Research Article

Lagrangian Dual Decomposition for Joint Resource Allocation Optimization Problem in OFDMA Downlink Networks

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1. Introduction

As millions of base stations and billions of user terminals are expected to operate simultaneously and consume huge amounts of energy, energy efficiency is becoming an increasingly important issue in wireless communications in order to reduce operational expenditure and carbon dioxide emissions [1, 2]. In [3, 4], power allocation strategies were investigated for both single-cell and multicell systems under the assumption of perfect channel state information (CSI). In [5], a resource allocation scheme was proposed for improving energy efficiency under imperfect CSI conditions.

The optimization of energy-efficient resource allocation usually appears as a nonconvex problem. The key issue in solving such a nonconvex problem is how to transform it into its convex alternative that can be solved with an acceptable complexity. For example, in [6], the energy-efficient power allocation problem for orthogonal frequency division multiplexing (OFDM) cognitive radio networks was transformed into a fractional programming problem [7]. Then, based on the fractional programming method, the energy-efficient power and subcarrier allocation scheme was proposed in [8] for multiuser OFDMA networks, where the requirements for quality of service (QoS) were not taken into account. However, simply improving the energy efficiency of the system may often degrade the QoS for users.

It is worth mentioning that, in practical cellular systems, the QoS requirements usually provide an important specification to justify the resource allocation schemes. To ensure necessary QoS for users, the minimum transmission rate constraint was incorporated into the energy-efficient resource allocation problem in [9] for single-cell OFDM systems. On the other hand, for multicell OFDMA systems, the optimal resource allocation problem presents an NP-hard problem [10] due to the existence of intercell interferences [11]. To relax the difficulty caused by intercell interferences, a tolerable interference (TI) constraint was introduced in [11, 12], whereas intercell interference coordination was investigated in [13]. Meanwhile, an interference-aware scheme was proposed in [14], and an iterative method was proposed in [15],
both in a noncooperative way. An heuristic scheduling approach was also investigated in [16] in order to achieve tradeoffs between QoS and throughout under the total power constraint.

Motivated by the works above, we have also investigated the resource allocation problems for multicell multiuser systems. First, by using a universal multiuser interference channel model [17], we investigated the power minimization and sum rate maximization problems under the constraints on system power budget and individual users’ QoS requirements, while taking into account wireless power transfer and physical layer security. Then, upon extending the multiuser interference channel model to a multicell multiuser massive MIMO system [18], we investigated the resource allocation problems for guaranteeing individual users’ QoS requirements with causal time-splitting wireless power transfer. The core part of our previous works was to reformulate the originally intractable nonconvex optimization problems by using the mathematical tools of semidefinite relaxation (SDR) and S-procedure for convex approximation, and thus to design iterative successive convex approximation (SCA) algorithms for efficiently solving these problems.

In this paper, we address an efficient iterative algorithm for solving the joint power and subcarrier allocation problem for multicell OFDMA downlink networks. Different from the works in [11–16], the proposed optimization scheme is aimed to maximize the energy efficiency under the total power constraint of the system, while ensuring the QoS requirements in terms of different minimum sum rates for each cell. To solve the resource allocation problem which is originally nonlinear and nonconvex, the following problem solving methods are considered:

1. To circumvent the difficulties caused by intercell cochannel interferences, the maximum TI thresholds are introduced practically for each interference channel, which makes the original problem remarkably simplified.

2. To decouple the optimization variables, the simplified resource allocation problem is decomposed into two iterative updating processes:

   (a) Subcarrier allocation based on optimal user scheduling.

   (b) Power allocation based on a convex parametric problem using the energy efficiency $\alpha$ as the parameter.

Since the convex parametric problem is a strictly decreasing function of $\alpha$, the optimal $\alpha^*$ is reached by converting the iterative updating processes into an efficient bisection search, during which the optimal solution to the resource allocation problem is accordingly obtained.

3. To solve the convex parametric problem for given $\alpha^*$, the convex parametric problem is converted into Lagrangian dual problems at two levels and solved in the way of distributive closed-form computations as follows:

   (a) The Lagrangian dual problem at the first level comprises a master problem and an inner problem, the latter being decomposed into independent inner subproblems with respect to cells.

   (b) Each inner subproblem from the first level is further rewritten as a master problem and an inner problem, the latter being decomposed into independent inner subproblems with respect to subcarriers. Then, each inner subproblem at the second level has an optimal solution that can be analytically determined.

According to the derivations and reformulations, the iterative algorithm, based on a bisection search in combination with the subgradient method, is presented for solving the joint resource allocation problem. Computer simulations are conducted to validate the proposed algorithm and examine its performance under various system settings.

In the remainder of the paper, Section 2 presents the system model together with the problem formulation, Section 3 presents the problem reformulations and the iterative algorithm, and Section 4 demonstrates simulation results to validate the iterative algorithm and examine its performances. Finally, the paper is concluded by Section 5.

### 2. System Model

Consider a multicell multiuser OFDMA downlink network composed of $N$ cells. Each cell $n \in \mathcal{N} = \{1, 2, \ldots, N\}$ consists of a base station (BS) and $K_n$ users $\mathcal{K}_n = \{1, 2, \ldots, K_n\}$. During each time interval, all the BS’s transmit signals to the users in respective cells simultaneously over the same set of orthogonal subcarriers $\mathcal{M} = \{1, 2, \ldots, M\}$.

