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Research Article

A New Entropy and Its Properties Based on the Improved Axiomatic Definition of Intuitionistic Fuzzy Entropy

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Regarding the problem of the existing intuitionistic fuzzy entropy formulas in ordering the partial entropy, the constraint condition that is consistent with the intuitionistic facts is proposed in this paper, the axiomatic definition of entropy which fully reflects the intuition and fuzziness of intuitionistic fuzzy sets is given, and the improved intuitionistic fuzzy entropy formula is constructed according to the entropy axiomatic definition and its properties are studied. Finally, we compare the improved formula with the existing intuitionistic fuzzy entropy formulas, and the result turns out that the improved formula can solve the problem in the entropy ordering theoretically and practically.

1. Introduction

Atanassov extended the fuzzy set theory given by Zadeh [1] to the intuitionistic fuzzy set theory [2]. Vague sets [3] and interval-valued fuzzy sets [4] are the other two generalizations of fuzzy sets which were proved to be equivalent to the intuitionistic fuzzy sets theoretically by Bustince and Burillo [5], Atanassov and Gargov [6], Cornelis, Atanassov, and Kerre [7], and Deschrijver and Kerre [8], respectively. At present, the intuitionistic fuzzy set theory is widely used in many fields, such as decision making [9], image [10], and medicine [11].

It is important to investigate the fuzzy entropy, which is used to describe the degree of uncertainty of fuzzy sets and is determined by the absolute deviation of membership and nonmembership degree of fuzzy sets. As the expansion of fuzzy sets, the degree of uncertainty of intuitionistic fuzzy sets includes not only the fuzziness of known information but also the intuition of unknown information. The ambiguity of known information is determined by the absolute deviation of membership degree and nonmembership degree, and the intuition of unknown information is determined by the degree of hesitation. The degree of uncertainty of intuitionistic fuzzy sets is described by intuitionistic fuzzy entropy which is first defined by Burillo and Bustince [12], while this axiom only describes the intuition of intuitionistic fuzzy sets but ignores its ambiguity. Based on the study of Burillo and Bustince, Szmidt and Kacprzyk proposed the axiomatic definition of intuitionistic fuzzy entropy which can reflect the intuition and fuzziness and constructed an entropy formula by using the geometric meaning of intuitionistic fuzzy sets [13], which provides different points of views for many scholars to build new intuitionistic fuzzy entropy formulas [14–17]. However, it is worth noting that the entropy formulas satisfying the entropy axiomatic definition of Szmidt and Kacprzyk demonstrate significant diversity in the sorting results under the same conditions of fuzziness in [13–17]. The reason lies in the fact that the axiomatic definition of entropy does not fully reflect the intuition of entropy under the same ambiguity. Therefore, the authors construct a new intuitionistic fuzzy entropy formula which fully reflects the fuzziness and intuition based on the pioneering study of Szmidt and Kacprzyk [13] and prove that the formula has two better properties in the following sections.

2. Preliminaries

In this section, we briefly review some basic notions and definitions related to intuitionistic fuzzy sets.

Definition 1 (see [2]). An intuitionistic fuzzy set A in $X = \{x_1, x_2, \cdots, x_n\}$ is given by Atanassov as follows:

$$A = \{\langle x, \mu_A(x), \upsilon_A(x)\rangle \mid x \in X\}, \quad (1)$$

where $\mu_A : X \longrightarrow [0,1]$ and $\upsilon_A : X \longrightarrow [0,1]$, with the condition $0 \le \mu_A(x) + \upsilon_A(x) \le 1$, $x \in X$. The numbers $\mu_A(x), \upsilon_A(x) \in [0,1]$ denote the degree of membership and nonmembership of x to A, respectively. For each intuitionistic fuzzy set in X, the intuitionistic fuzzy index of x in A is denoted by

$$\pi_A(x) = 1 - \mu_A(x) - \upsilon_A(x), \quad (2)$$

which is a hesitancy degree of x to A. It is obvious that

$$\begin{aligned} &\mu_A(x) \in [0,1], \\ &\upsilon_A(x) \in [0,1], \\ &\pi_A(x) \in [0,1], \\ &\forall x \in X. \end{aligned} \quad (3)$$

And the complement of intuitionistic fuzzy set A is represented by $A^c = \{\langle x, \upsilon_A(x), \mu_A(x)\rangle \mid x \in X\}$.

