Research Article

Joint Transportation and Inventory Strategy for Perishable Items with Weibull Distribution under Carbon Emission Regulations

Jianteng Xu,1 Xin Cui,1 Yuyu Chen,2 and Xin Zhang1

1School of Management, Qufu Normal University, Rizhao 276826, China
2College of Economics and Management, Nanjing University of Aeronautics and Astronautics, Nanjing 211106, China

Correspondence should be addressed to Jianteng Xu; jiantengxu@163.com

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This paper studies the optimal transportation and inventory strategy for perishable items under carbon cap-and-trade and carbon tax regulations. Nonlinear optimization models are constructed to maximize the total profits per unit time when the deterioration rate satisfies a two-parameter Weibull distribution and the demand rate is a linear function with respect to the current inventory level. The existence conditions of the optimal replenishment cycles under two carbon emission policies are proved. The characteristics of the optimal replenishment cycles, maximal total profits, and carbon emissions per unit time under two carbon emission policies are compared. Finally, a numerical test is provided to illustrate the theoretical results and Taguchi method is employed to analyze the sensitivity of total profits and carbon emissions per unit time with respect to the parameters of carbon emission policies, transportation time, and Weibull parameters. The results show that parameters of carbon emission regulations have greater effect on total profits per unit time, and the transportation time has the greatest effect on carbon emissions per unit time.

1. Introduction

The signature of Paris Agreement has set off a wave of carbon emissions control around the world. Many countries and governments have implemented policies to mitigate carbon emissions, such as United Kingdom, France, European Union, and China. The popular carbon policies are carbon cap-and-trade, carbon tax, and carbon cap policies, where carbon cap-and-trade and carbon tax policies are more flexible and acceptable to companies. Under carbon cap-and-trade policy, the government assigns a carbon emission quota to a firm; if carbon emissions generated from the firm’s operation exceed the quota, the firm needs to buy carbon emission rights from the carbon trade market. Otherwise, the firm can sell the remainder emission rights. Under carbon tax policy, firms pay tax to the government for their carbon emissions. These carbon emission policies have changed firms’ decision-making environment and decision makers begin to concentrate on the effect of carbon emission policies on firms’ operational decisions.

Perishable products that lose value, quality, and quantity over time are common products in human activities, such as fruits, vegetables, meat, aquatic products, and milchigs (Bai et al. [1] and Xu et al. [2]). The data show that, in the sales of products in Chinese supermarkets, the proportion of perishable products is more than 30%, and in some supermarkets even exceed 60% [3]. The deteriorative feature of perishable items requires additional treatments in storage and transportation to control the temperature and humidity, such as a mechanical refrigeration system and a humidifier. These treatments for perishable items consume more resources and emit more emissions than other normal items. Hence, the studies on effect of carbon emission regulations on operational decisions for perishable items and how to achieve high profits and low carbon emissions become highly concerned.
Three comprehensive reviews of recent literature on supply chain model for deteriorating items under carbon emission policies were undertaken by Goyal and Giri [4], Bakker et al. [5], and Janssen et al. [6]. Most of the literature studies the operational decisions for general products under carbon emission policies. Some of them study under the basic assumption of deterministic demand, such as Hua et al. [7], Chen et al. [8], Benjaafar et al. [9], Xu et al. [10], and Toptal and Cetinkaya [11], etc. Others study under the basic assumption of stochastic demand, such as Song and Lend [12], Gong and Zhou [13], Brandenburg [14], Bai and Chen [15], Purohit et al. [16], and He et al. [17], etc. There is little literature on the operational decisions of perishable products under carbon emission policies. Dye and Yang [18] consider trade credit and inventory strategies for perishable products with time varying deterioration rate under carbon cap-and-trade policy and analyze the impact of trade credit and carbon emission policy on inventory strategies. Hua et al. [19] propose two perishable inventory models under carbon tax and cap-and-trade regulations in which the deterioration rate is constant. They explore the characters of optimal solutions and discuss the effect of carbon emission regulations on inventory decisions and profits by numerical examples. Bai et al. [20] coordinate a two-echelon supply chain for perishable items with a constant deterioration rate by two contracts under cap-and-trade regulation. They show that cooperation can lead to higher profits and lower carbon emissions and compare the coordination effect of two contracts. Huang et al. [21] construct a Stackelberg game model for deteriorating food products in a three-level supply chain where the initial deterioration rate is constant. They provide an illustrative algorithm to solve the optimal pricing, order quantity and investment decisions, and analyze impact of critical factors on profits and carbon emissions.

When the above literature studies operational decisions of perishable products under carbon emission policies, they assume that the deterioration rate of perishable product is constant. However, Berrotoni [22] finds that leakage failure of dry cells and life expectancy of ethical drugs can be expressed by Weibull distribution when discussing the data-fitting problem. The degradation rate increases over time and all unused products fail in stock at some point. According to this result, Covert and Philpi [23] study the ordering decision for perishable items with deterioration rate following a two-parameter Weibull distribution on the basis of EOQ problem. Recently, some researchers study the inventory models of perishable items with deterioration rate following a two-parameter Weibull distribution, such as Pal et al. [24], Prasad et al. [25], and Pervin et al. [26], etc. Other literature considers the operational decisions for perishable items whose deterioration rate obeys three-parameter Weibull distribution, such as Yang [27] and Sanni et al. [28] The common characteristic of this literature is that when the authors study the operational decisions of perishable items, they consider the impact of demand, time, investment, and other factors inside firms, but they ignore the impact of transportation factor and the decision environment.