Figure 1 shows the channel model, where $g_{n,k}^m$ is the transmission channel power gain from BS $n$ to user $k$ over subcarrier $m$, and $g_{j,k}^m$ is the intercell interference channel power gain from BS $j \in \mathcal{N} \setminus n$ to user $k$ over subcarrier $m$. Note that $\mathcal{N} \setminus n$ denotes the subset of $\mathcal{N}$ with the element $n$ excluded. Specifically, $k^m$ represents the user of cell $n$ that will be scheduled to work on subcarrier $m$.

Let $p_{n,k}^m$ be the transmit power allocated to user $k \in \mathcal{K}_n$ working on subcarrier $m \in \mathcal{M}$ in cell $n \in \mathcal{N}$. Then, the signal-to-interference-plus-noise ratio (SINR) at this user is given by

![Figure 1: The channel model demonstrated with two cells $n$ and $j$.](image-url)
where \( \sigma^2_{n,k} \) is the associated additive white Gaussian noise (AWGN) power.

Let \( a_{n,k}^m \) be the subcarrier allocation indicator, i.e., \( a_{n,k}^m = 1 \) if user \( k_n \) is scheduled for subcarrier \( m \), and \( a_{n,k}^m = 0 \) otherwise. Then, the achievable transmission rate over the subcarrier is given by

\[
R^m_n = \sum_{k=1}^{K_n} a_{n,k}^m \log_2 \left( 1 + \gamma_{nk}^m \right)
\]

and the sum rate over all subcarriers in cell \( n \) is given by \( R_n = \sum_{m=1}^{M} R^m_n \).

For system design, denote the power allocation matrix by \( P = (P_1, P_2, \ldots, P_N) \) with column vectors \( P_n = (P_n^m)_{M \times 1} \), and denote the subcarrier allocation matrix by \( A_n = (a_{n,k}^m)_{M \times K_n} \), where \( n \in \mathcal{N} \). Let \( R_{n,\text{min}} \) be the minimum sum rate in cell \( n \) that measures the QoS requirement of the cell. Let \( P_n^\text{max} \) be the total transmit power threshold, and let \( P_c \) be the constant total circuit power consumption independent of the transmission power. With the objective of maximizing the energy efficiency, the joint resource allocation problem is formulated, under the constraints on the total transmit power of the system and the QoS requirements of all cells, as

\[
\max_{\mathcal{A}, \mathcal{P}} \rho = \frac{\sum_{n=1}^{N} R_n}{\sum_{n=1}^{N} \sum_{m=1}^{M} P_n^m + P_c}
\]

s.t. \( C_1 : \sum_{n=1}^{N} \sum_{m=1}^{M} P_n^m \leq P_n^\text{max} \)

\( C_2 : P_n^m \geq 0 \)

\( C_3 : R_n \geq R_{n,\text{min}} \)

\( C_4 : \sum_{k=1}^{K_n} a_{n,k}^m \leq 1 \)

for \( n \in \mathcal{N} \) and \( m \in \mathcal{M} \)

where \( \rho \) is defined as the energy efficiency of the system in “bit/Hz/Joule” [19], \( C_1 \) and \( C_2 \) represent the transmit power constraints, \( C_3 \) represents the QoS constraints for each cell, and \( C_4 \) indicates that each subcarrier in a cell can either be allocated exclusively to a user or stay idle depending upon user scheduling. Due to the coupling of the optimization variables \( P_n^m \) and \( a_{n,k}^m \) in the objective function, the problem is nonlinear and nonconcave and thus is intractable in its original form.

Remark 1. In the QoS constraints \( C_3 \), \( R_{n,\text{min}} \) can be chosen distinctly for different cells. Furthermore, \( C_3 \) can also be defined as \( R_n \geq R_{n,\text{min}} \) by using \( R_{n,\text{min}} \) to measure the QoS requirements of individual users.

### 3. Problem Reformulations and Solving Algorithm

In this section, problem (3) is first relaxed by introducing TIs to cope with intercell cochannel interferences, and the resultant problem is then decomposed into a user scheduling problem and a conditional power allocation problem that can be solved in an iterative manner. By using the energy efficiency \( \alpha \) as the parameter, the conditional power allocation problem is converted into a convex parametric problem. This strategy makes the joint optimization problem solvable by a bisection search for the parameter \( \alpha \) in combination with two-level iterative updates derived from double dual decomposition.

#### 3.1. Problem Relaxation by Introducing TIs

To relax the difficulties caused by intercell cochannel interferences [12], it is practical to introduce a set of preset thresholds \( \eta_{j,n}^m : j \in \mathcal{N} \setminus n \) for each pair \((n,m)\), where \( \eta_{j,n}^m \) represents the TI from cell \( j \) to cell \( n \) over subcarrier \( m \). Thus, under the constraints \( P_n^m a_{n,k}^m \leq \eta_{j,n}^m \), we obtain from (2) a lower bound on \( R_n^m \) as

\[
R_n^m = \sum_{k=1}^{K_n} a_{n,k}^m \log_2 \left( 1 + \frac{P_n^m g_{n,k}^m}{\eta_{j,n}^m} \right)
\]

where \( \eta_{n,k} = \sum_{j \in \mathcal{N} \setminus n} \eta_{j,n}^m + \sigma^2_{n,k} \) becomes a constant for any triple \((n,m,k_n)\).

