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Research Article

A Nonhomogeneous Multivariable Grey Prediction NMGM Modeling Mechanism and Its Application

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The purpose of this paper is to explore modeling mechanism of a nonhomogeneous multivariable grey prediction NMGM(1, m, k^{α}) model and its application. Although multi-variable grey prediction MGM(1, m) model has been employed in many fields, its prediction results are not always satisfactory. Traditional MGM(1, m) model is constructed on the hypothesis that original data sequences are in accord with homogeneous index trend; however, the nonhomogeneous index data sequences are the most common data existing in all systems, and how to handle multivariable nonhomogeneous index data sequences is an urgent problem. This paper proposes a novel nonhomogeneous multivariable grey prediction model termed NMGM(1, m, k^{α}) to deal with those data sequences that are not in accord with homogeneous index trend. Based on grey prediction theory, by least square method and solutions of differential equations, the modeling mechanism and time response function of the proposed model are expounded. A case study demonstrates that the novel model provides preferable prediction performance compared with traditional MGM(1, m) model. This work is an extension of the multivariable grey prediction model and enriches the study of grey prediction theory.

1. Introduction

Grey system theory (GST), with the superiority of dealing with uncertain problems that have partially unknown parameters, has been developed greatly since it was applied to system theory [1]. Traditional system analysis theory such as the qualitative theory of dynamic systems [2–18] is built on the assumption of acknowledging the system structure. However, with the rapid development of science and technology, it is impossible to completely master system structure as the system is becoming more complex and uncertain. Grey system theory has been adopted to various aspects of fields including systems analysis, forecasting, and decision-making due to its advantages in tackling semicomplex uncertainty problems.

Forecasting a future development is always of significant importance in energy [19], science and technology [20], and some other fields. A large number of studies on forecasting models and applications have been reported, such as garch-types models needing plenty of data, and sample size would limit the predictive accuracy of those methods [21]. Nevertheless, searching enough effective data is almost impossible either in physical system or in generalized system. Grey prediction models show excellent ability in dealing with small data problems since they were proposed by Professor Deng [22].

Grey forecasting models can be divided into two categories according to the number of variables, single-variable grey forecasting models where GM(1, 1) is the core [23–26] and multivariable grey forecasting models, represented by GM(1, n) [27–29] and MGM(1, m) [30]. Zeng et al. [27] pointed out that there were some structure deficiencies in GM(1, n), which may lead to greater errors. The MGM(1, m) model can reflect mutual relationship among systematic variables and performed better prediction accuracy compared with single variable prediction models. Compared to signal grey prediction models, multivariable grey prediction model MGM(1, m) is a distinct grey model being adequate for considering the mutual interactions of multiple variables in a system, and it is of vital importance in simulating and forecasting the multivariable data sequences.

The nonhomogeneous index data sequences are the most common data existing in all systems, Cui et al. [31] put forward a novel NGM(1, 1, k) model in order to solve the nonhomogeneous index function and established the foundation of our study. This grey model (NGM) is a novel tool to tackle the nonhomogeneous data sequence, which attracted considerable interest of research [32]. How to handle multivariable nonhomogeneous data sequences is an urgent problem; this paper expounds a novel nonhomogeneous multivariable grey prediction model termed NMGM(1, m, k^{α}) to tackle the multivariable nonhomogeneous index data sequences.

In recent years, great attention has been devoted to optimizing and expanding applications of MGM(1, m) model [33–38]. Xiong et al. [33] provided background values to improve the original MGM model, which can be used to eliminate the random fluctuations and errors of the observational data. Guo et al. [37] constructed SMGM through coupling self-memory principle of dynamic system to MGM; the example showed that SMGM had superior predictive performance over other traditional grey prediction models. Karaaslan and Özden [38] analyzed Turkey's ratings credit by multivariable grey prediction model and expanded the application scope of multivariable grey prediction model. Those studies illustrated that multivariable grey prediction model is widely used in many fields.