The data provided by the professional agencies of the United Nations show that among all energy-related carbon emissions, transportation accounts for a quarter of the carbon dioxide, accounting for about 15%-17% of total human carbon dioxide emissions [29]. It indicates that transportation is one of the main sources of carbon emissions. Under this recognition, Chen and Wang [30] consider the impact of carbon emission regulations on ordering and transportation mode selection decisions with stochastic demand. Konur et al. [31] consider an inventory control problem in which the order is split among multiple suppliers. Considering two delivery scheduling policies, they construct a biobjective optimization problem to minimize both the expected profits and the carbon emissions. The above literature investigates the impact of the transportation on the operational decisions and profits without considering deteriorating products.

Based on the above background, this paper studies the optimal transportation and inventory strategies for perishable products with fully considering the storage, deterioration, and carbon emissions during transportation. The deterioration rate obeys a two-parameter Weibull distribution. The objective is to determine the replenishment strategy to maximize the total profits per unit time under cap-and-trade and carbon tax policies. Different from the abovementioned literature that mainly uses one-parameter sensitivity analysis to explore the effect of carbon emission regulation or other parameters on the system performance when their authors study the operational decisions for perishable products, this paper employs Taguchi method to reveal the comprehensive influence on the system performance when multiple parameters fluctuate and to find the key factor that has the greatest influence on the system performance. Besides, the contribution of this paper lies on: (1) Among the existing research on perishable products under carbon emission regulations, the impact of transportation process and dynamic deterioration rate is fully considered and analyzed in this paper. (2) The optimal transportation and inventory strategies for perishable products under cap-and-trade and carbon tax policies are compared. Moreover, the sufficient condition that the total profits per unit time under carbon tax policy are larger than the ones under cap-and-trade policy is obtained. (3) Through Taguchi experiment, it is found that the parameters of carbon emission regulations have greater effect on total profits per unit time while the transportation time has the greatest effect on carbon emissions per unit time. A summary of the most related literature is shown in Table 1.

The rest of the paper is organized as follows. Section 2 describes the problems and gives some notations. Section 3 establishes the transportation and inventory optimization models for perishable products under two carbon emission policies. Some properties of optimal replenishment strategy and total profits per unit time are analyzed and compared among the cases under two carbon emission policies and without carbon emission regulations. The theoretical results are verified by numerical test in Section 4. Furthermore, the impact of key parameters on total profits and carbon emissions per unit time under two carbon emission policies are analyzed by Taguchi experiment. The optimal combination of key parameters is recommended to maximize total profits and minimize carbon emissions per unit time. Finally, the
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related conclusions and further research directions are given in Section 5.

2. Problem Description and Notations

The considered problem can be described as follows. A retailer orders a kind of perishable products from a supplier at the beginning of each replenishment cycle and entrusts the third-party logistics to ship products from the supplier to his location within a specified time. Arrived products are sold at a fixed price to meet the market demand with no shortages. The market demand is a linear function of the inventory level. The deterioration rate of the product follows a two-parameter Weibull distribution. The deteriorated products could not be repaired or replaced; in other words, the deteriorated products immediately leave the inventory system once they deteriorated. Carbon emissions are mainly produced in transportation and storage procedures. The retailer and the third-party logistics cooperate with each other to maximize the total profits and carbon emissions in one replenishment cycle. The total profits include order cost and transportation cost, inventory cost and deterioration cost during transportation and sale procedures. The total profits and carbon emissions in one replenishment cycle are

\[ TC(T) = A + c_1 Q + h_1 I_1 + h_2 I_2 + h_3 (B_1 + B_2), \]

\[ E(T) = \tilde{A} + \tilde{c}_1 Q + \tilde{h}_1 I_1 + \tilde{h}_2 I_2 + \tilde{h}_3 (B_1 + B_2). \]

Next, we will analyze the optimal replenishment cycles and corresponding carbon emissions under cap-and-trade and carbon tax regulations with the objective of maximizing total profits per unit time.

3. Mathematical Model and Analysis

3.1. Mathematical Model. According to the problem description, inventory changes in transportation and sale processes are shown in Figure 1.

