

Research Article

The Improved Combination Rule of D Numbers and Its Application in Radiation Source Identification

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The D numbers theory is a novel theory to express uncertain information. It successfully overcomes some shortcomings of Dempster-Shafer theory, such as the conditions of exclusiveness hypothesis and completeness constraint. However, the combination rule of D numbers does not satisfy the associative property, which leads to limitations in practical application for D numbers. In this paper, the improved D numbers theory is proposed to overcome the weakness based on the analysis of D numbers' combination rule. A new algorithm is constructed with the strict proof to simplify the combination rule. The similarities and differences among DS theory, D numbers, and the improved D numbers are introduced with the numerical analysis. An illustrative example of the radiation source identification is presented to demonstrate the effectiveness of the improved method.

1. Introduction

In real applications, uncertainty reasoning is widely applied to information fusion, risk assessment, pattern recognition, artificial intelligence, decision-making, and so forth. Many methods such as fuzzy sets theory, Dempster-Shafer theory, and possibility theory, have been proposed to model uncertain information.

Dempster [1] proposed the upper and lower probabilities that are induced by a multivalued mapping in the 1960s. His student Shafer introduced the concept of belief function and published the book [2], which is the sign of the formation of evidence theory. During the application process of evidence theory, many related problems have been studied in depth, such as the acquisition of basic probability assignments [3–6], information fusion of reliable sources [7, 8], conflict representation model [9, 10], and decision rule [11, 12]. Compared with the traditional Bayesian theory, DS theory, also called an evidence theory, needs weaker conditions and handles uncertain and incomplete information effectively, so it is often regarded as an extension of Bayesian theory. However, DS theory still has some shortcomings. When evidences acquired from different sources are highly conflicting, DS combination rule is defective in resolving evidence combination problem [13]. In addition, there are some strong hypotheses on the

frame of discernment. For example, the elements in the frame of discernment require mutual exclusiveness and the frame must be complete. To handle the existing shortcomings in the DS theory, a new theory is proposed by Deng [14], which is called D numbers.

The DS evidence theory combination rule cannot effectively deal with high-conflict information [9, 15]. A typical example is the zero trust paradox case proposed by Zadeh [13]; that is, 0 has one vote veto. For this problem, a large number of improved algorithms [16–21] made by relevant scholars are mainly divided into two categories: one is the improvement of combination rules and the other is the improvement of data sources [22–32]. The improvement of the combination rules is mainly divided into three categories. One is to recognize the multiplicative principle of the DS evidence theory and to study how to allocate the amount of conflict, such as Smets [25, 26], Yager [27, 28], and PCR1-6 [29–31]. The second type is the multiplicative rule of DS evidence theory, which gives additive combination rules, such as Murphy combination rules [32]. The third category is to change the recognition framework and extend to the generalized power set. On this basis, new combination rules are given, such as the DS_mT combination rule [31], which extends the recognition framework to the power set framework. The D number theory is a generalization of evidence

theory. It cancels the assumption that the propositions in the recognition framework are mutually independent. Related scholars have done lots of research on it. Li [33] proposed a novel distance function of D numbers. Deng [34] introduced the difference between D numbers and DS theory based on generalized evidence theory.

On the application side, Zuo [35] analyzed investment decision, Deng [36] analyzed bridge condition assessment, and Deng [37] analyzed environmental impact assessment. Meanwhile, some methods were modified or extended by D numbers and have been applied to engineering field. Fei and Bian [38, 39] analyzed human resources selection and failure mode based on D numbers and TOPSIS. Su, Deng, and Fan [40–42] proposed the dependence assessment in human reliability analysis, supplier selection, and curtain grouting efficiency assessment based on D numbers and AHP. Deng and Zhou [43, 44] proposed the D-CFPR and D-DEMATEL theory based on D numbers. Liao [45] introduced transformer condition assessment using game theory and modified evidence combination extended by D numbers. Wang [46] introduced a multicriteria decision-making method based on fuzzy entropy and evidential reasoning with linguistic D numbers.

Though D numbers theory overcomes some defects of DS theory and is widely used in different fields, the combination rule of D numbers does not satisfy the associative property. In order that multiple D numbers can be combined correctly and efficiently, a combination operation for multiple D numbers is developed in [37]. But it does not change the fact that D numbers' combination rule does not preserve the associative property. In this paper, the improved D numbers theory is proposed based on the analysis of D numbers' combination rule which satisfies the associative property. An illustrative example about radiation source identification is given to show the effectiveness of the proposed method.

The remainder of this paper is organized as follows. A brief introduction about the DS theory and D numbers is given in Section 2. Then, the concept of improved D numbers is depicted in Section 3. In Section 4, the similarities and differences among DS theory, D numbers, and the improved D numbers are represented. After that, an illustrative example is given to show the effectiveness of the proposed method in Section 5. Finally, conclusions are given in Section 6.

2. Materials and Methods Preliminaries

2.1. DS Theory. Let $\Omega = \{X_1, X_2, \dots, X_N\}$ be a set of exhaustive and exclusive hypotheses, satisfying,

$$X_i \cap X_j = \phi, \quad \forall i, j = \{1, \dots, N\}, \quad i \neq j \quad (1)$$

The power set of Ω is indicated by 2^Ω , and each element of 2^Ω is regarded as a proposition. It can be defined as

$$2^\Omega = \{A \mid A \subseteq \Omega\} \quad (2)$$

Based on the two conceptions, mass function is defined as below.