Let \( \bar{R}_n = \sum_{m=1}^{M} R_n^m \). Then, we simplify problem (3) as

\[
\max_{\mathcal{A}, \mathcal{P}} \alpha = \frac{\sum_{n=1}^{N} \bar{R}_n}{\sum_{n=1}^{N} \sum_{m=1}^{M} P_n^m + P_c}
\]

s.t. \( C_1 \), \( C_2 \)

\( C_4 : \sum_{k=1}^{K_n} a_{n,k}^m \leq 1 \)

\( C_5 : \bar{R}_n \geq R_{n,\text{min}} \)

\( C_6 : P_n^m g_{n,k}^m \leq \eta_{n,j}^m \)

for \( n \in \mathcal{N} \), \( m \in \mathcal{M} \), and \( j \in \mathcal{N} \setminus n \)

where \( \alpha \) is defined as the lower bound on the energy efficiency \( \rho \), \( C_3 \) can be viewed as a constricted form of \( C_5 \), and \( C_6 \) represents the newly added power constraints on \( P_n^m \) because of TIs.

#### 3.2. Iterative Processes of Subcarrier Allocation and Power Allocation

Consider now the subcarrier allocation by means of user scheduling. To maximize local energy efficiencies and hence the global one, the optimal user scheduling rule can be expressed as a function of power allocation \( \mathcal{P} \) below

\[
k_n^m (\mathcal{P}) = \arg \max_{k_j \in \mathcal{K}_n} \left[ \frac{\log_2 (1 + \gamma_{nk_j}^m)}{P_n^m + P_c} \right]
\]
where $p_{n,m}^*$ is the associated circuit power such that
\[ \sum_{n=1}^{N} \sum_{m=1}^{M} p_{n,m}^* = P_c. \]

Therefore, given any power allocation $P$, the optimal user scheduling rule reduces to
\[ k_n^* = \arg \max_{k_n \in \mathcal{K}_n} \gamma_{n,k_n}^*. \]  
(9)

thus yielding the optimal subcarrier allocation $A$ with $\alpha_{n,k_n}^* = 1$ and $\alpha_{n,k_n}^* = 0$ for all $k_n \in \mathcal{K}_n \setminus k_n^*$.

Conversely, given a subcarrier allocation $A$, the constraint $C_4$ in problem (6) is resolved, and hence we need to solve a conditional power allocation problem
\[ \max_P \alpha = \frac{\sum_{n=1}^{N} \overline{R}_n}{\sum_{n=1}^{N} \sum_{m=1}^{M} p_{n,m}^* + P_c} \]  
(10)

s.t. $C_1, C_2,$
\[ C_5 : \overline{R}_n \geq R_{n, \min} \]  
(11)
\[ C_6 : p_{n,m}^* \leq \eta_{n,m}^* \]  
for $n \in \mathcal{N}$, $m \in \mathcal{M}$, and $j \in \mathcal{N} \setminus n$

where $\overline{R}_n^*$ is simplified from (5) as
\[ \overline{R}_n^* = \log_2 \left(1 + \frac{p_{n,m}^* g_{n,k_n}^*}{\eta_{n,m}^*} \right). \]  
(12)

Based on the reformulations above, the joint resource allocation in problem (6) is solvable by iteratively performing updates in (9) and (10). When $P$ converges to the optimal $P^*$, $A$ must also converge to the optimal $A^*$, while the maximized objective function in (10) yields the optimal energy efficiency $\alpha = \alpha^*$.

Now that problem (10) remains nonconvex and difficult to directly solve. Fortunately, it is solvable by fraction programming [20]. To do this, we transform it into a convex parametric problem as
\[ F(\alpha) \equiv \max_P f(P, \alpha) \]  
(13)
\[ = \sum_{n=1}^{N} \overline{R}_n - \alpha \left( \sum_{n=1}^{N} \sum_{m=1}^{M} p_{n,m}^* + P_c \right) \]

s.t. $C_1, C_2, C_5, C_6$

where $\alpha > 0$ is treated as the positive parameter and $f(P, \alpha)$ is a strictly concave function of $P$ since it strictly holds that the second partial derivatives $\partial^2 f / \partial (p_{n,m}^*)^2 < 0$ for all $p_{n,m}^*$.

Let us discuss the equivalence between (10) and (13) as follows. It is observed that maximizing $\alpha$ in (10) is essentially equivalent to maximizing the numerator therein while minimizing the denominator of the fraction. The optimal point $\alpha = \alpha^*$ just reaches the best tradeoff between the maximizing and minimizing operations under the convex constraints. In (13), the intention is reflected by maximizing $f(P, \alpha)$ which expresses the fraction of (10) in the difference form. Therefore, the optimal solutions $A^*$ and $P^*$ to problem (6) can be equivalently approached by iterations between (9) and (13), for which the optimal $\alpha^*$ turns out to be the root of the nonlinear equation $F(\alpha) = 0$ [6].

