22	11(e2)	24	0	0	2	0	3 + 10	4 + 12	0	0	${}^4L$
23	11(e3)	24	0	0	2	0	3 + 10	4 + 12	0	0	${}^4L$
24	12(d1)	22 − 3	2	2	0	0	16	19	0	3	${}^2P_3 + {}^3L$
25	12(d2)	22 − 3	2	2	0	0	16	19	0	3	${}^2P_3 + {}^3L$
26	13(c)	25 − 6	4	2	0	0	16	20	0	6	${}^1P_1 + {}^2P_3$
27	13(f)	24 − 6	6	1	0	0	7 + 10	11 + 10	0	6	${}^1P_1 + {}^2P_3$
28	14(c)	20 − 6	6	0	0	0	4 + 12	7 + 12	0	6	2^2P_3
29	14(f)	25 − 6	4	2	0	0	6 + 12	10 + 12	0	6	2^2P_3
30	15(d)	22 − 3	2	2	0	0	14	17	3	3	${}^2P_3$

$M = 6(N - n - 1) + \sum f_i + \nu - \zeta = 5$, N: the numbers of links; n: the numbers of pairs; ν: the numbers of redundant constraints; ζ: passive DoF; f_i: local DoF of kinematic pair; $\sum f_i = 3n_s + 2n_u + 2n_c + n_p + n_r$; n_s, n_u, n_c, n_p, and n_r: the numbers of S, U, C, P, and R.

determined. The 31 valid DTGs (digital topology graphs) and revised DTGs can be derived from 17 different spatial 5-DoF associated linkages.

The subplanar mechanisms and/or parallel mechanisms in the 5-DoF parallel mechanisms can be transformed into simple equivalent limbs, and their equivalent relations and merits are determined.

The 30 different 5-DoF parallel mechanisms with different subserial or parallel mechanisms can be synthesized by utilizing the valid DTGs and revised DTGs. They can be simplified by replacing complicated subparallel mechanisms with their simple equivalent limbs.

The DoFs of all the synthesized parallel mechanisms are verified to be correct.

Many novel 5-DoF parallel mechanisms with subserial or parallel mechanisms can be synthesized by utilizing different valid DTGs, or by varying the order of the kinematic pairs and the orientations of the kinematic pairs.

Appendix

Let ${}^iP_M$ ($i \le 2$; $M = 1, 2, 3$) be the subplanar closed mechanisms with M DoFs and i actuators; see Figure 2. Four ${}^iP_M$ are represented in Table 3. In fact, 5-DoF PMs may include some other different ${}^iP_M$ ($i \le 2$; $M = 1, 2, 3$).

Let $\perp$ be the perpendicular constraint. Let "b, l_u, l_l, l_p, and l_c" be the binary link, the upper link, the lower link, the piston link, and the cylinder link, respectively.

${}^0P_1$ is formed by 4 links "$2b$, l_u, and l_c" in a planar loop chain connected in series by 4 mutually parallel R pairs; see Figure 2(a1).

TABLE 3: υ of ${}^{i}P_M$ ($i \le 2$; $M = 1, 2, 3$).

${}^{i}P_M$	i	M	N	n	n_p	n_r	ζ	ν
${}^{0}P_1$	0	1	4	4	0	4	0	3
${}^{1}P_1$	1	1	4	4	1	3	0	3
${}^{2}P_2$	2	2	5	5	2	3	0	3
${}^{2}P_3$	2	3	6	6	2	4	0	3

${}^{1}P_1$ is formed by 4 links "l_u, l_l, l_p, and l_c" in a planar loop chain connected in series by 1 R pair, 1 active P pair, and 2 R pairs; here 3 R pairs are parallel mutually and $P \perp R$ are satisfied; see Figure 2(b1).

${}^{2}P_2$ is formed by 5 links "l_u, l_l, $2l_p$, and l_c" in a planar loop chain connected in series by 1 R pairs, 1 active P pair, 2 R pairs, and 1 active P pair; here 3 R pairs are parallel mutually and $P \perp R$ are satisfied; see Figure 2(c1).

${}^{2}P_3$ is formed by 6 links "l_u, l_l, $2l_p$, and $2l_c$" in a planar loop chain connected in series by 2 R pairs, 1 active P pair, 2 R pairs, and 1 active P pair; here 4 R pairs are parallel mutually and $P \perp R$ are satisfied; see Figure 3(d1). Since each of ${}^{i}P_M$ ($i = 0, 1, 2$; $M = 1, 2, 3$) has no passive DoF, $\zeta = 0$ is satisfied. Thus, their redundant constraints υ are solved based on (1); see Table 1.

Since l_l and l_u in ${}^{i}P_M$ ($i = 1, 2$; $M = 1, 2, 3$) are required to provide 3 connection points: 2 for constructing ${}^{i}P_M$ and 1 for the connection point (a_l, a_u) at the two ends of ${}^{i}P_M$ for connecting with the 5-DoF PMs, both l_l and l_u must be the ternary link. In order to simplify the representation of the PMs, ${}^{i}P_M$ ($i = 1, 2$; $M = 1, 2, 3$) are represented by a line marked by ${}^{i}P_M$ and (a_l and a_u) at its two ends; see Figures 2(a2), 2(b2), 2(c2), and 2(d2).