Definition 1. A mass function is a mapping m from 2^Ω to $[0, 1]$, formally defined by

$$m : 2^\Omega \longrightarrow [0, 1], \quad (3)$$

which should satisfy the condition:

$$\begin{aligned} m(\phi) &= 0 \\ \sum_{A \in 2^\Omega} m(A) &= 1 \end{aligned} \quad (4)$$

A mass function is also called a basic probability assignment (BPA), which measures the support degree of the proposition A .

In real applications, for the same problem, there may be many different sources that acquire various evidences.

Definition 2. Considering two pieces of evidence from different and independent information sources, denoted by two BPAs m_1 and m_2 , the combination rule of DS theory is used to derive a new BPA from two BPAs, which is represented by $m_{12} = m_1 \oplus m_2$, and defined as follows:

$$\begin{aligned} m_{12}(C) &= m_1 \oplus m_2 \\ &= \begin{cases} \frac{\sum_{i,j,A_i \cap B_j = C} m_1(A_i) m_2(B_j)}{1 - k} & C \neq \phi \\ 0 & C = \phi \end{cases} \end{aligned} \quad (5)$$

with

$$k = \sum_{i,j,A_i \cap B_j = \phi} m_1(A_i) m_2(B_j) < 1, \quad (6)$$

where k is the conflict coefficient of two BPAs. Note that the combination rule of DS theory is only applicable to such two BPAs which satisfy the condition $k < 1$.

Definition 2'. Multiple pieces of evidence from different information sources m_j ($j = 1, 2, \dots, n$) also can be combined with the following formula. The result is a new piece of evidence, which incorporates the joint information acquired from various sources.

$$\begin{aligned} m(C) &= m_1 \oplus m_2 \oplus \dots \oplus m_n \\ &= \begin{cases} \frac{\sum_{\cap_i A_i = C} \prod_k m_k(A_i)}{1 - k} & C \neq \phi \\ 0 & C = \phi \end{cases} \end{aligned} \quad (7)$$

with

$$k = \sum_{\cap_i A_i = \phi} \prod_k m_k(A_i) < 1. \quad (8)$$

The factor k indicates the amount of evidential conflict. If $k = 0$, it shows that the sources are completely compatible. If $k = 1$, it shows that the sources are completely contradictory, and it is no longer possible to combine them.

2.2. D Numbers. D numbers theory is more suitable to the framework and is defined as follows.

Definition 3. Let Ω be a finite nonempty set, and D number is a mapping $D : \Omega \rightarrow [0, 1]$, with

$$\begin{aligned} D(\phi) &= 0 \\ \sum_{A \in \Omega} D(A) &\leq 1 \end{aligned} \quad (9)$$

The structure of the expression seems to be similar to mass function. However, the elements of Ω do not require to be mutually exclusive, and D numbers theory is suitable to incomplete information.

Let $\Omega = \{b_1, b_2, \dots, b_n\}$, $b_i \in N^+$, and a special form of D numbers can be defined as follows.

$$\begin{aligned} D(\{b_1\}) &= v_1 \\ D(\{b_2\}) &= v_2 \\ &\vdots \\ D(\{b_i\}) &= v_i \\ &\vdots \\ D(\{b_n\}) &= v_n, \end{aligned} \quad (10)$$

or simply denoted as

$$D = \{(b_1, v_1), (b_2, v_2), \dots, (b_i, v_i), \dots, (b_n, v_n)\} \quad (11)$$

where $v_i > 0$ and $\sum_{i=1}^n v_i \leq 1$.

Like the mass function, D numbers theory also has the combination rule to combine two D numbers.

Definition 4. Let D_1, D_2 be two D numbers, indicated by

$$\begin{aligned} D_1 &= \{(b_1^1, v_1^1), \dots, (b_i^1, v_i^1), \dots, (b_n^1, v_n^1)\} \\ D_2 &= \{(b_1^2, v_1^2), \dots, (b_i^2, v_i^2), \dots, (b_n^2, v_n^2)\} \end{aligned} \quad (12)$$

The combination of D_1, D_2 , indicated by $D = D_1 \oplus D_2$, is defined by

$$D(b) = v \quad (13)$$

with

$$\begin{aligned} b &= \frac{b_i^1 + b_j^2}{2}, \\ v &= \frac{(v_i^1 + v_j^2)/2}{c}, \\ c &= \begin{cases} \sum_{j=1}^n \sum_{i=1}^m \left(\frac{v_i^1 + v_j^2}{2} \right), & \sum_{i=1}^n v_i^1 = \sum_{j=1}^m v_j^2 = 1 \\ \sum_{j=1}^n \sum_{i=1}^m \left(\frac{v_i^1 + v_j^2}{2} \right) + \sum_{j=1}^m \left(\frac{v_c^1 + v_j^2}{2} \right), & \sum_{i=1}^n v_i^1 < 1, \sum_{j=1}^m v_j^2 = 1 \\ \sum_{j=1}^n \sum_{i=1}^m \left(\frac{v_i^1 + v_j^2}{2} \right) + \sum_{i=1}^n \left(\frac{v_i^1 + v_c^2}{2} \right), & \sum_{i=1}^n v_i^1 = 1, \sum_{j=1}^m v_j^2 < 1 \\ \sum_{j=1}^n \sum_{i=1}^m \left(\frac{v_i^1 + v_j^2}{2} \right) + \sum_{i=1}^n \left(\frac{v_i^1 + v_c^2}{2} \right) + \sum_{j=1}^m \left(\frac{v_c^1 + v_j^2}{2} \right) + \frac{v_c^1 + v_c^2}{2}, & \sum_{i=1}^n v_i^1 < 1, \sum_{j=1}^m v_j^2 < 1 \end{cases} \end{aligned} \quad (14)$$

where

$$\begin{aligned} v_c^1 &= 1 - \sum_{i=1}^n v_i^1 \\ v_c^2 &= 1 - \sum_{j=1}^m v_j^2. \end{aligned} \quad (16)$$

