

Research Article

Grey Relational Analysis for Hesitant Fuzzy Sets and Its Applications to Multiattribute Decision-Making

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Due to the superiority in expressing the uncertain and vague information, the hesitant fuzzy set (HFS) is regarded as an important tool to deal with multiattribute decision-making (MADM) problems. Quantitative and qualitative fuzzy measures have been proposed to solve such problems from different points. However, most of the existing information measures for HFSs are related to such fuzzy measures as distance, similarity, entropy, and correlation coefficients. The grey relational analysis is omitted. Besides, the existing grey relational analysis for HFSs only considers the range or distance between HFSs data which is only a partial measure of the HFSs. Therefore, in this paper, we improve the grey relational analysis for HFSs and explore a novel slope grey relational degree by considering another factor of HFSs data: the slope. Further, we combine both the distance and slope factors of HFSs data to construct a synthetic grey relational degree that describes the closeness and variation tendency of HFSs simultaneously, greatly enriching the fuzzy measures of HFSs. Furthermore, with the help of the TOPSIS method, we develop the grey relational based MADM methodology to solve the HFSs MADM problems. Finally, combining with two practical MADM examples about energy policy selection and multisensor target recognition, we obtain the most desirable decision results. Compared with the previous methods, the validity, comprehensiveness, and discrimination of the proposed synthetic grey relational degree for HFSs are demonstrated in detail.

1. Introduction

Multiattribute decision-making (MADM) is pervasive and active around human beings' practical activities. It is an effect and basic method to solve large quantitative and qualitative problems as information fusion, pattern recognition, alternatives selection and evaluation, clustering analysis, military applications, and so forth. However, with the increasing complexity of the decision-making's environments, the attributes tend to be more and more uncertain. The traditional decision-making methods cannot address these conditions. Therefore, the fuzzy multiattribute decision-making (FMADM) is introduced and widely used to tackle this uncertainty since Zadeh [1] initially proposed the theory of fuzzy set in 1965.

Many researchers have devoted themselves to the FMADM with different types of fuzzy sets, from the traditional fuzzy set to the intuitionistic fuzzy sets (IFSs) and hesitant fuzzy sets (HFSs). IFSs and HFSs are the two most popular fuzzy sets at present, which have been extensively paid attention to. Xu [2]

firstly introduced the intuitionistic fuzzy weighted averaging (IFWA) and intuitionistic fuzzy ordered weighted averaging (IFOWA) operators to make the decision. Further, Zhou et al. [3–6] extended these operators by different measures of the IFSs. Recently, Liu et al. [7–15] provided some newest achievements of IFWA and IFOWA operators for IFSs and their extensions by using different kinds of aggregation methods to deal with FMADM, which greatly perfect this theory.

As to the HFS and its extensions, it is a relative new type of fuzzy set which we mainly focus on in this paper. It also attracted great interest from scholars all over the world. Xu and Xia [16, 17], Farhadinia [18, 19], Zeng et al. [20], Zhao et al. [21], Chen et al. [22], and Liao et al. [23] provided a variety of distance, similarity, entropy, and correlation measures for HFSs. Xia et al. [24, 25] and Liao et al. [26, 27] also presented some basic operations and aggregation operators for HFSs. With the help of these basic information measures and aggregation operators, decision can be made with hesitant fuzzy information. He et al. [28]

first introduced the expected value and the geometric average value of hesitant multiplicative element (HME) to group decision-making problems. Xu and Zhang [29] developed a novel approach based on TOPSIS and the maximizing deviation method for solving MADM problems with hesitant fuzzy information. Further, Sun et al. [30] constructed an innovative TOPSIS based on synthetic correlation coefficient between HFSs which can handle negative values. Zhang and Xu [31] proposed an interval programming method for solving MAGDM problems with hesitant fuzzy alternatives based on LINMAP. Ashtiani and Azgomi [32] proposed a hesitant FMADM based computational trust model capable of taking into account the fundamental building blocks corresponding to the concept of trust. Ebrahimpour and Eftekhari [33] proposed an innovative method to deal with feature subset selection with HFSs based on maximum relevancy and minimum redundancy approach. Rodríguez et al. [34] introduced the concept of a hesitant fuzzy linguistic term set (HFLTS) to provide a linguistic and computational basis. Liao et al. [35] developed a method to solve the MCDM problem within the context of HFLTSs. Wang et al. [36] developed a likelihood-based TODIM approach for the selection and evaluation with multihesitant fuzzy linguistic information. Meng et al. [37] presented a similarity measure for uncertain linguistic hesitant fuzzy sets and constructed the optimal weight vector for multiattribute decision-making for evaluating corporate environmental performance. Feng et al. [38] proposed a consistency measure method based on the hesitant goal programming model to define the consistency hesitant fuzzy linguistic preference relations. Wu and Xu [39] proposed a large-scale group decision-making consensus model with possibility distribution based hesitant fuzzy preference. Li et al. [40] personalized individual semantics in group decision-making with hesitant fuzzy linguistic terms sets by consensus model. Zhang et al. [41] defined three kinds of additive consistency indices to measure the consistency level of hesitant fuzzy preference relation. Li et al. [42] used the modified TODIM and prospect theory to solve MADM issue of mineral resources evaluation and resources management with hesitant fuzzy linguistic information. Xue and Du [43] defined the multiplicative consistency of hesitant fuzzy preference relations to relax the same number for all elements in HFS and made the decision by fuzzy linear programming method. Liu et al. [44] used the continuous entropy weights and improved Hamacher information aggregation operators to aggregate interval-valued hesitant fuzzy information. Yang et al. [45] used the possibility degree to present a new comparative law for MADM problems with interval-valued hesitant fuzzy soft sets.

Through the above analysis, it is shown that HFSs and their extensions have been effective fuzzy tools for decision-making with uncertainty. However, despite the qualitative and quantitative studies of HFSs, the present work for them mainly focuses on such fuzzy measures as distance, similarity, entropy, and correlation coefficients. The grey relational analysis is omitted. Actually, the traditional grey relational analysis of the fuzzy set takes an important occupation in the fuzzy measure field. It can measure the closeness of two fuzzy sets just like the distance and similarity measures. Many

researchers have focused on the grey relational analysis of fuzzy set and proposed several approaches to solve MADM problems. Compared with IFSSs, Wei [46, 47], Zhang et al. [48], and Guo [49] had systematically established a series of grey relational analysis (GRA) methods to investigate the MADM problems with intuitionistic fuzzy information. Meanwhile the grey relational analysis for HFSs is limited. Although Li and Wei [50], Sun et al. [51], Wang [52], and Zang et al. [53] have applied GRA for HFSs, DHFSs, and IVDHFSs, the grey relational degrees of them are only a partial measure of the HFSs, which only consider the range or distance between HFSs data. The effects of these grey relational degrees are actually similar to the distance and similarity measures, which limit their applications. Sun et al. [51] considered the slope of the HFSs to form a synthetic grey relational degree, but the combination of this degree is simple and cannot reflect the influence of the whole index space. For these reasons, in this paper, we aim to improve the existing GRA for HFSs and develop a novel grey relational degree that considers both the distance and slope of the HFSs data. The improved grey relational degree satisfies the bidirectional deviation and reflects the influence of the whole index space, which is more reasonable in applications.