The inventory changes in the transportation process can be expressed as differential equation \( \frac{dI_1(t)}{dt} = -\theta(t)I_1(t), \) \( t \in [0, u], \) using the condition \( I_1(0) = Q, \) we can obtain the inventory level in transportation process at time \( t: \)

\[ I_1(t) = Qe^{-\int_{0}^{t} \theta(s)ds} = Qe^{-\alpha t}, \quad t \in [0, u]. \]

(1)

The inventory changes in the sale process can be expressed as differential equation \( \frac{dI_2(t)}{dt} = -D(t) - \delta(t)I_2(t) = -\alpha - [\beta + \theta(t)]I_2(t), \) \( t \in [0, T], \) using the initial condition \( I_2(u) = 0 \) and \( I_2(T) = Q_0, \) we can obtain the inventory level in the sale process at time \( t: \)

\[ I_2(t) = \int_{t}^{u} \alpha e^{-\int_{s}^{t} (\beta + \theta(\tau))d\tau} ds = \alpha e^{\int_{0}^{u} (\beta + \theta(s))ds} - \int_{0}^{u} \alpha e^{\int_{0}^{s} (\beta + \theta(\tau))d\tau} ds, \]

\[ t \in [0, u], \]

\[ I_2(t) = e^{\int_{0}^{T} (\beta + \theta(t)) ds} \int_{0}^{u} e^{-\alpha t - \int_{0}^{u} \beta ds} ds, \]

\[ t \in [u, T], \]

(2)

(3)

where the initial inventory level \( Q_0 \) (that is, reorder level) satisfies \( Q_0 = I_2(0) = \alpha \int_{0}^{u} e^{\int_{0}^{s} (\beta + \theta(\tau))d\tau} ds. \)

The inventory level in the transportation process \( I_1 \) and the deteriorated amount \( B_1 \) in the interval \( [0, u] \) are \( I_1 = \int_{0}^{u} I_1(t)dt = Q \int_{0}^{u} e^{-\alpha t} dt \) and \( B_1 = \int_{0}^{u} \theta(t)I_1(t)dt = Q(1 - e^{-\alpha u}). \)

Since the inventory level in the transportation process equals the one in the sale process at time \( u, \) we can obtain the order quantity from Eqs. (1) and (3); that is, \( Q = e^{\beta(T-\alpha)}(\theta(T)I_1(T)+\alpha \int_{0}^{u} e^{\int_{0}^{t} (\beta + \theta(s))ds} dt). \)

Using Eqs. (2) and (3), we can obtain the inventory level in the sale process \( I_2 \) and the deteriorated amount \( B_2 \) in the interval \( [0, T]; \) that is, \( I_2 = \int_{0}^{u} I_2(t)dt + \int_{u}^{T} I_2(t)dt \) and \( B_2 = \int_{0}^{u} \theta(t)I_2(t)dt + \int_{u}^{T} \theta(t)I_2(t)dt. \)

The total profits and carbon emissions in one replenishment cycle are

\[ TC(T) = A + c_1 Q + h_1 I_1 + h_2 I_2 + h_3 (B_1 + B_2), \]

\[ E(T) = \tilde{A} + \tilde{c}_1 Q + \tilde{h}_1 I_1 + \tilde{h}_2 I_2 + \tilde{h}_3 (B_1 + B_2). \]

3.2. Optimal Strategy under Cap-and-Trade Regulation. The optimization model with the objective of maximizing total profits per unit time (denoted by \( M_1 \)) can be stated as follows:

\[ (M_1) \quad \text{max} \quad \Pi_1(T) \]

\[ = \frac{P}{T} \int_{0}^{T} D(t) dt \]

\[ - \left[ \frac{1}{T} TC(T) + c_p \left( \frac{1}{T} E(T) - C \right) \right] \]

s.t. \( T \geq u. \)

In order to determine the optimal replenishment cycle \( T^* \) under cap-and-trade regulation, we analyze the properties of the optimal solutions.
Proof of Theorem 1. From Eq. (6), we can obtain 

\[ \Pi(t) = (1/T) p \beta I_1 + c_p E(T) + c_p (C + \rho \alpha) \]

The first-order derivative of \( \Pi(T) \) is \( \Pi'(T) = (1/T^2) [p \beta I_1 - (TC'(T) + c_p E(T))] - [p \beta I_2 - (TC(T) + c_p E(T))]. \) Let \( f_1(T) = T [p \beta I_1 - (TC'(T) + c_p E(T))] - [p \beta I_2 - (TC(T) + c_p E(T))], \) then \( \Pi'(T) = -T [TC''(T) + c_p E''(T) - p \beta I_1'], \) where

\[ TC''(T) + c_p E''(T) - p \beta I_1' \]

\[ = \left[ \alpha + Q_0 \left( \beta + abT^{b-1} \right) \right] \cdot \left[ (h_2 + c_p \tilde{h}_2 - p \beta) \cdot \Psi + \Lambda \right] + Q_0 ab (b-1) T^{b-2} \]  

(7)

\[ = \left[ (h_2 + c_p \tilde{h}_2 - p \beta) \cdot \left( e^{(\beta T + \lambda T^2)} \int_0^T e^{-(\beta T + \lambda T^2)} dt \right) + \Phi \right], \]

\[ \Psi = 1 + e^{(\beta T + \lambda T^2)} (\beta + ab T^{b-1}) \int_0^T e^{-(\beta T + \lambda T^2)} dt, \quad \Lambda = (h_3 + c_p \tilde{h}_3) ab T^{b-1} + e^{\beta T + \lambda T^2} (\beta + ab T^{b-1}) \cdot \left[ (c_1 + c_p \tilde{c}_1) e^{-\beta u} + (h_1 + c_p \tilde{h}_1) e^{-\beta u} \int_0^T e^{-\beta u} dt + (h_3 + c_p \tilde{h}_3) \right] \cdot \left( e^{\beta u} \int_0^T ab T^{b-1} e^{-\beta u} dt + \int_0^T ab T^{b-1} e^{-(\beta T + \lambda T^2)} dt \right), \]

and \( \Phi = (c_1 + c_p \tilde{c}_1) e^{-\beta u} + (h_1 + c_p \tilde{h}_1) e^{-\beta u} \int_0^T e^{-\beta u} dt + (h_3 + c_p \tilde{h}_3) [1 + (e^{\beta u} \int_0^T ab T^{b-1} e^{-\beta u} dt + \int_0^T ab T^{b-1} e^{-(\beta T + \lambda T^2)} dt)]). \]