The combination operation does not preserve the associative property. Based on formula (15), it is clear that, for $[D_1 \oplus D_2] \oplus D_3$,

$$b = \frac{((b_i^1 + b_j^2)/2) + b_k^3}{2} \quad (17)$$

while, for $D_1 \oplus [D_2 \oplus D_3]$,

$$b = \frac{b_i^1 + ((b_j^2 + b_k^3)/2)}{2} \quad (18)$$

As a result, a combination operation for multiple D numbers should be developed. Paper [17] represents a multiple D numbers' rule of combination.

Definition 5. Let $D_1, D_2, \dots, D_k, \dots, D_n$ be n D numbers, μ_j is an order variable for each D number D_j , indicated by tuple $\langle \mu_j, D_{\mu_j} \rangle$, and then the combination operation of multiple D numbers is a mapping f_D

$$f_D(D_1, D_2, \dots, D_i, \dots, D_n) = [\dots [D_{\lambda_1} \oplus D_{\lambda_2}] \oplus \dots \oplus D_{\lambda_n}] \quad (19)$$

where D_{λ_i} is D_{μ_j} of the tuple $\langle \mu_j, D_{\mu_j} \rangle$ which have the i th lowest μ_j .

In the meanwhile, for the special D number, an aggregation operator is defined as follows.

Definition 6. Let $D = \{(b_1, v_1), (b_2, v_2), \dots, (b_i, v_i), \dots, (b_n, v_n)\}$ be a D number, and the integration representation of the D number is defined as

$$I(D) = \sum_{i=1}^n b_i v_i. \quad (20)$$

3. An Improved Combination Rule of D Numbers

As a novel theory to express uncertain information, D numbers' combination rule does not preserve the associative property. For n D numbers, there are $n!$ outcomes. Formulas (17) and (18) can be extended to multiple D numbers, and the outcome b can be indicated by

$$b = \frac{b_{\lambda_1} + b_{\lambda_2}}{2^{n-1}} + \frac{b_{\lambda_3}}{2^{n-2}} + \dots + \frac{b_{\lambda_n}}{2} \quad (21)$$

where d_{λ_i} is the λ_i th parameter of the D numbers. From formula (21), it can be seen that the weight of the first combination number b is lower, which is $1/2^{n-1}$. And the weight of the latter combination number b is higher, which is $1/2$. The analysis shows that the weight is normalized, namely,

$$\frac{1}{2^{n-1}} + \frac{1}{2^{n-1}} + \frac{1}{2^{n-2}} + \dots + \frac{1}{2} = 1 \quad (22)$$

The main reason for the imbalance of weight is that formula (15) involves addition and multiplication, which do not satisfy the associative property. In the meanwhile, the combination rule uses serial method to combine multiple D numbers, as shown in Figure 1.

3.1. An Improved Algorithm for D Number Theory. In order for D numbers' combination rule to satisfy the associative property, in this paper, the improved D numbers' combination rule with parallel structure is proposed.

Before that, a novel algorithm \odot is proposed to facilitate the elaboration of the problem. Let

$$\begin{aligned} V_1 &= (v_1^1 + v_2^1 + \dots + v_{n_1}^1) \\ V_2 &= (v_1^2 + v_2^2 + \dots + v_{n_2}^2) \end{aligned} \quad (23)$$

be two summation formulas. With the new operation \odot , the combination of V_1, V_2 can be indicated by

$$\begin{aligned} V_1 \odot V_2 &= (v_1^1 + v_2^1 + \dots + v_{n_1}^1) \odot (v_1^2 + v_2^2 + \dots + v_{n_2}^2) \\ &= \sum_{i_2=1}^{n_2} \sum_{i_1=1}^{n_1} (v_{i_1}^1 + v_{i_2}^2) \end{aligned} \quad (24)$$

Then, the following valid theorem can be acquired.

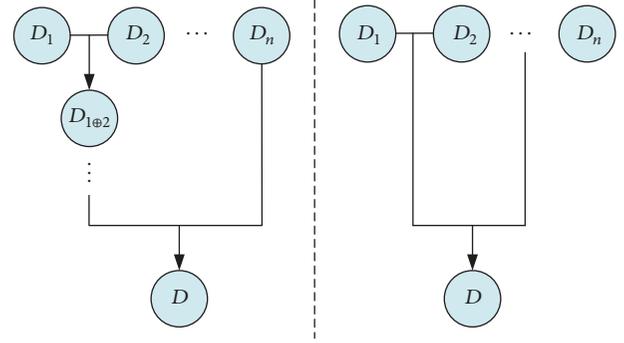


FIGURE 1: The combination rule of D numbers and improved D numbers.

