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Research Article

Numerical Study of the Zero Velocity Surface and Transfer Trajectory of a Circular Restricted Five-Body Problem

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We focus on a type of circular restricted five-body problem in which four primaries with equal masses form a regular tetrahedron configuration and circulate uniformly around the center of mass of the system. The fifth particle, which can be regarded as a small celestial body or probe, obeys the law of gravity determined by the four primaries. The geometric configuration of zero-velocity surfaces of the fifth particle in the three-dimensional space is numerically simulated and addressed. Furthermore, a transfer trajectory of the fifth particle skimming over four primaries then is designed.

1. Introduction

The restricted N-body problem has attracted the attention of many researchers from the fields of mathematics, astronomy, and mechanics because of its wide application in deep space exploration. Here we list some recent or interesting research results. Gao and Zhang [1] studied the existence of periodic orbits of the circular restricted three-body problem. According to the existing literature, the first type of Poincaré periodic orbit generally requires that the mass parameter μ of the system is sufficiently small, and the periodic orbit studied in this paper is applied to any μ between (0, 1), solving the problem that the first type of Poincaré's periodic orbit has always been considered to occur only when the masses of primaries are quite different. Baltagiannis and Papadakis [2] obtained the zero-velocity surfaces and corresponding equipotential curves in the planar restricted four-body problem where the primaries were always at the vertices of an equilateral triangle. Álvarez-Ramírez and Vidal [3] analyzed the zero-velocity surfaces and zero-velocity curves of the spatial equilateral restricted four-body problem. Singh [4] investigated the permissible regions of motion and the zero-velocity surfaces under the influence of small perturbations in the Coriolis and centrifugal forces in the restricted four-body problem. Mittal et al. [5] found the zero-velocity surfaces and regions of motion in the restricted four-body problem with variable mass where the three primaries formed an equilateral triangle. Asique et al. [6] drew the zero-velocity surfaces to determine the possible permissible boundary regions in the photogravitational restricted four-body problem.

However, limited research studies have been performed on circular restricted five-body problem while it is compared with related three-body and four-body problems. A spatial circular restricted five-body problem wherein the fifth particle (the small celestial body or probe) with negligible mass is moving under the gravity of the four primaries, which move in circular periodic orbits around their centers of mass fixed at the origin of the coordinate system. Because the mass of the fifth particle is small, it does not affect the motion of four primaries.

Kulesza et al. [7] observed the region of motion of the restricted rhomboidal five-body problem whose configuration is a rhombus using the Hamiltonian structure and proved the existence of periodic solutions. Albouy and Kaloshin [8] confirmed there were a finite number of isometry classes of planar central configurations, also called relative equilibria, in the Newtonian five-body problem. Marchesin and Vidal [9] determined the regions of possible motion in the spatial restricted rhomboidal five-body problem by using the Hamiltonian structure. Llibre and Valls [10] found that the unique cocircular central configuration is the regular 5-gon

with equal masses for the five-body problem. Bengochea et al. [11] studied the necessary and sufficient conditions for periodicity of some doubly symmetric orbits in the planar $1 + 2n$-body problem and studied numerically these types of orbits for the case n=2. Shoaib et al. [12] considered the central configuration of different types of symmetric five-body problems that have two pairs of equal masses; the fifth mass can be both inside the trapezoid and outside the trapezoid, but the triangular configuration is impossible. Xu et al. [13] discussed the prohibited areas of the Sun-Jupiter-Trojans-Greeks-Spacecraft system and designed a transfer trajectory from Jupiter to Trojans. Han et al. [14] obtained many new periodic orbits in the planar equal-mass five-body problem by using the variational method. Gao et al. [15] investigated the zero-velocity surfaces and regions of motion for specific configurations in the axisymmetric restricted five-body problem. Saari and Xia [16] verified that, without collisions, the Newtonian N-body problem of point masses could eject a particle to infinity in finite time and that three-dimensional examples exist for all $N \geqslant 5$.

In the present paper, it is assumed that the four primaries with equal masses constitute a regular tetrahedron configuration. A dynamic equation of the circular restricted five-body problem is established, and the relationship between energy surface structure of the fifth particle and the corresponding Jacobi constant is discussed. Moreover, the critical position of the fifth particle's permissible and forbidden regions of motion is also addressed. In addition, based on Matlab software, a transfer trajectory of the fifth particle skimming over four primaries is designed numerically. Because of the gravity of the four primaries, the transfer trajectory will reduce the consumption of fuel for the fifth particle effectively.

2. Equations of Motion

For a circular restricted five-body system, assume each of the four primaries, namely, M_1, M_2, M_3, and M_4, lie at one of the vertices of a regular tetrahedron. Furthermore, their motions are opposite to the center of mass. The small orbiter M is only subjected to the gravity of the four primaries, and the origin O of the inertial coordinate system XYZ is located at the center of mass of the four primaries, with one of them, say M_1, located on the Z-axis. The plane defined by the remaining three primaries is parallel to XOY plane. The origin o of the rotational coordinate system xyz coincides with O. The x-axis and y-axis of the rotational coordinate system are rotated counterclockwise around the centroid of the unit angular velocity relative to the X-axis and Y-axis of the inertial coordinate system XYZ, respectively.

