Research Article

Study on the Sound Quality of Steady and Unsteady Exhaust Noise

Falin Zeng¹ and Sunmin Sun²

¹Automobile Engineering Research Institute, Jiangsu University, Zhenjiang, China
²School of Automobile and Traffic Engineering, Jiangsu University, Zhenjiang, China

Correspondence should be addressed to Sunmin Sun; 731281380@qq.com

Received 17 January 2018; Accepted 10 June 2018; Published 28 June 2018

Academic Editor: Francesco Aymerich

In order to predict and study the sound quality of automobile exhaust noise, Zwicker steady-state and time-varying method were applied to calculate the psychoacoustic objective parameter values in terms of the exhaust noise of sample cars at uniform velocity and accelerated velocity; Thereby, a prediction model of GA-BP sound quality based on psychoacoustic objective parameters was established. At the same time, wavelet analysis was used to decompose the accelerated signal; in order to overcome the shortcomings such as Heisenberg uncertainty, the RNR (regularization nonstationary regression technique) was applied to compute the WVD distribution (RNR-WVD), therefrom obtaining the coefficient matrices of different-band signals after wavelet decomposition, and then A weighting was carried out on the coefficient matrices, so as to establish a new sound quality parameter SQP-WRW (sound quality parameter based on wavelet and then proceed to RNR-WVD) as the input of GA-BP model, and therefrom a sound quality prediction model was established. The results indicate that the model based on SQP-WRW has higher precision for predicting the sound quality of acceleration signal, and it can better reflect the characteristics of acceleration signal and sound quality.

1. Introduction

The research of automobile NVH has worked its way from noise control to the new stage emphasizing the design of noise and sound quality; the traditional research of vehicle noise aiming for sound pressure level can no longer satisfy the demand of the contemporary consumers.

The sound quality of automobiles reflects the subjective feeling of people towards the noise; the present research on sound quality is mostly based on subjective evaluation test, which can accurately and directly reflect the quality of voice, but it is time-consuming and labor-consuming. On the ground of that, domestic and foreign scholars put forward prediction models of automobile sound quality based on psychoacoustics parameters. Zhang et al. [1] optimized the genetic algorithm of support vector machines and established the prediction model of diesel engine sound quality based on psychoacoustic objective parameters. Based on the psychoacoustic objective parameters, Zuo Shuguang et al. [2] proposed the multiple linear regressions, neural network, and support vector machine, 3 prediction models for vehicle interior noise, and carried out comparison study. Bi Fengrong et al. [3] established the least squares support vector machine model on the basis of psychoacoustic objective parameters and EEMD signal features, thereby conducting the research of the acoustic quality of diesel engine radiated noise. Lee et al. [4] proposed the wavelet transform-based evaluation parameters of impact sound quality HFEC and the objective parameters with both roughness and volatility as multivariate linear regression model, which were applied to predict the acoustic quality of suspension system components.

To sum up, the present research on the sound quality of automobile mainly stays in the prediction model based on psychoacoustic objective parameters; however, the automobile noise mainly belongs to unsteady signal, and the single use of the analysis of time domain or frequency domain cannot accurately reflect the characteristic of vehicle noise. We should study and extract the signal features from time and frequency domains, while the sound quality prediction model based on advanced time frequency signal processing
(such as wavelets, EMD, and WVD) has not yet been widely used [5].

The exhaust noise of vehicles is one of the most important noise sources, the research on noise quality of which is of certain significance to noise pollution control. Firstly, this paper obtained relative psychoacoustic objective parameters of steady-state and acceleration signals in Artermis based on Zwicker steady-state and time-varying algorithm [6], and the noise quality GA-BP prediction model based on psychoacoustic parameters was established. Later on the methods of wavelet analysis and regularization unsteady regression were introduced to calculate the WVD distribution, and the characteristic parameters SQP-WRW of the acceleration exhaust signal were set up as the input of GA-BP model. The result suggests that the latter can predict the sound quality of nonstationary signal more accurately, which can provide a reference for the research of unsteady exhaust sound quality.