Definition 2 (see [13]). Let E be a set-to-point mapping $E : A \longrightarrow [0,1]$. $\boldsymbol{E}$ is called an entropy measure if it satisfies the following four constraints:

(1) $E(A) = 0$ iff A is nonfuzzy;

(2) $E(A) = 1$ iff $\mu_A(x) = \upsilon_A(x)$ for all x;

(3) $E(A) \le E(B)$ if A is less fuzzy than B, i.e.,

$$\begin{aligned} &\mu_A(x) \le \mu_B(x) \\ &\text{and } \upsilon_A(x) \ge \upsilon_B(x) \\ &\text{for } \mu_B(x) \le \upsilon_B(x) \end{aligned} \quad (4)$$

or $E(A) \ge E(B)$ if B is less fuzzy than A, i.e.,

$$\begin{aligned} &\mu_A(x) \ge \mu_B(x) \\ &\text{and } \upsilon_A(x) \le \upsilon_B(x) \\ &\text{for } \mu_B(x) \ge \upsilon_B(x); \end{aligned} \quad (5)$$

(4) $E(A) \ge E(A^c)$.

3. The Improved Entropy Axiomatic Definition

Mostly the existing study on intuitionistic fuzzy entropy formula is based on Definitions 1 and 2, among which Szmidt and Kacprzyk combined using the geometric meaning of intuitionistic fuzzy sets to give an intuitionistic fuzzy entropy formula as follows:

$$E_{SK}(A) = \frac{1}{n}\sum_{i=1}^{n}\frac{\max\text{Count}(A_i \cap A_i^c)}{\max\text{Count}(A_i \cup A_i^c)}, \quad (6)$$

where $A = \{A_1, A_2, \cdots, A_n\}$, $A_i = \langle x_i, \mu_A(x_i), \upsilon_A(x_i)\rangle$, and

$$\begin{aligned} &A_i \cap A_i^c \\ &= \langle \min\{\mu_A(x_i), \mu_{A^c}(x_i)\}, \max\{\upsilon_A(x_i), \upsilon_{A^c}(x_i)\}\rangle, \\ &A_i \cup A_i^c \\ &= \langle \max\{\mu_A(x_i), \mu_{A^c}(x_i)\}, \min\{\upsilon_A(x_i), \upsilon_{A^c}(x_i)\}\rangle, \end{aligned} \quad (7)$$

And max$Count(A_i) = \mu_A(x_i) + \upsilon_A(x_i)$ which is called the maximum potential. The following two intuitionistic fuzzy entropy formulas were given by Zeng and Li [15] and Wang and Lei [16], respectively.

$$E_{ZL}(A) = 1 - \frac{1}{n}\sum_{i=1}^{n}\left|\mu_A(x_i) - \upsilon_A(x_i)\right|, \quad (8)$$

$$E_{WL}(A) = \frac{1}{n}\sum_{i=1}^{n}\frac{\min\{\mu_A(x_i), \mu_A(x_i)\} + \pi_A(x_i)}{\max\{\mu_A(x_i), \mu_A(x_i)\} + \pi_A(x_i)}, \quad (9)$$

It was proved that formula (6) was equivalent to (9) by Wei, Wang et al. [18]. Vlachos and Sergiadis [14] gave two intuitionistic fuzzy entropy formulas as follows:

$$E_{VS1} = \frac{1}{n}\sum_{i=1}^{n}\frac{2\mu_A(x_i)\upsilon_A(x_i) + \pi_A^2(x_i)}{\mu_A^2(x_i) + \upsilon_A^2(x_i) + \pi_A^2(x_i)}, \quad (10)$$

and

$$E_{VS2} = E_{\text{fuzzy}}(A) + E_{\text{intuit}}(A), \quad (11)$$

where

$$\begin{aligned} E_{\text{fuzzy}}(A) = &-\frac{1}{n\ln 2}\sum_{i=1}^{n}(\mu_A(x_i)\ln\mu_A(x_i) \\ &+ \upsilon_A(x_i)\ln\upsilon_A(x_i) \\ &- (1-\pi_A(x_i))\ln(1-\pi_A(x_i))), \\ E_{\text{intuit}}(A) = &\frac{1}{n}\sum_{i=1}^{n}\pi_A(x_i). \end{aligned} \quad (12)$$

