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Research Article

A Fractional Derivative-Based Lateral Preview Driver Model for Autonomous Automobile Path Tracking

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The concept of focus point preview is proposed, and fractional calculus is introduced to driver model to build focus point preview driver model. A formula for calculating lateral error is given, where the weight coefficients of fractional calculus are designed to imitate the driver's focus preview property. The relationship between the speed and the order of fractional calculus is studied. A driver-vehicle-road simulation system is set up to illustrate the performances of the proposed preview model. The S-type road is used to test the model in the case of continuous small curvature turns and the Shanghai F1 track model is used as the case that automobile passing large curvature curve. The performances are evaluated from two aspects: path tracking effect and vehicle dynamic responses. It is concluded that, as the speed of the vehicle increases, the optimal order of fractional integral increases, so that the order is seen as the degree of driver's attention. What is more, in the case of large curvature, the path tracking performance is improved by increasing the corresponding fractional order. Simulations results also show that, compared with the single-point preview model, the performances of the focus point preview model are better. On the one hand, the proposed driver model can be used to control the vehicle steering and path tracking. On the other hand, fractional calculus is used to reveal the driver preview property and the order is given a certain physical meaning, which is conducive to the development of fractional calculus applications.

1. Introduction

The problem of path tracking control is one of the core control problems of unmanned vehicles. From the viewpoint of control theory, path tracking control strategies can be understood as the driver's attempt to diminish the lateral error [1]. The modeling of driver's steering behavior has developed for more than half a century, and, based on different applications and methods, there have been a lot of driver models published [2, 3]. Generally, driver steering behavior models can be divided into two categories, i.e., the compensation tacking model and the preview tacking model. The driver's preview function [4] is considered in the preview model, which is undoubtedly more in line with the reality than the compensation control model. Based on different visual steering mechanisms, preview models can be divided into single-point, two-point, and multiple-point model [5].

In 1953, Kondo [6] established a classic single-point preview model which many later models up to now are obliged [7–9]. In the case of single-point preview control, it is assumed that the driver focuses on a certain point called preview point ahead of the vehicle and based on which the preview lateral error can be obtained. It has been widely applied because of its simple structure and clear physical meaning [10]. Two-point preview models are built based on the far and near preview cues, assuming that drivers employ the near and far preview points to estimate the required steering wheel angle to control the vehicle [11–13]. Unlike the single-point and two-point preview model, the multiple-point model assumes that drivers focus on a front regional road to get multiple points of deviation [14]. Generally, parameter optimization method is used to make the tracking performance of the multipoint model better, but also it makes the physical concepts of the model unclear.

The above preview mechanism can reflect the driver's preview characteristics to a certain extent. However, we know that when human is driving, he always focuses on a point ahead of the vehicle (the basic assumption in the

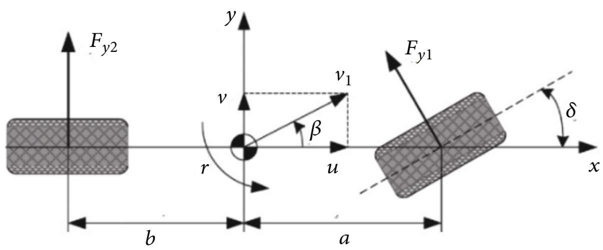


Figure 1: Two-DOF vehicle model.

single-point preview case) and gives consideration to the road information within the field of vision (the assumption in the multipoint preview case). Therefore, a novel preview assumption is proposed, called focus point preview assumption. The conception of "focus point" is in line with that in optics; that is, the information is the clearest at the focus point, becoming blurred along the directions away from the point. Here, fractional calculus is considered to build the focus point preview model. Although the idea of fractional calculus is as old as the idea of integer calculus, the application of fractional calculus has become more and more popular over the past few decades [15, 16]. It covers many research areas, including viscoelastic material, image processing, and automatic control. The fractional operators can be seen as the promotion of the common integer order operators, which can describe a more complex reality [17].

The rest of the paper is organized as follows. The lateral dynamic model of the vehicle is presented in Section 2. The focus point preview model is built in Section 3. The proposed driver model is used to the driver-vehicle-road closed-loop system, and the simulations are employed to verify the model in Section 4. Conclusions are given in Section 5.