2. Subjective Evaluation Model of Exhaust Noise

Based on GB1496-79 Measuring Method for Noise of Power-Driven Vehicles, this paper adopted LMS to test and collect the steady-state exhaust noise signals as well as the acceleration noise signals of 10 domestic vehicles. The signals with engine speed of 1000rpm, 2000rpm, 3000rpm, 4000rpm, and 5000rpm were collected, and signals in the entire process in which engine speed changes from 1000rpm to 5000rpm were collected. According to the research experience [7], loudness, sharpness, roughness, volatility, kurtosis, and A sound level were used to be the input parameters of the model. The steady-state noise test results of sample car 1 is shown in Figure 1; it can be concluded that the sound pressure level of noise generally increased with the increase of rotational speed.

2.1. Subjective Evaluation Test. Pair comparison method was utilized in this subjective test. 5s of steady-state signals was intercepted, and 15s of characteristic signals in acceleration was extracted. In order to eliminate the effect of time duration on the subjective evaluation, the steady-state signal samples were treated with delayed time processing. Samples were compared in pair, in which the sample with decent evaluation will obtain 1 point, and the ones with unsatisfying evaluation will gain nothing, so that each sample will get a definite value representing its sound quality, which is more intuitive and conducive for the modelling later on.

A total of 42 people participated in the subjective evaluation; they are postgraduates majoring in vehicle-related field from a university and related workers in research institutes. There are 24 males and 18 females aged roughly within 24–40. According to sample coincidence degree and consistency coefficient [8], the data of 4 reviewers were eliminated, the average coincidence degree of the samples was 0.783, and the average consistency coefficient was 0.928. Thereof the consistency coefficients were computed by Kendall method [9], and the psychoacoustic objective parameters of the samples and the results of the subjective tests are shown in Table 1. The normalization formula of satisfaction is showed as formula (2).

2.2. Correlation Analysis. In order to study the relationship between the subjective satisfaction degree of exhaust noise and the psychoacoustic objective parameters, a comparison study was carried out between the subjective satisfaction score and psychoacoustic objective parameters. SPSS.19 software was used to do the correlation analysis. The results are shown in Table 2. Because the sample contains unsteady noise, it will produce extreme values with the change of time. Therefore, we applied two-tailed Spearman rank correlation to carry on the correlation analysis, wherein the spearman rank correlation formula is as follows:

\[
\rho = 1 - \frac{6 \sum_{i=1}^{n} (U_i - V_i)^2}{n(n^2 - 1)}
\]  

In the formula: \(U_i\) and \(V_i\) are the ranks of the two variables, the role of which is to change the fixed-distance variable to nonfixed-distance, thereby reducing the effect of extreme values on the results; \(n\) is the sample number, and \(\rho\) is the spearman rank correlation coefficient.

According to the results of correlation analysis, it can be concluded that the correlations between loudness, sharpness, and satisfaction degree are relatively great. The coefficients have reached 0.875 and 0.682, respectively, the influence of these two parameters has a great weight on the noise satisfaction, and, except for the positive correlations between roughness, kurtosis, and subjective satisfaction degree, other parameters have inverse correlations with satisfaction degree.
Table 1: Objective parameters and satisfaction values of samples.

