Consensus Protocols for Heterogeneous Multiagent Systems with Disturbances via Integral Sliding Mode Control

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The existing results on consensus of multiagent systems are mainly based on homogeneous systems; that is, all agents have the same dynamic model. This paper focuses on consensus of heterogeneous multiagent systems, which consist of first-order and second-order integrator agents. Based on integral sliding mode control, the consensus protocols of heterogeneous multiagent systems with disturbances are investigated under the directed networks. Some sufficient conditions for finite-time consensus are obtained by utilizing Lyapunov stability theory. Finally, some examples verify the effectiveness of the proposed control schemes.

1. Introduction

Distributed cooperative control of multiagent systems (MAS) has attracted much attention due to its engineering applications in spacecraft, multiple mobile robots, transportation planning, and so forth [1–3]. An important problem in distributed coordination is to develop protocols, which specify the information exchange between agents, such that the group as a whole can reach an agreement. Such problem is usually called consensus problem [4]. The existing results on consensus mainly focus on homogeneous dynamic agents [5–9].

However, the agent dynamics may not be the same, and the study of heterogeneous MAS is more complex than the well-studied homogeneous MAS. The consensus of heterogeneous MAS has been studied in the last decade [10–18]. It is worth noting that these works do not consider the problem of disturbances.

Disturbances exist in almost all practical processes, which could have a negative effect on system performance. When the disturbances are unknown but bounded, an efficient approach is sliding mode control (SMC). SMC is a nonlinear control technique. It was regarded to have easy implementation and high robustness against uncertain but bounded disturbance. Numerous successful applications and theoretical extension of SMC have taken place in the last decades which include integral SMC (ISMC) [19–21]. Thus in this paper, we aim to study ISMC for heterogeneous MAS with disturbances.

It should be noted that the work [20] only focused on homogeneous second-order systems with disturbances, but our works focus on consensus of heterogeneous systems, which consist of first-order and second-order integrator agents. Compared with the results in [22], we only require that the speed feedback gain is positive. In [22], the largest in-degree of all agents was required in choosing the speed feedback gain. Moreover, the heterogeneous MAS draws back to a homogeneous MAS only composed of second-order integrator agents, and the obtained result is better than the one in [23] under the fixed topology.

According to whether there is a leading agent in the MAS, consensus problems can be divided into tracking and leaderless consensus problems [24]. In this paper, we deal with the leaderless consensus problem. The contributions are reflected as follows. First, we propose the novel finite-time consensus protocols for heterogeneous MAS, in which the communications among agents are described by a directed graph. Second, based on ISMC, the consensus protocols
ensure that the system has good robustness although the external disturbances exist.

The rest of this paper is arranged as follows. In Section 2, some basic ideas of MAS and other useful concepts are presented. The main results are presented in Section 3. Simulation results and conclusions are given in Sections 4 and 5, respectively.

2. Preliminaries and Problem Formulation

2.1. Preliminaries. In this subsection, some preliminary graph concepts and some lemmas are introduced. Let \( G = (V, E, A) \) be a directed graph of order \( n \) with the set of nodes \( V = \{ \zeta_1, \zeta_2, \ldots, \zeta_n \} \), the set of directed edges \( E \subseteq V \times V \), and an adjacency matrix \( A = [a_{ij}] \in \mathbb{R}^{m \times n} \). The set of neighbors of node \( \zeta_i \) is denoted by \( N_i = \{ \zeta_j \mid (\zeta_j, \zeta_i) \in E \} \). The Laplacian matrix of \( G \) is defined as \( L = D - A \). Denote \( D = \text{diag}(d_1, d_2, \ldots, d_n) \) by the in-degree matrix, where \( d_i = \sum_{j=1}^{n} a_{ij} = \sum_{j \in N_i} a_{ij} \). The eigenvalue 0 is algebraically simple and all other eigenvalues are with positive real parts if \( G \) has a spanning tree [6, 25].

Lemma 1 (see [26]). Consider the non-Lipschitz continuous nonlinear system

\[
\dot{x} = F(x), \\
F(0) = 0,
\]

\( x \in \mathbb{R}^n \)

Suppose that there exists positive definite function \( V(x) \) defined in a neighborhood of the origin. There exist real numbers \( c > 0 \), \( 0 < \alpha < 1 \), and \( V(x) + cV^\alpha(x) \leq 0 \). Then the system is finite-time stable. The upper bound of the settling time, depending on the initial state \( (0) = x_0 \), is satisfied with

\[
T(x_0) \leq \frac{V(x_0)^{1-\alpha}}{c(1-\alpha)}
\]

for all \( x_0 \) in some open neighborhood of the origin.