Symbols

DoF:	Degree of freedom
PM:	Parallel mechanism
AL:	Associated linkage
DTG:	Topology graph with digits
TG:	Topology graph
CG:	Contracted graph
(R, P, C, U, and S):	(revolute, prismatic, cylinder, universal, and spherical) kinematic pair
m:	Moving platform
B:	Fixed base
J:	Connection point with one-DoF
M:	DoF of m (output link)
N:	The number of the links including B
n:	The number of kinematic pairs
f_j:	The local DoF of the jth kinematic pair
ν:	The number of all the redundant constraints
ζ:	The number of passive DoFs
(n_s, n_u, n_c, n_p, and n_r):	The numbers of (S, U, C, P, and R) pairs
(b, t, q, p, and h):	(binary, ternary, quaternary, pentagonal, and hexagonal) link
n_i ($i = 2, \ldots, 6$):	The number of "b, t, q, p, and h
e_j:	Edge
$\perp$:	Perpendicular constraint
${}^{i}P_M$ ($i = 0, 1, 2$; $M = 1, 2, 3$):	The subplanar closed mechanisms with M DoFs and i actuators
${}^{i}L_s$:	Subserial mechanism with i actuators
${}^{i}L$ ($i = 3, 4$):	Subspatial i-DoF PM with i actuators
l_u, l_l:	Upper platform, lower platform
l_p, l_c:	Piston link, cylinder link.

Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this article.

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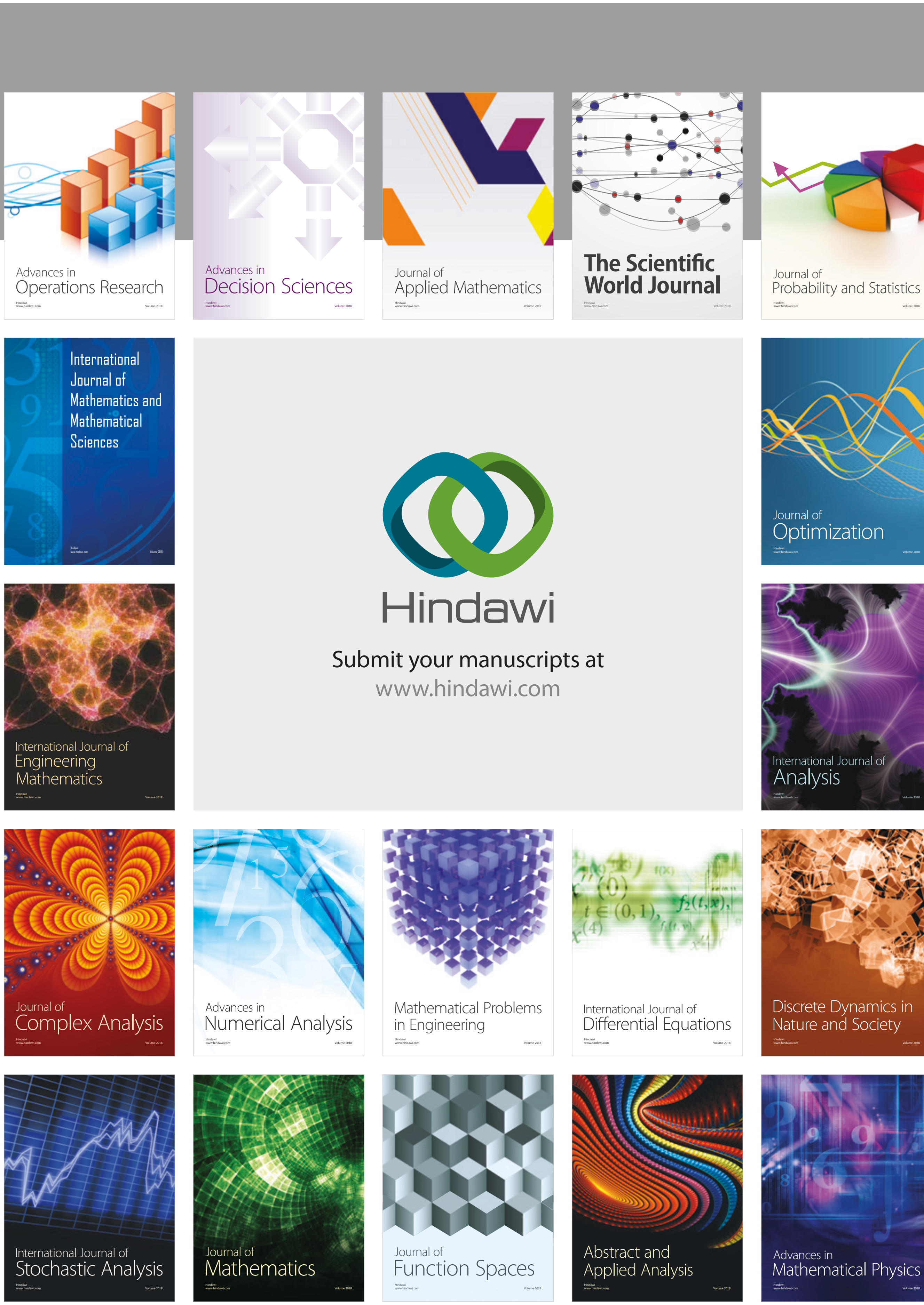
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