Clearly, $F(\alpha)$ is a strictly decreasing function of $\alpha$. There exists a feasible range $(\alpha_i, \alpha_f)$ such that $0 < \alpha_i < \alpha^* < \alpha_f$. It follows that $F(\alpha) > 0$ if $\alpha \in (\alpha_i, \alpha^*)$, and $F(\alpha) < 0$ if $\alpha \in (\alpha^*, \alpha_f)$. These properties imply an efficient bisection search for the root $\alpha^*$, and the feasible range can be determined loosely from (10) as
\[ \alpha_i = \frac{1}{P_{\max} + P_c} \sum_{n=1}^{N} R_{n, \min} \]  
(15a)
\[ \alpha_f = \frac{1}{P_{\max} + P_c} \sum_{n=1}^{N} \log_2 \left(1 + \frac{P_{\max}}{N M} \frac{g_{n,k_n}^*}{\eta_{n,m}^*} \right). \]  
(15b)

3.3. Double Dual Decomposition for Solving the Convex Parametric Problem. In this subsection, we cope with the parametric convex problem (13) with the parameter $\alpha$ by exploiting Lagrangian dualities at two levels with respect to cells and subcarriers, respectively.

3.3.1. Dual Decomposition at the First Level regarding Cells. To address the total power constraint $C_4$, we introduce a nonnegative dual variable $\lambda$, define a Lagrangian function $g(\alpha, \lambda, P)$, and convert problem (13) into a Lagrangian dual problem as follows:
\[ F(\alpha) = \min_{\lambda \geq 0} \quad G(\alpha, \lambda) \]  
(16a)
\[ G(\alpha, \lambda) = \max_P g(\alpha, \lambda, P) \]  
(16b)

s.t. $C_2, C_5, C_6$
\[ g(\alpha, \lambda, P) = f(P, \alpha) + \lambda \left( P_{\max} - \sum_{n=1}^{N} \sum_{m=1}^{M} p_{n,m}^* \right) \]  
(16c)
\[ = \sum_{n=1}^{N} \left( R_n - (\alpha + \lambda) \sum_{m=1}^{M} p_{n,m}^* \right) \]
\[ + \lambda P_{\max} - \alpha P_c \]

where the master problem (16a) explores the optimal $\lambda$ while seeking to minimize $G(\alpha, \lambda)$, whereas the inner problem (16b) explores the optimal $P$ so as to provide $G(\alpha, \lambda)$ as a candidate upper bound of $F(\alpha)$ for given $[\alpha, \lambda]$.

Since the item $\lambda P_{\max} - \alpha P_c$ in (16c) is irrelevant to $P$, it can be moved to the objective function of the master problem (16a). By decomposing the remaining summation item in (16c), we can reformulate the dual problem above into a master problem and $N$ independent inner subproblems as follows:
\[ F(\alpha) = \min_{\lambda \geq 0} \sum_{n=1}^{N} G_n(\alpha, \lambda) + \lambda P_{\max} - \alpha P_c \]  
(17a)
where subproblem (17b) for each \( n \) in \( N \) turns to explore \( \mathbf{p}_n \) while seeking to maximize \( G_n(\alpha, \lambda) \) under the related constraints.

The optimal value for \( \lambda \) that minimizes the objective function of (17a) can be approached by applying the heuristic subgradient method [20]

\[
G_n(\alpha, \lambda) = \min_{\mu_n, \Phi_n} H_n(\alpha, \lambda, \mu_n, \Phi_n)
\]

\[
H_n(\alpha, \lambda, \mu_n, \Phi_n) = \max_{\mathbf{p}_n} h_n(\alpha, \lambda, \mu_n, \Phi_n, \mathbf{p}_n)
\]

\[
\begin{align*}
s.t. & \quad \mathbf{p}_n^m \geq 0, \ m \in M \\
& \quad \mathbf{p}_n^m \geq \mathbf{R}_n^m \\
& \quad \mathbf{p}_n^m \geq \mathbf{R}_n^m - \mathbf{R}_n^m
\end{align*}
\]

\[
\begin{align*}
h_n(\alpha, \lambda, \mu_n, \Phi_n, \mathbf{p}_n) &= \mathbf{R}_n - (\alpha + \lambda) \sum_{m=1}^{M} \mathbf{p}_n^m + \mu_n (\mathbf{R}_n - \mathbf{R}_n^m) + \sum_{j \neq n} \sum_{m=1}^{M} \Phi_{n,j}^m (\eta_{n,j}^m - \mathbf{p}_n^m \mathbf{g}_{n,k_j}^m) \\
&= \sum_{m=1}^{M} \left[ (1 + \mu_n) \mathbf{R}_n^m - \mathbf{p}_n^m (\alpha + \lambda + \sum_{j \neq n} \sum_{m=1}^{M} \Phi_{n,j}^m \mathbf{g}_{n,k_j}^m) \right] - \mu_n \mathbf{R}_n^m + \sum_{j \neq n} \sum_{m=1}^{M} \Phi_{n,j}^m \mathbf{g}_{n,k_j}^m
\end{align*}
\]