<table>
<thead>
<tr>
<th>Sample number</th>
<th>loudness</th>
<th>sharpness</th>
<th>Roughness</th>
<th>fluctuation</th>
<th>A sound pressure</th>
<th>Kurtosis</th>
<th>satisfaction</th>
<th>Satisfaction normalization</th>
</tr>
</thead>
<tbody>
<tr>
<td>stable1</td>
<td>25.01</td>
<td>1.65</td>
<td>0.75</td>
<td>0.290</td>
<td>62.9</td>
<td>0.04</td>
<td>62.11</td>
<td>0.582</td>
</tr>
<tr>
<td>stable2</td>
<td>25.31</td>
<td>0.91</td>
<td>0.34</td>
<td>0.016</td>
<td>63.8</td>
<td>-0.06</td>
<td>87.86</td>
<td>0.859</td>
</tr>
<tr>
<td>stable3</td>
<td>29.69</td>
<td>1.96</td>
<td>0.89</td>
<td>0.01</td>
<td>64.2</td>
<td>-0.21</td>
<td>78.47</td>
<td>0.758</td>
</tr>
<tr>
<td>stable4</td>
<td>32.61</td>
<td>1.67</td>
<td>1.84</td>
<td>0.005</td>
<td>64.5</td>
<td>0.13</td>
<td>60.07</td>
<td>0.560</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>stable49</td>
<td>29.26</td>
<td>1.72</td>
<td>0.144</td>
<td>0.14</td>
<td>63.51</td>
<td>-0.11</td>
<td>69.57</td>
<td>0.662</td>
</tr>
<tr>
<td>stable50</td>
<td>43.68</td>
<td>1.64</td>
<td>0.26</td>
<td>0.12</td>
<td>67.98</td>
<td>-0.52</td>
<td>29.82</td>
<td>0.235</td>
</tr>
<tr>
<td>accele1</td>
<td>39.29</td>
<td>1.75</td>
<td>1.15</td>
<td>0.11</td>
<td>66.24</td>
<td>0.13</td>
<td>41.62</td>
<td>0.362</td>
</tr>
<tr>
<td>accele2</td>
<td>45.41</td>
<td>1.64</td>
<td>0.34</td>
<td>0.15</td>
<td>66.88</td>
<td>-0.35</td>
<td>30.63</td>
<td>0.246</td>
</tr>
<tr>
<td>accele3</td>
<td>30.59</td>
<td>1.24</td>
<td>2.18</td>
<td>0.06</td>
<td>63.21</td>
<td>-0.92</td>
<td>62.86</td>
<td>0.590</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>accele9</td>
<td>27.71</td>
<td>1.88</td>
<td>0.72</td>
<td>0.11</td>
<td>61.98</td>
<td>0.05</td>
<td>71.77</td>
<td>0.686</td>
</tr>
<tr>
<td>accele10</td>
<td>46.49</td>
<td>1.51</td>
<td>1.1</td>
<td>0.24</td>
<td>67.88</td>
<td>-0.22</td>
<td>34.97</td>
<td>0.291</td>
</tr>
</tbody>
</table>

Table 2: Correlation coefficients between satisfaction and subjective parameters.

<table>
<thead>
<tr>
<th>Acoustic parameters</th>
<th>loudness</th>
<th>sharpness</th>
<th>Roughness</th>
<th>fluctuation</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correlation coefficients</td>
<td>-0.875**</td>
<td>-0.682*</td>
<td>0.271*</td>
<td>-0.204</td>
<td>0.159*</td>
</tr>
</tbody>
</table>

(Note: ** indicates that when the bilateral confidence level is 0.01, the correlation is significant; * indicates that when the bilateral confidence level is 0.05, the correlation is significant.)

3. Establishment of GA-BP Prediction Model for Sound Quality

3.1. Establishment of Steady GA-BP Model. In this paper, BP (backpropagation) network was used to establish a complex nonlinear mapping relationship between psychoacoustic parameters and subjective satisfaction degree, and GA (genetic algorithms) was applied to optimize the weights and thresholds of neural networks. This not only can solve the problem that the nonlinear model of BP algorithm is easy to fall into local minimum but also can improve the efficiency of the calculation and the accuracy of the model. The model established is shown in Figure 2. The number of hidden layer nodes was calculated according to the formula $\sqrt{n_1 + n_2 + a}$ and the number of nodes was selected 7 according to the training results, wherein $n_1$ and $n_2$ are the number of input and output layer nodes, respectively. In order to improve the accuracy of acoustic quality prediction model, this paper used steady-state noises as the model's training samples; 46 steady-state samples were chosen as training samples, and the 4 remaining samples were used to validate the model's accuracy.

Tansig was chosen as the transfer function of the hidden layer, and purelin was chosen as the transfer function of the output layer for the model. Gradient descent algorithm traingd was chosen as network learning algorithm; the learning efficiency $\eta$ was set as 0.1, and the momentum coefficient $\mu$ was set as 0.9; the common mean square error (MSE) was used as the network training object function; the training target was set as 0.001; the maximum number of generations was 200, and the population size was 40, the generation gap (GGAP) was set as 0.85, the crossover rate was set as 0.7, and the mutation rate was set as 0.01. The input and output samples were all normalized before training. The normalized formula is as follows:

$$G = 0.8 \times \frac{(x_i - x_{\min})}{(x_{\max} - x_{\min})} + 0.1$$  \hspace{1cm} (2)

The training results of the model are shown in Figure 3. It can be seen from the training results that, with the increase of the iterative times of the genetic algorithm, the target value of the population is decreasing; that is, the adaptability is increasing, and it tends to be stable in the 50th iteration. After the optimization of the genetic algorithm, the object function value of the network training becomes smaller with the increase of training times. At the 100th iteration, the target value of the model tends to stabilize and the error reaches the set target. The value of fitting check of training result($R^2$) is 0.994; the sample expectation and the training value almost...
Table 3: Prediction results of accelerated signal satisfaction.