2. Vehicle Dynamics Model

A two-degrees-of-freedom (DOF) linear single track vehicle model is shown in Figure 1, which represents the lateral vehicle dynamics. For a full description of the vehicle model the reader is referred to [18, 19]. The equations of vehicle dynamics can be described as follows:

$$mu\left(\dot{\beta}+\gamma\right)=k_f\left(\delta-\beta-\frac{a}{u}\gamma\right)+k_r\left(\frac{b}{u}\gamma-\beta\right) \quad (1)$$

$$I_z\dot{\gamma}=ak_f\left(\delta-\beta-\frac{a}{u}\gamma\right)-bk_r\left(\frac{b}{u}\gamma-\beta\right), \quad (2)$$

where m is the vehicle mass, I_z is yaw moment of inertia about its mass center z-axis, a and b are the distances from the center of gravity to the front and rear axles, k_f and k_r are the stiffness of the front and rear tire, β is the side slip angle, γ is yaw rate of the vehicle, and δ is the front steering angle. v and u are the longitudinal velocity and lateral velocity of the vehicle.

Generally, the sideslip angle can be expressed as follows:

$$\beta=\frac{v}{u}. \quad (3)$$

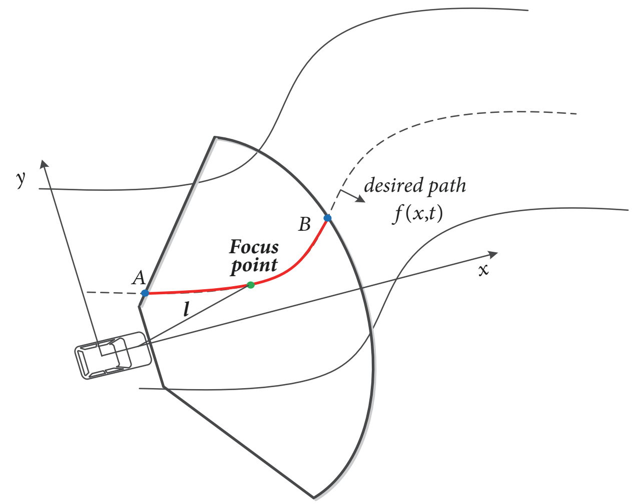


Figure 2: Focus point preview driver model.

Taking (3) into (1) and (2) and rearranging the equations, we have the state-space form

$$\begin{aligned}\dot{X}&=AX+Bu_1\\ Y&=CX,\end{aligned} \quad (4)$$

where $X=[\gamma,v]^T$, $A=\begin{pmatrix}-(a^2k_f+b^2k_r)/I_zu & -(ak_f-bk_r)/I_zu\\ -u-(ak_f+bk_r)/mu & -(k_f+k_r)/mu\end{pmatrix}$, $B=\begin{pmatrix}ak_f/I_z\\ k_f/m\end{pmatrix}$, $C=I_2$, $u_1=\begin{bmatrix}\delta\\0\end{bmatrix}$.

3. Focus Point Preview Driver Model

Driving a car is a complex activity. The preview function plays an important role in driving behavior, involving the way of processing the future path considering the current vehicle state [20]. Based on this, the drive controls the steering wheel to make the car follow the desired path.

In the process of the vehicle moving and steering, the driver is always looking at a finite interval of the future path. Based on these present driver preview assumptions, it is assumed that the driver will focus on a point of the target trajectory, where he sees the road information most clearly. In the interval, the road becomes blurred along the directions away from the point in driver's eyes at this moment. The assumption is consistent with the concept of focus in the optics, so we call it focus point preview assumption. The driver's field of vision is represented by fan-shaped region as shown in Figure 2, where a finite interval of the target trajectory can be obtained. At this moment, the focus point of driver's sight is located on the target trajectory with a distance l ahead of the vehicle.

The formula to calculate the vehicle positions in earth-fixed coordinates is

$$\begin{aligned}\dot{x}&=u\cos\theta-v\sin\theta\\ \dot{y}&=u\sin\theta+v\cos\theta,\end{aligned} \quad (5)$$

where (x, y) represents the position of the vehicle and θ is heading angle.

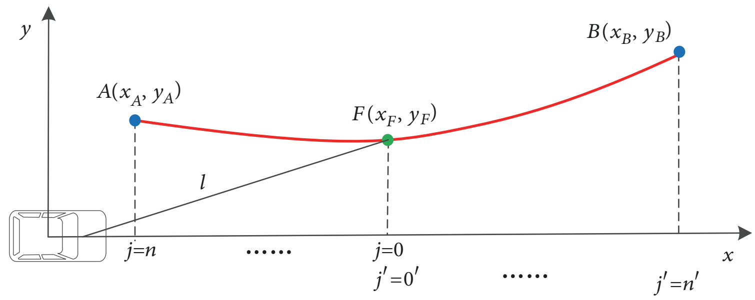


Figure 3: Driver's focus preview characteristics.