Through analyzing the existing research on MGM(1, m) model, we find that most of scholars optimized the model from the view of modeling parameters to better fit data sequences with index law rather than optimizing the model from modeling structure of MGM(1, m) model. Traditional MGM(1, m) model was constructed on the hypothesis that data sequences are suitable for grey exponential law resulted from accumulated generation operation. The simulated original data sequences are usually in the form of

$$\widehat{X}^{(0)}(k) = e^{Bk}A, \quad k \geq 2. \tag{1}$$

However, in multivariable system analysis, there are only a few data with characteristic of this hypothesis but more for other hypotheses that original data sequences are in accord with nonhomogeneous index trend. Therefore, it is necessary to propose a novel multivariable grey prediction model that is proper for data sequences that are not in accord with homogeneous index trend. This paper presents the novel model NMGM(1, m, k^{α}) to handle the nonhomogeneous index data sequences in the form of

$$\widehat{X}^{(0)}(k) = e^{Bk}A + C, \quad k \geq 2. \tag{2}$$

By least square method and differential equations, we obtain parameters identification values and time response function of the novel model. The prediction accuracy is theoretically analyzed and a case study is presented to illustrate the effectiveness of the proposed model. The remainder of this paper is organized as follows. A novel nonhomogeneous multivariable grey prediction model and its modeling mechanism are presented in Section 2. The precision analysis and a case study are adopted to demonstrate the effectiveness and practicality of the novel model in Section 3. Our conclusions and future work are given in Section 4.

2. Grey NMGM$(1, m, k^{\alpha})$ Model

Multivariable grey prediction model is one of the frequently used grey forecasting models. In this section, we present the modeling mechanism and time response function of the novel model NMGM(1, m, k^{α}). The constructing process of NMGM(1, m, k^{α}) is presented below.

Definition 1. Assume that $X^{(0)} = (X_1^{(0)}, X_2^{(0)}, \ldots, X_m^{(0)})^T$ is a nonnegative original data matrix and the original nonnegative data vector $X_j^{(0)}$ is

$$X_j^{(0)} = \left(x_j^{(0)}(1), x_j^{(0)}(2), \ldots, x_j^{(0)}(n)\right), \quad j = 1, 2, \ldots, m. \tag{3}$$

The data matrix $X^{(1)} = (X_1^{(1)}, X_2^{(1)}, \ldots, X_m^{(1)})^T$ is called the first-order accumulated generation vector, where

$$X_j^{(1)} = \left(x_j^{(1)}(1), x_j^{(1)}(2), \ldots, x_j^{(1)}(n)\right), \quad j = 1, 2, \ldots, m \tag{4}$$

and

$$x_j^{(1)}(k) = \sum_{i=1}^{k} x_j^{(0)}(i), \quad k = 1, 2, \ldots, n. \tag{5}$$

The adjacent neighbour average sequence is $Z^{(1)} = (Z_1^{(1)}, Z_2^{(1)}, \ldots, Z_m^{(1)})^T$ and

$$Z_j^{(1)} = \left(z_j^{(1)}(1), z_j^{(1)}(2), \ldots, z_j^{(1)}(n)\right), \quad j = 1, 2, \ldots, m, \tag{6}$$

where

$$z_j^{(1)}(k) = 0.5\left(x_j^{(1)}(k) + x_j^{(1)}(k-1)\right), \quad k = 2, 3, \ldots, n. \tag{7}$$