<table>
<thead>
<tr>
<th>sample</th>
<th>sample</th>
<th>sample</th>
<th>sample</th>
<th>sample</th>
<th>sample</th>
<th>sample</th>
<th>sample</th>
<th>sample</th>
<th>sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>car1</td>
<td>car2</td>
<td>car3</td>
<td>car4</td>
<td>car5</td>
<td>car6</td>
<td>car7</td>
<td>car8</td>
<td>car9</td>
<td>car10</td>
</tr>
<tr>
<td>satisfaction</td>
<td>0.362</td>
<td>0.246</td>
<td>0.59</td>
<td>0.467</td>
<td>0.558</td>
<td>0.621</td>
<td>0.49</td>
<td>0.652</td>
<td>0.686</td>
</tr>
<tr>
<td>Prediction value</td>
<td>0.336</td>
<td>0.234</td>
<td>0.551</td>
<td>0.442</td>
<td>0.511</td>
<td>0.552</td>
<td>0.509</td>
<td>0.610</td>
<td>0.621</td>
</tr>
<tr>
<td>error (%)</td>
<td>7.2</td>
<td>5.1</td>
<td>6.7</td>
<td>5.4</td>
<td>8.5</td>
<td>11.1</td>
<td>-3.9</td>
<td>6.4</td>
<td>9.5</td>
</tr>
</tbody>
</table>

Figure 3: Training results of GA-BP model based on psychoacoustic parameters.

perfectly coincide. The psychoacoustic objective parameter values of the remaining four steady noises were used as the input of the training GA-BP model, and the validation errors’ percentages are 2.3%, 1.8%, 1.6%, and 3.5%, respectively, and the average verification error is only 2.3%, which proves that the GA-BP network model has higher precision and can satisfy the requirements of the sound quality research and prediction.

The GA-BP model established was used to predict the sound quality of the acceleration noise of the 10 vehicle samples, and the predicted results are shown in Table 3. It can be found that the prediction errors of the acceleration noise signals of the 10 vehicles are basically greater than 5%, and the error RMS value is 7.13%.

3.2. Establishment of Unsteady GA-BP Model. The sound quality prediction model based on steady-state sample training is not ideal when it comes to predicting the sound quality of the acceleration exhaust noise, and, in order to study whether the training samples result in the difference in the model established and lead to insufficient accuracy, 8 groups of the acceleration unsteady noise samples were used as the
training samples of GA-BP model, and the remaining two groups were used in the prediction of the new model.

The network structure of the model is still 6-7-1; the training parameters, transfer functions, network training methods, and genetic algorithm parameters are all consistent with model 1. After the training of the model, the objective parameter values of the predicting samples were introduced into the predicting model, and the comparison results of the two predictions are shown in Figure 4.

From the comparison results, it can be perceived that the application of GA-BP sound quality model trained by acceleration noise samples improved the precision compared with the previous model, but the prediction accuracy was still not satisfying.

4. Based on WVT-RNR-WVD Sound Quality Model

4.1. Based on WVT-RNR-WVD Signal Analysis. Wavelet analysis, as a nearly mature analysis method for signal time-frequency, can effectively process acceleration signals, but the classical wavelet analysis is rooted in the superposition of specific basis functions, and it is difficult to approximate the local signal characteristics of the small wave functions derived by a single basis function on different levels. This may result in the loss of nonstationary signal characteristics [10]. In this regard, we can introduce WVD (Wigner-Ville) which is a bilinear time frequency distribution with a higher time frequency resolution and certain noise suppression capability, to improve the accuracy of signal analysis. However, the WVD spectral decomposition has the disadvantage of cross noise, so the regularization unsteady-state regression (RNR) [11, 12] was proposed in this paper to compute WVD spectral decomposition. Based on the above methods, a new acoustic quality evaluation parameter SQP-WRW (sound quality parameter based on wavelet and then proceed) was proposed to evaluate the sound quality of the acceleration exhaust noise; the process chart is shown in Figure 5.