Consequently, the motivation of this paper is to explore a novel synthetic grey relational degree for HFSs and develop a methodology based on TOPSIS to solve MADM problems with HFSs information. The novelty of this paper lies in three aspects: (1) exploring the slope grey relational coefficient and slope grey relational degree of the HFSs, (2) constructing the synthetic grey relational coefficient and synthetic grey relational degree of the HFSs by taking into account the distance and slope of the HFSs data, and (3) developing a grey relational based MADM methodology with HFSs information.

The rest of the paper is organized as follows: Section 2 briefly reviews the concepts of HFSs and grey relational analysis theory. In Section 3, we define the grey relational coefficient and grey relational degree for HFSs. We also improve the slope grey relational degree by defining the difference and slope of the HFSs. With the help of the grey relational degree and slope grey relational degree, we construct the synthetic grey relational degree by combining them. In Section 4, we develop a hesitant fuzzy MADM methodology based on the grey relational analysis for HFSs. In Section 5, we apply the proposed grey relational hesitant fuzzy MADM methodology to the practical MADM problems. Finally, the paper ends with some concluding remarks and future challenges in Section 6.

2. Preliminaries

In this section, we recall the HFSs and grey relational analysis theory.

2.1. Hesitant Fuzzy Sets. When an expert makes a decision, he may hesitate to choose the exact membership degree in $[0, 1]$. For such a circumstance where there are several membership degrees of one element to a set, Torra [54] developed the hesitant fuzzy set (HFS), which is a kind of generalized fuzzy

set where the membership degree of an element to a certain set can be illustrated as several different values between 0 and 1. HFS is good at dealing with the situations where people have disagreements or hesitancy when deciding something.

Definition 1 (see [54]). Suppose that $X = \{x_1, x_2, \dots, x_n\}$ is a reference set; a hesitant fuzzy set (HFS) A on X is defined in terms of a function $h_A(x)$ which is when applied to X returns a subset of $[0, 1]$; that is,

$$A = \{\langle x, h_A(x) \rangle \mid x \in X\}, \quad (1)$$

where $h_A(x)$ is a set of some different values in $[0, 1]$, representing the possible membership degrees of the element $x \in X$ to the set A . For convenience, Xia and Xu [24] call $h_A(x)$ a hesitant fuzzy element (HFE), which is a basic unit of HFS.

2.2. Grey Relational Analysis Theory. Grey relational theory was originally introduced by Deng [55]. The traditional grey relational theory describes the closeness of two variables, which is necessary in the decision-making fields. It has been widely applied to decision-making, pattern recognition, and some other problems under uncertainty, particularly under the discrete data and fuzzy information.

Definition 2 (see [55]). For reference set $X_0 = (x_0(j), j = 1, 2, \dots, k)$ and $X_i = (x_i(j), j = 1, 2, \dots, k)$, the grey relational coefficient is defined by

$$r(x_0(j), x_i(j)) = \frac{\min_i \min_j |x_0(j) - x_i(j)| + \rho \cdot \max_i \max_j |x_0(j) - x_i(j)|}{|x_0(j) - x_i(j)| + \rho \cdot \max_i \max_j |x_0(j) - x_i(j)|}, \quad (2)$$

$$r(h_A(x_i), h_{B_j}(x_i)) = \frac{\min_j \min_i \{d(h_A(x_i), h_{B_j}(x_i))\} + \rho \cdot \max_j \max_i \{d(h_A(x_i), h_{B_j}(x_i))\}}{d(h_A(x_i), h_{B_j}(x_i)) + \rho \cdot \max_j \max_i \{d(h_A(x_i), h_{B_j}(x_i))\}}, \quad (5)$$

where $d(h_A(x_i), h_{B_j}(x_i))$ is the distance between HFEs $h_A(x_i)$ and $h_{B_j}(x_i)$, which can be calculated according to following equation:

$$d_{hne}(h_A(x_i), h_{B_j}(x_i)) = \left[\left(\frac{1}{l_{Ai}} \sum_{k=1}^{l_{Ai}} |\gamma_{Aik} - \gamma_{B_j ik}|^2 \right) \right]^{1/2}; \quad (6)$$

actually, a variety of distance measures for HFEs have been proposed; for more details, please refer to [16–20].

Based on grey relational coefficient between HFEs, the grey relational degree between HFSs A and B_j is defined as

$$\gamma(A, B_j) = \frac{1}{n} \cdot \sum_{i=1}^n r(h_A(x_i), h_{B_j}(x_i)). \quad (7)$$

where ρ is the identification coefficient, $\rho \in [0, 1]$, and generally we let $\rho = 0.5$.

The grey relational degree is defined as

$$\gamma(X_0, X_i) = \frac{1}{k} \cdot \sum_{j=1}^k r(x_0(j), x_i(j)). \quad (3)$$

Take the weight into consideration, let the weight vector of X_i be $\mathbf{w} = (w_1, w_2, \dots, w_k)^T$, $\sum_{j=1}^k w_j = 1$, $j = 1, 2, \dots, k$, and the grey relational degree is extended to the weighted grey relational degree:

$$\gamma(X_0, X_i) = \sum_{j=1}^k w_j \cdot r(x_0(j), x_i(j)). \quad (4)$$

3. Grey Relational Analysis for HFSs

In this section, we apply grey relational theory to the HFSs and define some novel HFSs expressions based on grey relational theory.