Suppose the masses of four primaries are 1/4, the mass of the fifth particle is m, the angle that the system rotates around its center of mass is θ, and the two coordinate systems coincide with each other when the time t is 0. Hence, we obtain θ is equal to t. The dimensionless distance between each two primaries is 1, and the distances between four primaries and the center of mass are $\sqrt{6}/4$. Thus, the coordinates of four primaries are

$$\begin{gathered} M_1\left(0, 0, \frac{\sqrt{6}}{4}\right), \\ M_2\left(-\frac{\sqrt{3}}{3}\cos\left(\frac{\pi}{6}-t\right), \frac{\sqrt{3}}{3}\sin\left(\frac{\pi}{6}-t\right), -\frac{\sqrt{6}}{12}\right), \\ M_3\left(\frac{\sqrt{3}}{3}\cos\left(\frac{\pi}{6}+t\right), \frac{\sqrt{3}}{3}\sin\left(\frac{\pi}{6}+t\right), -\frac{\sqrt{6}}{12}\right), \\ M_4\left(\frac{\sqrt{3}}{3}\cos\left(\frac{2\pi}{3}-t\right), -\frac{\sqrt{3}}{3}\sin\left(\frac{2\pi}{3}-t\right), -\frac{\sqrt{6}}{12}\right). \end{gathered} \tag{1}$$

In the coordinate system xyz, we hypothesize the coordinate of M is (x, y, z); thus, the coordinates of four primaries are

$$\begin{gathered} M_1\left(0, 0, \frac{\sqrt{6}}{4}\right), \\ M_2\left(-\frac{1}{2}, \frac{\sqrt{3}}{6}, -\frac{\sqrt{6}}{12}\right), \\ M_3\left(\frac{1}{2}, \frac{\sqrt{3}}{6}, -\frac{\sqrt{6}}{12}\right), \\ M_4\left(0, -\frac{\sqrt{3}}{3}, -\frac{\sqrt{6}}{12}\right). \end{gathered} \tag{2}$$

Suppose that the coordinate of the fifth particle is (X, Y, Z) in the inertial coordinate system; thus, the Lagrange function satisfies the following relationship:

$$L = \frac{1}{2}\left(\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2\right) - U(X, Y, Z, t), \tag{3}$$

where the gravitational potential energy $U(X, Y, Z)$ is defined as

$$U(X, Y, Z, t) = -\frac{1}{4}\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right), \tag{4}$$

and $R_i (i = 1, 2, 3, 4)$ denotes the distance between the fifth particle and one primary:

$$R_1 = \sqrt{X^2 + Y^2 + \left(Z - \frac{\sqrt{6}}{4}\right)^2},$$

$$R_2 = \sqrt{\left(X + \frac{\sqrt{3}}{3}\cos\left(\frac{\pi}{6}-t\right)\right)^2 + \left(Y - \frac{\sqrt{3}}{3}\sin\left(\frac{\pi}{6}-t\right)\right)^2 + \left(Z + \frac{\sqrt{6}}{12}\right)^2},$$

$$R_3 = \sqrt{\left(X - \frac{\sqrt{3}}{3}\cos\left(\frac{\pi}{6} + t\right)\right)^2 + \left(Y - \frac{\sqrt{3}}{3}\sin\left(\frac{\pi}{6} + t\right)\right)^2 + \left(Z + \frac{\sqrt{6}}{12}\right)^2},$$

$$R_4 = \sqrt{\left(X - \frac{\sqrt{3}}{3}\cos\left(\frac{2\pi}{3} - t\right)\right)^2 + \left(Y + \frac{\sqrt{3}}{3}\sin\left(\frac{2\pi}{3} - t\right)\right)^2 + \left(Z + \frac{\sqrt{6}}{12}\right)^2}. \tag{5}$$

By substituting (3) into the following Euler-Lagrange equation,

$$\frac{d}{dt} \cdot \frac{\partial L}{\partial q_i} - \frac{\partial L}{\partial q_i} = 0, \tag{6}$$

we obtain the dimensionless equations of motion of the fifth particle in the inertial coordinate system

$$\begin{aligned} \ddot{X} &= -U_X, \\ \ddot{Y} &= -U_Y, \\ \ddot{Z} &= -U_Z, \end{aligned} \tag{7}$$

where U_X, U_Y, and U_Z denote the derivative of U with respect to X, Y, and Z, respectively.

Suppose that the coordinates of the orbiter M in the rotating coordinate system are (x, y, z). The relationship between the two coordinate systems is

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = R_t \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \tag{8}$$

where R_t is

$$R_t = \begin{pmatrix} \cos t & -\sin t & 0 \\ \sin t & \cos t & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{9}$$

Thus, the dimensionless equations of motion of the fifth particle in the rotational coordinate system are

$$\begin{aligned} \ddot{x} - 2\dot{y} &= \Omega_x, \\ \ddot{y} + 2\dot{x} &= \Omega_y, \\ \ddot{z} &= \Omega_z, \end{aligned} \tag{10}$$

where the generalized potential energy $\Omega(x, y, z)$ is defined as

$$\Omega(x, y, z) = \frac{1}{2}\left(x^2 + y^2\right) + \frac{1}{4}\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \frac{1}{r_4}\right), \tag{11}$$

and

$$\begin{aligned} r_1 &= \sqrt{x^2 + y^2 + \left(z - \frac{\sqrt{6}}{4}\right)^2}, \\ r_2 &= \sqrt{\left(x + \frac{1}{2}\right)^2 + \left(y - \frac{\sqrt{3}}{6}\right)^2 + \left(z + \frac{\sqrt{6}}{12}\right)^2}, \\ r_3 &= \sqrt{\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{\sqrt{3}}{6}\right)^2 + \left(z + \frac{\sqrt{6}}{12}\right)^2}, \\ r_4 &= \sqrt{x^2 + \left(y + \frac{\sqrt{3}}{3}\right)^2 + \left(z + \frac{\sqrt{6}}{12}\right)^2}. \end{aligned} \tag{12}$$

Thus, a first Jacobi type integral is

$$2\Omega(x, y, z) - \left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right) = C, \tag{13}$$

where $V = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}$ is the motion velocity of the fifth particle and C is the Jacobi constant. The permissible motion region and prohibited region are defined by $V \geqslant 0$ and $V < 0$, respectively. Therefore, when the velocity of the fifth particle is zero, the curve shown by the above equation is called zero-velocity curve on the plane and is called zero-velocity surface in space.