4.1.1. Wavelet Transform. Wavelet is a function with oscillatory attenuation. Given a small wave function \( \psi(t) \), the wavelet transform sequence function is a function family [13] transferred from a single primary image wavelet through expansion and translation, as shown in the following form:

\[
\psi_{a,b}(t) = \frac{1}{|a|^{1/2}} \psi\left(\frac{t-b}{a}\right)
\]

wherein, \( a \) is an expansion factor, reflecting the scale or width of the function, and \( b \) is a translation factor, reflecting the translation position of the function along the time axis. \( \Psi_{a,b}(t) \) is the function family obtained by the expansion and translation of wavelet function \( \psi(t) \) under the continuous change of \( a \) and \( b \). Given a square-integrable signal \( x(t) \), then the wavelet transform is as follows:

\[
W_x(a,b) = \int x(t) |a|^{-1/2} \psi\left(\frac{t-b}{a}\right) dt = \langle x, \psi(a,b) \rangle
\]

\( W_x(a,b) \) contains the information of \( x(t) \) and \( \Psi_{a,b}(t) \), so the selection of the wavelet function is very important.

Based on the experience, the Daubechies (dBN) wavelets with orthogonality, compactness, and approximate symmetry was selected to analyze the acceleration signals. Firstly, the frequency below 20hz of signals was filtered by Pasteurized high-pass filter, so as to eliminate the influence of infrasound, and then high-frequency denoising process was carried out to the filtered noise signals based on Shannon Criterion. Figure 6 shows the contrast spectrum before and after processing.
6 Mathematical Problems in Engineering

Accelerated exhaust signal

Signal preprocessing

Wavelet decomposition

analytic wavelet 1
analytic wavelet 2
...
analytic wavelet 3

RNR-WVD calculating and A-weighting

Model training with the calculating values

Sound quality prediction model of accelerated noise built

Figure 5: The calculation flow process chart of the parameter.

20HZ high pass filter

Figure 6: Filter processing spectrum diagram.
From the frequency spectrum of the acceleration signals, it can be observed that its energy is basically concentrated in low-frequency range below 500Hz, but the sharpness was calculated based on Zwicker which focuses on the proportion of medium-high-frequency components in the entire signals. Roughness and fluctuation reflect the feelings of the human ear on the modulation frequency and amplitude of the sound. Kurtosis reflects the signal abrupt or flat degree; according to Table 2, the roughness, fluctuation, and kurtosis are less related to subjective satisfaction. However, the unsteady-state acceleration signal is very volatile and unordered, so these parameters calculated based on Zwicker could not express the fluctuation and steepness of the unstable signal accurately.

CARRYING OUT THE WAVELET TRANSFORM TO THE PROCESSED NOISE SIGNALS BASED ON THE DAUBECHIES IN MATLAB IS THEN FOLLOWED BY RECONSTRUCTING THE OBTAINED WAVELET TRANSFORM COEFFICIENTS BY WRCOEF FUNCTION. THEREBY THE RESULT OF WAVELET DECOMPOSITION CAN BE OBTAINED, AS SHOWN IN FIGURE 7. WE CAN SEE THAT THE ORIGINAL SIGNAL IS DECOMPOSED INTO 8-LAYER DETAIL WAVELET DI (i=1,.8) AND APPROXIMATE WAVELET A8 BY WAVELET TRANSFORM. THROUGH THE WAVELET TRANSFORM, THE ORIGINAL SIGNAL WAS PROCESSED WITH MULTISCALE DECOMPOSITION, AND THE RELATIONSHIP BETWEEN THE ORIGINAL SIGNAL AND DECOMPOSITION WAVELETS CAN BE EXPRESSED IN THE FOLLOWING FORMULA:

\[ S = d_1 + d_2 + d_3 + d_4 + d_5 + d_6 + d_7 + d_8 + a_8 \]  

\[ \text{(5)} \]