3.1. Grey Relational Definition of HFEs and HFSs

Definition 3. For two hesitant fuzzy sets on the fixed set $X = \{x_1, x_2, \dots, x_n\}$, $A = \{\langle x_i, h_A(x_i) \rangle \mid x_i \in X, i = 1, 2, \dots, n\}$ and $B_j = \{\langle x_i, h_{B_j}(x_i) \rangle \mid x_i \in X, i = 1, 2, \dots, n, j = 1, 2, \dots, m\}$, with $h_A(x_i) = \{\gamma_{Ai1}, \gamma_{Ai2}, \dots, \gamma_{Ail_{Ai}}\}$ and $h_{B_j}(x_i) = \{\gamma_{B_j i1}, \gamma_{B_j i2}, \dots, \gamma_{B_j il_{B_j i}}\}$, $i = 1, 2, \dots, n, j = 1, 2, \dots, m$; then we define the grey relational coefficient between HFEs $h_A(x_i)$ and $h_{B_j}(x_i)$ as

In practical applications, the elements $\{x_1, x_2, \dots, x_n\}$ in the universe X have different weights. Take the weight into consideration and let the weight vector of X be $\mathbf{w} = (w_1, w_2, \dots, w_n)^T$, $\sum_{i=1}^n w_i = 1$, $i = 1, 2, \dots, n$, and we extend the HFSs grey relational degree to the weighted HFSs grey relational degree as

$$\gamma_w(A, B_j) = \sum_{i=1}^n w_i \cdot r(h_A(x_i), h_{B_j}(x_i)). \quad (8)$$

3.2. Slope Grey Relational Definition for HFEs and HFSs. The HFSs grey relational degree in Section 3.1 mainly focuses on the closeness of the two HFSs; here we improve a novel HFSs grey relational degree called HFSs slope grey relational degree to represent the linear fashion of HFSs. As a departure, we introduce two new concepts of HFEs and HFSs based on Sun et al.'s [51] definition of the difference and slope of the HFSs.

Definition 4. For hesitant fuzzy sets $A = \{\langle x_i, h_A(x_i) \rangle \mid x_i \in X, i = 1, 2, \dots, n\}$ with $h_A(x_i) = \{\gamma_{Ai1}, \gamma_{Ai2}, \dots, \gamma_{Ai l_{Ai}}\}$ on the fixed set $X = \{x_1, x_2, \dots, x_n\}$, we define the difference of the HFSs as

$$\Delta A = \{\langle x_i, \Delta h_A(x_i) \rangle \mid x_i \in X, i = 1, 2, \dots, n\}, \quad (9)$$

where $\Delta h_A(x_i)$ denotes the difference of the HFEs.

$$\Delta h_A(x_i) = \{\Delta \gamma_{Ai1}, \Delta \gamma_{Ai2}, \dots, \Delta \gamma_{Aik}, \dots, \Delta \gamma_{Ai l_{Ai}-1}\}, \quad (10)$$

where

$$\Delta \gamma_{Aik} = \gamma_{Aik+1} - \gamma_{Aik}, \quad k = 1, 2, \dots, l_{Ai} - 1. \quad (11)$$

Definition 5. For hesitant fuzzy sets $A = \{\langle x_i, h_A(x_i) \rangle \mid x_i \in X, i = 1, 2, \dots, n\}$ with $h_A(x_i) = \{\gamma_{Ai1}, \gamma_{Ai2}, \dots, \gamma_{Ai l_{Ai}}\}$ on the fixed set $X = \{x_1, x_2, \dots, x_n\}$, the difference of A is $\Delta A = \{\langle x_i, \Delta h_A(x_i) \rangle \mid x_i \in X, i = 1, 2, \dots, n\}$ with $\Delta h_A(x_i) = \{\Delta \gamma_{Ai1}, \Delta \gamma_{Ai2}, \dots, \Delta \gamma_{Aik}, \dots, \Delta \gamma_{Ai l_{Ai}-1}\}$; we define the slope of the HFSs as

$$A' = \{\langle x_i, h_{A'}(x_i) \rangle \mid x_i \in X, i = 1, 2, \dots, n\}, \quad (12)$$

where $h_{A'}(x_i)$ denotes the slope of the HFEs.

$$h_{A'}(x_i) = \{\gamma_{A'i1}, \gamma_{A'i2}, \dots, \gamma_{A'ik}, \dots, \gamma_{A'il_{A'}-1}\}, \quad (13)$$

where

$$\gamma_{A'ik} = \frac{\Delta \gamma_{Aik}}{\bar{h}_A(x_i)} = \frac{\gamma_{Aik+1} - \gamma_{Aik}}{\bar{h}_A(x_i)}, \quad (14)$$

$$k = 1, 2, \dots, l_{Ai} - 1,$$

$$\xi_s(\Delta h_A(x_i), \Delta h_{B_j}(x_i)) = \frac{1 + |\gamma_{A'ik}|}{1 + |\gamma_{A'ik}| + |\gamma_{A'ik} - \gamma_{B'jk}|}$$

$$= \frac{1 + |(\gamma_{Aik+1} - \gamma_{Aik}) / \bar{h}_A(x_i)|}{1 + |(\gamma_{Aik+1} - \gamma_{Aik}) / \bar{h}_A(x_i)| + |(\gamma_{Aik+1} - \gamma_{Aik}) / \bar{h}_A(x_i) - (\gamma_{B'jk+1} - \gamma_{B'jk}) / \bar{h}_{B_j}(x_i)|}. \quad (17)$$

Based on HFSs slope grey relational coefficient, the HFSs slope grey relational degree is defined as

$$\gamma_s(A, B_j) = \frac{1}{n} \cdot \sum_{i=1}^n r_s(h_A(x_i), h_{B_j}(x_i)). \quad (18)$$

Take the weight into consideration; let the weight vector of X be $\mathbf{w} = (w_1, w_2, \dots, w_n)^T$, $\sum_{i=1}^n w_i = 1$, $i = 1, 2, \dots, n$, and we extend the HFSs slope grey relational degree to the weighted HFSs slope grey relational degree as

where $\bar{h}_A(x_i)$ is the mean of the HFE $h_A(x_i)$.

$$\bar{h}_A(x_i) = \frac{1}{l_{Ai}} \sum_{k=1}^{l_{Ai}} \gamma_{Aik}, \quad i = 1, 2, \dots, n. \quad (15)$$

The difference and slope of the HFSs can denote the linear fashion of the HFSs clearly, which are useful in practice. With the help of these definitions, we can give the definition of the slope grey relational degree.