3. Zero-Velocity Surfaces

The diagrams of relationship between the zero-velocity surfaces and the Jacobi constant C are shown in the following.

For a given value of C, we can obtain the zero-velocity surfaces and the zero-velocity curves of the circular restricted five-body problem on the xoy, xoz, and yoz planes, as shown in Figures 1(a)–1(c), respectively.

We now turn to discuss the zero-velocity surfaces of the fifth particle.

For the Jacobi type integral (13) of the system, when the velocity of the fifth particle is zero, the relationship between the zero-velocity surface and the values of Jacobi constant C is shown in Figures 1(a)–1(c). The smaller the C value, the greater the energy of the system. In addition, the area of the zero-velocity surface of the fifth particle decreases, while the permissible motion region of the fifth particle increases.

Figure 2 shows the evolution of prohibited area of the fifth particle when C is 3.4. The fifth particle can only skim over

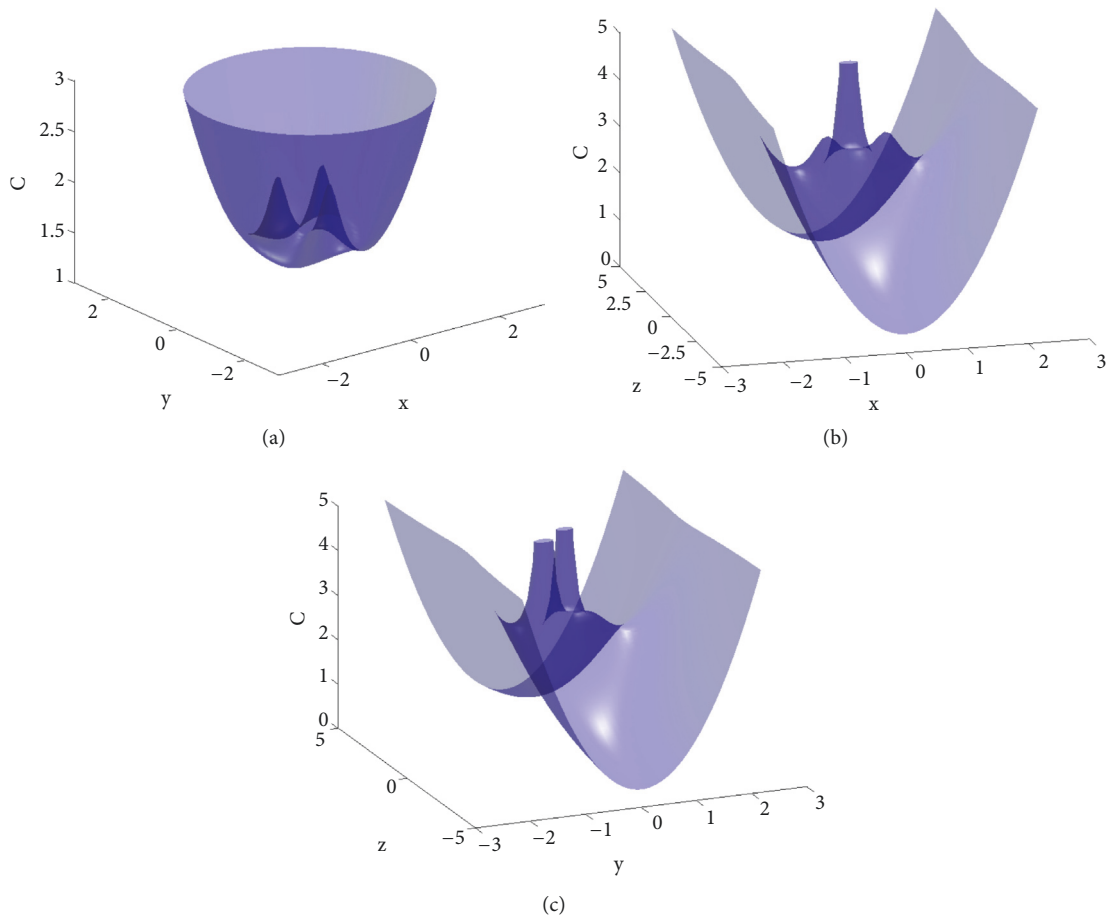


Figure 1: (a), (b), and (c) are the diagrams of the relationships between the zero-velocity surfaces of the circular restricted five-body problem and the Jacobi constant C on the xoy, xoz, and yoz planes, respectively.

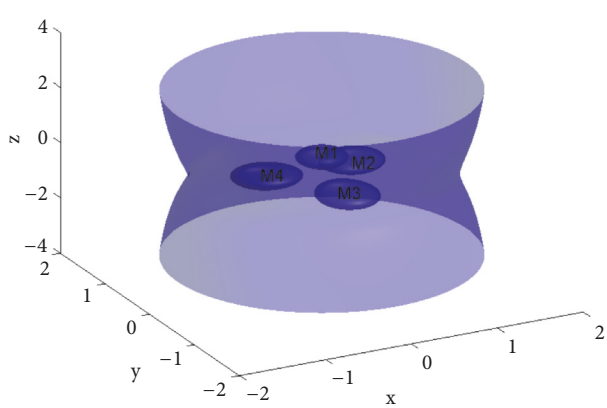


Figure 2: Zero-velocity surface of the fifth particle: $C = 3.4$.

four primaries under the gravity, rather than shuttle among them.

When C is 3.4, as shown in Figure 3, the regions that the fifth particle can move around is the neighborhood of those three small circles on xoy and xoz planes under the gravity of four primaries, while it moves around those two small circles on the yoz plane.

When C is 3.2829, as shown in Figures 4(a) and 4(b), the permissible regions of the two primaries, M_2 and M_3, are interconnected to create "channel". Therefore, "channel A" appears, through which fifth particle can pass through. However, it cannot fly across the permissible area of M_1. The prohibited areas of the fifth particle decrease.