Similarly, the item \(-\mu_n \mathbf{R}_n^m + \sum_{j \neq n} \sum_{m=1}^{M} \Phi_{n,j}^m \eta_{n,j}^m \) on the last line of (20c) is irrelevant to \( \mathbf{p}_n \), whereas each bracketed summand in the first summation of (20c) is related to a unique \( \mathbf{p}_n^m \) for \( m \in M \). Therefore, we can rewrite the dual problem expressed by (20a)–(20c) into the form of a master problem connected with \( M \) independent inner subproblems as follows:

\[
G_n(\alpha, \lambda) = \min_{\mu_n, \Phi_n} \sum_{m=1}^{M} H_n^m(\alpha, \lambda, \mu_n, \Phi_n^m) - \mu_n \mathbf{R}_n^m
\]

\[
H_n^m(\alpha, \lambda, \mu_n, \Phi_n^m) = \max_{\mathbf{p}_n^m} h_n^m(\alpha, \lambda, \mu_n, \Phi_n^m, \mathbf{p}_n^m)
\]

\[
\begin{align*}
\text{s.t.} & \quad \mathbf{p}_n^m \geq 0 \\
& \quad \mathbf{p}_n^m \geq (1 + \mu_n) \mathbf{R}_n^m \log_2 \left( 1 + \frac{\mathbf{p}_n^m \mathbf{g}_{n,k_n}^m}{\mathbf{R}_n^m \mathbf{g}_{n,k_n}^m} \right) \\
& \quad - \mathbf{p}_n^m \left( \alpha + \lambda + \sum_{j \neq n} \sum_{m=1}^{M} \Phi_{n,j}^m \mathbf{g}_{n,k_j}^m \right)
\end{align*}
\]

where the master problem (21a) seeks to optimize \((\mu_n, \Phi_n)\) while achieving the least upper bound \( G_n(\alpha, \lambda) \) with \( [\alpha, \lambda, n] \) given and subproblem (16b) seeks to maximize \( h_n^m(\alpha, \lambda, \mu_n, \Phi_n^m, \mathbf{p}_n^m) \) by finding the optimal \( \mathbf{p}_n^m \) \( \geq 0 \) with \([\alpha, \lambda, n, m, \mu_n, \Phi_n^m]\) given.

It is readily shown that \( h_n^m(\alpha, \lambda, \mu_n, \Phi_n^m, \mathbf{p}_n^m) \) is a strictly concave function of \( \mathbf{p}_n^m \) as it is differentiable with the strictly negative second derivative \( \partial^2 h_n^m / \partial (\mathbf{p}_n^m)^2 < 0 \) existing in the real domain. By taking the first derivative \( \partial h_n^m / \partial \mathbf{p}_n^m \) and letting it be zero, the maximum point for \( \mathbf{p}_n^m \geq 0 \) is derived as

\[
\mathbf{p}_n^{m*} = \left[ 1 + \frac{1}{\mu_n} \left( \mathbf{p}_n^{m*} \mathbf{g}_{n,k_n}^m \mathbf{R}_n^m \mathbf{g}_{n,k_n}^m \right)^{1/2} \right] + \left( \mathbf{R}_n^m \mathbf{g}_{n,k_n}^m \right)
\]

thus yielding the optimal solution to subproblem (21b) as

\[
H_n^m(\alpha, \lambda, \mu_n, \Phi_n^m) = h_n^m(\alpha, \lambda, \mu_n, \Phi_n^m, \mathbf{p}_n^{m*})
\]
By substituting solution (23), the master problem (21a) for each \( n \) can be solved by using the heuristic subgradient method in the following form:

\[
\mu_n \left[ t_2 + 1 \right] = \left[ \mu_n \left[ t_2 \right] - \Delta \left( \overline{R}_n \left[ t_2 \right] - R_{\min} \right) \right]^+
\]

(24a)

\[
\varphi_{m,j}^n \left[ t_2 + 1 \right] = \left[ \varphi_{m,j}^n \left[ t_2 \right] - \Delta \left( \eta_{m,j}^n - F_{m,j} \left[ t_2 \right] g_{m,j}^n \right) \right]^+
\]

(24b)

where \( t_2 \) indexes the iterations at the second level, and the expressions enclosed in the parentheses are the derived subgradients, and \( \Delta \) is the same searching step size as in (19). As \( t_2 \) grows, the dual variables in \( \{\mu_n[t_2], \Phi_n[t_2]\} \) will reach their optimal values when \( P \) updated by (22) converges to a fixed point for given \( (\alpha, \lambda, n) \).

3.4. Iterative Algorithm for Solving Problem. Figure 2 shows a framework for solving the concave parametric problem (13) based on the double dual decomposition above. Given a value of \( \alpha, F(\alpha) \) in (17a) is minimized by exploring \( \lambda \) while demanding \( \{G_n(\alpha, \lambda) : n \in \mathcal{N}\} \) from (21a). Given any instance of \( (\alpha, \lambda) \), each \( G_n(\alpha, \lambda) \) in (21a) is minimized by exploring \( \{\mu_n, \varphi_{m,j}^n : m \in \mathcal{M}\} \) while demanding \( \{H_{m,j}(\alpha, \lambda, \mu_n, \varphi_{m,j}^n) : m \in \mathcal{M}\} \) from (21b). Given an instance of \( (\alpha, \lambda, \mu_n, \varphi_{m,j}^n) \), the optimal \( p_{m,j}^n \) and hence \( H_{m,j}^n \) are computed by (22) and (23), respectively. By using the heuristic subgradient methods based on (19) and (24a), the concave parametric problem (13) is gradually approached by two-level iterations running over (17a) and (21a).