Theorem 7. The parameter c of formula (15) can be rewritten as

$$c = \frac{1}{2} (v_1^1 + v_2^1 + \dots + v_{n_1}^1 + v_c^1) \odot (v_1^2 + v_2^2 + \dots + v_{n_2}^2 + v_c^2) \quad (25)$$

where

$$\begin{aligned} v_c^1 &= 1 - \sum_{i=1}^n v_i^1 \\ v_c^2 &= 1 - \sum_{j=1}^m v_j^2 \end{aligned} \quad (26)$$

Proof. When $\sum_{i=1}^n v_i^1 = \sum_{j=1}^m v_j^2 = 1$, then $v_c^1 = v_c^2 = 0$, and

$$\begin{aligned} c &= \frac{1}{2} (v_1^1 + v_2^1 + \dots + v_{n_1}^1) \odot (v_1^2 + v_2^2 + \dots + v_{n_2}^2) \\ &= \frac{1}{2} \sum_{i_2=1}^{n_2} \sum_{i_1=1}^{n_1} (v_{i_1}^1 + v_{i_2}^2) = \sum_{i_2=1}^{n_2} \sum_{i_1=1}^{n_1} \left(\frac{v_{i_1}^1 + v_{i_2}^2}{2} \right); \end{aligned} \quad (27)$$

when $\sum_{i=1}^n v_i^1 < 1$, $\sum_{j=1}^m v_j^2 = 1$, then $v_c^1 = 0$, and

$$\begin{aligned} c &= \frac{1}{2} (v_1^1 + v_2^1 + \dots + v_{n_1}^1 + v_c^1) \odot (v_1^2 + v_2^2 + \dots + v_{n_2}^2) \\ &= \frac{1}{2} \left(\sum_{i_2=1}^{n_2} \sum_{i_1=1}^{n_1} (v_{i_1}^1 + v_{i_2}^2) + \sum_{i_2=1}^{n_2} (v_c^1 + v_{i_2}^2) \right) \\ &= \sum_{i_2=1}^{n_2} \sum_{i_1=1}^{n_1} \left(\frac{v_{i_1}^1 + v_{i_2}^2}{2} \right) + \sum_{i_2=1}^{n_2} \left(\frac{v_c^1 + v_{i_2}^2}{2} \right); \end{aligned} \quad (28)$$

when $\sum_{i=1}^n v_i^1 = 1$, $\sum_{j=1}^m v_j^2 < 1$, then $v_c^2 = 0$, and

$$\begin{aligned} c &= \frac{1}{2} (v_1^1 + v_2^1 + \dots + v_{n_1}^1) \odot (v_1^2 + v_2^2 + \dots + v_{n_2}^2 + v_c^2) \\ &= \frac{1}{2} \left(\sum_{i_2=1}^{n_2} \sum_{i_1=1}^{n_1} (v_{i_1}^1 + v_{i_2}^2) + \sum_{i_1=1}^{n_1} (v_c^2 + v_{i_1}^1) \right) \\ &= \sum_{i_2=1}^{n_2} \sum_{i_1=1}^{n_1} \left(\frac{v_{i_1}^1 + v_{i_2}^2}{2} \right) + \sum_{i_1=1}^{n_1} \left(\frac{v_c^2 + v_{i_1}^1}{2} \right); \end{aligned} \quad (29)$$

when $\sum_{i=1}^n v_i^1 < 1$, $\sum_{j=1}^m v_j^2 < 1$, then

$$\begin{aligned}
 c &= \frac{1}{2} (v_1^1 + v_2^1 + \dots + v_{n_1}^1 + v_c^1) \odot (v_1^2 + v_2^2 + \dots + v_{n_2}^2 \\
 &+ v_c^2) = \frac{1}{2} \left(\sum_{i_2=1}^{n_2} \sum_{i_1=1}^{n_1} (v_{i_1}^1 + v_{i_2}^2) + \sum_{i_2=1}^{n_2} (v_c^1 + v_{i_2}^2) \right. \\
 &+ \left. \sum_{i_1=1}^{n_1} (v_c^2 + v_{i_1}^1) + (v_c^1 + v_c^2) \right) \\
 &= \sum_{i_2=1}^{n_2} \sum_{i_1=1}^{n_1} \left(\frac{v_{i_1}^1 + v_{i_2}^2}{2} \right) + \sum_{i_2=1}^{n_2} \left(\frac{v_c^1 + v_{i_2}^2}{2} \right) \\
 &+ \sum_{i_1=1}^{n_1} \left(\frac{v_c^2 + v_{i_1}^1}{2} \right) + \left(\frac{v_c^1 + v_c^2}{2} \right)
 \end{aligned} \tag{30}$$

In summary, expression (25) and expression (15) are completely equivalent. \square

Theorem 8. The parameter c of formula (25) can be further simplified, indicated by

$$c = \frac{n_1' + n_2'}{2}, \tag{31}$$

where

$$n_i' = \begin{cases} n_i & \sum_{j=1}^{n_i} v_j^i = 1 \\ n_i + 1 & \sum_{j=1}^{n_i} v_j^i < 1. \end{cases} \tag{32}$$