When C is 3.2828, as shown in Figures 5(a) and 5(b), there are two new channels, namely, "channel B" and "channel C", through which the fifth particle can fly from the permissible area of M_4 into M_2 and M_3, respectively. The prohibited regions of the fifth particle decrease.

The prohibited area of the fifth particle when C reduces to 3.2774 is presented in Figures 6(a) and 6(b). "Channel D" and "channel E" are the channels through which the fifth particle can fly from the permissible area of M_1 into M_2, M_3, respectively.

When C is 3.2772, there is a "channel F", through which the fifth particle can fly from the permissible area of M_1 into M_3, as shown in Figures 7(a) and 7(b). At this point, the fifth particle can travel through the above six channels between the permissible regions of each two interconnected primaries.

When C is 3.2675, with the larger "channel D" and "channel E", a new "channel G" is formed, as shown in Figures 8(a) and 8(b). Moreover, if C is further reduced, the fifth

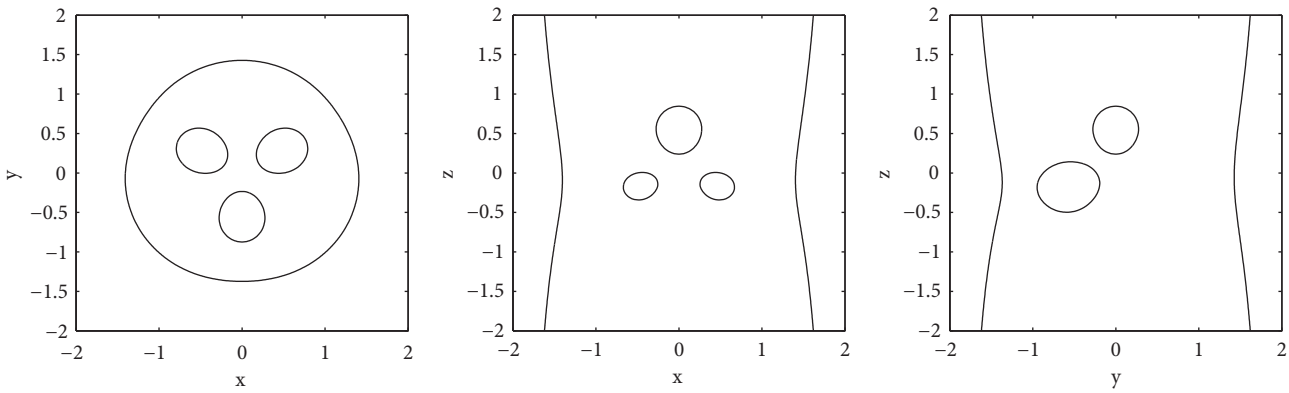
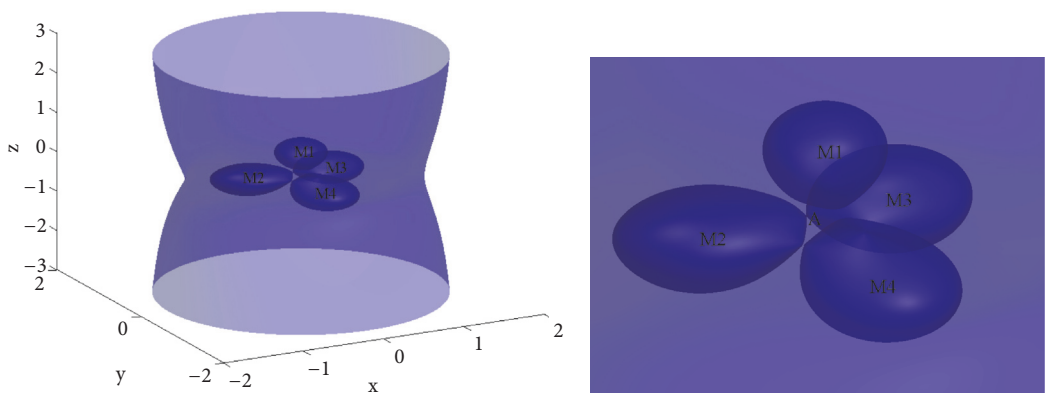


Figure 3: Zero-velocity curves of the fifth particle for $C = 3.4$.



(a) (b)

Figure 4: (a). Zero-velocity surface of the fifth particle: C=3.2829. (b) Magnified view of the zero-velocity surface: C=3.2829.

particle will move from the permissible region of M_2 to M_3 directly, without any help of other channels.

As shown in Figures 9(a) and 9(b), "channel H" and "channel I" exist when C is 3.2674. Thus, with C further declining, the fifth particle will move from the permissible area of M_4 to M_2 or M_3 through M_1; i.e., its prohibited areas are reduced.

When C is 3.2674, as shown in Figure 10, the fifth particle can move from the inside of these "petals", except the center of the outer ring, to periphery of the "petals" on *xoy* plane and move from the inside of the "clover" to the periphery of the "clover" directly on the *xoz* plane. On the *yoz* plane, there is a point of intersection, that is, "fortress T", where fifth particle can fly through it. Moreover, the prohibited areas of the fifth particle decrease with the decrease of C value.

Figures 11(a) and 11(b) show the prohibited area of the fifth particle when C is 3.2662. The "region J" will disappear with the decrease of C. At this case, the fifth particle will be in the permissible region of four primaries within the shuttle flight, without any assistance of the "channels". However, it still cannot fly into outer space.