The iterative algorithm for solving the joint resource allocation problem (6) is presented in Algorithm 1, which mainly comprises three loops of “repeat-until” as follows:

(i) **Loop 1**, including lines 2–34, performs the bisection search for \( \lambda \) in the range \( (\alpha_1, \alpha_2) \) given by (15a) and (15b), where the exiting condition is expressed as \( |F(\lambda)| \leq \varepsilon \), and \( \varepsilon \) is the accuracy.

(ii) **Loop 2**, including lines 6–27, performs the subgradient search for \( \lambda \) according to (19), while updating \( F(\lambda) \) with the results of \( \{G_n : n \in \mathcal{N}\} \) from Loop 3, where the exiting condition is expressed as \( \delta_1 \leq \varepsilon \), and \( \delta_1 \) is the mean squared error (MSE) metric defined as

\[
\delta_1 \left( P \left[ t_1 + 1 \right], P \left[ t_1 \right] \right) = \frac{1}{NM} \sum_{n=1}^{N} \sum_{m=1}^{M} \left( p_{m,j}^n \left[ t_1 + 1 \right] - p_{m,j}^n \left[ t_1 \right] \right)^2 .
\]

(25)

(iii) **Loop 3**, including lines 11–21, performs the subgradient search for \( \mu_n \) and \( \Phi_n \) according to (24a) and (24b), while updating \( \{G_n : n \in \mathcal{N}\} \) with the results of \( \{p_{m,j}^n, H_{m,j}^n : m \in \mathcal{M}\} \) from (22) and (23), where the exiting condition is expressed as \( \delta_2 \leq \varepsilon \), and \( \delta_2 \) is the MSE metric defined as

\[
\delta_2 \left( \Phi_n \left[ t_2 + 1 \right], \Phi_n \left[ t_2 \right] \right) = \frac{1}{M} \sum_{m=1}^{M} \left( p_{m,j}^n \left[ t_2 + 1 \right] - p_{m,j}^n \left[ t_2 \right] \right)^2 .
\]

(26)

We briefly analyze the computational complexity of the algorithm. By (9), the subcarrier allocation problem is a complexity of order \( \Theta(\sum_{n=1}^{N} K_n M) \). For power allocation, there are \( NM \) optimization variables \( p_{m,n}^j \) in the following form:

\[
\mathbf{P} = \{ \mu_n \} \quad \text{and} \quad \mathbf{Q} = \{ \varphi_{m,j}^n \}.
\]

(27)

where \( \mathbf{P} \) and \( \mathbf{Q} \) are the dual variables in (17a), and \( 1 + (N - 1)M \) dual variables \( \mu_n \) and \( \varphi_{m,j}^n \) in (21a). Thus, the power allocation yields a complexity of order \( \Theta(N^2 M + N + 1) \). To achieve the accuracy \( \varepsilon \) for the bisection search, the computational complexity is estimated of polynomial order \( \Theta((\sum_{n=1}^{N} K_n M + N^2 M + N + 1)\log(1/\varepsilon)) \) in the network scale parameters and thus acceptable for practical applications.

4. Simulation Results

In this section, we use computer simulations to validate Algorithm 1 and examine its performance. For demonstration purpose, we consider a small scaled OFDMA downlink network with the number of cells \( N = 2 \), the number of users in each cell \( K_1 = K_2 = 4 \), and the number of subcarriers \( M = 10 \). Upon assuming that all the channels experience independent identically distributed (i.i.d) Rayleigh fading, the intracell and intercell average channel gains are given by \( E(g_{m,n}^j) = 1 \) and \( E(g_{m,j}^n) = 1/3 \), respectively, for all \( m \in \mathcal{M} \) and \( j \neq n \in \mathcal{N} \). Without loss of generality, we assume identically the circuit powers \( P_c = 0.05 \) W and \( P_{c,n} = P_c/(NM) = 0.025 \) W, the noise powers \( \sigma_n^2 = \sigma^2 \), the minimum cell sum rates \( R_{\min} = R_{\min} \), and the TI thresholds \( \eta_{m,j}^n = \eta \) for all \( m, j \).

For performance comparison for different parameters and different schemes, we define the average signal-to-noise ratio (SNR) as \( \text{SNR} = P_{\max}/(NM\sigma^2) \) per subcarrier. To determine the total transmit power threshold \( P_{\max} \) such that the QoS requirements can be satisfied for all cells, we define a satisfaction index (SI) metric in percentage as

\[
\text{SI} = \frac{1}{N} \sum_{n=1}^{N} \min \left\{ 1, \frac{R_n}{R_{\min}} \right\}
\]

(27)

where \( R_n \) is the sum rate of cell \( n \) computed from (2), and \( R_{\min} \) represents the QoS requirements for all cells. Clearly, we have SI = 100% if and only if \( R_n \geq R_{\min} \) for all \( n \in \mathcal{N} \).