Eq. (7) indicates that \( f_1(T) \) is a strictly decreasing function of \( T \) if \( h_2 + c_p \tilde{h}_2 - p \beta > 0 \). Hence, there is a unique \( T_1^* \in (u, +\infty) \) such that \( f_1(T_1^*) = 0 \) when \( f_1(u) \geq 0 \). In this case, \( \Pi_1(T) \) is a concave function, so the replenishment cycle \( T_1^* \) determined by \( f_1(T_1^*) = 0 \) is the unique one maximizing the total profits per unit time. If \( f_1(u) < 0 \), according to the monotonicity of \( f_1(T) \), we have \( f_1(T) < f_1(u) < 0 \) and \( \Pi_1(T)/\Pi_1(T) \leq T < 0 \). It means that \( \Pi_1(T) \) is a strictly decreasing function of \( T, T \geq u \). Hence, if \( f_1(u) < 0 \), the total profits per unit time are maximized when \( T_1^* = u \). □

Theorem 1 shows the existence condition of the optimal strategy under cap-and-trade regulation. Note that, under carbon cap-and-trade regulation, the retailer has to buy carbon emission permits from the carbon trading market when carbon emissions with maximal profits exceed the quota given by government. The following corollary concluded from Theorem 1 shows the condition under which the retailer has to buy carbon emission permits from the carbon trading market.

Corollary 2. In model \( M_1 \), the retailer can sell extra carbon emission permits if \( E(T_1^*) > C \); otherwise, the retailer needs to buy absent carbon emission permits.

Theorem 1 and Corollary 2 show that the optimal replenishment cycle is not affected by the carbon emission quota but the carbon trading price. However, the total profits per unit time increase as the carbon emission quota increases. Hence,
on one hand, the retailer should determine the optimal replenishment strategy according to the carbon trading price; on the other hand, the retailer hopes that the government gives higher carbon emission quota.

In order to explore the upper bound and lower bound of the maximal total profits per unit time under cap-and-trade regulation, we discuss the optimization model (denoted by \( M_0 \)) of perishable products without carbon emission regulations. The optimization model \( M_0 \) can be expressed as follows:

\[
(M_0) \max \quad \Pi_0(T) = \frac{1}{T} \int_0^T D(t) dt - \frac{1}{T} TC(T)
\]

s.t. \( T \geq u \).

**Lemma 3.** In model \( M_0 \), there exists a unique replenishment cycle \( T_0^* \in [u, +\infty) \) to maximize the total profits per unit time if \( h_2 - p\beta > 0 \).

**Proof of Lemma 3.** The total profits per unit time without carbon emission regulations are \( \Pi_0(T) = \frac{1}{T}[p\beta L_2 - TC(T)] + px \). Let \( f_0(T) = T[p\beta L_2 - TC'(T)] - [p\beta L_2 - TC(T)] \); the first-order derivatives of \( \Pi_0(T) \) and \( f_0(T) \) are \( \frac{\partial \Pi_0(T)}{\partial T} = (1/T^2)f_0(T) \) and \( f_0'(T) = -[TC''(T) - p\beta L_2'] \). Similar to the proof of Theorem 1, \( f_0(T) \) is a strictly decreasing function of \( T \) if \( h_2 - p\beta > 0 \). Hence, there exists a unique \( T_0^* \in [u, +\infty) \) satisfying \( f_0'(T_0^*) = 0 \) when \( f_0'(u) < 0 \). If \( f_0'(u) = 0 \), we have \( f_0'(T) < f_0'(u) < 0 \) and \( \Pi_0(T)/\partial T < 0 \). It means that \( \Pi_0(T) \) is a strictly decreasing function of \( T; T \geq u \), and the total profits per unit time are maximized when \( T_0^* = u \).

Lemma 3 shows the existence condition of the optimal replenishment cycle \( T_0^* \) of model \( M_0 \). It indicates that the transportation factor makes the optimal strategy for the perishable products more difficult to determine.

Comparing the maximal total profits per unit time under cap-and-trade regulation with the ones without carbon emission regulations, we can obtain the following conclusion.

**Theorem 4.** \( \Pi_1(T_1^*) \geq \Pi_0(T_0^*) \) holds if \( E_0(T_0^*)/T_0^* \leq C \), and \( \Pi_1(T_1^*) \leq \Pi_0(T_0^*) \) holds if \( E_1(T_1^*)/T_1^* \geq C \).