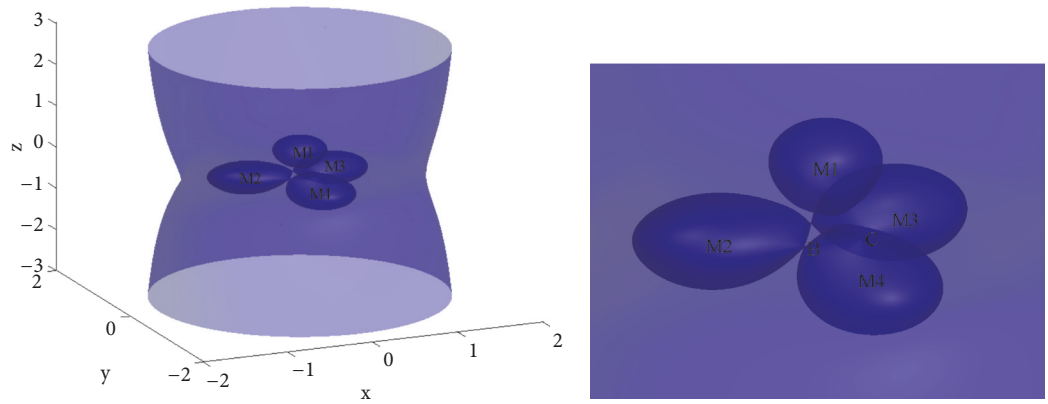
As shown in Figures 12(a) and 12(b), when C is 3.2085, the permissible regions of M_1, M_2, and M_3 communicate the feasible regions of the system, growing three new "channels", namely, K, N, and L, through which the fifth particle can fly to outer space.

When C is 2.8383, the prohibited areas of the fifth particle are further reduced in Figures 13(a) and 13(b). With C value further decreasing in Figure 13(b), the prohibited areas P, Q, and R will disappear, and the fifth particle will not need to crossover any "channels" to fly into outer space. Thus, the prohibited areas will gradually shrink to a point and finally disappear. The fifth particle will be free of the influences of the four primaries and be able to fly into outer space.

As shown in Figure 14, the prohibited areas of the fifth particle appear as three circles on the *xoy* plane, and the prohibited areas on the *xoz* plane are two parts of upper and lower concave and convex curves. In addition, a new "fortress W", through which the fifth particle can fly to outer space, appears on the *yoz* plane when C is 2.8383. As C decreases, the prohibited areas on these three planes will disappear. Finally, the fifth particle will not be bound by the primaries and will be able to fly freely on these planes.

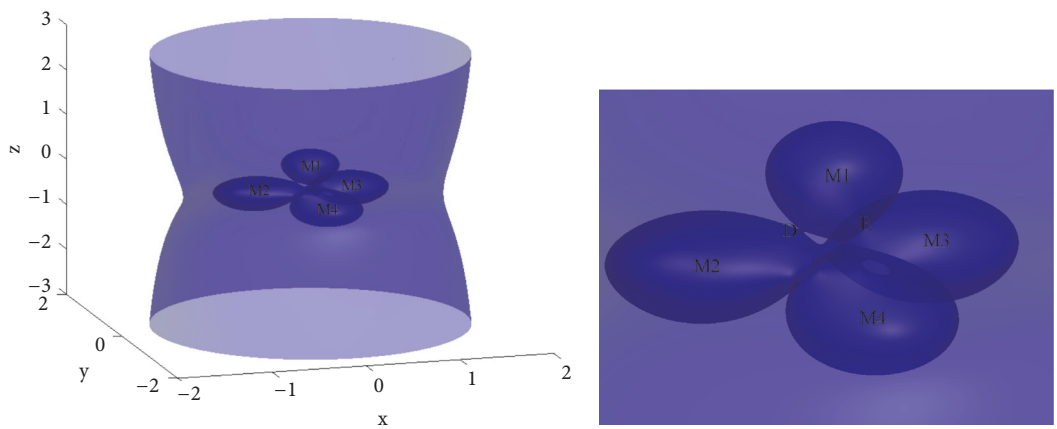
4. Numerical Simulation of the Transfer Trajectory

In deep space exploration, Figure 9 plays a role in the circular restricted five-body problem, because when Jacobi constant



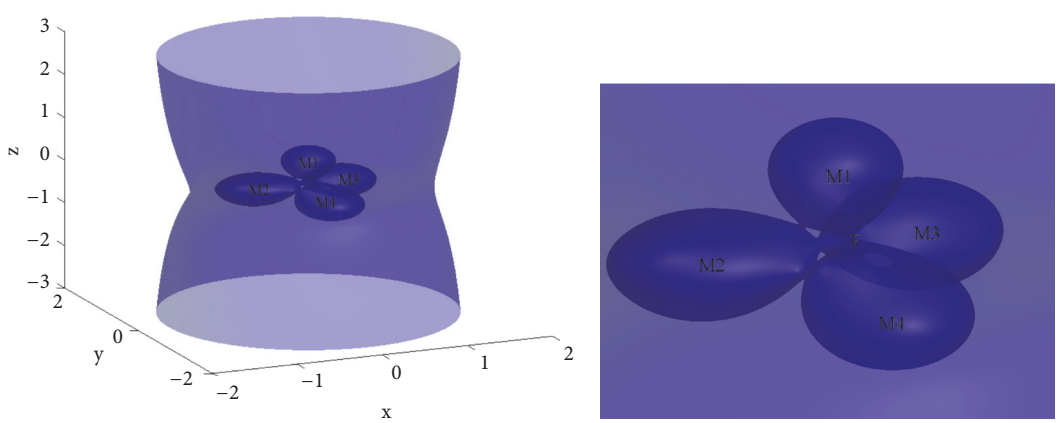
(a) (b)

Figure 5: (a). Zero-velocity surface of the fifth particle: C=3.2828. (b) Magnified view of the zero-velocity surface: C=3.2828.