The original form of nonhomogeneous multivariable grey prediction model abbreviated NMGM(1, m, k^{α}) is defined as follows:

$$\begin{aligned}
\frac{dx_1^{(1)}(t)}{dt} &= a_{11}x_1^{(1)}(t) + a_{12}x_2^{(1)}(t) + \cdots + a_{1m}x_m^{(1)}(t) \\
&\quad + b_1 t^{\alpha}, \\
\frac{dx_2^{(1)}(t)}{dt} &= a_{21}x_1^{(1)}(t) + a_{22}x_2^{(1)}(t) + \cdots + a_{2m}x_m^{(1)}(t) \\
&\quad + b_2 t^{\alpha}, \\
&\vdots \\
\frac{dx_m^{(1)}(t)}{dt} &= a_{m1}x_1^{(1)}(t) + a_{m2}x_2^{(1)}(t) + \cdots + a_{mm}x_m^{(1)}(t) \\
&\quad + b_m t^{\alpha},
\end{aligned} \tag{8}$$

where $\alpha \geq 0$. We denote the notation

$$A = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1m} \\ a_{21} & a_{22} & \cdots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mm} \end{pmatrix} \text{ and } \quad B = \begin{pmatrix} b_1 \\ \vdots \\ b_m \end{pmatrix}. \tag{9}$$

For the convenience of the reader, (8) can be written in matrix, which is

$$\frac{dX^{(1)}(t)}{dt} = AX^{(1)}(t) + Bt^{\alpha}. \tag{10}$$

Definition 2. Assume that $X^{(0)}$ is nonnegative original data matrix, $X^{(1)}$ is the first-order accumulated generation sequences, and $Z^{(1)}$ is adjacent neighbour average sequences. The differential equation

$$\frac{dX^{(1)}(t)}{dt} = AZ^{(1)}(t) + Bt^{\alpha} \tag{11}$$

is said to be basic form of nonhomogeneous multivariable grey prediction model abbreviated as NMGM(1, m, k^{α}). The differential equation

$$\frac{dX^{(1)}(t)}{dt} = AX^{(1)}(t) + Bt^{\alpha} \tag{12}$$

is said to be whitening differential equation of grey NMGM(1, m, k^{α}) model. The discrete form of NMGM(1, m, k^{α}) model is

$$x_j^{(0)}(k) = \sum_{l=1}^{m} a_{jl} z_l^{(1)}(k) + b_j k^{\alpha}, \qquad j = 1, 2, \ldots, m, \ k = 1, 2, \ldots, n. \tag{13}$$

The novel model NMGM(1, m, k^{α}) contains a nonlinear correction term Bk^{α}, and Bk^{α} is said to be time term of the model. The restored values of original data sequences can be adjusted through their coefficients Bk^{α}, which is more suitable for time series prediction. It is easy to see that MGM(1, m) model is a special case of NMGM(1, m, k^{α}) model when $\alpha = 0$.

In the following, we illustrate the modeling mechanism of NMGM(1, m, k^{α}) model.

Theorem 3. *Assume that $X^{(0)}$ is nonnegative original data matrix, $X^{(1)}$ is first-order accumulated generation sequences, and $Z^{(1)}$ is adjacent neighbour average sequences. If $a_j = (a_{j1}, \ldots, a_{jm}, b_j)^T$, then*

$$(\hat{a}_1, \hat{a}_2, \ldots, \hat{a}_m) = \left(P^T P\right)^{-1} P^T (Q_1, Q_2, \ldots, Q_m), \tag{14}$$

where

$$P = \begin{pmatrix} z_1^{(1)}(2) & z_2^{(1)}(2) & \ldots & z_m^{(1)}(2) & 2^{\alpha} \\ z_1^{(1)}(3) & z_2^{(1)}(3) & \ldots & z_m^{(1)}(3) & 3^{\alpha} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ z_1^{(1)}(n) & z_2^{(1)}(n) & \ldots & z_m^{(1)}(n) & n^{\alpha} \end{pmatrix} \tag{15}$$

and

$$Q_j = \left(x_j^{(0)}(2), x_j^{(0)}(3), \ldots, x_j^{(0)}(n)\right)^T, \qquad j = 1, 2, \ldots, m. \tag{16}$$