**Proof of Theorem 4.** Two cases are considered to prove the conclusion.

**Case 1** \( T_1^* = T_0^* \). In this case, we have \( \Pi_1(T_1^*) - \Pi_0(T_0^*) = -cE_0(T_0^*)/T_0^* + c_0C \). It means that \( \Pi_1(T_1^*) \geq \Pi_0(T_0^*) \) holds if \( E_0(T_0^*)/T_0^* \leq C \), and \( \Pi_1(T_1^*) \leq \Pi_0(T_0^*) \) holds if \( E_0(T_0^*)/T_0^* \geq C \).

**Case 2** \( T_1^* \neq T_0^* \). Using the optimality of \( T_1^* \), we have \( \Pi_1(T_1^*) > \Pi_0(T_0^*) \) and \( \Pi_1(T_1^*) - \Pi_0(T_0^*) > \Pi_0(T_0^*) - \Pi_0(T_1^*) \) = \( -c_0E_0(T_1^*)/T_1^* + c_0C \). Hence, when \( E_0(T_0^*)/T_0^* \leq C \), we have \( \Pi_1(T_1^*) > \Pi_0(T_0^*) \). On the other hand, using the optimality of \( T_0^* \), we have \( \Pi_1(T_1^*) - \Pi_0(T_0^*) < \Pi_1(T_1^*) - \Pi_0(T_1^*) = -c_0E_0(T_1^*)/T_1^* + c_0C \). Hence, \( \Pi_1(T_1^*) \leq \Pi_0(T_0^*) \) holds when \( E_1(T_1^*)/T_1^* \geq C \).

Theorem 4 shows the condition under which the total profits per unit time under cap-and-trade regulation and the ones without carbon emission regulation can be compared. It implies that the total profits per unit time under cap-and-trade regulation are not always larger than the ones without carbon emission regulation. The retailer needs to adjust carbon emission parameters according to the emission quota to obtain larger total profits.

### 3.3. Optimal Strategy under Carbon Tax Regulation

Using Eqs. (4) and (5), we can formulate the optimization model (denoted by \( M_2 \)) for maximizing the total profits per unit time under carbon tax regulation:

\[
(M_2) \max \quad \Pi_2(T) = \frac{1}{T} \int_0^T D(t) dt
\]

s.t. \( T \geq u \).

Obviously, \( \Pi_2(T) \) has the same structure as \( \Pi_1(T) \) except for the part \( -c_0C\beta \). Hence, the existence condition of the optimal replenishment cycle \( T_2^* \) in model \( M_2 \) satisfies the following theorem.

**Theorem 5.** In model \( M_2 \), there exists a unique replenishment cycle \( T_2^* \in [u, +\infty) \) to maximize the total profits per unit time when \( h_2 + c_0\beta - p\beta > 0 \).

Theorems 1 and 5 imply that the optimal replenishment cycle is affected by the carbon emission price and irrelevant to the carbon quota. The total profits per unit time under carbon tax regulation satisfy the following relation.

**Theorem 6.** \( \Pi_2(T_2^*) < \Pi_0(T_0^*) \).

**Proof of Theorem 6.** Two cases are considered to prove this conclusion.

**Case 1** \( T_2^* = T_0^* \). In this case, we have \( \Pi_2(T_2^*) - \Pi_2(T_2^*) = c_0E(T_2^*)/T_2^* > 0 \) and further \( \Pi_0(T_0^*) > \Pi_2(T_2^*) \).

**Case 2** \( T_2^* \neq T_0^* \). According to the optimality of \( T_0^* \), we have \( \Pi_0(T_0^*) > \Pi_2(T_2^*) \) and \( \Pi_0(T_0^*) - \Pi_2(T_2^*) \). It implies that the total profits per unit time under carbon tax regulation are always less than the ones without carbon emission regulations. Hence, carbon tax regulation is not the retailer’s favorite regulation. The government should compensate the retailer via some subsidy policies to stimulate them to curb carbon emissions under carbon tax regulation.

### 3.4. Comparison of the Optimal Strategies under Two Carbon Emission Regulations

To investigate the relation of optimal replenishment cycles and carbon emissions under two
carbon emission regulations, we first analyze the optimal replenishment cycle without carbon emission regulation by minimizing the emissions per unit time.

Let $M_3$ denote the optimization model whose objective is to minimize emissions per unit time without carbon emission regulations; then it can be expressed as

$$(M_3) \min E_1(T) = \frac{E(T)}{T}$$

s.t. $T \geq u$.

Let $P_1(T)$ be the corresponding total profits per unit time; we have $P_1(T) = P_3(T)$. The optimal replenishment cycle in model $M_3$ satisfies the following lemma.

**Lemma 7.** In model $M_3$, there exists a unique replenishment cycle $T^*_3 \in [u, +\infty)$ such that the total emissions per unit time are minimized.

**Proof of Lemma 7.** Let $f_3(T) = TE(T) - E(T)$; then $f_3(T)$ is a strictly increasing function in $[u, +\infty)$ and $\partial E_1(T)/\partial T = (1/T^2)f_3(T)$ if $T^* > 0$, there exists a unique $T^*_3 \in [u, +\infty)$ in such that $f_3(T^*_3) = 0$. $E_1(T)$ first decreases and then increases in $[u, +\infty)$. Hence, $T^*_3$ is the uniquely optimal replenishment cycle to minimize the emissions per unit time. If $f_3(u) \geq 0$, we have $\partial E_1(T)/\partial T > 0$; in this case, the emissions per unit time are minimized when $T^*_3 = u$.