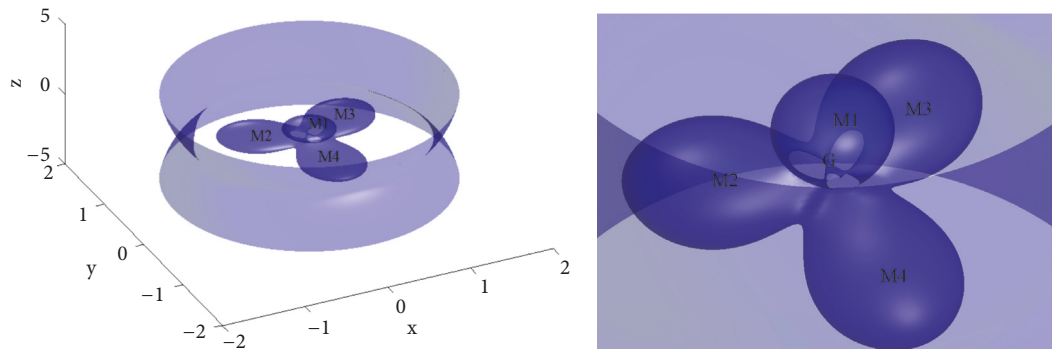
(a) (b)

Figure 6: (a) Zero-velocity surface of the fifth particle: C = 3.2774. (b) Magnified view of the zero-velocity surface: C = 3.2774.



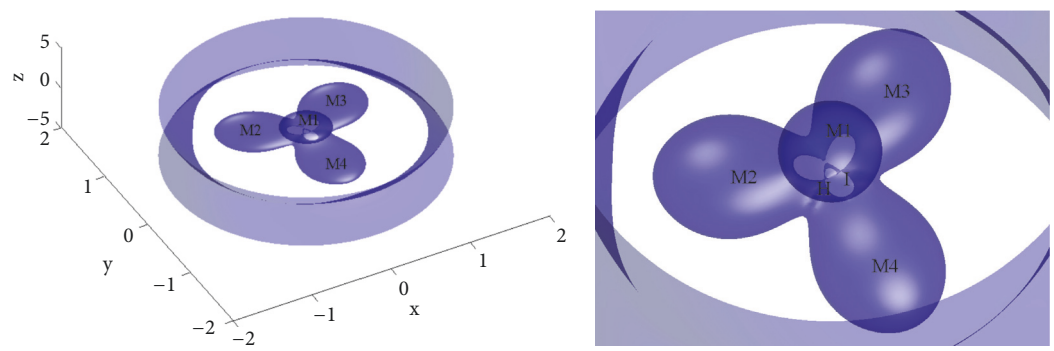
(a) (b)

Figure 7: (a). Zero-velocity surface of the fifth particle: C = 3.2772. (b) Magnified view of the zero-velocity surface: C = 3.2772.



(a) (b)

Figure 8: (a) Zero-velocity surface of the fifth particle: $C = 3.2675$. (b). Magnified view of the zero-velocity surface: $C = 3.2675$.



(a) (b)

Figure 9: (a) Zero-velocity surface of the fifth particle: $C = 3.2674$. (b) Magnified view of the zero-velocity surface: $C = 3.2674$.

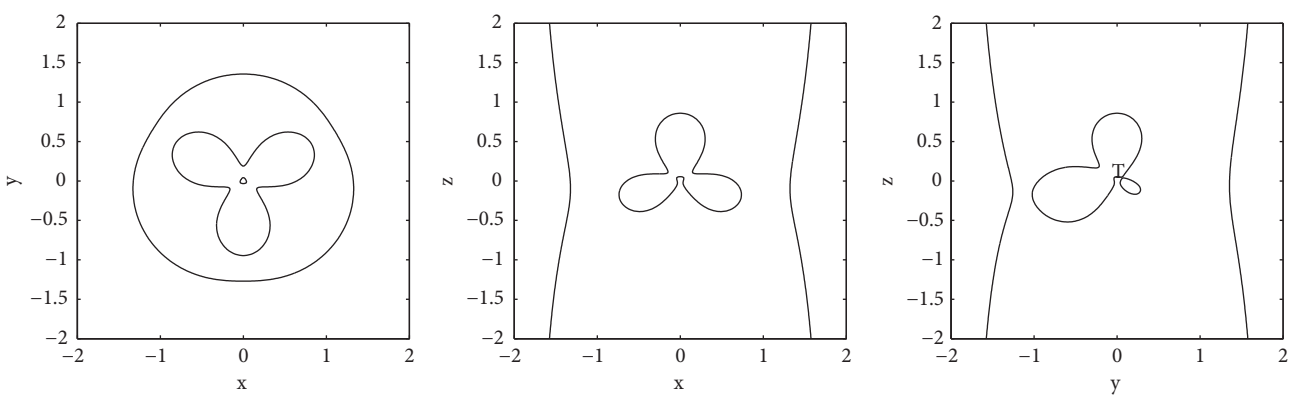
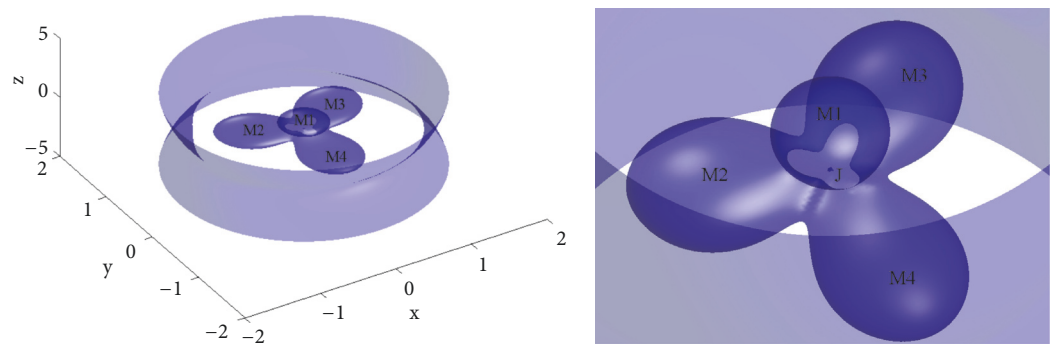


Figure 10: Zero-velocity curves of the fifth particle for $C = 3.2674$.

value C is greater than that in Figure 9, the fifth particle must fly from the permissible area of one primary to another through the corresponding "fortresses". With the decrease of the C value, the permissible areas of the fifth particle will increase, allowing the fifth particle to be able to shuttle between the permissible regions of the four primaries.