Proof. Substituting all data values into the discrete form of the model, we obtain

$$\begin{aligned} x_1^{(0)}(k) &= a_{11} z_1^{(1)}(k) + a_{12} z_2^{(1)}(k) + \cdots + a_{1m} z_m^{(1)}(k) + b_1 k^{\alpha}, \\ x_2^{(0)}(k) &= a_{21} z_1^{(1)}(k) + a_{22} z_2^{(1)}(k) + \cdots + a_{2m} z_m^{(1)}(k) + b_2 k^{\alpha}, \\ &\vdots \\ x_m^{(0)}(k) &= a_{m1} z_1^{(1)}(k) + a_{m2} z_2^{(1)}(k) + \cdots + a_{mm} z_m^{(1)}(k) + b_m k^{\alpha}. \end{aligned} \tag{17}$$

For one of the fixed equations j, setting $k = 2, 3, \ldots, n$, we have

$$\begin{aligned} x_j^{(0)}(2) &= a_{j1} z_1^{(1)}(2) + a_{j2} z_2^{(1)}(2) + \cdots + a_{jm} z_m^{(1)}(2) + 2^{\alpha} b_j, \\ x_j^{(0)}(3) &= a_{j1} z_1^{(1)}(3) + a_{j2} z_2^{(1)}(3) + \cdots + a_{jm} z_m^{(1)}(3) + 3^{\alpha} b_j, \\ &\vdots \\ x_j^{(0)}(n) &= a_{j1} z_1^{(1)}(n) + a_{j2} z_2^{(1)}(n) + \cdots + a_{jm} z_m^{(1)}(n) + n^{\alpha} b_j. \end{aligned} \tag{18}$$

The matrix form of the above equations is $Q_j = Pa_j$, $j = 1, 2, \ldots, m$.

In order to get parameters vector $\hat{a}_j = (\hat{a}_{j1}, \ldots, \hat{a}_{jm}, \hat{b}_j)^T$ $(j = 1, 2, \ldots, m)$, substituting $\sum_{l=1}^{m} a_{jl} z_l^{(1)}(k) + b_j k^{\alpha}$ with $\sum_{l=1}^{m} \hat{a}_{jl} z_l^{(1)}(k) + \hat{b}_j k^{\alpha}$ $(k = 2, 3, \ldots, n,\ j = 1, 2, \ldots, m)$, we obtain the error sequence $\varepsilon_j = Q_j - P\hat{a}_j$. Let

$$\begin{aligned} S_j &= \varepsilon_j^T \varepsilon_j = \left(Q_j - P\hat{a}_j\right)^T \left(Q_j - P\hat{a}_j\right) \\ &= \sum_{k=2}^{n} \left[x_j^{(0)}(k) - \sum_{l=1}^{m} \hat{a}_{jl} z_l^{(1)}(k) - \hat{b}_j k^{\alpha} \right]^2. \end{aligned} \tag{19}$$

Then the parameters vector $\widehat{a}_j = (\widehat{a}_{j1}, \ldots, \widehat{a}_{jm}, \widehat{b}_j)^T$ making S_j minimum should satisfy

$$\begin{aligned}
\frac{\partial S_j}{\partial \widehat{a}_{j1}} &= \sum_{k=2}^{n}\left[\widehat{b}_j k^{\alpha} + \sum_{l=1}^{m}\widehat{a}_{jl} z_l^{(1)}(k) - x_j^{(0)}(k)\right] z_1^{(1)}(k) \\
&= 0, \\
\frac{\partial S_j}{\partial \widehat{a}_{j2}} &= \sum_{k=2}^{n}\left[\widehat{b}_j k^{\alpha} + \sum_{l=1}^{m}\widehat{a}_{jl} z_l^{(1)}(k) - x_j^{(0)}(k)\right] z_2^{(1)}(k) \\
&= 0, \\
&\vdots \\
\frac{\partial S_j}{\partial \widehat{b}_j} &= \sum_{k=2}^{n}\left[\widehat{b}_j k^{\alpha} + \sum_{l=1}^{m}\widehat{a}_{jl} z_l^{(1)}(k) - x_j^{(0)}(k)\right] k^{\alpha} = 0.
\end{aligned} \tag{20}$$