Generally, the target trajectory function is always recognized by the drivers as a relative function $y = f_r(x, t)$ based on vehicle oriented coordinates [21]. The relations between the earth coordinates and the vehicle oriented coordinates are as follows:

$$\begin{bmatrix} x_r \\ y_r \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x - x_0 \\ y - y_0 \end{bmatrix}. \quad (6)$$

In the vehicle oriented coordinates, consider using fractional calculus to simulate the driver focus point preview feature.

There are many definitions of fractional derivatives [22, 23]. A commonly used representation of discrete fractional derivatives is the Grünwald-Letnikov (G-L) formula, which is

$$\begin{aligned} D^{\alpha}_{\mathrm{GL}} f(x) &= \lim_{h \longrightarrow 0} \frac{1}{h^{\alpha}} \sum_{j=0}^{[(b-a)/h]} (-1)^j \binom{\alpha}{j} f(x - jh) \\ &= \frac{1}{h^{\alpha}} \sum_{j=0}^{[(b-a)/h]} w_j^{(\alpha)} f(x - jh), \end{aligned} \quad (7)$$

where $x \in [a, b]$, α is the order of the fractional derivative, h represents the space step, and $w_j^{(\alpha)} = (-1)^j \binom{\alpha}{j}$ denotes the normalized weights [24], which can be directly obtained by the following recursive formula:

$$\begin{aligned} w_0^{(\alpha)} &= 1, \\ w_j^{(\alpha)} &= \left(1 - \frac{\alpha + 1}{j}\right) w_{j-1}^{(\alpha)}, \\ &\qquad j = 1, 2, \ldots n. \end{aligned} \quad (8)$$

Equation (8) shows that, for different α, we have different weight coefficients. When $\alpha < 0$, it can be seen as fractional integral. As $\alpha \in (-1, 0)$, the weight coefficient is maximum for j=0; i.e., the function values $f(b)$ has the maximum coefficient $w_0^{(\alpha)} = 1$ in the whole operation process. According to the recursive formula, as j increases, we have $w_j^{(\alpha)} < w_{j-1}^{(\alpha)}$; i.e., the weights of the function values $f(b\text{-}jh)$ become smaller and smaller.

In the vehicle coordinates, the desired path is shown in Figure 3, points A and B are the near and far point, and F is the focus point. Based on the above analysis, fractional calculus is used to process the path information in the current field of vision. Here, the formula for calculating lateral preview error y_d is given as follows:

$$y_d = \frac{{}^{G}_{x_A}D^{\alpha}_{x_F} f(x) + {}^{G}_{x_F}D^{\alpha'}_{xB} f(x')}{x_B - x_A}, \quad (9)$$

where α, $\alpha' \in (-1, 0)$, $f(x)$ is the path function from point A to F, and $f(x')$ is the path function from point F to B. In the fractional integral process, the maximum weight coefficient corresponds to the function value at point F, while the weight coefficients gradually become smaller along the directions of $F \longrightarrow A$ and $F \longrightarrow B$. Therefore, (9) is designed ingeniously to make the obtained lateral error y_d contain the information of an interval of the future path and it complies with the focus point preview assumption.

Take the vehicle's state into account, and then the actual preview error is

$$y_{\varepsilon} = y_d - l_1 \beta, \quad (10)$$

where β can be obtained from the vehicle's internal sensor or observer.

After the preview error is obtained, the desired steering angle is determined by [25]

$$\delta(t) = K_m y_{\varepsilon}(t - T_s), \quad (11)$$

where T_s is the driver response delay time and K_m is the system gain.

By Laplace transform, we have

$$\delta(s) = \frac{K_m y_{\varepsilon}(s)}{e^{T_s s}}. \quad (12)$$

Submit the Taylor series expansion of $e^{T_s s}$ into (12), and the first two terms of the series are considered due to T_s being small. We have

$$(1 + T_s s)\,\delta(s) = K_m y_{\varepsilon}(s). \quad (13)$$

By the inverse Laplace transform, we have

$$\dot{\delta}(t) = -\frac{1}{T_s}\delta(t) + \frac{1}{T_s} K_m y_{\varepsilon}(t). \quad (14)$$

Table 1: Parameters of the vehicle model.