Lemma 2 (see [27]). For \( x_i \in \mathbb{R}, i = 1, 2, \ldots, n \), \( 0 < \rho \leq 1 \), then \( (\sum_{i=1}^{n} |x_i|)^{\rho} \leq \sum_{i=1}^{n} |x_i|^\rho \).

2.2. Problem Formulation. In this paper, we pay attention to the heterogeneous MAS of \( n \) agents, comprising \( m \)-second-order integrator agents and \( n-m \) first-order integrator agents. The communications among agents in the heterogeneous system are described by a directed graph.

The dynamic of each agent is given by

\[
\dot{x}_i = v_i \\
\dot{v}_i = u_i + \delta_1 \\
\dot{x}_l = u_l + \delta_2
\]

where \( x_i, v_i, u_i \in \mathbb{R} \) denote position, velocity, and control input of second-order agent \( i \), \( x_l, u_l \in \mathbb{R} \) represent position and control input of first-order agent \( l \), and \( \delta_1, \delta_2 \) are disturbances.

The main goal of this paper is to study consensus protocols for the heterogeneous MAS with disturbances such that the state consensus can be achieved in finite time.

3. Main Results

For second-order agents in system (3), the consensus protocol based on ISMC is designed as follows:

\[
s_i = v_i - v_i (0) - \int_{0}^{t} k_1 \sum_{j \in N_i} a_{ij} (x_j - x_i) - k_2 v_i d\tau
\]

\[
u_i = k_1 \sum_{j \in N_i} a_{ij} (x_j - x_i) - k_2 v_i - k_3 \text{sgn}(s_i)
\]

\[
i = 1, 2, \ldots, m
\]

where \( s_i \) denotes the sliding mode state, \( v_i(0) \) denotes the initial value of \( v_i \), \( \text{sgn}() \) represents the sign function, and \( k_1 > 0, k_2 > 0, k_3 > 0 \) are the feedback gains.

For first-order agents in system (3), the consensus protocol is designed as follows:

\[
s_l = x_l - \int_{0}^{t} k_3 \sum_{j \in N_i} a_{ij} (x_j - x_l) d\tau
\]

\[
u_l = k_3 \sum_{j \in N_i} a_{ij} (x_j - x_l) - k_4 \text{sgn}(s_l)
\]

\[
l = m + 1, m + 2, \ldots, n
\]

where \( s_l \) denotes the sliding mode state and \( k_3 > 0 \) is the feedback gain.

Assumption 3. The disturbances \( \delta_1, \delta_2 \) are bounded, and the feedback gain \( k_4 > \max(|\delta_1|, |\delta_2|) \).

Assumption 4. The digraph \( G \) has a directed spanning tree.

Theorem 5. Finite-time consensus in heterogeneous MAS (3) with protocols (4) and (5) is said to be achieved if Assumptions 3 and 4 are satisfied.

Proof. Take a Lyapunov function for (3) as

\[
V = V_1 + V_2
\]

where \( V_1 = (1/2)s_i^T s_i, V_2 = (1/2)s_i^T s_i \).

Differentiating \( V \), we obtain

\[
\dot{V} = s_i^T \dot{s}_i + s_i^T \dot{s}_i
\]

\[
= s_i^T \left[ \dot{v}_i - k_1 \sum_{j \in N_i} a_{ij} (x_j - x_i) + k_2 v_i \right]
\]

\[
+ s_i^T \left[ \dot{x}_i - k_3 \sum_{j \in N_i} a_{ij} (x_j - x_i) \right]
\]

3. Mathematical Problems in Engineering
From Assumption 3, it is easy to show that
\[
\dot{V} \leq - (k_4 - \delta_1) \|s_i\|_2 - (k_4 - \delta_2) \|s_i\|_2
\]
and
\[
\dot{V} \leq - (k_4 - \delta_1) \sqrt{2}V_1^{1/2} - (k_4 - \delta_2) \sqrt{2}V_2^{1/2}
\]
Set
\[
c = \min\left((k_4 - \delta_1) \sqrt{2}, (k_4 - \delta_2) \sqrt{2}\right)
\]
Then
\[
\dot{V} \leq -cV_1^{1/2} - cV_2^{1/2}
\]
According to Lemma 2
\[
(V_1 + V_2)^{1/2} \leq V_1^{1/2} + V_2^{1/2}
\]
Then
\[
\dot{V} \leq -c(V_1 + V_2)^{1/2} = -cV^{1/2}
\]
Thus
\[
\dot{V} + c(V)^{1/2} \leq 0
\]
By Lemma 1, we conclude that heterogeneous MAS (3) with protocols (4) and (5) can achieve consensus in a finite time.