Lemma 7 indicates that there always exists an optimal replenishment cycle to minimize the carbon emissions in model $M_3$. If the optimal replenishment cycles $T^*_3$ exists in models $M_i$, $i = 0, 1, 2, 3$, they satisfy the following relation.

**Theorem 8.** The optimal replenishment cycles $T^*_i$ and $T^*_2$ in models $M_i$ and $M_2$ satisfy that if $c_p \geq c_i$, $T^*_3 \leq T^*_1 \leq T^*_2 \leq T^*_0$ or $T^*_2 \leq T^*_1 \leq T^*_0$ holds. Otherwise, $T^*_1 \leq T^*_2 \leq T^*_0$ holds.

**Proof of Theorem 8.** According to the proofs of Lemmas 3–7 and Theorems 1–5, we have $f_2(T_1) = f_0(T_1) + (c_p - c_i)f_1(T_1)$ and $f_3(T) = (c_p - c_i)f_0(T)$ if $f_0(T)$ and $f_3(T) = (c_p - c_i)f_0(T)$ if $f_3(T)$ is an increasing function in $[u, +\infty)$ and $f_3(T)$ is an increasing function in $[u, +\infty)$. According to the monotonicity of $T^*_2$, we have $f_2(T^*_2) = f_3(T^*_2) = (c_p - c_i)f_0(T^*_2)$ if $f_0(T)$ and $f_3(T)$ are decreasing functions in $[u, +\infty)$ and $f_3(T)$ is an increasing function in $[u, +\infty)$. According to the monotonicity of $T^*_3$, we have $f_2(T^*_3) = f_3(T^*_3) = (c_p - c_i)f_0(T^*_3)$ if $f_0(T)$ and $f_3(T)$ are decreasing functions in $[u, +\infty)$ and $f_3(T)$ is an increasing function in $[u, +\infty)$. Further, $f_2(T^*_3) \leq f_2(T^*_2)$ if $T^*_3 \leq T^*_2$ holds. Hence, $T^*_3 \geq T^*_2 \geq T^*_0$ holds. On the other hand, if $f_2(T^*_3) = 0$, we have $f_3(T^*_3) \geq 0$. Similarly, we can obtain $T^*_3 \leq T^*_1 \leq T^*_2 \leq T^*_0$.

**4. Numerical Test**

This section will illustrate the above theoretical results and analyze the impact of key system parameters on replenishment cycles, carbon emissions, and profits per unit time. The values of parameters are set as follows: $A = 2500$, $c_1 = 7$, $h_1 = 5$, $h_2 = 11$, $h_3 = 0.5$, $\lambda = 2000$, $c_2 = 5$, $h_2 = 3$, $h_3 = 2$, $\beta = 1.5$, $\alpha = 6000$, $c_1 = 0.5$, and $p = 26$. We will reveal how to adopt the optimal replenishment cycles to maximize the total profits per unit time under cap-and-trade and carbon tax regulations. How do different carbon emission regulations affect the optimal replenishment cycles, total profits, and carbon emissions per unit time? Which parameter has the greatest impact on the
replenishment cycle, total profits, and carbon emissions per unit time, respectively.

According to the theoretical analysis, we can obtain the optimal replenishment cycle \( T^*_i \), total profits per unit time \( \Pi_i(T^*_i) \), carbon emissions per unit time \( E(T^*_i)/T^*_i \), and unit time demand \( D(T^*_i)/T^*_i \) in models \( M_i \), \( i = 0, 1, 2, 3 \), as shown in Table 3.

Table 3: The calculation results.

<table>
<thead>
<tr>
<th>( T^*_i )</th>
<th>( M_0 )</th>
<th>( M_1 )</th>
<th>( \Pi_i(T^*_i) )</th>
<th>( E(T^<em>_i)/T^</em>_i )</th>
<th>( D(T^<em>_i)/T^</em>_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.04 (0%)</td>
<td>1.18 (+13.46%)</td>
<td>1.21 (+6.73%)</td>
<td>1.36 (+30.77%)</td>
<td>3446.50 (-5.39%)</td>
<td>575.19 (+3.45%)</td>
</tr>
</tbody>
</table>

Table 4: The orthogonal table and SN ratio under cap-and-trade regulation.

<table>
<thead>
<tr>
<th>No.</th>
<th>( u )</th>
<th>( \beta )</th>
<th>( b )</th>
<th>( C )</th>
<th>( c_i )</th>
<th>( \Pi_i(T^*_i) )</th>
<th>( \Pi_i(T^*_i) )</th>
<th>( E(T^<em>_i)/T^</em>_i )</th>
<th>SN ratio of ( \Pi_i(T^*_i) )</th>
<th>SN ratio of ( E(T^<em>_i)/T^</em>_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>8542.8</td>
<td>8608.7</td>
<td>5772.6</td>
<td>78.70</td>
<td>-75.23</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2940.5</td>
<td>3814.5</td>
<td>5778.9</td>
<td>71.63</td>
<td>-75.24</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>8279.9</td>
<td>8383.8</td>
<td>5598.3</td>
<td>78.47</td>
<td>-74.96</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2529.6</td>
<td>3462.8</td>
<td>5626.8</td>
<td>70.79</td>
<td>-75.01</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>4613.9</td>
<td>5246.5</td>
<td>6542.0</td>
<td>74.40</td>
<td>-76.31</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>-1270.1</td>
<td>211.2</td>
<td>6534.6</td>
<td>46.50</td>
<td>-76.30</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>4215.2</td>
<td>4905.3</td>
<td>6224.4</td>
<td>73.81</td>
<td>-75.88</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>-1366.4</td>
<td>128.8</td>
<td>6184.5</td>
<td>42.20</td>
<td>-75.83</td>
</tr>
</tbody>
</table>