Parameter	Value
m / Kg	1700
I_z / kg·m^2	2200
a / m	1.2
b / m	1.6
K_f / N/deg	960
K_r / N/deg	1100

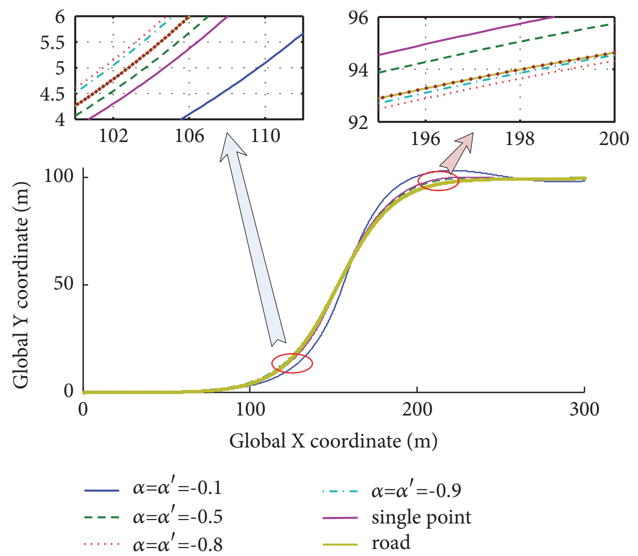


Figure 4: When v_1=20m/s, the comparison results of path tracking in the cases of different orders.

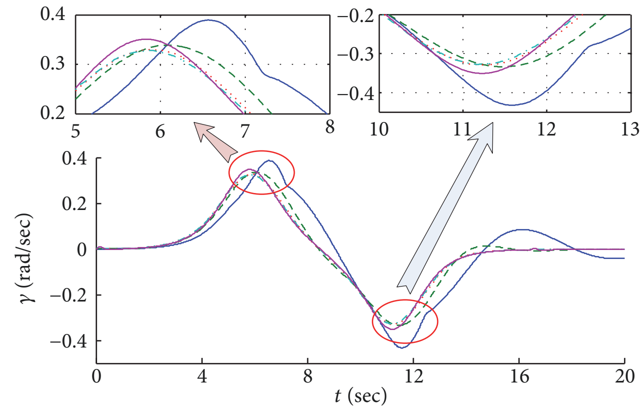


Figure 5: When v_1=20m/s, the yaw rate response.

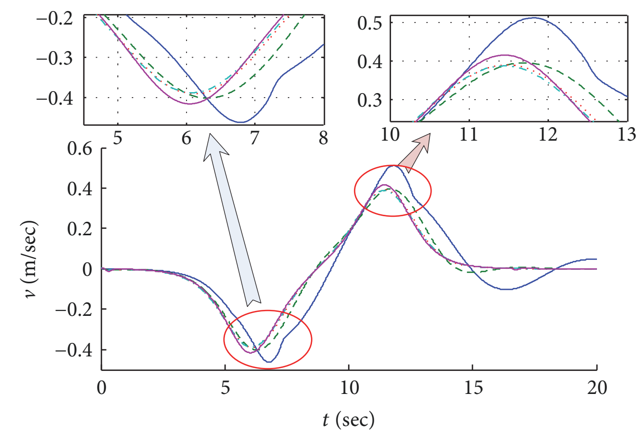


Figure 6: When v_1=20m/s, the lateral speed responses of the vehicle.

4. Simulation Results and Discussion

4.1. S-Type Road. A driver-vehicle-road simulation system [26] is built to examine the effect of the focus point preview model. When vehicle is moving in straight roads, there is little difference between various preview models. Therefore, the simulations are carried out on the S-shaped road, which can reflect the process of continuous turning. The performances of the model are evaluated from two aspects: the path tracking performances and the vehicle dynamic responses. The path tracking performance is evaluated according to the total square tracking error, and the vehicle dynamic responses are the responses of yaw angle and lateral velocity. The simulation results are compared with the performances of the single-point preview model. It is assumed that the vehicle runs at a constant speed and vehicle parameters are presented in Table 1. For illustrating the role of fractional orders α, α' in the model, the following simulations are carried out. Vehicle speed v_1 is set as 20 m/s; α increases from -1 to -0.1 by step 0.1, α=α'.

Figures 4–6 show that in the case of different fractional orders, the vehicle has different path tracking performances and the dynamic responses. The total square tracking error S is shown in Table 2. We can see that, at the speed of 20 m/s, in the case of α=α' ⟶-0.1, the performances of the focus point preview model are poor. Overall, when α=α'=-0.9, the path tracking performances and the dynamic responses are better than single-point preview model and other selected α, α'. Therefore, when v=20m/s, we choose α=α'=-0.9 as the optimal order.