Based on Trigonometric Function, an intuitionistic fuzzy entropy formula was given by Wei, Gao, and Guo [17] as follows:

$$E_W(A) = \frac{1}{n}\sum_{i=1}^{n}\left(\cos\frac{\mu_A(x_i) - \upsilon_A(x_i)}{2(1+\pi_A(x_i))}\pi\right). \quad (13)$$

Example 3. Use the above six entropy formulas and calculate the following intuitionistic fuzzy sets which have the same fuzziness ($|\mu_A(x) - \upsilon_A(x)|$):

$$\begin{aligned} A_1 &= \langle 0.4, 0.5\rangle, \\ A_2 &= \langle 0.3, 0.4\rangle, \\ A_3 &= \langle 0.1, 0.2\rangle. \end{aligned} \quad (14)$$

The results are shown in Table 1.

Table 1: Comparison of the results calculated using the existing entropy formulas.

	E_{SK}	E_{ZL}	E_{WL}	E_{VS1}	E_{VS2}	E_{WHG}
A_1	0.8333	0.9	0.8333	0.9762	0.9920	0.9898
A_2	0.8571	0.9	0.8571	0.9706	0.9897	0.9927
A_3	0.8889	0.9	0.8889	0.9815	0.9755	0.9957

As can be seen from Table 1, with the same ambiguity, the ranking results are quite different when different entropy formulas are adopted. The entropy formula E_{ZL}, which has the same entropy, i.e., 0.9, does not take into account the effect of intuition ($\pi_A(x)$). Regarding the entropy formulas E_{VS1} and E_{VS2}, the entropy is smaller when the intuition is larger ($\pi_{A_2}(x) > \pi_{A_1}(x)$ while $E(A_2) < E(A_1)$), which is obviously not consistent with the subjective understanding. The results of the entropy formulas E_{SK}, E_{WL}, and E_{WHG} are in line with intuitive facts. Thus, it can be seen that although the above six formulas all satisfy the four axioms of Szmidt and Kacprzyk, they show different sorting results. The authors find out that the reason is due to the fact that the axiomatic definition of entropy does not fully reflect the intuition of entropy when the fuzziness is the same. So on the basis of Definition 2, the constraint condition (5) is strengthened to be as follows: when the fuzziness of the intuitionistic fuzzy sets is the same, the entropy increases monotonically with the intuition. Therefore, the improved axiomatic definition of entropy proposed in this paper is defined as follows.

Definition 4. Let E be a set-to-point mapping $E : A \longrightarrow [0,1]$, and E is called an entropy measure if it satisfies the following five axioms:

(1) $E(A) = 0$ iff A is nonfuzzy;

(2) $E(A) = 1$ iff $\mu_A(x) = \upsilon_A(x)$ for all x;

(3) $E(A) \leq E(B)$ if A is less fuzzy than B, i.e.,

$$\mu_A(x) \leq \mu_B(x) \text{ and } \upsilon_A(x) \geq \upsilon_B(x) \text{ for } \mu_B(x) \leq \upsilon_B(x) \tag{15}$$

or $E(A) \geq E(B)$ if B is less fuzzy than A, i.e.,

$$\mu_A(x) \geq \mu_B(x) \text{ and } \upsilon_A(x) \leq \upsilon_B(x) \text{ for } \mu_B(x) \geq \upsilon_B(x); \tag{16}$$

(4) $E(A) \geq E(A^c)$;

(5) $E(A) \leq E(B)$ if $|\mu_A(x) - \upsilon_A(x)| = |\mu_B(x) - \upsilon_B(x)|$ for $\pi_A(x) \leq \pi_B(x)$.