Definition 6. For two hesitant fuzzy sets on the fixed set $X = \{x_1, x_2, \dots, x_n\}$, $A = \{\langle x_i, h_A(x_i) \rangle \mid x_i \in X, i = 1, 2, \dots, n\}$ and $B_j = \{\langle x_i, h_{B_j}(x_i) \rangle \mid x_i \in X, i = 1, 2, \dots, n, j = 1, 2, \dots, m\}$ with $h_A(x_i) = \{\gamma_{Ai1}, \gamma_{Ai2}, \dots, \gamma_{Ai l_{Ai}}\}$ and $h_{B_j}(x_i) = \{\gamma_{B_ji1}, \gamma_{B_ji2}, \dots, \gamma_{B_ji l_{B_ji}}\}$, $i = 1, 2, \dots, n, j = 1, 2, \dots, m$; the differences of them are $\Delta A = \{\langle x_i, \Delta h_A(x_i) \rangle \mid x_i \in X, i = 1, 2, \dots, n\}$, $\Delta h_A(x_i) = \{\Delta \gamma_{Ai1}, \Delta \gamma_{Ai2}, \dots, \Delta \gamma_{Aik}, \dots, \Delta \gamma_{Ai l_{Ai}-1}\}$, and $\Delta \gamma_{Aik} = \gamma_{Aik+1} - \gamma_{Aik}$, $k = 1, 2, \dots, l_{Ai} - 1$, and $\Delta B_j = \{\langle x_i, \Delta h_{B_j}(x_i) \rangle \mid x_i \in X, i = 1, 2, \dots, n, j = 1, 2, \dots, m\}$, $\Delta h_{B_j}(x_i) = \{\Delta \gamma_{B_ji1}, \Delta \gamma_{B_ji2}, \dots, \Delta \gamma_{B_jik}, \dots, \Delta \gamma_{B_ji l_{B_ji}-1}\}$, and $\Delta \gamma_{B_jik} = \gamma_{B_jik+1} - \gamma_{B_jik}$, $k = 1, 2, \dots, l_{B_ji} - 1$; the slopes of them are $A' = \{\langle x_i, h_{A'}(x_i) \rangle \mid x_i \in X, i = 1, 2, \dots, n\}$ with $h_{A'}(x_i) = \{\gamma_{A'i1}, \gamma_{A'i2}, \dots, \gamma_{A'ik}, \dots, \gamma_{A'il_{A'}-1}\}$ and $B'_j = \{\langle x_i, h_{B'_j}(x_i) \rangle \mid x_i \in X, i = 1, 2, \dots, n\}$ with $h_{B'_j}(x_i) = \{\gamma_{B'_ji1}, \gamma_{B'_ji2}, \dots, \gamma_{B'_jik}, \dots, \gamma_{B'_ji l_{B'_ji}-1}\}$; we define the slope grey relational coefficient between the HFEs $h_A(x_i)$ and $h_{B_j}(x_i)$ as

$$r_s(h_A(x_i), h_{B_j}(x_i)) = \frac{1}{l_{Ai} - 1} \sum_{k=1}^{l_{Ai}-1} \xi_s(\Delta h_A(x_i), \Delta h_{B_j}(x_i)), \quad (16)$$

where

$$\gamma_{sw}(A, B_j) = \sum_{i=1}^n w_i \cdot r_s(h_A(x_i), h_{B_j}(x_i)). \quad (19)$$

3.3. Synthetic Grey Relational Definition for HFSs

Definition 7. For two hesitant fuzzy sets on the fixed set $X = \{x_1, x_2, \dots, x_n\}$, $A = \{\langle x_i, h_A(x_i) \rangle \mid x_i \in X, i = 1, 2, \dots, n\}$ and $B_j = \{\langle x_i, h_{B_j}(x_i) \rangle \mid x_i \in X, i = 1, 2, \dots, n, j = 1, 2, \dots, m\}$ with $h_A(x_i) = \{\gamma_{Ai1}, \gamma_{Ai2}, \dots, \gamma_{Ai l_{Ai}}\}$ and $h_{B_j}(x_i) = \{\gamma_{B_ji1}, \gamma_{B_ji2}, \dots, \gamma_{B_ji l_{B_ji}}\}$; the slopes of them

are $A' = \{ \langle x_i, h_{A'}(x_i) \rangle \mid x_i \in X, i = 1, 2, \dots, n \}$ with $h_{A'}(x_i) = \{ \gamma_{A'i1}, \gamma_{A'i2}, \dots, \gamma_{A'ik}, \dots, \gamma_{A'il_{A_i-1}} \}$ and $B'_j = \{ \langle x_i, h_{B'_j}(x_i) \rangle \mid x_i \in X, i = 1, 2, \dots, n \}$ with $h_{B'_j}(x_i) =$

$\{ \gamma_{B'_ji1}, \gamma_{B'_ji2}, \dots, \gamma_{B'_jik}, \dots, \gamma_{B'_jil_{A_i-1}} \}, i = 1, 2, \dots, n, j = 1, 2, \dots, m;$ then we define the synthetic grey relational coefficient between HFEs $h_A(x_i)$ and $h_{B_j}(x_i)$ as

$$r_c(h_A(x_i), h_{B_j}(x_i)) = \frac{1 + \xi \cdot \max_j \max_i \{ d(h_A(x_i), h_{B_j}(x_i)) \} + \eta \cdot \max_j \max_i \{ d(h_{A'}(x_i), h_{B'_j}(x_i)) \}}{1 + \lambda_1 \cdot d(h_A(x_i), h_{B_j}(x_i)) + \lambda_2 \cdot d(h_{A'}(x_i), h_{B'_j}(x_i)) + \xi \cdot \max_j \max_i \{ d(h_A(x_i), h_{B_j}(x_i)) \} + \eta \cdot \max_j \max_i \{ d(h_{A'}(x_i), h_{B'_j}(x_i)) \}}, \quad (20)$$

where $\lambda_1, \lambda_2 > 0$, which indicates the importance of the closeness and linear fashion of HFSs, respectively, which satisfied $\lambda_1 + \lambda_2 = 1$, ξ and η denote the distinguished coefficient of the closeness and linear fashion, $d(h_A(x_i), h_{B_j}(x_i))$ and $d(h_{A'}(x_i), h_{B'_j}(x_i))$ are the distance between HFEs $h_A(x_i)$ and $h_{B_j}(x_i)$ and the distance between the slopes of HFEs $h_A(x_i)$ and $h_{B_j}(x_i)$, respectively, and $d(h_{A'}(x_i), h_{B'_j}(x_i))$ can be calculated by

$$d_{hne}(h_{A'}(x_i), h_{B'_j}(x_i)) = \left[\left(\frac{1}{l_{A_i} - 1} \sum_{k=1}^{l_{A_i}-1} |\gamma_{A'ik} - \gamma_{B'_jik}|^2 \right) \right]^{1/2}. \quad (21)$$

If the numbers of values in different HFEs of HFSs are different, we have to extend the shorter one until both of them have the same length when we compare them. We can extend them according to the optimistic or the pessimistic methods in [7, 8].