Table 5: The orthogonal table and SN ratio under carbon tax regulation.

<table>
<thead>
<tr>
<th>No.</th>
<th>( u )</th>
<th>( \beta )</th>
<th>( b )</th>
<th>( C )</th>
<th>( c_i )</th>
<th>( \Pi_i(T^*_i) )</th>
<th>( \Pi_i(T^*_i) )</th>
<th>( E(T^<em>_i)/T^</em>_i )</th>
<th>SN ratio of ( \Pi_i(T^*_i) )</th>
<th>SN ratio of ( E(T^<em>_i)/T^</em>_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>843.1</td>
<td>2130.7</td>
<td>5784.5</td>
<td>66.57</td>
<td>-75.25</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2578.5</td>
<td>3634.7</td>
<td>5788.4</td>
<td>71.21</td>
<td>-75.25</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2075.0</td>
<td>398.3</td>
<td>5682.1</td>
<td>70.10</td>
<td>-75.09</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>374.6</td>
<td>172.7</td>
<td>5650.1</td>
<td>64.73</td>
<td>-75.04</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>994.2</td>
<td>2261.6</td>
<td>6572.2</td>
<td>67.09</td>
<td>-76.35</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>-958.1</td>
<td>569.7</td>
<td>6548.7</td>
<td>55.11</td>
<td>-76.32</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>-1478.8</td>
<td>118.4</td>
<td>62579</td>
<td>41.47</td>
<td>-75.93</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>413.8</td>
<td>1758.6</td>
<td>6275.6</td>
<td>64.90</td>
<td>-75.95</td>
</tr>
</tbody>
</table>

To investigate the effect of key parameters (including transportation time, elasticity coefficients, and carbon emission regulations) on the optimal replenishment strategy, profits, and carbon emissions, we carry out the Taguchi experiment for parameters \( u, \beta, b, C, c, c_i \) (detailed instructions about Taguchi method can be found in [32]). We choose orthogonal tables \( L_4(2^3) \) and \( L_4(2^4) \) to test the effect of key parameters under cap-and-trade and tax regulations. The results are arranged in Tables 4–7 and Figures 2 and 3. In Tables 4–7, levels 1 and 2, respectively, represent the values that float +30% and −30% on the basis of original ones. In order to meet the nonnegative requirement of Taguchi method, we convert the original data of the total profits per...
Table 6: Response table for SN ratios of total profits per unit time under two carbon emission regulations.

(a) Cap-and-trade regulation

<table>
<thead>
<tr>
<th>Level</th>
<th>( u )</th>
<th>( \beta )</th>
<th>( b )</th>
<th>( C )</th>
<th>( c_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>37.48</td>
<td>36.45</td>
<td>36.22</td>
<td>37.65</td>
<td>35.42</td>
</tr>
<tr>
<td>2</td>
<td>35.16</td>
<td>36.19</td>
<td>36.42</td>
<td>34.99</td>
<td>37.22</td>
</tr>
</tbody>
</table>

Delta 2.32 0.26 0.20 2.66 1.81

Rank 2 4 5 1 3

(b) Carbon tax regulation

<table>
<thead>
<tr>
<th>Level</th>
<th>( u )</th>
<th>( \beta )</th>
<th>( b )</th>
<th>( C )</th>
<th>( c_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>68.15</td>
<td>65.00</td>
<td>61.31</td>
<td>56.97</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>57.14</td>
<td>60.30</td>
<td>63.99</td>
<td>68.32</td>
<td></td>
</tr>
</tbody>
</table>

Delta 11.01 4.69 2.68 11.35

Rank 2 3 4 1

Table 7: Response table for SN ratios of carbon emissions per unit time under two carbon emission regulations.

(a) Cap-and-trade regulation

<table>
<thead>
<tr>
<th>Level</th>
<th>( u )</th>
<th>( \beta )</th>
<th>( b )</th>
<th>( C )</th>
<th>( c_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>−75.11</td>
<td>−75.77</td>
<td>−75.54</td>
<td>−75.60</td>
<td>−75.58</td>
</tr>
<tr>
<td>2</td>
<td>−76.08</td>
<td>−75.42</td>
<td>−75.65</td>
<td>−75.59</td>
<td>−75.61</td>
</tr>
</tbody>
</table>

Delta 0.97 0.35 0.10 0.00 0.03

Rank 1 2 3 5 4

(b) Carbon tax regulation

<table>
<thead>
<tr>
<th>Level</th>
<th>( u )</th>
<th>( \beta )</th>
<th>( b )</th>
<th>( C )</th>
<th>( c_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>−75.16</td>
<td>−75.79</td>
<td>−75.65</td>
<td>−75.63</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>−76.14</td>
<td>−75.50</td>
<td>−75.64</td>
<td>−75.66</td>
<td></td>
</tr>
</tbody>
</table>

Delta 0.97 0.35 0.10 0.00 0.03

Rank 1 2 3 4 3

unit time to a nonnegative value which is denoted as \( \Pi_1(T^*_1) \) in Table 4 and \( \Pi_1(T^*_2) \) in Table 5 and accordingly calculate the Signal-to-Noise ratio (SN ratio).