4. An Intuitionistic Fuzzy Entropy Formula with Improved Constraints

Definition 4 fully reflects the ambiguity of the known information and the intuition of the unknown information of the intuitionistic fuzzy sets, where the fuzziness is reflected by the constraint condition (3), and the intuition is embodied by the constraint condition (5). However, the following intuitionistic fuzzy entropy formula given by Zhao, Wang, and Hao [19] produces counterintuitive results in sorting the partial intuitionistic fuzzy entropy (see Example 10 for details):

$$E_{ZWH}(A) = \frac{1}{n} \cdot \sqrt{\sum_{i=1}^{n} \frac{\mu_A(x_i)\upsilon_A(x_i) + \pi_A^2(x_i)}{\mu_A^2(x_i) + \upsilon_A^2(x_i) - \mu_A(x_i)\upsilon_A(x_i) + \pi_A(x_i)}}. \tag{17}$$

Therefore, formula (17) can be modified to be a new intuitionistic fuzzy entropy formula such that the new formula satisfies the constraint condition (5) in Definition 4.

Theorem 5. *For any intuitionistic fuzzy set A, let*

$$E(A) = \frac{1}{n} \cdot \sum_{i=1}^{n} \frac{\mu_A(x_i)\upsilon_A(x_i) + \pi_A(x_i)}{\mu_A^2(x_i) + \upsilon_A^2(x_i) - \mu_A(x_i)\upsilon_A(x_i) + \pi_A(x_i)}, \tag{18}$$

then $E(A)$ is an entropy of the intuitionistic fuzzy set A. It can be proved that it satisfies the 5 constraint conditions of the axiomatic definition of entropy.

Proof. Let

$$E(A_i) = \frac{\mu_A(x_i)\upsilon_A(x_i) + \pi_A(x_i)}{\mu_A^2(x_i) + \upsilon_A^2(x_i) - \mu_A(x_i)\upsilon_A(x_i) + \pi_A(x_i)}. \tag{19}$$

To prove Theorem 5, it is only necessary to prove that $E(A_i)$ satisfies 5 constraint conditions in Definition 4.

According to Definition 1, we obtain

$$\begin{aligned} &0 \leq \mu_A(x_i) \leq 1, \\ &0 \leq \upsilon_A(x_i) \leq 1, \\ &0 \leq \pi_A(x_i) = 1 - \mu_A(x_i) - \upsilon_A(x_i) \leq 1. \end{aligned} \tag{20}$$

So $0 \leq E(A_i) \leq 1$.

(1) Let

$$E(A_i) = 0, \tag{21}$$

that is

$$\frac{\mu_A(x_i)\upsilon_A(x_i) + \pi_A(x_i)}{(\mu_A(x_i) - \upsilon_A(x_i))^2 + \mu_A(x_i)\upsilon_A(x_i) + \pi_A(x_i)} = 0, \tag{22}$$

which is equivalent to

$$\begin{aligned} &\mu_A(x_i) = 0, \\ &\upsilon_A(x_i) = 1 \\ \text{or } &\mu_A(x_i) = 1, \\ &\upsilon_A(x_i) = 0. \end{aligned} \tag{23}$$

This implies that A is nonfuzzy.

(2) Suppose

$$E(A_i) = 1, \tag{24}$$

which is equivalent to

$$\frac{\mu_A(x_i)\upsilon_A(x_i) + \pi_A(x_i)}{(\mu_A(x_i) - \upsilon_A(x_i))^2 + \mu_A(x_i)\upsilon_A(x_i) + \pi_A(x_i)} = 1, \tag{25}$$

if and only if

$$(\mu_A(x_i) - \upsilon_A(x_i))^2 = 0. \tag{26}$$

So it means that

$$\mu_A(x_i) = \upsilon_A(x_i). \tag{27}$$

(3) Let

$$f(\mu, \upsilon) = \frac{\mu\upsilon + (1 - \mu - \upsilon)}{\mu^2 + \upsilon^2 - \mu\upsilon + (1 - \mu - \upsilon)}, \tag{28}$$

where $\mu, \upsilon \in [0, 1]$ and $1 - \mu - \upsilon \in [0, 1]$. To prove that $E(A_i)$ satisfies the constraint condition (3) in Definition 4, it only needs to prove that $f(\mu, \upsilon)$ is monotonically increasing relative to μ and monotonically decreasing relative to υ if $\mu < \upsilon$, and $f(\mu, \upsilon)$ is monotonically decreasing relative to μ, and monotonically increasing relative to υ if $\mu > \upsilon$. The proof is as follows.