Based on HFEs synthetic grey relational coefficient, the HFSs synthetic grey relational degree is defined as

$$\gamma_c(A, B_j) = \frac{1}{n} \cdot \sum_{i=1}^n r_c(h_A(x_i), h_{B_j}(x_i)). \quad (22)$$

Take the weight into consideration; let the weight vector of X be $\mathbf{w} = (w_1, w_2, \dots, w_n)^T, \sum_{i=1}^n w_i = 1, i = 1, 2, \dots, n$, and we extend the HFSs synthetic grey relational degree to the weighted HFEs synthetic grey relational degree as

$$\gamma_{cw}(A, B_j) = \sum_{i=1}^n w_i \cdot r_c(h_A(x_i), h_{B_j}(x_i)). \quad (23)$$

The HFSs synthetic grey relational degree takes the considerations of both the closeness and linear fashion of HFSs together, which can better represent the fuzzy measure between the HFSs.

4. The Grey Relational Based MADM Methodology with HFSs Information

In this section, we investigate the grey relational analysis to MADM problems with HFSs information and propose the grey relational based MADM methodology with the help of the TOPSIS method.

Suppose that a hesitant fuzzy MADM problem has m alternatives $A_i (i = 1, 2, \dots, m)$; each alternative has n hesitant fuzzy attribute $C_j (j = 1, 2, \dots, n)$; $h_{A_i}(C_j) = \{ \gamma_{A_i1}, \gamma_{A_i2}, \dots, \gamma_{A_ik}, \dots, \gamma_{A_il_{ij}} \}$ represents the hesitant fuzzy information of the alternatives A_i on the attribute C_j ; l_{ij} is the number of the membership values in $h_{A_i}(C_j)$; let $\mathbf{w} = (w_1, w_2, \dots, w_n)^T$ be the relative weight vector of the attribute, satisfying the normalization conditions: $0 \leq w_j \leq 1$ and $\sum_{j=1}^n w_j = 1$. Then all the hesitant fuzzy information can be concisely expressed in matrix format as

$$A = \begin{bmatrix} h_{A_1}(C_1) & h_{A_1}(C_2) & \cdots & h_{A_1}(C_n) \\ h_{A_2}(C_1) & \ddots & \cdots & h_{A_2}(C_n) \\ \vdots & \vdots & h_{A_i}(C_j) & \vdots \\ h_{A_m}(C_1) & h_{A_m}(C_2) & \cdots & h_{A_m}(C_n) \end{bmatrix}_{m \times n}. \quad (24)$$

Then, according to the TOPSIS approach, we propose the grey relational based MADM methodology with hesitant fuzzy matrix as follows.

Step 1. Determine the positive ideal solution (PIS) and negative ideal solution (NIS) of each attribute in the normalized hesitant fuzzy decision matrix to form two new positive and negative HFSs:

$$A^+ = \{ \langle C_j, h_A^+(C_j) \rangle \mid C_j \in C, j = 1, 2, \dots, n \} \quad (25)$$

$$A^- = \{ \langle C_j, h_A^-(C_j) \rangle \mid C_j \in C, j = 1, 2, \dots, n \},$$

where $h_A^+(C_j)$ and $h_A^-(C_j)$ are the new positive and negative HFEs:

$$h_A^+(C_j) = \{ \gamma_1^+, \gamma_2^+, \dots, \gamma_k^+, \dots, \gamma_l^+ \},$$

$$\gamma_k^+ = \left(\max_{1 \leq i \leq m} \{ \gamma_{A_ik} \} \text{ if } \gamma_{A_ik} \in \Omega_b, \min_{1 \leq i \leq m} \{ \gamma_{A_ik} \} \text{ if } \gamma_{A_ik} \in \Omega_c \right) \quad (26)$$

$$h_A^-(C_j) = \{ \gamma_1^-, \gamma_2^-, \dots, \gamma_k^-, \dots, \gamma_l^- \},$$

$$\gamma_k^- = \left(\min_{1 \leq i \leq m} \{ \gamma_{A_ik} \} \text{ if } \gamma_{A_ik} \in \Omega_b, \max_{1 \leq i \leq m} \{ \gamma_{A_ik} \} \text{ if } \gamma_{A_ik} \in \Omega_c \right),$$

where Ω_b and Ω_c are related to benefit attribute and cost attribute, l_j^+ and l_j^- are the numbers of the membership values in the new positive and negative HFEs, respectively, and $l_j^+ = l_j^-$.

Step 2. Calculate the HFSs positive and negative grey relational degrees between each alternative and the PIS and NIS, respectively. Here, we can calculate them by the proposed three grey relational degrees: the HFSs grey relational degree, HFSs slope grey relational degree, and HFSs synthetic grey relational degree.

$$\begin{aligned} \gamma_w^+(A_i, A^+) &= \sum_{j=1}^n w_j \cdot r(h_{A_i}(C_j), h_A^+(C_j)) \\ \gamma_w^-(A_i, A^-) &= \sum_{j=1}^n w_j \cdot r(h_{A_i}(C_j), h_A^-(C_j)) \\ \gamma_{sw}^+(A_i, A^+) &= \sum_{j=1}^n w_j \cdot r_s(h_{A_i}(C_j), h_A^+(C_j)) \\ \gamma_{sw}^-(A_i, A^-) &= \sum_{j=1}^n w_j \cdot r_s(h_{A_i}(C_j), h_A^-(C_j)) \\ \gamma_{cw}^+(A_i, A^+) &= \sum_{j=1}^n w_j \cdot r_c(h_{A_i}(C_j), h_A^+(C_j)) \\ \gamma_{cw}^-(A_i, A^-) &= \sum_{j=1}^n w_j \cdot r_c(h_{A_i}(C_j), h_A^-(C_j)), \end{aligned} \tag{27}$$

where (27) can be obtained according to Definitions 3, 6, and 7.

Step 3. Construct the relative closeness to the ideal solution based on the calculated positive and negative grey relational degree. The relative closeness of the alternative A_i ($i = 1, 2, \dots, m$) with respect to the ideal solution is defined as

$$\begin{aligned} \eta_i &= \frac{\gamma_w^+(A_i, A^+)}{\gamma_w^+(A_i, A^+) + \gamma_w^-(A_i, A^-)}, \quad i = 1, 2, \dots, m \\ \eta_{si} &= \frac{\gamma_{sw}^+(A_i, A^+)}{\gamma_{sw}^+(A_i, A^+) + \gamma_{sw}^-(A_i, A^-)}, \quad i = 1, 2, \dots, m \\ \eta_{ci} &= \frac{\gamma_{cw}^+(A_i, A^+)}{\gamma_{cw}^+(A_i, A^+) + \gamma_{cw}^-(A_i, A^-)}, \quad i = 1, 2, \dots, m. \end{aligned} \tag{28}$$

Step 4. Rank the alternatives according to the decreasing order of their relative closeness. That is, the best alternative is the one with the greatest relative closeness to the ideal solution.