**4.1.2. WVD Distribution and Regularization Theories.** WVD is proposed by Wigner in the study of quantum mechanics in 1932, which was later applied by Ville in signal analysis; WVD is a quadratic distribution, which can meet many mathematical properties expected by time frequency analysis, and its transformation form is relatively simple, which belongs to the time frequency analysis of strict sense [14]. Given a signal \( x(t) \), its Wigner-Ville distribution is defined as

\[ WVD_x(t,f) = \int_{-\infty}^{+\infty} z(t+\frac{\tau}{2})z^*(t-\frac{\tau}{2})e^{-j2\pi f\tau}d\tau \]  

\[ \text{(6)} \]

wherein \( z(t) \) is the analytic signal; in the time domain, the analytic signal \( z(t) \) is defined as

\[ z(t) = x(t) + jH[x(t)] \]  

\[ \text{(7)} \]

wherein \( x(t) \) is the real signal, \( t \) is the time, \( \tau \) is the delayed time, \( f \) is the frequency, \( z^* \) is the transpose of \( z \), and \( H[x(t)] \) is the Hilbert transformation [15] of the real signal \( x(t) \). From the above formula, it can be found that product term appears in the WVD distribution, which leads to cross interference in the analysis of multifrequency signals, and this cross interference will affect the readability of the signals.

In order to eliminate the cross interference caused by the WVD distribution, regularization regression technique was introduced to compute WVD, the core of which is the shaping regularization theory; firstly, the theory of shaping regularization is briefly expounded.

Regularization technique is a practical mathematical method, which aims to strengthen the limit of estimated model, so as to solve the problem of ill-posed inverse problem. The most commonly used regularization method is Tikhonov regularization [16]. Fomel [17] proposes shaping regularization theory by considering the function of shaping operator. The method can be used to select the regularization operator in a simple way, such as Gaussian smooth operator and band-pass filter operator. Subsequently, Fomel extends
the theory of shaping regularization to nonlinear inverse problem, thereby establishing the theoretical basis of shaping regularization in inverse problem. One assumes that the vector \( d \) represents data, \( m \) represents model parameters, and the relationship between the data and the model is defined by forward-modelling operator \( L \), which can be expressed as

\[
d = Lm
\]

With the least square method, the optimization problem can be solved as follows:

\[
\min \| d - Lm \|_2
\]

wherein \( \| \cdot \|_2 \) indicates the norm of \( l_2 \). The aim of least squares optimization method is to estimate the optimal solution \( m \) under the condition of known data \( d \). When the condition number of the operator \( L \) is large, the direct solution of the inverse problem to calculate \( m \) is unstable; considering Tikhonov regularization method to be used to strengthen the constraint of the model \( m \), there are

\[
\min \| d - Lm \|_2 \quad \min \| eDm \|_2
\]

In the formula, \( D \) is the regularization term of Tikhonov, and then the optimization problem of the above formula has the following theoretical solution:

\[
\bar{m} = (L^TL + \epsilon^2D^TD)^{-1}L^Td
\]

Smooth operator was taken into account in the shaping regularization; in general, smooth operator can be regarded as the mapping of the constraint model in an acceptable space, which is called shaping by Fomel [18]. The shaping operator can be written as

\[
s = (I + \epsilon^2D^TD)^{-1}
\]

wherein \( s \) is shaping operator, from which we can derive that

\[
\epsilon^2D^TD = s^{-1} - I
\]

Introducing the above formula into formula (11), we can obtain the theoretical solution under shaping regularization:

\[
\bar{m} = (L^TL + s^{-1} - I)^{-1}L^Td
\]

\[
= [I + s(L^TL - I)]^{-1}sl^Td
\]

Set the discrete WVD distribution as

\[
\text{WVD}_d(n,k) = \sum_{m=0}^{N-1} 2z(n+m)z^*(n-m)e^{-j(\epsilon m n k/N)}
\]

Its cross-correlation function \( R(n,m) \) is defined as

\[
R(n,m) = \begin{cases} 
2z(n+m)z^*(n-m) & |m| \leq \theta \\
0 & |m| \geq \theta 
\end{cases}
\]

wherein \( \theta = \min(n, N-n) \), and the inverse transformation of WVD can be deduced as follows:

\[
R(n,m) = \sum_{k=0}^{N-1} \text{WVD}_d(n,k) e^{-j(\epsilon m n k/N)}
\]

The least squares optimal solution of the formula above is

\[
\min \| R(n,m) - \sum_{k=0}^{N-1} \text{WVD}_d(n,k) e^{-j(\epsilon m n k/N)} \|_2
\]

The correlation functions \( R(n,m) \) and \( \text{WVD}_d(n,k) \) in the above formula are complex and real, respectively, the above optimization problem is an ill-posed problem in mathematics, because the unknown quantity is more than the constraint equation, and therefore the shaping regularization algorithm is introduced to solve this problem.