Hence, we deduce that

$$\begin{aligned}
&\begin{pmatrix} \sum_{k=2}^{n}\left(z_1^{(1)}(k)\right)^2 & \cdots & \sum_{k=2}^{n} k^{\alpha} z_1^{(1)}(k) \\ \vdots & \ddots & \vdots \\ \sum_{k=2}^{n} k^{\alpha} z_1^{(1)}(k) & \cdots & \sum_{k=2}^{n} k^{2\alpha} \end{pmatrix} \begin{pmatrix} \widehat{a}_{j1} \\ \vdots \\ \widehat{a}_{jm} \\ \widehat{b}_j \end{pmatrix} \\
&= \begin{pmatrix} z_1^{(1)}(2) & \cdots & z_1^{(1)}(n) \\ \vdots & \ddots & \vdots \\ z_m^{(1)}(2) & \cdots & z_m^{(1)}(n) \\ 2^{\alpha} & \cdots & n^{\alpha} \end{pmatrix} \begin{pmatrix} x_j^{(0)}(2) \\ \vdots \\ x_j^{(0)}(n) \end{pmatrix}.
\end{aligned} \tag{21}$$

Thus, we obtain the desired solution

$$\widehat{a}_j = \left(P^T P\right)^{-1} P^T Q_j, \tag{22}$$

where P and Q_j are defined in Theorem 3. We can get the results of $\widehat{A}$ and $\widehat{B}$ by letting $j = 1, 2, \ldots, m$.

The NMGM$(1, m, k^{\alpha})$ model reduces to MGM$(1, m)$ when $\alpha = 0$, and NMGM$(1, m, k^{\alpha})$ model becomes NMGM$(1, m, k)$ model when $\alpha = 1$. In the following, we give the time response function of MGM$(1, m)$, NMGM$(1, m, k)$, and NMGM$(1, m, k^{\alpha})$, respectively. □

Theorem 4. *Suppose that $X^{(0)}$ is nonnegative original data matrix, $X^{(1)}$ is first-order accumulated generation sequences and $Z^{(1)}$ is adjacent neighbour average sequences. The parameters A and B are obtained by Theorem 3. Then*

(1) *The time response function of MGM(1, m) model is*

$$\widehat{X}^{(1)}(k) = e^{A(k-1)}\left(X^{(1)}(1) + A^{-1}B\right) - A^{-1}B, \quad k \geq 2. \tag{23}$$

(2) *The inverse accumulated generation is*

$$\widehat{X}^{(0)}(k) = \widehat{X}^{(1)}(k) - \widehat{X}^{(1)}(k-1), \quad k \geq 2. \tag{24}$$

Theorem 5. *Assume that $X^{(0)}$, $X^{(1)}$, and $Z^{(1)}$ are defined as Theorem 4. The parameters A and B are obtained by Theorem 3. Then*

(1) *The time response sequence of discrete NMGM(1, m, k) model is*

$$\begin{aligned}
\widehat{X}^{(1)}(k) &= \left(x_1^{(1)}(k), x_2^{(1)}(k), \ldots, x_m^{(1)}(k)\right)^T \\
&= e^{A(k-1)}\left(X^{(0)}(1) + A^{-1}B + \left(A^{-1}\right)^2 B\right) \\
&\quad - \left(A^{-1}Bk + \left(A^{-1}\right)^2 B\right), \quad k \geq 2.
\end{aligned} \tag{25}$$

(2) *The inverse accumulated generation is*

$$\widehat{X}^{(0)}(k) = \widehat{X}^{(1)}(k) - \widehat{X}^{(1)}(k-1), \quad k \geq 2. \tag{26}$$