Proof. When $\sum_{i=1}^n v_i^1 = \sum_{j=1}^m v_j^2 = 1$, then

$$\begin{aligned}
 c &= \sum_{i_2=1}^{n_2} \sum_{i_1=1}^{n_1} \left(\frac{v_{i_1}^1 + v_{i_2}^2}{2} \right) = \sum_{i_2=1}^{n_2} \left(n_1 \frac{v_{i_2}^2}{2} + \sum_{i_1=1}^{n_1} \left(\frac{v_{i_1}^1}{2} \right) \right) \\
 &= n_1 \sum_{i_2=1}^{n_2} \left(\frac{v_{i_2}^2}{2} \right) + \sum_{i_2=1}^{n_2} \left(\frac{1}{2} \right) = \frac{n_1 + n_2}{2};
 \end{aligned} \tag{33}$$

when $\sum_{i=1}^n v_i^1 < 1$, $\sum_{j=1}^m v_j^2 = 1$, then

$$\begin{aligned}
 c &= \sum_{i_2=1}^{n_2} \sum_{i_1=1}^{n_1} \left(\frac{v_{i_1}^1 + v_{i_2}^2}{2} \right) + \sum_{i_2=1}^{n_2} \left(\frac{v_c^1 + v_{i_2}^2}{2} \right) \\
 &= \sum_{i_2=1}^{n_2} \left(n_1 \frac{v_{i_2}^2}{2} + \sum_{i_1=1}^{n_1} \left(\frac{v_{i_1}^1}{2} \right) + \frac{v_c^1 + v_{i_2}^2}{2} \right) \\
 &= \sum_{i_2=1}^{n_2} \left((n_1 + 1) \cdot \frac{v_{i_2}^2}{2} + \frac{\sum_{i_1=1}^{n_1} v_{i_1}^1 + v_c^1}{2} \right) \\
 &= (n_1 + 1) \cdot \sum_{i_2=1}^{n_2} \left(\frac{v_{i_2}^2}{2} \right) + \sum_{i_2=1}^{n_2} \left(\frac{1}{2} \right) = \frac{n_1 + n_2 + 1}{2};
 \end{aligned} \tag{34}$$

when $\sum_{i=1}^n v_i^1 = 1$, $\sum_{j=1}^m v_j^2 < 1$, then

$$\begin{aligned}
 c &= \sum_{i_2=1}^{n_2} \sum_{i_1=1}^{n_1} \left(\frac{v_{i_1}^1 + v_{i_2}^2}{2} \right) + \sum_{i_1=1}^{n_1} \left(\frac{v_c^2 + v_{i_1}^1}{2} \right) \\
 &= \sum_{i_1=1}^{n_1} \left(n_2 \frac{v_{i_1}^1}{2} + \sum_{i_2=1}^{n_2} \left(\frac{v_{i_2}^2}{2} \right) + \frac{v_c^2 + v_{i_1}^1}{2} \right) \\
 &= \sum_{i_1=1}^{n_1} \left((n_2 + 1) \cdot \frac{v_{i_1}^1}{2} + \frac{\sum_{i_2=1}^{n_2} v_{i_2}^2 + v_c^2}{2} \right) \\
 &= (n_2 + 1) \cdot \sum_{i_1=1}^{n_1} \left(\frac{v_{i_1}^1}{2} \right) + \sum_{i_1=1}^{n_1} \left(\frac{1}{2} \right) = \frac{n_1 + n_2 + 1}{2};
 \end{aligned} \tag{35}$$

when $\sum_{i=1}^n v_i^1 < 1$, $\sum_{j=1}^m v_j^2 < 1$, then

$$\begin{aligned}
 c &= \sum_{i_2=1}^{n_2} \sum_{i_1=1}^{n_1} \left(\frac{v_{i_1}^1 + v_{i_2}^2}{2} \right) + \sum_{i_2=1}^{n_2} \left(\frac{v_c^1 + v_{i_2}^2}{2} \right) \\
 &+ \sum_{i_1=1}^{n_1} \left(\frac{v_c^2 + v_{i_1}^1}{2} \right) + \left(\frac{v_c^1 + v_c^2}{2} \right) \\
 &= \sum_{i_2=1}^{n_2} \left(\sum_{i_1=1}^{n_1} \left(\frac{v_{i_1}^1 + v_{i_2}^2}{2} \right) + \frac{v_c^1 + v_{i_2}^2}{2} \right) \\
 &+ \sum_{i_1=1}^{n_1} \left(\frac{v_c^2 + v_{i_1}^1}{2} \right) + \left(\frac{v_c^1 + v_c^2}{2} \right) \\
 &= \sum_{i_2=1}^{n_2} \left(\frac{1}{2} + \sum_{i_1=1}^{n_1} \left(\frac{v_{i_2}^2}{2} \right) + \frac{v_{i_2}^2}{2} \right) + \sum_{i_1=1}^{n_1} \left(\frac{v_c^2 + v_{i_1}^1}{2} \right) \\
 &+ \left(\frac{v_c^1 + v_c^2}{2} \right) \\
 &= \sum_{i_2=1}^{n_2} \left(\frac{1}{2} + (n_1 + 1) \frac{v_{i_2}^2}{2} \right) + (n_1 + 1) \frac{v_c^2}{2} \\
 &+ \sum_{i_1=1}^{n_1} \left(\frac{v_{i_1}^1}{2} \right) + \left(\frac{v_c^1}{2} \right) \\
 &= \frac{1}{2} n_2 + (n_1 + 1) \sum_{i_2=1}^{n_2} \left(\frac{v_{i_2}^2}{2} \right) + (n_1 + 1) \frac{v_c^2}{2} + \frac{1}{2} \\
 &= \frac{1}{2} (n_2 + 1) + \frac{1}{2} (n_1 + 1) = \frac{1}{2} (n_1 + n_2 + 2)
 \end{aligned} \tag{36}$$

In summary, expression (25) can be further simplified by expression (31). \square

Definition 9. The improved D numbers' combination rule is defined as follows.