The partial derivative of f with respect to μ is

$$f_\mu(\mu, \upsilon) = \frac{(\mu - \upsilon)(-\upsilon^2 - \mu\upsilon + \mu + 3\upsilon - 2)}{(\mu^2 + \upsilon^2 - \mu\upsilon + (1 - \mu - \upsilon))^2}. \tag{29}$$

Let

$$g(\mu, \upsilon) = -\upsilon^2 - \mu\upsilon + \mu + 3\upsilon - 2, \tag{30}$$

then

$$\begin{aligned} g_u(u, \upsilon) &= 1 - \upsilon \geq 0, \\ g_\upsilon(u, \upsilon) &= 3 - 2\upsilon - u \geq 0. \end{aligned} \tag{31}$$

Thus

$$\begin{aligned} \max \mathrm{g}(\mu, \upsilon) &= \mathrm{g}(1, 1) = -1, \\ g(\mu, \upsilon) &\leq g(1, 1) = -1 < 0. \end{aligned} \tag{32}$$

Therefore we get $f_\mu(\mu, \upsilon) > 0$ if $\mu < \upsilon$, and $f_\mu(\mu, \upsilon) < 0$ if $\mu > \upsilon$.

Similarly, it can be proved that $f_\upsilon(\mu, \upsilon) > 0$ if $\mu > \upsilon$, and $f_\upsilon(\mu, \upsilon) < 0$ if $\mu < \upsilon$.

In summary, we obtain

$$\begin{aligned} &f_\mu(\mu, \upsilon) > 0 \\ \text{and } &f_\upsilon(\mu, \upsilon) < 0 \\ &\qquad \text{if } \mu < \upsilon, \\ &f_\mu(\mu, \upsilon) < 0 \\ \text{and } &f_\upsilon(\mu, \upsilon) > 0 \\ &\qquad \text{if } \mu > \upsilon. \end{aligned} \tag{33}$$

Hence we get the conclusion that $E(A_i)$ satisfies the constraint condition (3) in Definition 4.

(4) For any $1 \leq i \leq n$, it holds that

$$\begin{aligned} E(A_i^c) &= \frac{\mu_A(x_i)\upsilon_A(x_i) + \pi_A(x_i)}{\upsilon_A(x_i)^2 + \mu_A(x_i)^2 - \mu_A(x_i)\upsilon_A(x_i) + \pi_A(x_i)} \\ &= E(A_i). \end{aligned} \tag{34}$$

(5) Setting $|\mu_A(x_i) - \upsilon_A(x_i)| = a$, $a \in (0, 1)$. Notice that

$$\pi_A(x_i) = 1 - \mu_A(x_i) - \upsilon_A(x_i), \tag{35}$$

then we can get

$$\begin{aligned} \mu_A(x_i) &= \frac{1 - \pi_A(x_i) + a}{2} \\ \upsilon_A(x_i) &= \frac{1 - \pi_A(x_i) - a}{2} \\ \text{or } \mu_A(x_i) &= \frac{1 - \pi_A(x_i) - a}{2} \\ \upsilon_A(x_i) &= \frac{1 - \pi_A(x_i) + a}{2}. \end{aligned} \tag{36}$$

Thus, we obtain

$$E(A_i) = \frac{(1 - \pi_A(x_i))^2 - a^2 + 4\pi_A(x_i)}{(1 - \pi_A(x_i))^2 + 3a^2 + 4\pi_A(x_i)}. \tag{37}$$

Let

$$f(\pi_A(x_i)) = \frac{(1 - \pi_A(x_i))^2 - a^2 + 4\pi_A(x_i)}{(1 - \pi_A(x_i))^2 + 3a^2 + 4\pi_A(x_i)}, \tag{38}$$

then it is easy to see that

$$\begin{aligned} f'(\pi_A(x_i)) &= \frac{8a^2(3 - \pi_A(x_i))}{((1 - \pi_A(x_i))^2 + 3a^2 + 4\pi_A(x_i))^2} \\ &> 0, \end{aligned} \tag{39}$$

which shows that $E(A_i) < E(B_i)$ if $|\mu_A(x_i) - \upsilon_A(x_i)| = |\mu_B(x_i) - \upsilon_B(x_i)|$ for $\pi_A(x_i) < \pi_B(x_i)$.