Figure 3 shows the tendencies of SI versus SNR for various \( R_{\min} \) values, where equal power allocation per carrier is considered, and the noise power is set to \( \sigma^2 = 0.01 \) W. For each \( R_{\min} \) value, it is seen that SI tends to reach the maximum of 100% when SNR increases beyond some critical point. Meanwhile, it is in nature to observe that the critical SNR value increases with \( R_{\min} \). Therefore, for simulations regarding a specific \( R_{\min} \) value, we can use the associated critical SNR value and the used \( \sigma^2 \) value to estimate \( P_{\max} = \text{SNR} \cdot NM\sigma^2 \) as the minimum required system power. For example, the critical SNR value associated with \( R_{\min} = 16 \) bps/Hz is approximately 12 dB by the figure, which corresponds to \( P_{\max} = 3.17 \) W when \( \sigma^2 = 0.01 \) W.

Figure 4 shows the simulation results on energy efficiency performance of Algorithm 1 versus \( R_{\min} \) within the range from 1 to 10 bps/Hz, where SNR = 10 dB, and \( \eta = 10\sigma^2 \).
Figure 2: Problem solving framework of two-level iterations based on double dual decomposition.

Figure 3: QoS satisfaction index versus SNR for various $R_{\text{min}}$ values based on equal power allocation.

Figure 4: Comparison of energy efficiencies versus $R_{\text{min}}$ for variant $P_{\text{max}}$ with respect to SNR = 10 dB and $\eta = 10\sigma^2$.

under various channel conditions $\sigma^2 = 0.01, 0.02, 0.03, 0.04$ W. Accordingly, the total transmit power threshold is determined to be $P_{\text{max}} = 2, 4, 6, 8$ W. The simulation results are analyzed as follows: leftmargin=*;labelsep=4.9mm.

(1) The effects of the QoS requirements $R_{\text{min}}$ for each given $\sigma^2$ and hence $P_{\text{max}}$, it is observed that the energy efficiency reaches the maximum over a lower region of $R_{\text{min}}$ and becomes decreasing as $R_{\text{min}}$ increases over the relatively higher region. This behavior is relevant to the effects of intercell interferences. To achieve a large $R_{\text{min}}$ value, each cell definitely needs to consume a high power, which can generate severer interferences to other cells. To overwhelm the interferences, an extra amount of power is in turn required, thus degrading the energy efficiency.
could result in large achievable rates for users with good
OFDMA networks under the total power constraint, which
maximize the system throughput for multicell multiuser
throughput between the proposed scheme and the scheme in
system performance in energy efficiency, total power, and

Next, we present a comprehensive comparison of the
effects of the channel conditions $\sigma^2$: for a fixed
$R_{\text{min}}$, it is seen that a worse channel condition $\sigma^2$
results in an increased $P_{\text{max}}$ and hence a degraded
energy efficiency. For the instance of $R_{\text{min}} = 1$ bps/Hz,
when $\sigma^2$ is increased from 0.01 to 0.04 W such that
$P_{\text{max}}$ is increased from 2 to 8 W, the energy
efficiency is reduced from 26.5 to 12.5 bits/Hz/joule at a
reduction of approximately 53%. This behavior can be
similarly analyzed in that more power consumptions are
required to overwhelm large noise powers and increased intercell interferences.

Next, we present a comprehensive comparison of the
system performance in energy efficiency, total power, and
throughput between the proposed scheme and the scheme in
[16]. Note that the heuristic scheduling in [16] was designed
to maximize the system throughput for multicell multiuser
OFDMA networks under the total power constraint, which
could result in large achievable rates for users with good
channel conditions, but smaller achievable rates for users
with worse channel conditions. However, in the proposed
scheme, the throughput can be guaranteed by presetting $R_{\text{min}}$
for each cell.

Figure 5 compares the energy efficiency performance
versus SNR between Algorithm 1 (noted “EE-opt”) and the
algorithm proposed in [16] (noted “SE-opt”) for $R_{\text{min}} = 1$ bps/Hz. To examine the effects of TI thresholds, two
different settings of $\eta = 10\sigma^2$ and $\eta = 20\sigma^2$ are considered.
It is observed that increasing SNR can improve the energy
efficiency for both algorithms, while the proposed EE-opt
algorithm outperforms the SE-opt one with an increasing
performance gap. On the other hand, increasing the TI
threshold from $\eta = 10\sigma^2$ to $\eta = 20\sigma^2$ reduces the energy
efficiency for each algorithm. The performance loss tends
to become larger as SNR increases. This observation can be
explained by the fact that the increased TI threshold allows
for severer intercell interferences, and thus more power is
allocated to maintain the required $R_{\text{min}}$.