Let

$$\begin{aligned} D_1 &= \{(b_1^1, v_1^1), \dots, (b_i^1, v_i^1), \dots, (b_{n_1}^1, v_{n_1}^1)\} \\ D_2 &= \{(b_1^2, v_1^2), \dots, (b_i^2, v_i^2), \dots, (b_{n_2}^2, v_{n_2}^2)\} \\ &\vdots \\ D_m &= \{(b_1^m, v_1^m), \dots, (b_i^m, v_i^m), \dots, (b_{n_m}^m, v_{n_m}^m)\} \end{aligned} \quad (37)$$

be m D numbers. The combination of m D numbers, indicated by $D = D_1 \oplus D_2 \oplus \dots \oplus D_m$, is defined by

$$D(b) = v, \quad (38)$$

with

$$\begin{aligned} b &= \frac{b_{i_1}^1 + b_{i_2}^2 + \dots + b_{i_m}^m}{m}, \\ v &= \frac{(v_{i_1}^1 + v_{i_2}^2 + \dots + v_{i_m}^m)/m}{c}, \end{aligned} \quad (39)$$

$$c = \frac{1}{m} (v_1^1 + v_2^1 + \dots + v_{n_1}^1 + v_c^1) \odot (v_1^2 + v_2^2 + \dots + v_{n_2}^2 + v_c^2) \dots \odot (v_1^m + v_2^m + \dots + v_{n_m}^m + v_c^m), \quad (40)$$

where $v_c^k = 1 - \sum_{i=1}^{n_k} v_i^k$.

When $m = 2$, then formula (40) can be reduced to formula (25).

Theorem 10. Similarly, the parameter c of formula (40) can be further simplified, indicated by

$$c = \frac{\sum_{i=1}^m (n_1' n_2' \dots n_m' / n_i')}{m} = \frac{n_1' n_2' \dots n_m'}{m} \cdot \sum_{i=1}^m \frac{1}{n_i'} \quad (41)$$

where

$$n_i' = \begin{cases} n_i & \sum_{j=1}^{n_i} v_j^i = 1 \\ n_i + 1 & \sum_{j=1}^{n_i} v_j^i < 1; \end{cases} \quad (42)$$

when $m = 2$, then formula (41) can be reduced to formula (31). The proof of Theorem 10 can be carried out in a similar manner.

3.2. Evidence Combination of Time Series Based on Improved D Number Theory. In the case of obtaining n D numbers at the same time, the proposed method is able to satisfy the associative law. Unfortunately, the more common situation in practical engineering applications is that we first get a part of D numbers and then obtain the others and calculate their fusion results step by step. The following is a study of the D number combination rule for time series evidence.

In order to facilitate the explanation of the problem, it is worthwhile to record the parameter $b \triangleq [b]_{12 \dots m}$, $(v_{i_1}^1 + v_{i_2}^2 + \dots + v_{i_m}^m)/m \triangleq [v_u]_{12 \dots m}$, $c = [c]_{12 \dots m}$ in formula (40) in which the m pieces of evidence are combined by the D number. Then, formula (40) can be simplified as

$$D([b]_{12 \dots m}) = \frac{[v_u]_{12 \dots m}}{[c]_{12 \dots m}}. \quad (43)$$

So, the problem of studying the D number combination rule of the time series evidence is converted to the known D number combination rule of the first $n-1$ moments,

$$D([b]_{12 \dots (n-1)}) = \frac{[v_u]_{12 \dots (n-1)}}{[c]_{12 \dots (n-1)}}, \quad (44)$$

and the evidence at the n th moment,

$$D_n = \{(b_1^n, v_1^n), \dots, (b_i^n, v_i^n), \dots, (b_{n_2}^n, v_{n_2}^n)\} \quad (45)$$

Solving the result of the D number combination rule at the first n time,

$$D([b]_{12 \dots n}) = \frac{[v_u]_{12 \dots n}}{[c]_{12 \dots n}}. \quad (46)$$

It can be obtained by formula (40) that

$$\begin{aligned} [b]_{12 \dots n} &= \frac{b_{i_1}^1 + b_{i_2}^2 + \dots + b_{i_n}^n}{n} \\ &= \frac{(b_{i_1}^1 + b_{i_2}^2 + \dots + b_{i_{n-1}}^{n-1}) / (n-1) \cdot (n-1) + b_{i_n}^n}{n} \\ &= \frac{[b]_{12 \dots (n-1)} \cdot (n-1) + b_{i_n}^n}{n}. \end{aligned} \quad (47)$$

Similarly,

$$\begin{aligned} [v_u]_{12 \dots n} &= \frac{v_{i_1}^1 + v_{i_2}^2 + \dots + v_{i_n}^n}{n} \\ &= \frac{(v_{i_1}^1 + v_{i_2}^2 + \dots + v_{i_{n-1}}^{n-1}) / (n-1) \cdot (n-1) + v_{i_n}^n}{n} \\ &= \frac{[v_u]_{12 \dots (n-1)} \cdot (n-1) + v_{i_n}^n}{n}. \end{aligned} \quad (48)$$

TABLE 1: Similarities and differences of D numbers and DS theory.

	DS theory	D numbers	
Differences	The elements in the frame of discernment	Mutually exclusive \	
	Basic probability assignment	Complete \	
	Major application fields	An extension of probability theory and Bayesian theory	Multiattribute decision-making
	Combination rule	Satisfies the associative property	Doesn't satisfy the associative property
Similarities	A theory to express uncertain information		

For the parameter c , it can be obtained by formula (24) that the multiplication-sum operation satisfies the commutative law. And so,

$$[c]_{12\dots n} = \frac{1}{n} (v_1^1 + v_2^1 + \dots + v_{n_1}^1 + v_c^1) \odot (v_1^2 + v_2^2 + \dots + v_{n_2}^2 + v_c^2) \dots \odot (v_1^n + v_2^n + \dots + v_{n_n}^n + v_c^n) = \frac{n-1}{n} [c]_{12\dots(n-1)} \odot (v_1^n + v_2^n + \dots + v_{n_n}^n + v_c^n) \quad (49)$$

By formulas (47)-(49), a general flow chart of the combination rule of the D number of time series evidence is given, as shown in Figure 2.