According to the above analysis, the relationship between the speed and the order is studied. Table 2 shows the optimal orders chosen in the case of different speeds. It is found that when the speed increases, the optimal order α⟶-1. As is shown in (8), when α⟶-1, in the process of fractional calculus, the weight coefficient of every polynomial is larger compared with that in the case of α⟶0. More intuitively, when α⟶-1, in Figure 3, the weights of these points on the curve are larger than the weights of the same points in the case of α⟶0. Therefore, the fractional order integral result in the region $x \in (x_A, x_B)$ is larger when α⟶-1, and the lateral error y_d obtained by (9) gradually becomes larger when α⟶-1. In summary, as we know, when a vehicle passes through a certain curve at a certain speed, there is optimal steering angle to make the performance of the vehicle better, while, based on the above analysis, an appropriate y_d that determines the steering angle can be obtained by adjusting the order α; therefore, in this model, when the speed increases, the optimal order α⟶-1. On the other hand, when α changes, the fractional order integral result is mainly influenced by

Table 2: The total square tracking error S in the cases of different orders.

α,α'	-0.1	-0.2	-0.3	-0.4	-0.5	-0.6	-0.7	-0.8	-0.9	-1.0	Single point
S	30414	15995	2607	2455	849.1	251.9	339.3	170.4	93.0	274.8	616.9

Table 3: Parameter selection.

Speed (m/s)	5	7	10	13	15	17	20	23	25
Order	-0.5	-0.6	-0.6	-0.7	-0.7	-0.8	-0.9	-1.0	-1.0

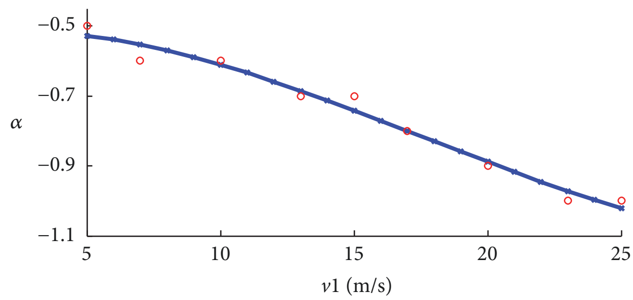


Figure 7: The rule for selecting optimal orders under different speeds.

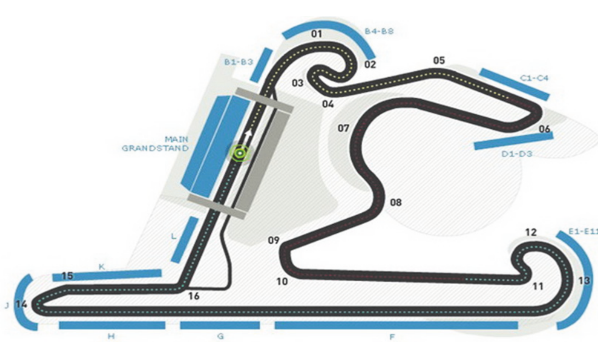


Figure 8: Shanghai F1 track.

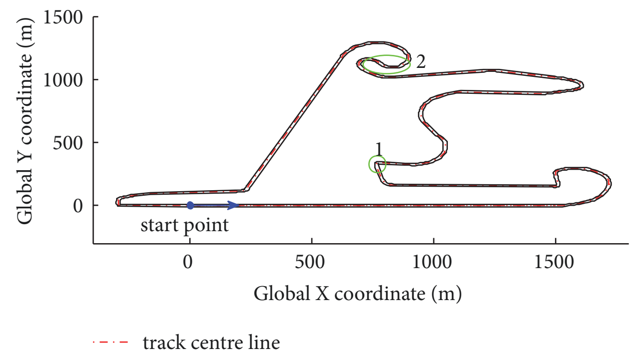


Figure 9: The F1 track simulation model.

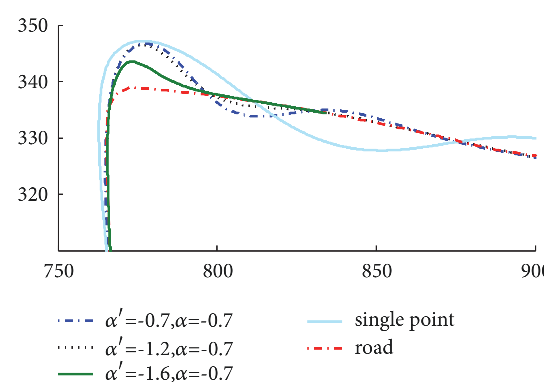
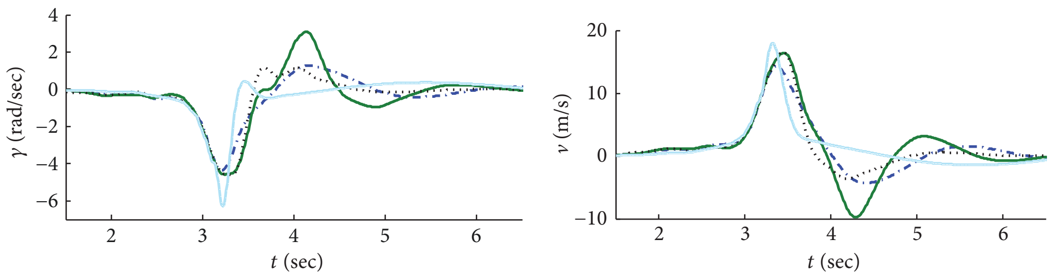