Theorem 5 is proved. And the new entropy formula has the following properties. □

Table 2: Comparison of the results calculated using the $E_{ZWH}(A)$ and $E(A)$.

	A_1	A_2	A_3	A_4	A_5
E_{ZWH}	0.8729	0.8231	0.6988	0.8358	0.7143
E	0.9524	0.9677	0.9767	0.9863	1.0000

Property 6. Let $\Phi = \{\langle x, \mu_\Phi(x), \upsilon_\Phi(x)\rangle \mid x \in X\}$ be an intuitionistic fuzzy set satisfying the condition $\pi_\Phi(x) = 1 - \mu_\Phi(x) - \upsilon_\Phi(x) = b$ ($b \in (0,1)$).Then $E(\Phi)$ is monotonically decreasing relative to $|\mu_\Phi(x) - \upsilon_\Phi(x)|$.

Proof. To prove Property 6, it is only necessary to prove that when $\pi_C(x) = \pi_D(x)$ $(\forall C, D \in \Phi)$, $E(C) < E(D)$ if $|\mu_C(x) - \upsilon_C(x)| > |\mu_D(x) - \upsilon_D(x)|$.

To prove the above, it is only necessary to prove that when $\pi_C(x_i) = \pi_D(x_i)$, $E(C_i) < E(D_i)$ if $|\mu_C(x_i) - \upsilon_C(x_i)| > |\mu_D(x_i) - \upsilon_D(x_i)|$.

In fact, we have

$$\begin{gathered}\pi_C(x_i) = \pi_D(x_i), \\ |\mu_C(x_i) - \upsilon_C(x_i)| > |\mu_D(x_i) - \upsilon_D(x_i)|,\end{gathered} \tag{40}$$

which is equivalent to

$$\mu_C(x_i)\upsilon_C(x_i) < \mu_D(x_i)\upsilon_D(x_i), \tag{41}$$

if and only if

$$\begin{aligned}0 &< \mu_C(x_i)\upsilon_C(x_i) + \pi_C(x_i) \\ &< \mu_D(x_i)\upsilon_D(x_i) + \pi_D(x_i),\end{aligned} \tag{42}$$

and

$$\begin{aligned}&\mu_C^2(x_i) + \upsilon_C^2(x_i) - \mu_C(x_i)\upsilon_C(x_i) + \pi_C(x_i) \\ &\quad > \mu_D^2(x_i) + \upsilon_D^2(x_i) - \mu_D(x_i)\upsilon_D(x_i) + \pi_D(x_i) \\ &\quad > 0.\end{aligned} \tag{43}$$

Then

$$\begin{aligned}&E(C_i) \\ &\quad = \frac{\mu_C(x_i)\upsilon_C(x_i) + \pi_C(x_i)}{\mu_C^2(x_i) + \upsilon_C^2(x_i) - \mu_C(x_i)\upsilon_C(x_i) + \pi_C(x_i)} \\ &\quad < E(D_i).\end{aligned} \tag{44}$$

Hence, the property is proved. □

Remark 7. Property 6 demonstrates that if the intuitionistic fuzzy sets have the same intuition, the greater the fuzziness is, i.e., the smaller the absolute deviation of membership and nonmembership is, the greater the intuitionistic fuzzy entropy is.

Property 8. Let $IFS(X)$ be a set of intuitionistic fuzzy sets on a nonempty set X. If the set-to-point mapping $E : IFS(X) \longrightarrow [0,1]$ is the intuitionistic fuzzy entropy of the intuitionistic fuzzy set A, then we have $E(A) \geq \pi(A)$.