4. The Relationship among DS Theory, D Numbers, and Improved D Numbers

4.1. *The Relationship between DS Theory and D Numbers.* Though the D numbers derive from DS theory, the two theories solve different problems. The specific similarities and differences of the two theories are summarized as shown in Table 1.

Example 11. To make a comprehensive assessment of a student, we can get the following basic probability assignment based on DS theory.

$$\begin{aligned} m(\{a_1\}) &= 0.3 \\ m(\{a_2\}) &= 0.6 \\ m(\{a_1, a_2, a_3\}) &= 0.1, \end{aligned} \quad (50)$$

where $\{a_1\}$ is $[0, 30)$, $\{a_2\}$ is $[30, 70)$, $\{a_3\}$ is $[70, 100]$, and $\{a_1, a_2, a_3\}$ is the frame of discernment. The elements $\{a_1\}$ $\{a_2\}$ $\{a_3\}$ are mutually exclusive.

If we apply the D numbers theory to analyze the problem, we can get the expression as follows.

$$\begin{aligned} D(\{b_1\}) &= 0.3 \\ D(\{b_2\}) &= 0.5 \\ D(\{b_1, b_2, b_3\}) &= 0.1, \end{aligned} \quad (51)$$

where $\{b_1\}$ is $[0, 35)$, $\{b_2\}$ is $[30, 80)$, $\{b_3\}$ is $[70, 100]$, and $\{b_1, b_2, b_3\}$ is the frame of discernment. It is clear that the elements $\{b_1\}$ $\{b_2\}$ $\{b_3\}$ are not mutually exclusive.

4.2. *The Contrast between D Numbers and Improved D Numbers.* As we can see from formulas (17) and (18), the D numbers' combination rule does not satisfy the associative property. However, for the improved D numbers, $D_1 \oplus [D_2 \oplus D_3]$ and $[D_1 \oplus D_2] \oplus D_3$ have the same expression.

$$b = \frac{b_{i_1}^1 + b_{i_2}^2 + b_{i_3}^3}{3}, \quad (52)$$

namely,

$$D_1 \oplus [D_2 \oplus D_3] = [D_1 \oplus D_2] \oplus D_3. \quad (53)$$

From formula (21), it can be seen that the weight of the first combination number b is lower, which is $1/2^{n-1}$, and the weight of the latter combination number b is higher, which is $1/2$. However, for the improved D numbers, the weights of the first combination number and the latter combination number b are the same, which is $1/n$.

Example 12. Let

$$\begin{aligned} D_1 &= \{(0.6, 0.5), (0.7, 0.2), (0.8, 0.3)\} \\ D_2 &= \{(0.7, 0.7), (0.5, 0.1), (0.6, 0.2)\} \\ D_3 &= \{(0.2, 0.5), (0.1, 0.4), (0.0, 0.1)\} \end{aligned} \quad (54)$$

be 3 D numbers.

Then, we use the combination rules of D numbers and the improved D numbers separately to calculate the combined D number's integration representation $I(D)$. And the results are shown in Table 2.

It can be seen from Table 2 that the algorithm can satisfy the commutative law, and the original D number theory does not satisfy the commutative law.

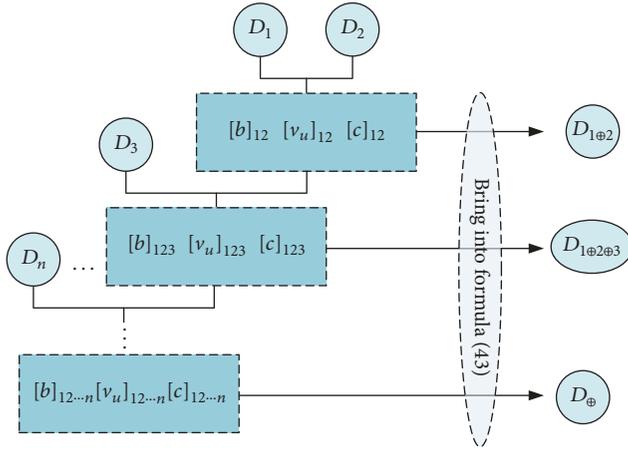


FIGURE 2: Flowchart of evidence D combination rule flow chart.

TABLE 2: Combined D number's integration representation.

$I(D)$	D numbers	Improved D numbers
$[D_1 \oplus D_2] \oplus D_3$	0.1719	0.4756
$D_1 \oplus [D_2 \oplus D_3]$	0.2267	0.4756
$D_2 \oplus [D_1 \oplus D_3]$	0.2302	0.4756

5. An Application in the Radiation Source Identification

In this section, in order to illustrate the application of the improved D numbers, a simple example about radiation source identification is given.

Radiation source identification mainly includes the analysis and decision-making of the detected signals. The detected signals of radiation source are radio frequency information, which have added up the noise and been modulated by the atmosphere. As a result, the radiation source identification can be regarded as an uncertainty reasoning process.