Next, we will compute the settling time. According to (2) and (13)
\[
T \leq \frac{V_0^{1/2}}{c(1 - 1/2)}
\]
Substitute (9) to (14)
\[
T \leq \frac{2V_0^{1/2}}{\min\left((k_4 - \delta_1) \sqrt{2}, (k_4 - \delta_2) \sqrt{2}\right)}
\]
Synthesize (4), (5), and (6)
\[
V_0 = \frac{1}{2} x^T (0) x_i (0)
\]
Then
\[
T \leq \frac{\left|x^T (0) x_i (0)\right|^{1/2}}{\min\left((k_4 - \delta_1), (k_4 - \delta_2)\right)}
\]

4. Numerical Examples and Simulations

Example 1. The heterogeneous MAS is composed of five agents, where agents 1-3 are governed by second-order integrators and agents 4-5 are governed by first-order integrators. The direct topology of the system is described by Figure 1.

The adjacency matrix $A$ is given by
\[
A = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}
\]

Then the Laplacian matrix $L$ is described as
\[
L = \begin{bmatrix} 1 & 0 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 \\ -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & -1 & 2 \end{bmatrix}
\]

The five eigenvalues of $L$ are 0, 1, 2, 2 + i, 2 – i. Assumption 4 holds obviously.

The initial conditions are given as $x(0) = [1.5, 2, 2.5, 3, 3.5]$ and $v(0) = [1.5, 2, 3]$. The values of the parameters are designed by $k_1 = 15$, $k_2 = 5$, $k_3 = 50$, $k_4 = 4$. $\delta_1$ and $\delta_2$ are stochastic disturbances. $\delta_1 \in (0, 2)$, $\delta_2 \in (0, 2)$. The simulation results are illustrated in Figures 2–5. Figures 2 and 3 depict the trajectories of position error ($x_i - \bar{x}$) and velocity error ($v_i - \bar{v}$) under the consensus protocols in document [15]. Here, $\bar{x} = \sum_{i=1}^{n} x_i / n$, $\bar{v} = \sum_{i=1}^{m} v_j / m$. The protocols in document [15] are rewritten as follows.

\[
u_i = k_1 \sum_{j \in N_i} a_{ij} (x_j - x_i) - k_2 v_i \quad i = 1, 2, \ldots, m
\]

\[
u_l = k_3 \sum_{j \in N_l} a_{ij} (x_j - x_l) \quad l = m + 1, m + 2, \ldots, n
\]

Figures 4 and 5 depict the trajectories of position error ($x_i - \bar{x}$) and velocity error ($v_i - \bar{v}$) under the consensus protocols of
this paper. Compared with protocols in document [15], the protocols (4) and (5) given in this paper can still achieve perfect performance in spite of the disturbances.

**Example 2.** The systems considered here are the same as those in Example 1. In this example, we will compare our work with the so-called saturated consensus protocols proposed in document [12].

The initial states are chosen as $x(0) = [1.5, 2, 2.5, 3, 3.5]$, and $v(0) = [1.5, 2, 3]$. The values of the parameters are designed by $k_1 = 15, k_2 = 5, k_3 = 50, k_4 = 4$. We assume the disturbances $\delta_1 = \delta_2 = 2 \sin(t)$. The simulation results are illustrated in Figures 6–9. Figures 6 and 7 depict the trajectories of position errors $(x_i - \bar{x})$ and velocity errors $(v_i - \bar{v})$ under the saturated consensus protocols proposed in document [12]. Figures 8 and 9 depict the trajectories of position errors $(x_i - \bar{x})$ and velocity errors $(v_i - \bar{v})$.
under the consensus protocols (4) and (5). We can see that the consensus protocols based on ISMC can suppress the disturbances effectively.

5. Conclusions

In this paper, the finite-time consensus problem for heterogeneous MAS with disturbances has been studied under a directed graph. The performance and effectiveness of the theoretical results are illustrated by numerical examples. Compared with the consensus protocols in [12, 15], the proposed protocols in this paper can suppress the disturbances effectively.

Data Availability

The program codes of this paper are available from the corresponding author upon request (Hongtao Ye: yehongtao@126.com).
Conflicts of Interest
The authors declare that there are no conflicts of interest regarding the publication of this paper.

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