Proof. To prove that $E(A) \geq \pi(A)$, it only needs to prove that $E(A_i) \geq \pi_A(x_i)$. By using the reduction to absurdity, its proof can be done as follows.

Assume $E(A_i) < \pi_A(x_i)$, then

$$\begin{aligned}&\frac{\mu_A(x_i)\upsilon_A(x_i) + \pi_A(x_i)}{\mu_A^2(x_i) + \upsilon_A^2(x_i) - \mu_A(x_i)\upsilon_A(x_i) + \pi_A(x_i)} \\ &\quad < \pi_A(x_i),\end{aligned} \tag{45}$$

if and only if

$$\begin{aligned}\mu_A(x_i)\upsilon_A(x_i) &< \pi_A(x_i)(\mu_A(x_i)(\mu_A(x_i) - 1) \\ &\quad - \mu_A(x_i)\upsilon_A(x_i) + \upsilon_A(x_i)(\upsilon_A(x_i) - 1)).\end{aligned} \tag{46}$$

So it is easy to get that

$$\mu_A(x_i)\upsilon_A(x_i) < 0, \tag{47}$$

which contradicts with $\mu_A(x_i)\upsilon_A(x_i) \geq 0$. Hence, the property is proved. □

Remark 9. Property 8 implies that the degree of hesitation is the infimum of intuitionistic fuzzy entropy.

Example 10. Use the formulas $E_{ZWH}(A)$ and $E(A)$ to calculate the entropy of the following intuitionistic fuzzy sets:

$$\begin{aligned}A_1 &= \langle 0, 0.2\rangle, \\ A_2 &= \langle 0.4, 0.5\rangle, \\ A_3 &= \langle 0.3, 0.4\rangle, \\ A_4 &= \langle 0.1, 0.2\rangle, \\ A_5 &= \langle 0.3, 0.3\rangle.\end{aligned} \tag{48}$$

The results are shown in Table 2.

According to Table 2, the following conclusions can be drawn:

① The comparison of intuitionistic fuzzy entropy of A_2, A_3, and A_4 is based on the same fuzziness, which means that the value of $|\mu_A(x) - \upsilon_A(x)|$ of the three sets is equal. And the results calculated by formula (18) show that the greater the intuition of the unknown information is, the greater the intuitionistic fuzzy entropy is; that is, $E(A_4) > E(A_3) > E(A_2)$. However, this property is not available in formula (8), (10), (11), and (17).

② For the comparison of intuitionistic fuzzy entropy of A_1 and A_4, they have different intuitions and ambiguities while they meet the same condition, i.e., $\mu_A < \upsilon_A$. Calculated using the formula (18), it can be seen that $E(A_4) > E(A_1)$

because $\mu_{A_1} < \mu_{A_4}$ and $\upsilon_{A_1} = \upsilon_{A_4}$, which conforms to the relationship between the intuitionistic fuzzy sets, but it can not be seen in formula (17).

③ The intuitionistic fuzzy set A_5 has the same degree of membership and nonmembership, which implies that there is no enough information to support or oppose a proposition. In this way, the entropy reaches a maximum of 1, that is $E(A_5) = 1$, but it is not seen in E_{ZWH}. In summary, the results calculated by the entropy formula proposed in this paper are consistent with intuitionistic facts, which shows that it is better than the entropy formula (17).

5. Conclusion

The entropy of the intuitionistic fuzzy set is used to describe the degree of uncertainty of the intuitionistic fuzzy set, including the fuzziness of known information and the intuition of unknown information. Firstly this paper analyzes the existing intuitionistic fuzzy entropy formulas comprehensively and explores the reason why the sorting of some entropies of intuitionistic fuzzy sets is not consistent with intuitionistic facts. Secondly, the constraint conditions are given in accordance with the intuitionistic facts based on the analysis of the differences in entropy formulas, and then a new intuitionistic fuzzy entropy formula is constructed and its properties are analyzed. Finally, the comparison analysis shows that the method and the formula proposed in this paper can better solve the problem of entropy ordering.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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