The process of the grey relational based MADM methodology with hesitant fuzzy information is shown in Figure 1.

5. MADM Applications

In this section, we apply the proposed grey relational based MADM methodology to deal with MADM problems with HFSs information.

5.1. Apply the Proposed Grey Relational Based MADM Methodology to Energy Policy Selection. In this section, the MADM example with hesitant fuzzy information concerning energy policy selection in [16, 18, 29, 56] is used to validate the proposed grey relational based MADM methodology.

Example 8. Suppose that there are five alternatives (energy projects) A_i ($i = 1, 2, \dots, 5$) to be selected and four attributes to be considered: P_j ($j = 1, 2, 3, 4$). The attribute weight vector is $w = (0.15, 0.3, 0.2, 0.35)^T$. The attribute values are described in hesitant fuzzy sets, shown in Table 1. We want to get the most desirable alternative from the hesitant fuzzy information.

Because the numbers of values in different HFEs of HFSs are different, we cannot directly process this hesitant fuzzy information. We have to extend the shorter one until both of them have the same length when we compare them. In this example, we extend these data in the pessimistic view as [29] does. Actually, the extension of the HFSs plays an important role in the ultimate decision-making; different extension methods may obtain different results, so how to select the appropriate extension methods remains to be further research. The extended hesitant fuzzy decision-making information is shown in Table 2.

Step 1. All the attributes are of benefit type; we select each maximum HFE in the five alternatives HFSs on the four attributes to construct the hesitant fuzzy PIS A^+ and each minimum HFE to construct the hesitant fuzzy NIS A^- :

$$\begin{aligned} A^+ &= [\{0.9, 0.7, 0.6, 0.6, 0.6\}, \{0.9, 0.8, 0.7, 0.6, 0.6\}, \\ &\quad \{0.9, 0.8, 0.7, 0.7\}, \{0.9, 0.8, 0.6, 0.6\}], \\ A^- &= [\{0.5, 0.3, 0.3, 0.3, 0.1\}, \{0.7, 0.4, 0.2, 0.1, 0.1\}, \\ &\quad \{0.5, 0.1, 0.1, 0.1\}, \{0.6, 0.4, 0.3, 0.3\}]. \end{aligned} \tag{29}$$

Step 2. Calculate the three types of HFSs positive and negative grey relational degrees between each alternative and the PIS and NIS, respectively. We assume that $\rho = 0.5$, $\lambda_1 = \lambda_2 = 0.5$, and $\xi = \eta = 0.5$. The results are shown in Table 3.

Step 3. Construct the relative closeness to the ideal solution based on the above calculated three types of positive and negative grey relational degree. The relative closeness of the alternative A_i ($i = 1, 2, \dots, m$) with respect to the ideal solution is shown in Table 4 and the compared effect of the three methods is shown in Figure 2.

Step 4. Rank the alternatives according to the decreasing order of the three types of relative closeness, shown in Table 4 too.

Consequently, we select the best one with the greatest relative closeness to the ideal solution and all the three types of relative closeness indicate that the decision result is alternative A_5 .

Comparing the decision results, we can see that the decision result is consistent with the decision result in [16, 18, 28, 56], derived from different distance and similarity measures. It is proven that the three types of grey relation

TABLE 1: The hesitant fuzzy decision-making information for the 4 criteria of 5 alternatives.

Alternatives	Attributes			
	Technological	Environmental	Sociopolitical	Economic
A_1	{0.5, 0.4, 0.3}	{0.9, 0.8, 0.7, 0.1}	{0.5, 0.4, 0.2}	{0.9, 0.6, 0.5, 0.3}
A_2	{0.5, 0.3}	{0.9, 0.7, 0.6, 0.5, 0.2}	{0.8, 0.6, 0.5, 0.1}	{0.7, 0.4, 0.3}
A_3	{0.7, 0.6}	{0.9, 0.6}	{0.7, 0.5, 0.3}	{0.6, 0.4}
A_4	{0.8, 0.7, 0.4, 0.3}	{0.7, 0.4, 0.2}	{0.8, 0.1}	{0.9, 0.8, 0.6}
A_5	{0.9, 0.7, 0.6, 0.3, 0.1}	{0.8, 0.7, 0.6, 0.4}	{0.9, 0.8, 0.7}	{0.9, 0.7, 0.6, 0.3}

TABLE 2: The extended hesitant fuzzy decision-making information for the 4 criteria of 5 alternatives.

Alternatives	Attributes			
	Attribute 1	Attribute 2	Attribute 3	Attribute 4
A_1	{0.5, 0.4, 0.3, 0.3, 0.3}	{0.9, 0.8, 0.7, 0.1, 0.1}	{0.5, 0.4, 0.2, 0.2}	{0.9, 0.6, 0.5, 0.3}
A_2	{0.5, 0.3, 0.3, 0.3, 0.3}	{0.9, 0.7, 0.6, 0.5, 0.2}	{0.8, 0.6, 0.5, 0.1}	{0.7, 0.4, 0.3, 0.3}
A_3	{0.7, 0.6, 0.6, 0.6, 0.6}	{0.9, 0.6, 0.6, 0.6, 0.6}	{0.7, 0.5, 0.3, 0.3}	{0.6, 0.4, 0.4, 0.4}
A_4	{0.8, 0.7, 0.4, 0.3, 0.3}	{0.7, 0.4, 0.2, 0.2, 0.2}	{0.8, 0.1, 0.1, 0.1}	{0.9, 0.8, 0.6, 0.6}
A_5	{0.9, 0.7, 0.6, 0.3, 0.1}	{0.8, 0.7, 0.6, 0.4, 0.4}	{0.9, 0.8, 0.7, 0.7}	{0.9, 0.7, 0.6, 0.3}