Table 6 shows that the effect order of the key parameters on total profits per unit time under cap-and-trade regulation is \( C > u > c_p > \beta > b \) while the one under carbon tax regulation is \( c_p > u > \beta > b \). It means that (i) the effect of parameters of carbon emission regulations and transportation time on total profits per unit time is greater than the one of elasticity coefficients. (ii) The effect of carbon trading price in cap-and-trade regulation on total profits per unit time is less than the one in carbon tax regulation. In cap-and-trade regulation, carbon cap has the greatest effect on the total profits per unit time.

Table 7 shows that the effect order of the key parameters on carbon emissions per unit time under cap-and-trade regulation is \( u > \beta > b > c_p > C \) while the one under carbon tax regulation is \( u > \beta > c_p > b \). It means that (i) the transportation time \( u \) has the greatest effect on carbon emissions per unit time under both carbon emission regulations. (ii) The effect of carbon emission price \( c_p \) on carbon emissions per unit time is changed by different carbon emission regulations. (iii) The effect of the key parameters on carbon emissions per unit time is less than their effect on total profits per unit time.

From Figures 2 and 3, we can obtain the following observations.

(1) Figures 2(a) and 3(a) show that the optimal combination of the key parameters to maximize the total profits per unit time under cap-and-trade regulation is \( u \) at level 1, \( \beta \) at level 1, \( b \) at level 2, \( C \) at level 1, and \( c_p \) at level 2, while the optimal combination under carbon tax regulation is \( u \) at level 1, \( \beta \) at level 1, \( b \) at level 2, and \( c_p \) at level 2. It means that, under both carbon emission regulations, increasing transportation time \( u \) and elasticity coefficient \( \beta \) and decreasing Weibull parameter \( b \) and carbon emission price \( c_p \) (or \( c_t \)) can bring more profits per unit time.

(2) Figures 2(b) and 3(b) show that the optimal combination of the key parameters to minimize carbon emissions per unit time under cap-and-trade regulation is \( u \) at level 1, \( \beta \) at level 2, \( b \) at level 1, \( C \) at level 2, and \( c_p \) at level 1, while the optimal combination under carbon tax regulation is \( u \) at level 1, \( \beta \) at level 2, \( b \) at level 2, and \( c_p \) at level 1. It means that increasing transportation time \( u \) and carbon emission price \( c_p \) (or \( c_t \)) and decreasing elasticity coefficient \( \beta \) can bring less carbon emissions per unit time under both cap-and-trade and carbon tax regulations.

(3) When other key parameters do not change, increasing transportation time \( u \) can bring higher total profits per unit time and less carbon emissions per unit time.

5. Conclusion

This paper studies the optimal transportation and inventory strategy of perishable products under cap-and-trade and carbon tax regulations. The deterioration rate of products satisfies a two-parameter Weibull distribution. The existence condition of the optimal transportation and inventory strategies with given transportation time is provided, and the properties of optimal replenishment strategy, profits, and carbon emissions per unit time under two carbon emission regulations are compared. The analysis results indicate that: (1) The intervals of optimal replenishment cycle under two carbon emission regulations are all between \( T_1^* \) and \( T_2^* \). (2) When the carbon emissions per unit time satisfy certain conditions, the total profits per unit time under carbon tax regulation will exceed the ones under cap-and-trade regulation. The results of numerical test indicate that the parameters of carbon emission regulations have greater effect on total profits per unit time while the transportation time has the greatest effect on carbon emissions per unit time. When other key parameters do not change, increasing transportation time can bring higher total profits per unit time and less carbon emissions per unit time. These results advise the decision-maker that (i) the total profits per unit time under carbon tax regulation are not always lower than the ones under cap-and-trade regulation. (ii) More attention should be paid to control the transportation time if lower carbon emissions are preferred.
This paper considers the transportation time as a constant and is to find the joint operational strategy of a single firm; it will be an interesting problem to study the coordination of low-carbon supply chains and the case when the transportation time is limited to a range.

**Data Availability**

The data used to illustrate the findings of this study are randomly selected based on the reality. Since the models are abstracted from reality problem, the objective of the experimental data is to illustrate the theoretical results and investigate the impact of key system parameters on the replenishment cycles, carbon emissions, and profits per unit time. In this study, we mainly use the relative number to reveal the impact of key system parameters and verify the theoretical results. Hence, the only requirement for the experimental data is to satisfy the assumptions of the models.

**Conflicts of Interest**

The authors declare that they have no conflicts of interest.

**Acknowledgments**

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**References**