Proof. (1) Multiplying the whitening differential equation of NMGM$(1, m, k)$ model by e^{-At} we have

$$e^{-At}\frac{dX^{(1)}(t)}{dt} - e^{-At}AX^{(1)}(t) = e^{-At}Bt, \tag{27}$$

which is equivalent to

$$\frac{d\left(e^{-At}X^{(1)}(t)\right)}{dt} = e^{-At}Bt. \tag{28}$$

Integrating the above equation from t_0 to t, we get

$$\begin{aligned}
&e^{-At}X^{(1)}(t) - e^{-At_0}X^{(1)}(t_0) \\
&\quad = -e^{-At}A^{-1}Bt - e^{-At}\left(A^{-1}\right)^2 B + e^{-At_0}A^{-1}Bt_0 \\
&\quad + e^{-At_0}\left(A^{-1}\right)^2 B.
\end{aligned} \tag{29}$$

Multiplying the equation by e^{At}, we obtain

$$\begin{aligned}
X^{(1)}(t) &= e^{A(t-t_0)}\left(X^{(0)}(t_0) + A^{-1}Bt_0 + \left(A^{-1}\right)^2 B\right) \\
&\quad - \left(A^{-1}Bt + \left(A^{-1}\right)^2 B\right).
\end{aligned} \tag{30}$$

Set $t_0 = 1$ and $t = k$. We get time response function of discrete NMGM$(1, m, k)$ model

$$\begin{aligned}
\widehat{X}^{(1)}(k) &= \left(x_1^{(1)}(k), x_2^{(1)}(k), \ldots, x_m^{(1)}(k)\right)^T \\
&= e^{A(k-1)}\left(X^{(0)}(1) + A^{-1}B + \left(A^{-1}\right)^2 B\right) \\
&\quad - \left(A^{-1}Bk + \left(A^{-1}\right)^2 B\right), \quad k \geq 2.
\end{aligned} \tag{31}$$

(2) The restored data is easy to obtain by the definition of first-order accumulated data sequences and so is omitted. □

Theorem 6. *Assume that $X^{(0)}$, $X^{(1)}$, and $Z^{(1)}$ are defined as Theorem 4. The parameters A and B are obtained by Theorem 3. The time response function of NMGM(1, m, k^{α}) model is*

$$\widehat{X}^{(1)}(t) = e^{At}\left(\int t^{\alpha} e^{-At} dt\right) B. \tag{32}$$

Table 1: The original data of X_1 and X_2 (unit: yuan).

	Time						
Variable	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$	$k = 7$
$X_1^{(0)}$	7375	8178	9262	10482	12319	14084	16378
$X_2^{(0)}$	3785	3996	4239	4754	5276	5813	6561

3. Precision Analysis

In this part, we compare the precision of MGM(1, m) and NMGM(1, m, k) model to illustrate the practicality of our results.

3.1. Theoretical Analysis. The predictive performance is of great importance in multivariable grey prediction when constructing a new model. Though many scholars conducted much research to improve the precision of MGM(1, m) model and obtained some progress, their findings are not always satisfactory. Most researchers investigated MGM(1, m) model based on the hypothesis that original data sequences are in accord with homogeneous sequences. The time response function of MGM(1, m) model indicates that the restored data of MGM(1, m) model is

$$\begin{aligned}\widehat{X}^{(0)}(k) &= \widehat{X}^{(1)}(k) - \widehat{X}^{(1)}(k-1) \\ &= e^{A(k-2)}\left(e^{A} - I\right)\left(X^{(1)}(1) + A^{-1}B\right), \\ &\qquad k \geq 2,\end{aligned} \tag{33}$$

where I is unit matrix. Similarly, we obtain the restored values of NMGM (1, m, k) model

$$\begin{aligned}\widehat{X}^{(0)}(k) &= \widehat{X}^{(1)}(k) - \widehat{X}^{(1)}(k-1) \\ &= e^{A(k-2)}\left(e^{A} - I\right)\left(X^{(0)}(1) + A^{-1}B + \left(A^{-1}\right)^{2}B\right) \\ &\quad - A^{-1}B, \quad k \geq 2.\end{aligned} \tag{34}$$

Compared with restored data functions of MGM and NMGM model, we find that the restored function of MGM(1, m) model is a data sequence with pure exponential growth law. The simulation and prediction function of NMGM(1, m, k) model is in accordance with nonhomogeneous grey exponential law.