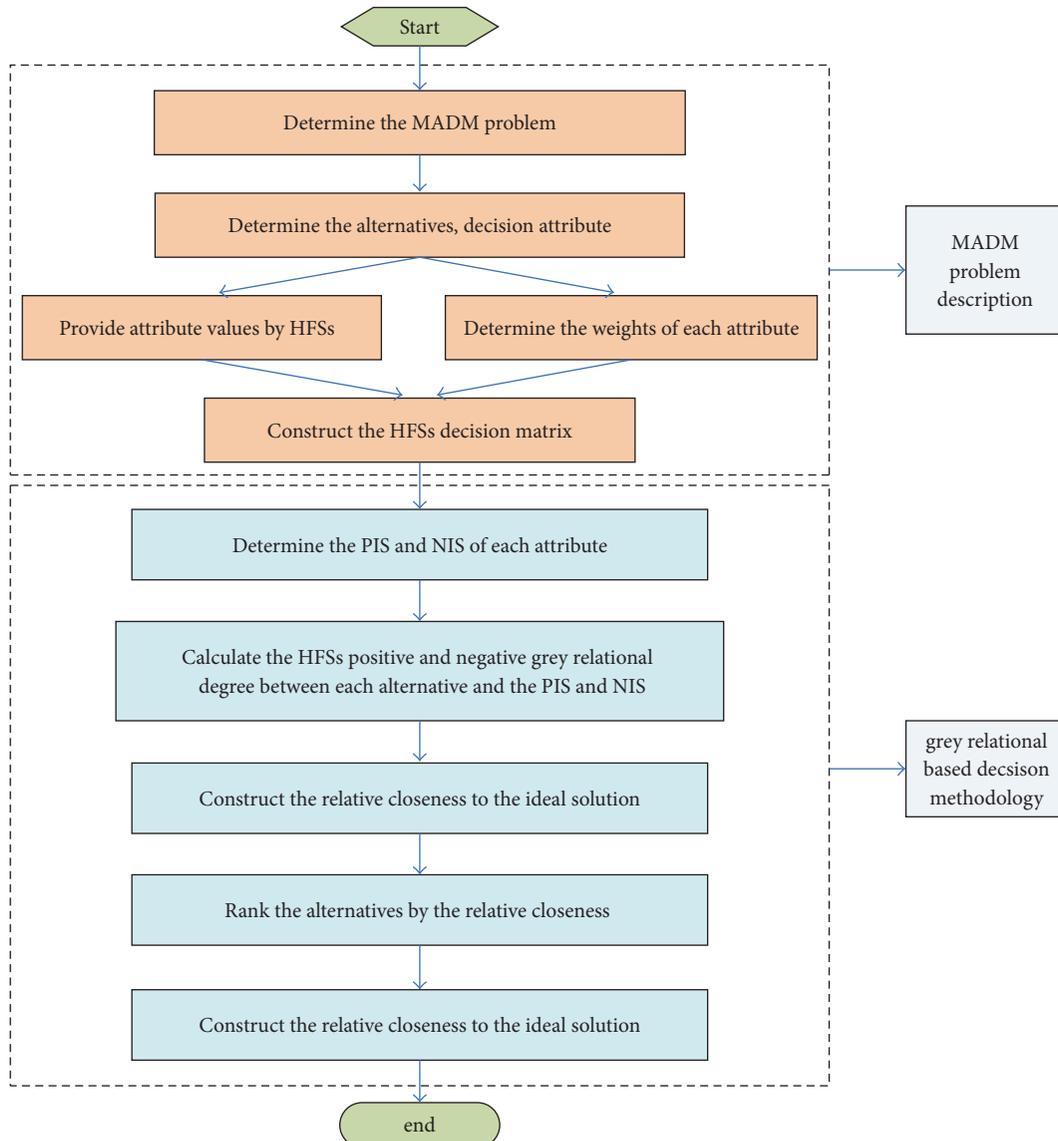


FIGURE 1: The process of the grey relational based MADM methodology with HFSs information.

TABLE 3: Positive and negative grey relational degree from the PIS and the NIS.

Methods	Relational degrees	Alternatives				
		A_1	A_2	A_3	A_4	A_5
HFSs grey relational degree	γ_w^+	0.4934	0.4984	0.5922	0.6250	0.6949
	γ_w^-	0.6942	0.7630	0.6867	0.7325	0.5583
HFSs slope grey relational degree	γ_{sw}^+	0.8854	0.8882	0.8901	0.9021	0.9107
	γ_{sw}^-	0.8086	0.8288	0.8265	0.8929	0.8188
HFSs synthetic grey relational degree	γ_{cw}^+	0.8640	0.8676	0.9054	0.8658	0.9251
	γ_{cw}^-	0.8366	0.8605	0.8493	0.8883	0.8160

TABLE 4: The three types of grey relative closeness of the 5 alternatives to the ideal solution.

Relative closeness	Alternatives					Rankings
	A_1	A_2	A_3	A_4	A_5	
HFSs grey relative closeness	0.4155	0.3951	0.4631	0.4604	0.5545	$A_5 > A_3 > A_4 > A_1 > A_2$
HFSs slope grey relative closeness	0.5227	0.5173	0.5185	0.5026	0.5266	$A_5 > A_1 > A_3 > A_2 > A_4$
HFSs synthetic grey relative closeness	0.5081	0.5020	0.5160	0.4936	0.5313	$A_5 > A_3 > A_1 > A_2 > A_4$

TABLE 5: The hesitant fuzzy decision-making information for detected target and known targets.

Alternatives	Attributes		
	PRF	CF	PW
Known target 1	{0.1, 0.2, 0.3}	{0.2, 0.4}	{0.1, 0.3, 0.4}
Known target 2	{0.15, 0.3, 0.45}	{0.3, 0.6}	{0.15, 0.45, 0.6}
Known target 3	{0.3, 0.4, 0.5}	{0.6, 0.7}	{0.4, 0.5, 0.7}
Known target 4	{0.1, 0.5, 0.6}	{0.3, 0.6}	{0.3, 0.4, 0.9}
Known target 5	{0.2, 0.3, 0.7}	{0.2, 0.5}	{0.4, 0.5, 0.9}
Detected target	{0.2, 0.4, 0.6}	{0.4, 0.8}	{0.2, 0.6, 0.8}

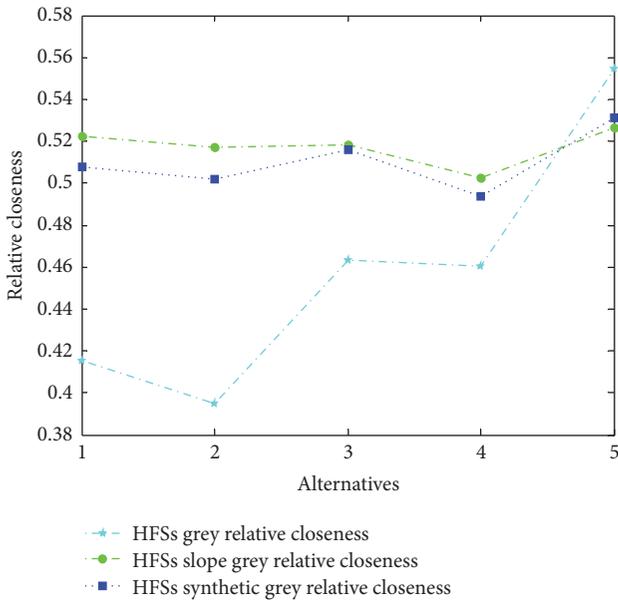


FIGURE 2: The three types of relative closeness.

methods are effective in the hesitant fuzzy decision problems, successfully applying the grey relation analysis theory into the hesitant fuzzy domain.