In this paper, the Gaussian smooth operator with adjustable size was used as the shaping operator. What needs to be emphasized is that although WVD expressions are mathematical real functions, the WVD expressions are complex after using RNR iterations. Therefore, the absolute values of WVD were taken as time frequency distribution characteristic quantity of WVD. Figure 8 shows the analytic wavelet d8 analysis results of sample car 2 by WVD and RNR-WVD. The results show that there are a lot of “burr” in WVD analysis, which is mainly due to the cross noise caused by WVD decomposition algorithm. Through contrasting contour graphs, it can be clearly observed that the interference of cross noises can be effectively overcome by RNR-WVD method which is introduced with smooth operator. And this can bring the signal a better smoothness and clearer time frequency resolution. Some false characteristic interference in the signal analysis was also eliminated, and this can make the signal characteristic extraction more accurate.

4.2. Establishment of Sound Quality Model Based on SQP-WRW. Based on the above analysis, a new sound quality parameter SQP-WRW was established by using wavelet analysis and RNR-WVD method, and it was taken as input parameter to establish sound quality prediction model. And a comparison study was carried out with the GA-BP model based on psychoacoustic parameter of the acceleration signals. The main establishment steps of SQP-WRW model are as follows.

\textbf{Step 1. Wavelet Decomposition.} After filtering and denoising the collected acceleration exhaust signals, the wavelet transform was used to decompose the processed signals, and 9 analytic wavelets were obtained, which are an approximate signal and 8 detail signals.

\textbf{Step 2. RNR-WVD Transform.} Processing the analytic wavelets obtained by wavelet decomposition based on RNR-WVD and then k matrices \( RW_{k,m,n} \) \((k=9)\) with m rows and n columns were obtained, in which \( K \) is the number of the decomposition wavelets.

\textbf{Step 3 (weighting).} In order to simulate the acoustic filtering characteristic of human ear, the matrices \( RW_{k,m,n} \) were
Table 4: SQP-WRW values of accelerated samples.

<table>
<thead>
<tr>
<th>Sample car</th>
<th>a8</th>
<th>d8</th>
<th>d7</th>
<th>d6</th>
<th>d5</th>
<th>d4</th>
<th>d3</th>
<th>d2</th>
<th>d1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sample car 1</td>
<td>109.55</td>
<td>14.01</td>
<td>15.97</td>
<td>22.29</td>
<td>77.08</td>
<td>79.41</td>
<td>541.54</td>
<td>3753.46</td>
<td>1183.79</td>
</tr>
<tr>
<td>Sample car 2</td>
<td>93.21</td>
<td>13.23</td>
<td>13.41</td>
<td>45.01</td>
<td>82.24</td>
<td>99.11</td>
<td>438.91</td>
<td>4107.78</td>
<td>2228.20</td>
</tr>
<tr>
<td>Sample car 3</td>
<td>58.32</td>
<td>10.34</td>
<td>11.22</td>
<td>12.39</td>
<td>40.03</td>
<td>41.77</td>
<td>312.22</td>
<td>2737.98</td>
<td>1186.82</td>
</tr>
<tr>
<td>Sample car 4</td>
<td>77.67</td>
<td>6.73</td>
<td>16.21</td>
<td>34.22</td>
<td>56.81</td>
<td>101.18</td>
<td>337.53</td>
<td>4263.80</td>
<td>2718.52</td>
</tr>
<tr>
<td>Sample car 5</td>
<td>11.70</td>
<td>10.69</td>
<td>13.23</td>
<td>25.79</td>
<td>39.29</td>
<td>92.49</td>
<td>509.88</td>
<td>2599.73</td>
<td>1237.62</td>
</tr>
<tr>
<td>Sample car 6</td>
<td>121.72</td>
<td>15.57</td>
<td>17.74</td>
<td>24.77</td>
<td>85.64</td>
<td>88.23</td>
<td>601.71</td>
<td>4170.51</td>
<td>1315.98</td>
</tr>
<tr>
<td>Sample car 7</td>
<td>96.97</td>
<td>13.22</td>
<td>15.34</td>
<td>30.23</td>
<td>41.05</td>
<td>117.02</td>
<td>824.16</td>
<td>4523.57</td>
<td>2924.15</td>
</tr>
<tr>
<td>Sample car 8</td>
<td>83.85</td>
<td>11.76</td>
<td>13.92</td>
<td>40.01</td>
<td>73.11</td>
<td>88.79</td>
<td>390.14</td>
<td>3651.37</td>
<td>1980.62</td>
</tr>
<tr>
<td>Sample car 9</td>
<td>60.53</td>
<td>9.62</td>
<td>11.91</td>
<td>23.21</td>
<td>35.36</td>
<td>83.24</td>
<td>458.89</td>
<td>2439.72</td>
<td>1113.83</td>
</tr>
<tr>
<td>Sample car 10</td>
<td>79.34</td>
<td>10.82</td>
<td>12.55</td>
<td>24.73</td>
<td>33.59</td>
<td>95.74</td>
<td>674.23</td>
<td>3692.91</td>
<td>2392.50</td>
</tr>
</tbody>
</table>