To demonstrate the practicability and maneuverability of nonhomogeneous multivariable grey prediction NMGM model, a case study is employed to compare the predictive performance of MGM and NMGM model.

3.2. Case Study. Per-capita net income of rural households and per-capita disposable income of urban residents are considered to be two important indicators that can reflect people's living standard and economic level. Therefore, it is necessary and helpful to control the variation trend of per-capita income. However, making such a prediction is challenging because per-capita net income of rural households and per-capita disposable income of urban residents are influenced by many factors. Grey prediction model is fairly appropriate for this problem and shows excellent ability in solving such problems [39, 40]. Zhao et al. [39] forecasted per-capita annual net income of rural households in China by optimized grey GM(1, 1) model, which demonstrated that grey prediction model can be used effectively. In the following, the per-capita net income of rural households and per-capita disposable income of urban residents of Jiangsu province are forecasted by multivariable grey prediction model.

Jiangsu is a representative province of urban development in China, which enters into a new phase of economic development and undergoes a rapid development. Per-capita net income of rural households and per-capita disposable income of urban residents from 2001 to 2013 in Jiangsu province are chosen [41], which are obtained from Jiangsu Province Statistical Yearbook. The average annual growth rate of per-capita net income of rural households is 13.7%, and the average annual growth rate of per-capita disposable income of urban residents is only 11.25%. The per-capita net income of rural households and per-capita disposable income of urban residents are all affected by some certain factors, and there exist interaction and interrelation between them. The data set 2001–2013 is stable, which is suitable for constructing grey prediction model.

The data is divided into two groups: the data set 2001–2007 are used as original data while those 2008–2013 as test. Assume that 2001 is $k = 1$ and so on as shown in Table 1. The input data sequence is the per-capita net income of rural households and per-capita disposable income of urban residents from 2001 to 2007 of Jiangsu province; then parameters of MGM and NMGM model can be determined. On the basis of this, the forecasting results of 2008–2013 can be calculated by Theorems 4 and 5.

In the following, we construct MGM(1, m) model and NMGM(1, m, k) model to compare the prediction accuracy. Let X_1 be the per-capita disposable income of urban residents and X_2 be the per-capita net income of rural households.

By traditional MGM(1, m) model, we construct MGM(1, 2) model

$$\begin{aligned}\frac{dx_1^{(1)}(k)}{dk} &= 0.2732x_1^{(1)}(k) - 0.3044x_2^{(1)}(k) \\ &\quad + 6772.164, \\ \frac{dx_2^{(1)}(k)}{dk} &= 0.08618x_1^{(1)}(k) - 0.096x_2^{(1)}(k) \\ &\quad + 3526.2495.\end{aligned} \tag{35}$$

By NMGM(1, m, k) proposed in this paper, we construct NMGM(1, 2, k) model

Table 2: The actual and prediction values of X_1 and X_2 (unit: yuan).

	Actual values		MGM		NMGM	
Time	$X_1^{(0)}$	$X_2^{(0)}$	$\widehat{X}_1^{(0)}$	$\widehat{X}_2^{(0)}$	$\widehat{X}_1^{(0)}$	$\widehat{X}_2^{(0)}$
$k = 8$	18680	7357	19011.8	7385.7	10547.9	4025.6
$k = 9$	20965	8108	22235.2	8402.1	20863.7	7974.6
$k = 10$	23217	8980	26083.5	9615.7	24427.2	9100
$k = 11$	26341	10805	30677.9	11064.5	28672.5	10441.5
$k = 12$	29677	12202	36163.4	12794.3	33730	12039
$k = 13$	32538	13598	42712.4	14859.5	39755.1	13942.7

Table 3: The relative errors of MGM and NMGM (unit: %).