5.2. Apply the Proposed Grey Relational Based MADM Methodology to Multisensor Target Recognition

Example 9. Consider a multisensor target recognition problem. Three sensors detect an uncertain target with three different electromagnetic radiation features, respectively: pulse repetition frequency (PRF), carrier frequency (CF), and pulse width (PW). For the uncertainty of these data, these data are reported by HFSs. We need to find which target in the database matches the detected one. It is a MADM problem. We use the proposed grey relational based MADM methodology to recognize it in the database. The detected target and known targets data in the database are shown in Table 5.

The attribute weight vector of the electromagnetic radiation features is $w = (0.4, 0.2, 0.4)^T$. In order to show the advantages of the proposed method, we use five HFSs measurements methods to compare the recognition result: the Hamming distance in [16], the correlation coefficient in [22], the grey relative degree, the slope grey relative degree, and synthetic grey relative degree proposed in our paper. The recognition data and recognition effect of these five methods are shown in Table 6 and Figure 3.

From Table 6 and Figure 3, we can see that different measurements methods result in different results. The similarity method and grey relative degree method regard known

TABLE 6: The hesitant fuzzy measurement degree for 5 alternatives with 5 different methods.

Methods	Alternatives				
	Known target 1	Known target 2	Known target 3	Known target 4	Known target 5
Similarity	0.7533	0.8767	0.8900	0.8900	0.8700
Correlation coefficient	1.0000	1.0000	0.9714	0.9174	0.9768
Grey relational degree	0.5694	0.8261	0.8718	0.8718	0.8363
Slope grey relational degree	1.0000	1.0000	0.8075	0.8174	0.8629
Synthetic grey relational degree	0.9152	0.9557	0.8578	0.8541	0.8719

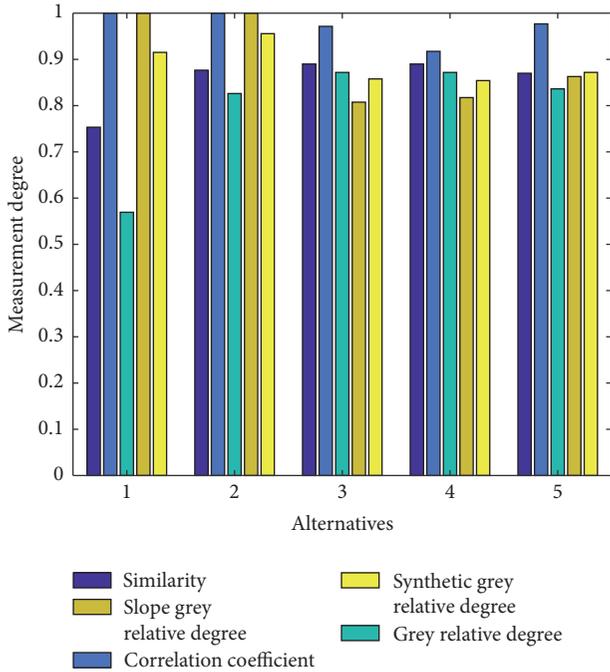


FIGURE 3: Recognition effect with different measurements.

targets 3 and 4 as the best recognition result, while the correlation coefficient and slope grey relative degree refer the best recognition result to known targets 1 and 2. These two kinds of result are completely different. The reason is that the similarity method and grey relative degree method pay more attention to the closeness of the data and the correlation coefficient and slope grey relative degree method pay more attention to the linear relationship of the data. If the closeness of the data is equal, the similarity method and grey relative degree method get the same result. However, the patterns of known targets 3 and 4 are not the same. Under this condition, the similarity method and grey relative degree method cannot recognize which one is the recognition result. Similarly, if linear relationship of the data is equal, the correlation coefficient and slope grey relative degree method get the same result. But the patterns of known targets 1 and 2 are not the same. Under this condition, the correlation coefficient and slope grey relative degree method cannot get the correct recognition result. These two kinds of methods only consider one aspect of the real data, either the closeness

or the linear relationship. They cannot distinguish which one is the correct result. Obviously, this will result in the unreasonable result, which is only partial one.

However, the synthetic grey relative degree takes the two factors, closeness and linear relationship, into consideration; it determines that only known target 2 is the recognition result. This result is more convincing than the others with its superiority in comprehensiveness and discrimination. Therefore, according to the above analysis and comparison, we make the decision that the detected target is recognized to be known target 2.

Combined with two practical MADM examples about energy policy selection and multisensor target recognition with HFSs information, we can see that the proposed grey relational based MADM methodology can deal with the HFSs MADM problems well. In addition, the decision results are the same as the previous methods, which demonstrates the grey relational based MADM methodology's effectiveness. Furthermore, it is the first time to construct the synthetic grey relative degree for HFSs by considering both the closeness and the slope factors of HFSs data, which is significant in the development of the HFSs by enriching their fuzzy measures.

6. Conclusion

In this paper, we apply the grey relational analysis theory to the HFSs domain. We improve the traditional grey relational degree for HFSs by a novel slope grey relational degree which considers the slope factor of the HFSs data. We further construct the synthetic grey relational degree for HFSs which takes both the distance and slope factors of the HFSs data into account. This synthetic grey relational degree can better describe the fuzzy measures than the traditional grey relational degree. Additionally, we also develop the HFSs grey relational based MADM methodology based on the TOPSIS method to solve the HFSs MADM problems. Finally, combined with two practical MADM examples about energy policy selection and multisensor target recognition, we obtain the appropriate decision results. Comparing the decision results with the previous methods, the validity and accuracy of the proposed MADM methodology based on the synthetic grey relational degree are illustrated.

In the future, we will apply the proposed synthetic grey relational degree to other HFSs extensions as the interval hesitant fuzzy sets, dual hesitant fuzzy sets, hesitant fuzzy linguistic sets, and so forth.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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