Figure 10: Path tracking performances at the bend 1 in the case of driver models with different orders, v_1=30m/s.

the weight of every point on the curve. As mentioned above, when $\alpha \in (-1, 0)$, the operator can be seen as the fractional integral with the order $|\alpha|$. Therefore, $|\alpha|$ can be seen as the degree of driver's attention. This is also in line with reality; when the speed increases, the driver has to pay more attention to preview the desired path, and increasing $|\alpha|$ means the weight of every point becomes larger. Therefore, it is found that the higher the speed, the larger the $|\alpha|$. Table 3 shows the selected optimal orders under different speeds conditions. Accordingly, the rule for selecting optimal orders in the case of different speeds is shown in Figure 7.

4.2. F1 Track. In order to illustrate the performance of the driver focus preview model in the case of vehicle passing a curve road with large curvature, the Shanghai F1 track is used, as shown in Figure 8. The simplified F1 track model used for simulation is shown in Figure 9, where (0, 0) is set as the start point. Considering the F1 track is long and the lateral movement is mainly studied here, the marked curve roads are chosen to present the simulation results. In the closed system, the initial position parameters and state parameters of the vehicle are set to the same before vehicle passing the curves. In the simulation process, we selected a variety of different α and α', but to make it easier for readers to understand, Figures 10 and 11 show the simulation results at the curve 1 in the following three different cases: α=-0.7, α'=-0.7; α=-0.7, α'=-1.4; α=-0.7, α'=-1.8. Figures 12 and 13 show the results at the curve 2 in the following three different cases: α=-0.7, α'=-0.7; α=-0.7, α'=-1.2; α=-0.7, α'=-1.6.

As is said above, $|\alpha|$ can be seen as the degree of driver's attention. In the case of small curvature, for imitating the focus preview property, it is set that $\alpha=\alpha'$, which means, at both sides of this point along the path, the driver's attention is symmetrically distributed. However, in the case of large curvature, the real driver will pay more attention to the front road information to cope with the upcoming turn. Therefore, $|\alpha'|$ is set larger than $|\alpha|$. After simulations with different



(a) (b)

Figure 11: The vehicle responses at bend 1 in the case of driver models with different orders, v_1=30m/s. (a) The yaw rate responses. (b) The lateral speed responses.

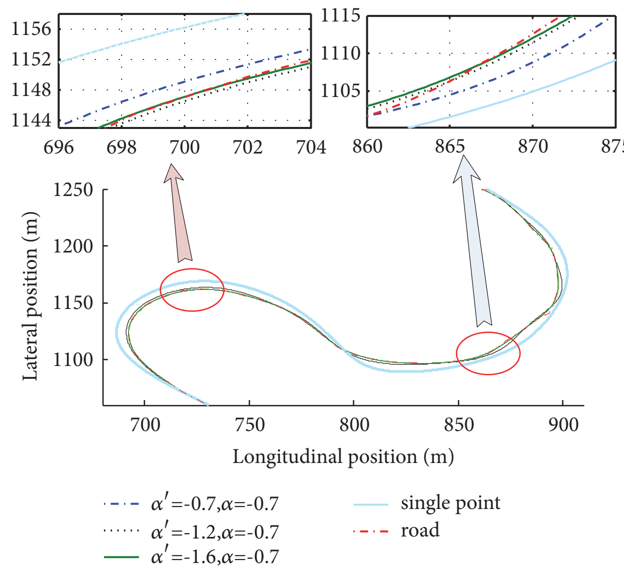
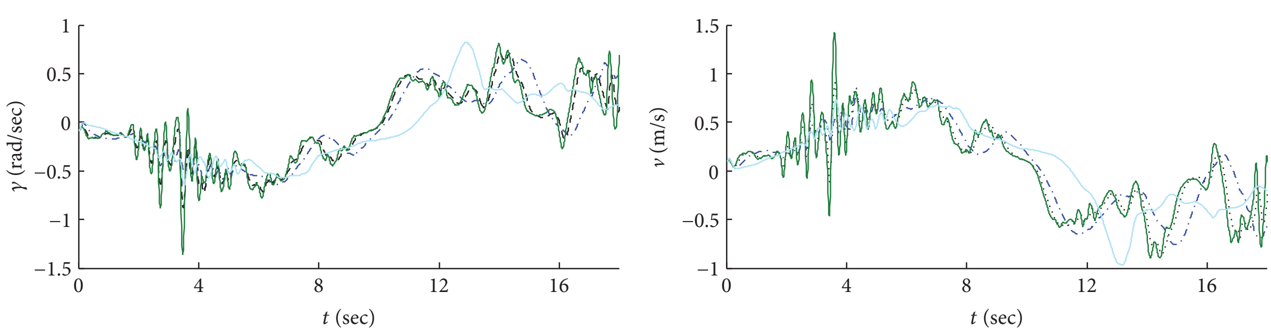