	X_1		X_2	
Time	MGM	NMGM	MGM	NMGM
$k = 8$	1.78	4.32	0.39	4.44
$k = 9$	6.06	0.48	3.62	1.65
$k = 10$	12.34	5.2	7.07	1.34
$k = 11$	16.46	8.8	2.4	3.36
$k = 12$	21.86	12.01	4.85	1.33
$k = 13$	31.27	22.18	9.27	2.53
MRE	14.96	8.83	4.6	2.44

$$\begin{aligned} \frac{dx_1^{(1)}(k)}{dk} &= 3.7136x_1^{(1)}(k) - 11.2009x_2^{(1)}(k) \\ &\quad + 15103.6306k, \\ \frac{dx_2^{(1)}(k)}{dk} &= 1.8827x_1^{(1)}(k) - 5.7843x_2^{(1)}(k) \\ &\quad + 7878.2759k. \end{aligned} \tag{36}$$

Actual values and prediction values of MGM(1, m) and NMGM(1, m, k) model are presented in Table 2. As can be seen from Table 2, the prediction values of NMGM(1, 2, k) are more close to actual values compared with MGM(1, 2) prediction values.

To analyze reliability of the model, the accuracy of models should be tested. Two criteria are employed to measure the model, including relative error (RE) and mean relative error (MRE), by which we can investigate the effectiveness of the proposed model, where

$$\begin{aligned} \mathrm{RE} &= \left| \frac{x_i(k) - \widehat{x}_i(k)}{x_i(k)} \right| \times 100\%, \\ \mathrm{MRE} &= \frac{1}{n} \sum_{k=1}^{n} \left| \frac{x_i(k) - \widehat{x}_i(k)}{x_i(k)} \right| \times 100\%. \end{aligned} \tag{37}$$

The relative errors of per-capita disposable income of urban residents and per-capita net income of rural households predicted by MGM(1, 2) and NMGM(1, 2, k) model are listed in Table 3. The mean relative error (MRE) indicates that NMGM(1, 2, k) model is more accurate than traditional MGM(1, 2) model. From the comparative analysis, we know that NMGM(1, m, k) model can better follow the tendency of per-capita disposable income of urban residents and per-capita net income of rural households. The case study shows that the novel nonhomogeneous NMGM(1, m, k^α) model has certain advantages compared with traditional multiple grey prediction model.

4. Conclusions and Future Research

Forecasting a future development is always an important issue in system analysis; however, because of the limitation of information and knowledge, only part of system structure could be fully realized. Grey forecasting models demonstrated its superiority in dealing with problems that partial information known and partial information unknown.

This paper put forward a novel model named NMGM(1, m, k^α) to tackle the nonhomogeneous multivariable data sequences, and the novel model makes up the deficiency of traditional MGM(1, m) model. By least square method and differential equations, we present the modeling mechanism and time response function of the proposed model. The case study shows that the improved model is more accurately than traditional MGM(1, m) model.

The proposed NMGM(1, m, k^α) model is fairly appropriate for the systems that are affected by other relative factors and their characteristic values are not in accordance with exponential law completely, and the novel NMGM(1, m, k^α) model can improve the adaptability of the traditional grey model.

There are also some problems needed to be solved in our future work, such as the optimization of background value, properties of novel model, finding the best α for some specific cases, and integrating other kinds of optimization techniques with the novel model in order to further improve the prediction accuracy. This work is an extension of the multivariable grey prediction MGM(1, m) model and a

great contribution to the development of multivariable grey prediction theory.

Data Availability

The data used to support the findings of this study are from Jiangsu Province Statistical Yearbook and the data are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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