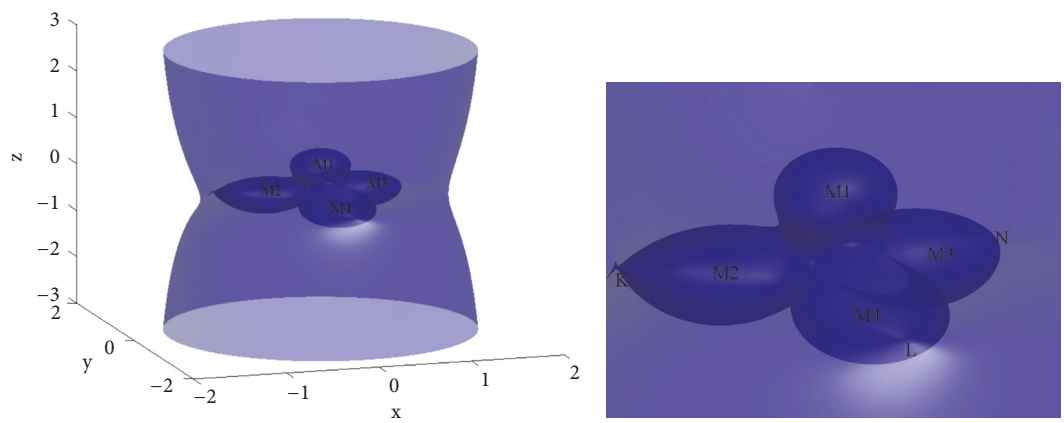
The calculation shows that the four primaries M_1, M_2, M_3, and M_4 locate at $(0, 0, 0.61237)$, $(-0.5, 0.288675, -0.204124)$, $(0.5, 0.288675, -0.204124)$, and $(0, -0.57735, -0.204124)$, respectively. With appropriate initial conditions used, the numerical method is adopted to simulate the motions of the circular restricted five-body problem based on Matlab 2012a and the algorithm of ode45 which gives the fifth-order accurate method. Based on the simulations, the following transfer trajectory is designed.

As shown in Figure 15, the fifth particle starts from the point A (i.e., M_2) in the permissible region of primary M_2, passes through B, C (two interior points of the primary M_3), D, E (two interior points of the primary M_4), and F (an interior point of the primary M_1), and finally flies to G (a nearby point of M_1). The specific analysis is as follows: first, for the primary M_2 (i.e., the point A), we consider



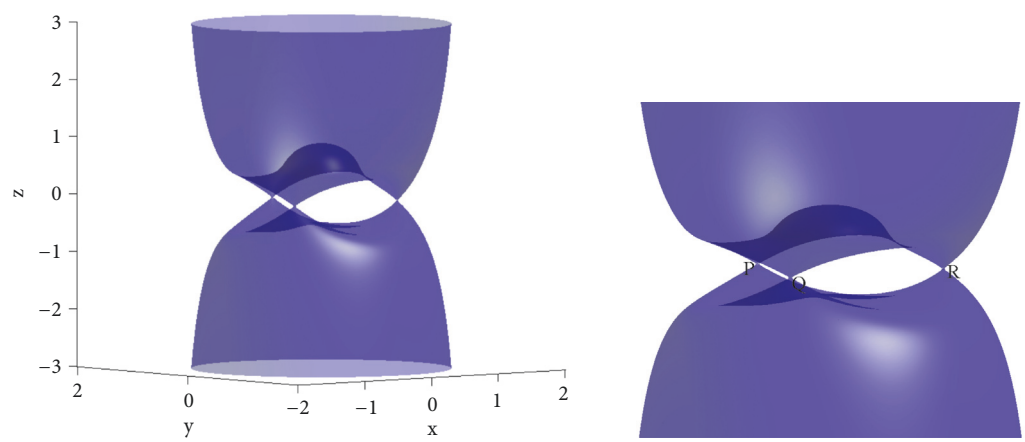
(a) (b)

Figure 11: (a) Zero-velocity surface of the fifth particle: $C = 3.2662$. (b) Magnified view of the zero-velocity surface: $C = 3.2662$.



(a) (b)

Figure 12: (a) Zero-velocity surface of the fifth particle: $C = 3.2085$. (b) Magnified view of the zero-velocity surface: $C = 3.2085$.



(a) (b)

Figure 13: (a) Zero-velocity surface of the fifth particle: $C = 2.8383$. (b) Magnified view of the zero-velocity surface: $C = 2.8383$.

the initial conditions $x0 = [-0.5, 0.288675, -0.204124]$, $v0 = [0.76, 0.009, -0.02]$, where $x0$ is the initial position coordinates of the fifth particle, and $v0$ is its initial velocity. By numerical simulation, we obtain a transfer trajectory, on which the fifth particle can fly from $M_2(-0.5, 0.288675, -0.204124)$, starting at the initial speed $(0.76, 0.009, -0.02)$, to finally reach the permissible region of M_3, i.e., a track from point A to point B. Second, selecting $x0 = [0.4705, 0.07041, 0.04673]$, $v0 = [0.06, 0.09, 0.02]$, a permissible area of the internal transfer trajectory in the primary M_3 is the fifth particles track from B to C. At this case, C is very close to M_3. Third, choosing $x0 =$