Figure 12: Path tracking performances at bend 2 in the case of driver models with different orders, v_1=30m/s.



(a) (b)

Figure 13: The vehicle responses at bend 2 in the case of driver models with different orders, v_1=30m/s. (a) The yaw rate responses. (b) The lateral speed responses.

combinations of α and α', it is found that, with a larger $|\alpha'|$, the path tracking performances are better, as shown in Figures 10 and 12. In Figure 10, when $|\alpha|$=0.7, in the case of $|\alpha'|$=1.8, the path tracking performance is better than that in the cases of $|\alpha'|$=1.4 and $|\alpha'|$=0.7. However, in Figure 11, the vehicle dynamic responses are not in that case. When $|\alpha'|$=1.8, the yaw rate responses and the lateral speed responses are even worse than those in the case of single-point preview model. This phenomenon is more obvious in Figure 13. Consequently, in the case of large curvature, take into account the path tracking effect and vehicle dynamic response with different weights for meeting different requirements. It makes the model flexible due to the relative relationship between $|\alpha|$ and $|\alpha'|$.

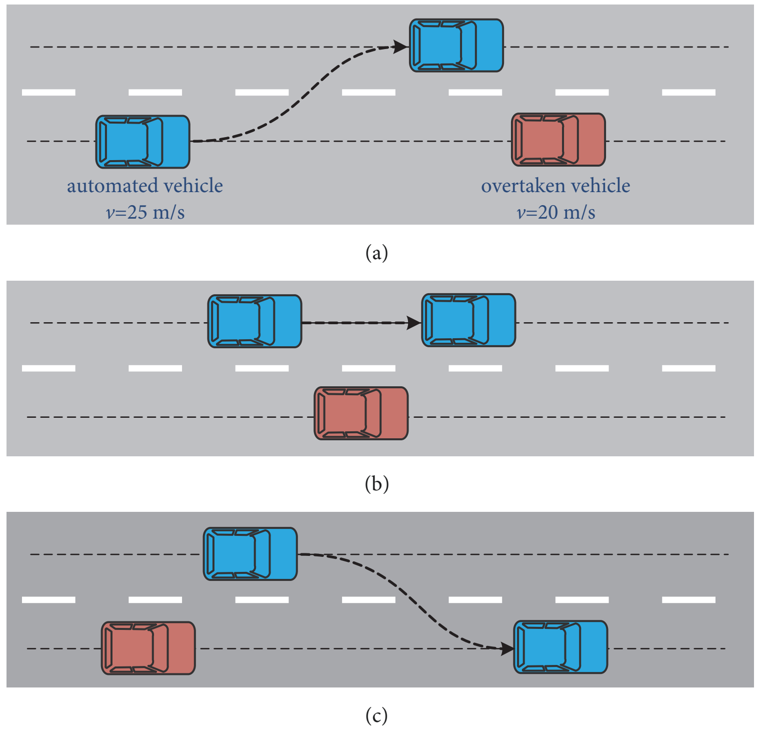


Figure 14: Overtaking maneuver phases. (a) First lane changes to the contiguous left lane. (b) Circulation in the left lane. (c) Second lane changes to the right lane.

As we know, a car with four-wheel steering performs well when the lane curvature is very large. However, this kind of car has a complex mechanical structure and needs to solve the problem of stability of control algorithm. The proposed model improves the performances by improving the preview function of the driver model, which means the same effect can be obtained by adjusting the parameters.