![Figure 8: WVD and RNR-WVD analysis results of signal d8 of sample car 2.](image)

Treated with A weighting, and the weighting coefficient matrices $R_{W_{k,m,n}}$ can be obtained.

**Step 4 (SQP-WRW calculation).** Calculating the acceleration noise quality parameters SQP-WRW based on WVT and RNR-WVD, the formula is as follows:

$$SQP - WRW_k = \sqrt{\sum_{i=1}^{m} \sum_{j=1}^{n} \left[ RW_{m,n}(i,j) - RMS[RW_{m,n}] \right]}$$

(19)

wherein K is set within 1–9, which is the number of analytic wavelets. RMS[] is used to obtain the valid value of each coefficient matrix; the SQP-WRW values of each acceleration sample are shown in Table 4.

Unsteady-state signal is different from steady-state signal. Its distribution parameter and distribution rule will change dramatically with time. The SQP-WRW parameter presented in this paper can reflect the signal intensity, disorder degree, and jitter degree in time and frequency domain. A coefficient matrix will be obtained after the RNR-WVD procession, which records the characteristics of the signal. Taking one-dimensional time and one-dimensional frequency as an example, but the actual coefficient matrix is m rows and n columns, the schematic diagram is shown in Figure 9. Different curve has different max and min value, different distribution rule and parameters, and different SQP-WRW followed. The parameter can almost describe the characteristics of nonstationary signals.

**Step 5 (modelling).** Normalizing the SQP-WRW$_K$ (k=1,2,..., 9), the results of which were taken as the inputs of the GA-BP model, and a 9-10-1 sound quality prediction model was established. The algorithm parameters and the objective function were consistent with the model based on psychoacoustic parameters. The 1-8 acceleration signals were used as the
5. Conclusions

(1) Based on Zwicker steady-state and unsteady-state algorithm, the psychoacoustic objective parameters of steady-state exhaust noise and acceleration exhaust noise were calculated, and the GA-BP sound quality prediction models were established by steady-state and unsteady-state samples, respectively, which were applied to predict the unsteady acceleration exhaust noise.

(2) By combining WVT and RNR-WVD methods, the coefficient matrices featuring the information of time frequency signal characteristics in different frequency range can be obtained, and the obtained coefficient matrices were processed with A weighting, thereby obtaining a new sound quality parameter SQP-WRW, which was used as GA-BP input to train the sound quality model. And a comparison study was conducted with the prediction model based on the psychoacoustic objective parameters. The results suggest that the model trained by SQP-WRW has higher precision in terms of the sound quality prediction of the unsteady signal, and the parameter SQP-WRW built in this paper can be used to extract the characteristics of the unsteady noise signal, and it is suitable to be used as the training parameter of the prediction model of sound quality, which can provide certain reference for the future research on the sound quality of unsteady noise signal.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.
References