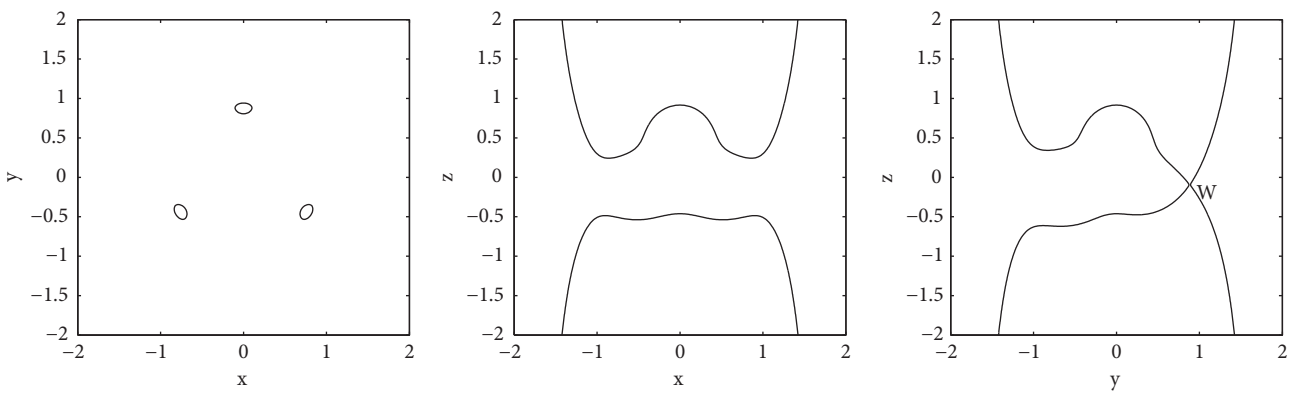
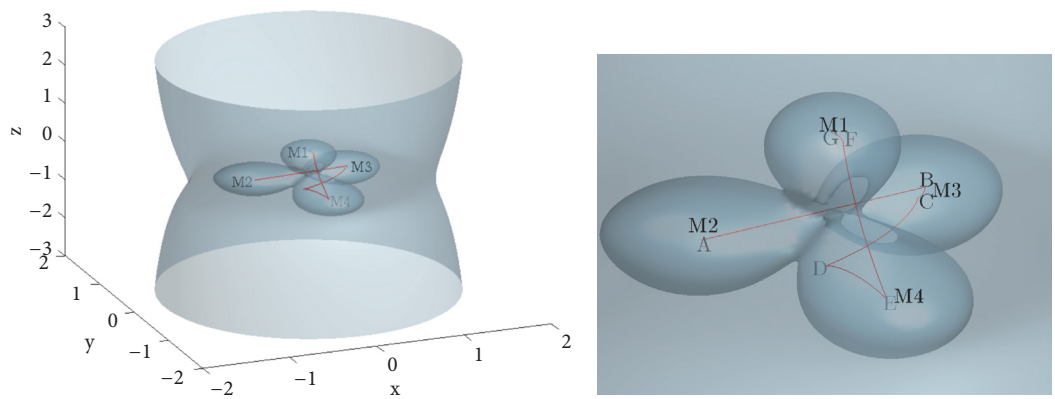


Figure 14: Zero-velocity curves of the fifth particle for $C = 2.8383$.



(a) (b)

Figure 15: (a) The orbits of motion of the fifth particle skimming over four primaries. (b) Magnified view of these orbits.

$[0.5233, 0.2849, -0.1971]$, $v0 = [-1.3, -5.7, 1.9]$ corresponds to a transfer trajectory from C to D; at the same time, the fifth particle flies from the permissible regions of M_3 to M_4. Fourth, for $x0 = [-0.2277, -0.4345, 0.008506]$, $v0 = [0.2, 0.07, 0.008]$, we obtain a transfer trajectory from D to E inside the active area of the primary M_4, with the point E and M_4 being very close. Fifth, for $x0 = [0.002021, -0.5811, -0.2055]$, $v0 = [7.3, 5.9, 9.8]$, the fifth particle can fly from E to F at the flying track; i.e., the fifth particle moves from the permissible region of M_4 to M_1. Sixth, with $x0 = [0.1188, 0.1583, 0.3905]$, $v0 = [0.01, 0.001, 0.3]$, we obtain an orbit where the fifth particle flies from F to G inside the permissible area of primary M_1, and point G is very close to M_1. Finally, a complete transfer trajectory is obtained, when the flyby occurs between the fifth particle and the four primaries.

5. Conclusions

In this paper, we introduced and established the dynamic equations of the circular restricted five-body problem, and the geometric configurations of zero-velocity surfaces and zero-velocity curves of the problem with different Jacobi constants of C values are also discussed in detail. Through the analysis and discussion of zero-velocity surfaces and zero-velocity curves of the system, we found that, when the C value is large, the performance of the fifth particle is not very active. When C decreased, the prohibited areas of the fifth particle decreased, while the permissible regions expanded, ultimately eliminating the possibility of the four primaries flying freely in outer space. Based on these numerical results, the case of Figure 9 is found to be worthy of further study, because the fifth particle shuttled in the permissible regions within the primaries under the gravity, without too much extra energy. The fifth particle can skim over to four primaries to explore them. Therefore, we simulated the corresponding transfer trajectories numerically, starting from the primary M_2, followed by M_3, M_4, and finally to fly around M_1, to explore the transfer trajectories of four primaries; this approach will not only improve the exploration efficiency but also save resources.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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Supplementary Materials

There are three MATLAB code files in the supplementary material, namely, "eqns.m", "fifth-body.m", and "trajectory.m". The file "Eqn.m" contains the governing equation of the fifth particle, the file "fifth-body.m" includes an extrapolation method for this problem, and the main file "trajectory.m" is used to design the transfer trajectories between the four primaries. *(Supplementary Materials)*

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