4.3. Overtaking Maneuver. Overtaking is a common behavior for drivers. For verifying the performances of the focus point preview model during overtaking, a simple scene is designed, as is shown in Figure 14. It is assumed that there are only two cars running in the same lane, and the initial position is 100 m apart. The front car runs at the speed of 20 m/s, while the speed of the rear car is 25 m/s. When the distance between the two cars becomes 25 m, the rear car begins to overtake at a constant speed. The lateral safety distance is set as 3 m. The fractional order is set as $\alpha=\alpha'$=-0.9. The performances of the focus preview model are compared with those of the single-point preview model.

Figure 15 shows the vehicle trajectory in the case of focus preview model is smoother than that of the single-point preview model when overtaking. Figures 16 and 17 show the vehicle dynamic response of the focus preview model is much better than that of the single-point preview model when overtaking.

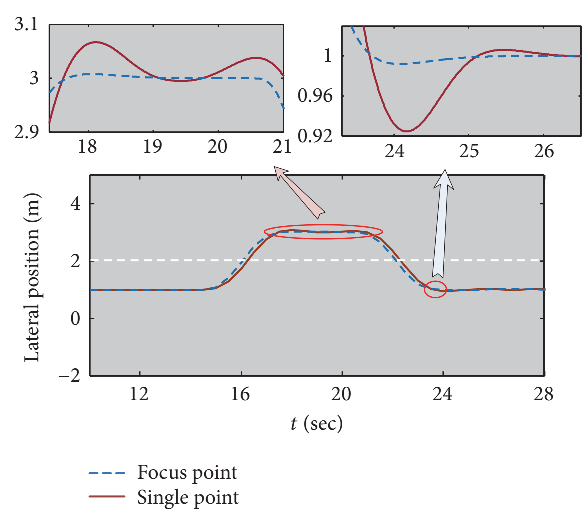


Figure 15: Comparison of overtaking results between single preview model and focus preview model.

5. Conclusions

In this paper, the focus point preview assumption is proposed, which is more in line with the reality than other preview assumptions. Fractional calculus theory is used to simulate the driver focus point preview property. A formula for calculating the lateral error is designed ingeniously, where the weight coefficients of the fractional calculus with a certain order are designed to be in line with the conception of focus point, and the error includes the information of a finite interval of desired path. The order can be understood as the degree of driver's attention, when the speed of the car increases, the optimal order is larger, which means the driver needs to pay more attention to preview the path. In the case

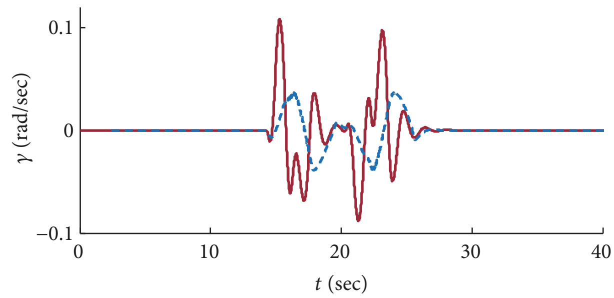


FIGURE 16: Comparison of the velocity response of the yaw angle during overtaking.

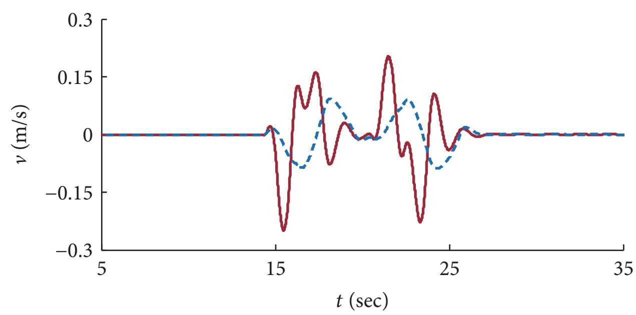


FIGURE 17: Comparison of lateral velocity response of vehicle during overtaking.

of large curvature, the driver preview model with a larger $|\alpha'|$ shows better path tracking performance. It means the driver pay more attention to the coming large turning condition. Therefore, the physical conception of this model is clear. The performances of path tracking and the dynamic responses of the vehicle are both improved compared with the single-point preview model, especially in the case of overtaking. The model is firstly proposed, there are still much work that needs to be done:

(1) A more complete vehicle model that contains more degrees of freedom will be built, especially, building a racing car model to better test the performances of this proposed driver model on the F1 track.

(2) Use machine learning methods to select parameters and study the relationship between parameters to perfect the model.

(3) A more complex overtaking maneuver will be studied in the following work.

In summary, it not only can be used to a closed driver-vehicle-road system, but also is conducive to reveal the driver preview property by using fractional calculus.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

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