There are many theories to express uncertain information. In this paper, we will use the improved theory to identify the source of radiation. The problem is described in detail below.

There are 3 sources of information to investigate the target separately, indicated as S_1 S_2 S_3 . The 3 sources give corresponding decision based on different attributes. Among them, the measurable attributes of the source S_1 have distance C_1 , direction C_2 , and speed C_3 . The measurable attributes of the source S_2 have carrier frequency C_1 , pulse width C_2 , and pulse repetition period C_3 . The measurable attributes of the source S_3 have data chain C_1 , the frequency of communication C_2 , and modulation C_3 . Those three sources make enemy plane A_1 , friend plane A_2 , passenger plane A_3 , and unknown plane A_4 decisions. The decision results are shown as Table 3.

The accuracy and detection range of the source can vary with the battlefield environment and climate conditions. Table 4 shows the weights of sources in a specific time period.

TABLE 3: The decision matrix of multiple source.

Attributes	S_1			S_2			S_3		
	C_1	C_2	C_3	C_1	C_2	C_3	C_1	C_2	C_3
A_1	0.5	0.3	0.4	0.2	0.5	0.6	0.2	0.7	0.2
A_2	0.4	0.5	0.2	0.4	0.4	0.1	0.1	0.0	0.7
A_3	0.1	0.1	0.1	0.0	0.1	0.1	0.5	0.1	0.1
A_4	0.0	0.1	0.3	0.4	0.0	0.2	0.2	0.2	0.0

TABLE 4: The weights of different sources.

	S_1	S_2	S_3
C_1	0.4	0.6	0.5
C_2	0.3	0.1	0.4
C_3	0.3	0.3	0.1

TABLE 5: The expression of D numbers for A_1 .

A_1	D numbers
S_1	$D_{A_1}^{S_1} = \{(0.5, 0.4), (0.3, 0.3), (0.4, 0.3)\}$
S_2	$D_{A_1}^{S_2} = \{(0.2, 0.6), (0.5, 0.1), (0.6, 0.3)\}$
S_3	$D_{A_1}^{S_3} = \{(0.2, 0.5), (0.7, 0.4), (0.2, 0.1)\}$

In this paper, we use the improved D numbers to solve this problem. The application process is presented as follows.

Step 1. Express these assessment results of Table 3 in the forms of D numbers.

Take alternative A_1 as an example. For A_1 , in the opinion of source S_1 , the assessment results are indicated as b , and the weights of attributes are indicated as v . Then apply formula (11) to express these assessment results in the forms of D numbers. The expression is denoted by

$$D_{A_1}^{S_1} = \{(0.5, 0.4), (0.3, 0.3), (0.4, 0.3)\}. \quad (55)$$

Other sources' expressions can be required similarly, listed in Table 5.

Step 2. Apply the improved D numbers' combination rule to combine the D numbers of Table 5. The fusion results are listed in Table 6.

Step 3. Apply formula (20) to calculate the integration representation of $D_{A_1} = D_{A_1}^{E_1} \oplus D_{A_1}^{E_2} \oplus D_{A_1}^{E_3}$.

Step 4. Calculate and rank the integration representations of $D_{A_2} D_{A_3} D_{A_4}$ similarly. The ranking results are listed in Table 7.

Step 5. Analyze the data of Table 5 and make decisions. According to Table 7, the ranking of alternative is obtained, which is $A_1 > A_2 > A_4 > A_3$, where $>$ represents "better than". It is clear that the target is the enemy plane A_1 .

From Table 7 it can be seen that the target A_1 A_3 of the original D number theory has similar results. As can be seen from the raw data, goal A_1 is more likely to be the

TABLE 6: The results of $D_{A_1}^{S_1} \oplus D_{A_1}^{S_2} \oplus D_{A_1}^{S_3}$.

b	ν	b	ν	b	ν
0.3000	0.0556	0.2333	0.0519	0.2667	0.0519
0.4667	0.0519	0.4000	0.0481	0.4333	0.0481
0.3000	0.0407	0.2333	0.0370	0.2667	0.0370
0.4000	0.0370	0.3333	0.0333	0.3667	0.0333
0.5667	0.0333	0.5000	0.0296	0.5333	0.0296
0.4000	0.0222	0.3333	0.0185	0.3667	0.0185
0.4333	0.0444	0.3667	0.0407	0.4000	0.0407
0.6000	0.0407	0.5333	0.0370	0.5667	0.0370
0.4333	0.0296	0.3667	0.0259	0.4000	0.0259

TABLE 7: The ranking of decisions.

D number theory	Ranking	Proposed algorithm	Ranking
A_1	0.2314	1	0.3956
A_2	0.1210	3	0.2963
A_3	0.2012	2~1	0.1378
A_4	0.0984	4	0.1704

final decision result. The improved algorithm proposed in this paper can effectively identify the target.

6. Conclusions

In this paper, the improved D numbers theory is proposed for radiation source identification. In the improved theory, a novel combination rule of D numbers theory is represented to satisfy the associative property. Meanwhile, a new algorithm is constructed with the strict proof to simplify the combination rule. For the application of radiation source identification, the novel theory is simpler and more effective. An illustrative example has shown the improved theory's effectiveness.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare no conflicts of interest.

Authors' Contributions

Xin Guan and Haiqiao Liu conceived the concept and performed the research. Haiqiao Liu conducted the experiments to evaluate the performance of the proposed information fusion algorithm based on the improved D numbers method. Xiao Yi and Jing Zhao reviewed the manuscript. All authors have read and approved the final manuscript.

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