<table>
<thead>
<tr>
<th>Algorithm 1: Iterative algorithm for solving problem (6).</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Input:</strong> $N, M, {K_n^m, \eta_{nj}^m : n \in \mathcal{N}, m \in \mathcal{M}, j \in \mathcal{N} \setminus n}, P_{\text{max}}, \alpha, \beta, \lambda, \mu, \varphi, \Delta, \varepsilon$</td>
</tr>
<tr>
<td><strong>Output:</strong> the subcarrier allocation $A$, and the power allocation $P$</td>
</tr>
<tr>
<td>1: $p_{0}^n \leftarrow P_{\text{max}}/(N,M)$ for $n \in \mathcal{N}$ and $m \in \mathcal{M}$</td>
</tr>
<tr>
<td>2: repeat * the bisection search for $\alpha^*$ *</td>
</tr>
<tr>
<td>3: * the search for $\lambda^*$ *</td>
</tr>
<tr>
<td>4: $\alpha \leftarrow (\alpha + \alpha_0)/2$</td>
</tr>
<tr>
<td>5: $t_1 \leftarrow 0, F(\alpha) \leftarrow \infty, \lambda \leftarrow \lambda_0$</td>
</tr>
<tr>
<td>6: repeat * the search for $\lambda^*$ *</td>
</tr>
<tr>
<td>7: $P'[0] \leftarrow P[t_1]$</td>
</tr>
<tr>
<td>8: for $n = 1$ to $N$ do</td>
</tr>
<tr>
<td>9: $t_2 \leftarrow 0, G_n \leftarrow \infty, \mu_{[0]} \leftarrow \mu_0$</td>
</tr>
<tr>
<td>10: $q_{[0]} \leftarrow \varphi_0$ for $m \in \mathcal{M}$ and $j \in \mathcal{N} \setminus n$</td>
</tr>
<tr>
<td>11: repeat * the search for $(\mu_n, \Phi_n)^*$ *</td>
</tr>
<tr>
<td>12: for $m = 1$ to $M$ do</td>
</tr>
<tr>
<td>13: $\text{compute } p_{[m]}^{n}\text{ by (22) with } [\alpha, \lambda^{[t_1]}, \mu_n^{[t_2]}, \varphi_n^{[t_2]}]$</td>
</tr>
<tr>
<td>14: $\text{compute } H_{n}^{m}\text{ by (23) with } [\alpha, \lambda^{[t_1]}, \mu_n^{[t_2]}, \varphi_n^{[t_2]}]$</td>
</tr>
<tr>
<td>15: end for</td>
</tr>
<tr>
<td>16: update $G_n$ by (21a) with $[\mu_n^{[t_2]}, \varphi_n^{[t_2]}], H_{[m]}^{n} : m \in \mathcal{M}$</td>
</tr>
<tr>
<td>17: $t_2 \leftarrow t_2 + 1$</td>
</tr>
<tr>
<td>18: $p_{[m]}^{n} \leftarrow {p_{[m]}^{n} : m \in \mathcal{M}}$</td>
</tr>
<tr>
<td>19: update $\mu_n^{[t_2]}$ by (24a) with $p_{[n]}^{[t_2]}$</td>
</tr>
<tr>
<td>20: update $\varphi_n^{[t_2]}$ by (24b) with $p_{[n]}^{[t_2]}$</td>
</tr>
<tr>
<td>21: until $\delta_{\lambda}(p_{[n]}^{[t_2]}, p_{[n]}^{[t_2-1]} - 1) \leq \varepsilon$</td>
</tr>
<tr>
<td>22: end for</td>
</tr>
<tr>
<td>23: update $F(\alpha)$ by (17a) with $[\alpha, \lambda^{[t_1]}, G_n : n \in \mathcal{N}]$</td>
</tr>
<tr>
<td>24: $t_1 \leftarrow t_1 + 1$</td>
</tr>
<tr>
<td>25: $P[t_1] \leftarrow P[t_1]$</td>
</tr>
<tr>
<td>26: update $\lambda^{[t_1]}$ by (19) with $P[t_1]$</td>
</tr>
<tr>
<td>27: until $\delta_{\lambda}(P[t_1], P[t_1-1]) \leq \varepsilon$</td>
</tr>
<tr>
<td>28: if $F(\alpha) &gt; 0$ then</td>
</tr>
<tr>
<td>29: $\alpha_0 = \alpha$</td>
</tr>
<tr>
<td>30: else if $F(\alpha) &lt; 0$ then</td>
</tr>
<tr>
<td>31: $\alpha_0 = \alpha$</td>
</tr>
<tr>
<td>32: end if</td>
</tr>
<tr>
<td>33: $P[0] \leftarrow P[t_1]$</td>
</tr>
<tr>
<td>34: until $</td>
</tr>
<tr>
<td>35: return $A, P[0]$</td>
</tr>
</tbody>
</table>
Figure 5: Comparison of energy efficiencies for two different settings of TI thresholds $\eta$ and two different algorithms with $R_{\text{min}} = 1 \text{ bps/Hz}$. 

Figure 6 compares the total power consumption and the achievable rate for the system under the settings $R_{\text{min}} = 1 \text{ bps/Hz}$, $P_{\text{max}} = 2 \text{ W}$, $\eta = 10\sigma^2$, and the appropriate $\sigma^2$ values determined with respect to the range of SNR values considered. It is observed that the proposed EE-opt algorithm is more advantageous over the SE-opt algorithm in power efficiency, although it suffers from a loss in achievable rate. This can be explained by the different objectives of the two algorithms. With the objective of maximizing the energy efficiency, the EE-opt algorithm may allocate only a partial amount of $P_{\text{max}}$ to satisfy the QoS constraint of $R_{\text{min}}$. However, the SE-opt algorithm is aimed to maximize the throughput by allocating fully the amount of $P_{\text{max}}$. This also interprets the observation that the SE-opt algorithm yields a constant power consumption of $P_{\text{max}}$. Therefore, from Figures 5 and 6, it is concluded that the proposed EE-opt algorithm can be employed to provide a good tradeoff between energy efficiency and the system throughput compared with the existing algorithm.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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